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Assignment 2

(1) I loaded data for t and y and ploted it.
Then I defined a fitting function as a $f_a = A e^{-t/\tau}$

I guessed parameters A and τ and put $A = 5$ and $\tau = 3$.
I ploted $f_a(A, \tau, t) = 5 \cdot e^{-t/3}$ on the same graph 1

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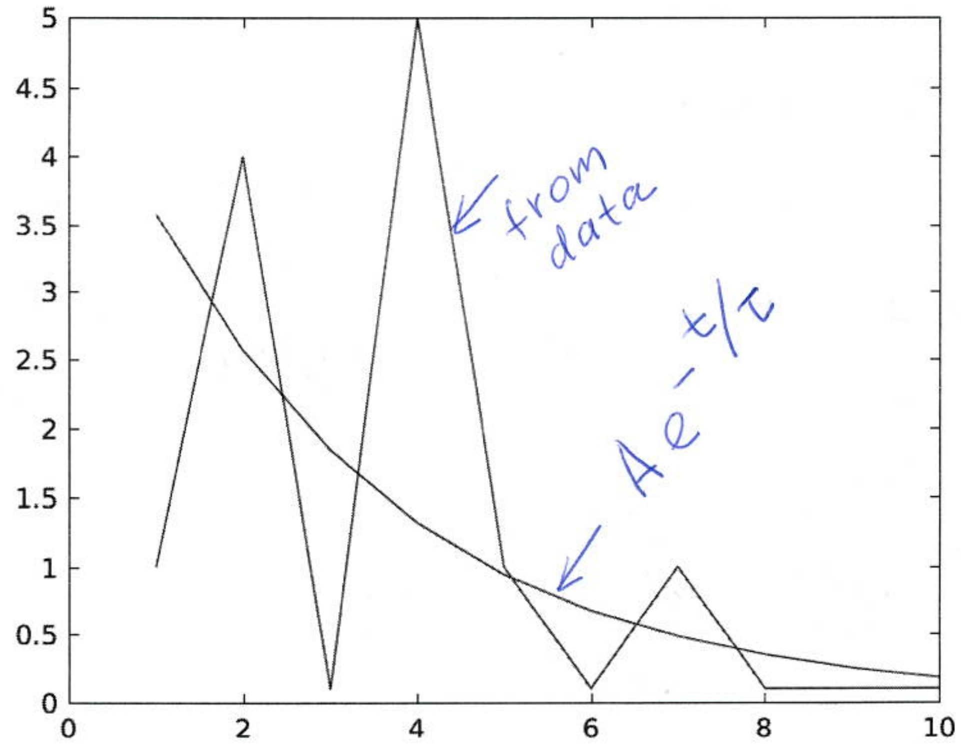
(2) For getting the best estimates A and τ parameters, we will use log-likelihood fitting equalent to minimise
$-2 \ln(L(A, \tau))$



Pic 1

$$-2\ln L = 2\sum_i \left(f_a(x_i) - y_i \ln f_a(x_i) + \right.$$
$$\left. + \ln(y_i!) \right)$$

Where $f_a(x_i) = Ae^{-t_i/\tau}$

y_i is actual data, So,

$$-2\ln L = 2\sum_i \left(Ae^{-t_i/\tau}\right) -$$
$$- y_i \ln\left(Ae^{-t_i/\tau}\right) + \ln(y_i).$$

For example for $i=1$,
$t_i = 1$, $y_i = 1$ we can write.

$$-2\ln L = 2\left(Ae^{-1/\tau}\right) - 1 \cdot \ln\left(A \cdot e^{-1/\tau}\right)$$
$$+ \ln(1!)$$

However we have 10 points,
so we need to sum up

$$-2\ln L = \underline{\sum_i -2\ln L_i}$$

I defined function $(-2\ln L(A,\tau))$ on matlab

However, I had difficulties to find a minimum of this function.

I estimated approximatelly values of A and τ when $(-2\ln L)$ has a minimum.

$$\boxed{\begin{aligned} A &\approx 4.3 \pm 0.3 \\ \tau &\approx 3.1 \pm 0.3 \\ (-2\ln L) &\approx 29.0 \pm 0.1 \end{aligned}}$$

So at $A \approx 4.3 \pm 0.3$ and $\tau \approx 3.1 \pm 0.3$

function $f_a(t) = A \cdot e^{-t/\tau}$

will be the best fitting function for our data.

$$\boxed{f_a(t) \approx 4.3 * e^{-\frac{t}{3.1}}}$$

Is the best fitting function

2 Covarience matrix

$$cov^{-1}(a_i, a_j) = -\frac{\partial^2 \ln L}{\partial a_i \partial a_j}\bigg|_{\bar{a}=\dot{\bar{a}}}$$

will look

$$cov^{-1}(a_i, a_j) = \begin{vmatrix} \frac{\partial^2 \ln L}{\partial A^2} & \frac{\partial^2 \ln L}{\partial A \partial \tau} \\ \frac{\partial^2 \ln L}{\partial A \partial \tau} & \frac{\partial^2 \ln L}{\partial \tau^2} \end{vmatrix}$$

I estimated this matrix.

$$cov^{-1}(A, \tau) = \begin{pmatrix} 1.2 \pm 0.1 & 1.5 \pm 0.1 \\ 1.5 \pm 0.1 & 1.8 \pm 0.2 \end{pmatrix}$$