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Assignment

① 1) The mean of the function

$$f(x,\sigma) = \frac{x}{\sigma^2} e^{-x^2/(2\sigma^2)}$$

can be calculated as integral using matlab.

$$M1 = \frac{\int_0^\infty x f(x,\sigma)\,dx}{\int_0^\infty f(x,\sigma)\,dx} =$$

$$= \frac{\int_0^\infty \frac{x^2 e^{-x^2/(2\cdot(0,5)^2)}}{(0,5)^2}}{\int_0^\infty \frac{x\ e^{-x^2/(2\cdot(0,5)^2)}}{(0,5)^2}} = \underline{0.6267.}$$

$$\left\{ \int_0^\infty x e^{-x^2/(2(0,5)^2)}\,dx = 1 \right\}$$

$$\left\{ \int_0^\infty x^2 e^{-x^2/(2(0,5)^2)}\,dx = 0.6267 \right.$$

(2) The variance of $f(x,\sigma)$ can be defined as

$$\sigma^2 = M_2 - M_1^2$$

$$M_2 = \int_0^\infty x^2 f(x,\sigma)\,dx$$

Using matlab

$$M_2 = \int_0^\infty x^2 \cdot \frac{x}{(0,5)^2} e^{-x^2/(2\cdot(0.5)^2)}\,dx$$

$$= \int_0^\infty \frac{x^3}{(0.5)^2} e^{-x^2/(2\cdot(0.5)^2)}\,dx$$

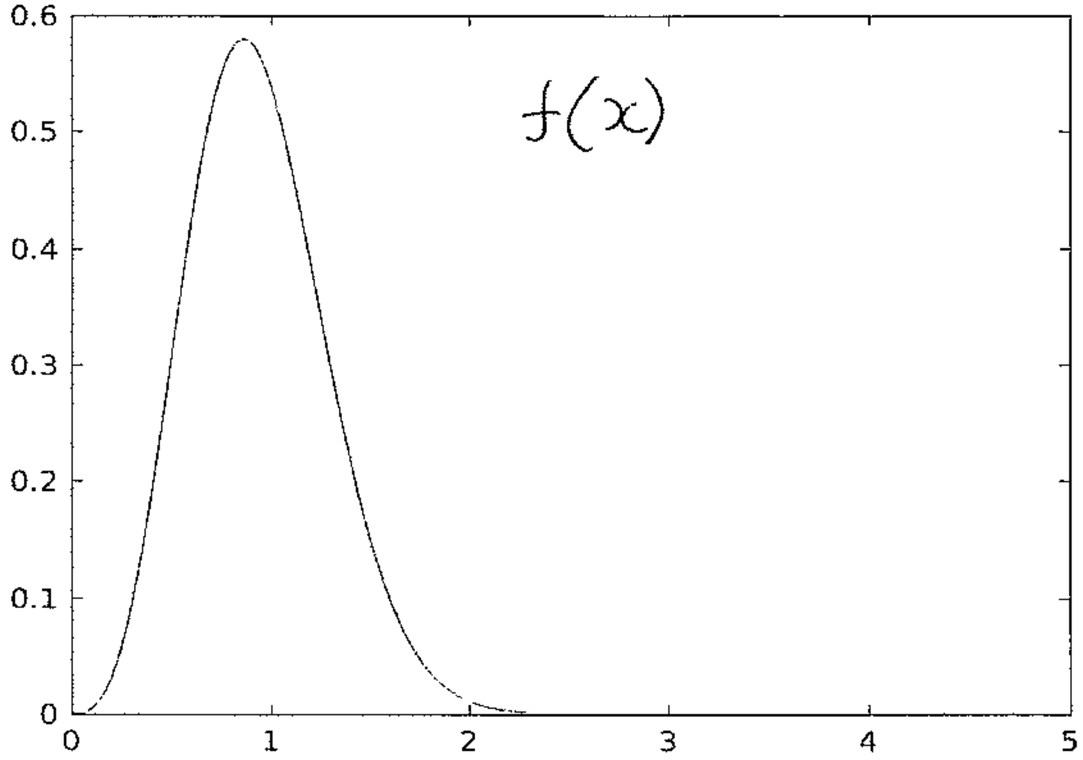
$$= 0.500$$

So, $\sigma^2 = M2 - M1 = (0.500 - (0.6267)^2)$

$= \underline{0.1073}$

or $V = \sqrt{\sigma^2} = \underline{0.3276}$

(3) $$f(x,\sigma) = \frac{x}{(0.5)^2} e^{-\frac{x^2}{2(0.5)^2}}$$



② (1) Expression for covarience matrix

$$V = \begin{pmatrix} \sigma_L^2 & 0 & 0 \\ 0 & \sigma_{T_1}^2 & 0 \\ 0 & 0 & \sigma_{T_2}^2 \end{pmatrix}$$

or

$$V = 10^{-6} \begin{pmatrix} 2^2 & 0 & 0 \\ 0 & (0.3)^2 & 0 \\ 0 & 0 & (0.4)^2 \end{pmatrix}$$

$$V = 10^{-6} \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0.09 & 0 \\ 0 & 0 & 0.16 \end{pmatrix}$$

Because T_1 and T_2 are independant variablees.

② 2)

$$g_1 = \frac{4\pi^2 L}{T^2}$$

$$g_2 = \frac{4\pi^2 L}{T_1^2} \; ; \; g_2 = \frac{4\pi^2 L}{T_2^2}$$

$$G = \begin{pmatrix} \frac{\partial g_1}{\partial L} & \frac{\partial g_1}{\partial T_1} & \frac{\partial g}{\partial T_2} \\ \frac{\partial g_2}{\partial L} & \frac{\partial g_2}{\partial T_1} & \frac{\partial g_2}{\partial T_2} \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{4\pi^2}{T_1^2} & -\frac{8\pi^2 L}{T_1^3} & 0 \\ \frac{4\pi^2}{T_2^2} & 0 & -\frac{8\pi^2 L}{T_2^3} \end{pmatrix}$$

$T_1 = 1.1433$

$T_2 = 1.1446$

$L = 0{,}325$

$$G = \begin{pmatrix} 30.196 & -17.167 & 0 \\ 30.127 & 0 & -17.109 \end{pmatrix}$$

$$\tilde{G} = \begin{pmatrix} 30.127 & 30.196 \\ 0 & -17.167 \\ -17.109 & 0 \end{pmatrix}$$

$$V_f = G V \tilde{G} =$$

$$\begin{pmatrix} 30.196 & -17.167 & 0 \\ 30.127 & 0 & -17.109 \end{pmatrix} \times 10^{-6} \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0.09 & 0 \\ 0 & 0 & 0.16 \end{pmatrix} \times \begin{pmatrix} 30.127 & 30.196 \\ 0 & -17.167 \\ -17.109 & 0 \end{pmatrix}$$

<u>Results from matlab</u>

$$V_f = G V \tilde{G} = \begin{pmatrix} 3.6374 & 3.6747 \\ 3.6267 & 3.6843 \end{pmatrix} \cdot 10^{-6}$$

$$\tilde{g} = \frac{4\pi^2 L}{\tilde{T}^2}$$

$$V = G V_f \tilde{G}$$

$$V = \begin{pmatrix} \frac{\partial g}{\partial L} & \frac{\partial g}{\partial T} \end{pmatrix} \begin{pmatrix} V_f \end{pmatrix} \begin{pmatrix} \frac{\partial g}{\partial L} \\ \frac{\partial g}{\partial T} \end{pmatrix}$$

$$\begin{pmatrix} \frac{\partial g}{\partial L} & \frac{\partial g}{\partial T} \end{pmatrix} = \begin{pmatrix} \frac{4\pi^2}{\tilde{T}^2} & \frac{\cancel{\theta}-2 \times 4\pi^2 \cdot L}{\tilde{T}^3} \end{pmatrix}$$

$$= \begin{pmatrix} 30.1196 ; & -17.167 \end{pmatrix}$$

$$\begin{pmatrix} \frac{\partial g}{\partial L} \\ \frac{\partial g}{\partial T} \end{pmatrix} = \begin{pmatrix} 30.1196 \\ -17.167 \end{pmatrix}$$

Results from Matlab

$$V = \begin{pmatrix} 30{,}1196 & -17{,}167 \end{pmatrix} 10^{-6} \times \begin{pmatrix} 3.6374 & 3{,}6747 \\ 3{,}667 & 3{,}6893 \end{pmatrix} \begin{pmatrix} 30{,}1196 \\ -17{,}167 \end{pmatrix} =$$

$$= 10^{-6} \times 620{,}3 = 6{,}203 \cdot 10^{-4}$$

$$\boxed{\sqrt{V} = \sqrt{6.203 \cdot 10^{-4}} = 2{,}45 \times 10^{-2}}$$

$$\tilde{g} = \frac{g_1 + g_2}{2} = 9.8040$$

$$g = \tilde{g} \pm \sqrt{V}$$

$$g = 9.8040 \pm 0.0245$$

$$\boxed{g = 9.804 \pm 0.025}$$