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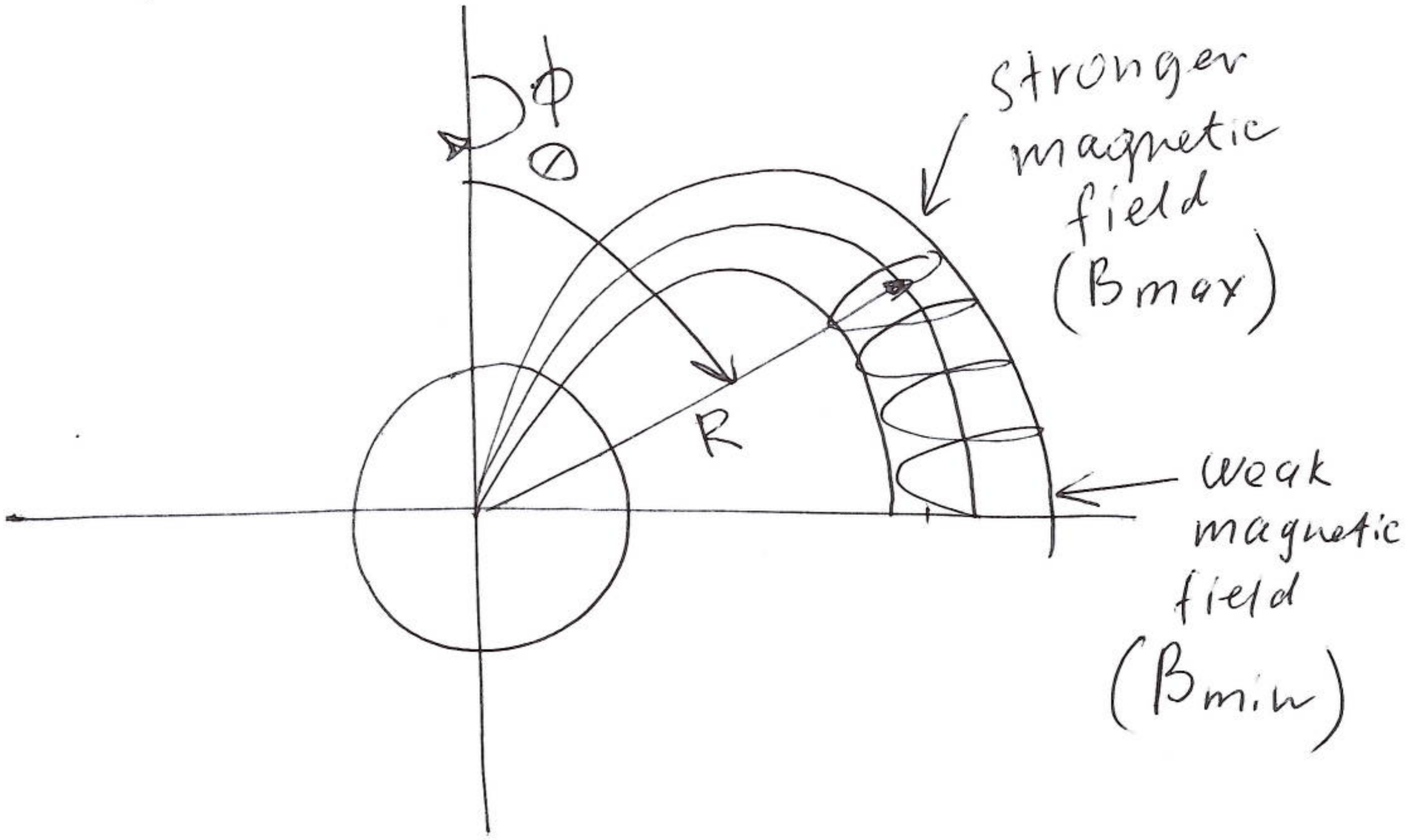
Assignment 1

a)



The particle will be trapped because it will be reflected by magnetic mirrors. A magnetic mirror is when a particle moves from a weak magnetic field to a strong magnetic field and pushed back by Lorentz force.

$$\frac{v_\perp^2}{v_0^2} = \frac{v_\perp^2}{v_\perp^2 + v_\parallel^2} = \frac{9}{10} > \frac{B_{min}}{B_{max}} \ll 2 \text{ times}$$

As $|B| \sim \frac{1}{r^3}$, Near poles $|B|$ decreases significantly.

Also we can calculate dif. between B

$$\frac{B_{min}}{B_{max}} = \frac{\sqrt{1+3\sin^2\theta}}{\sqrt{1+\sin^2\theta}}\Bigg|_{\theta=90^\circ} =$$

$$= \frac{\sqrt{1+0}}{\sqrt{1+3}} = \frac{1}{2}$$

$$\cancel{\tfrac{1}{2}}\ \frac{V_\perp^2}{v_0^2} = \frac{9}{10} > \frac{B_{min}}{B_{max}} = \frac{1}{2}$$

So the proton moving in Earth magnetic field, from equator towards poles, will be reflected.

b) Vectors of the magnetic field

$$\vec{B} = \frac{\mu_0 M_E}{4\pi r^3}(\hat{r}\, 2\cos\theta + \hat{\theta}\sin\theta)$$ and

differential displacement

$$\vec{dl} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\, d\phi\,\hat{\phi}$$

should have the same direction.

If ϕ is fixed. $d\phi = 0$.

$$\frac{dr}{2\cos\theta} = \frac{r\,d\theta}{\sin\theta}$$

$$\frac{2\cos\theta}{\sin\theta}\,d\theta = \frac{dr}{r}$$

$$2\int\frac{\cos\theta}{\sin\theta}\,d\theta = \frac{dr}{r}$$

$$2\ln\sin\theta = \ln r + C$$

$r = C\sin^2\theta$, From initial condition we find that $C = r_0$

$\boxed{r = r_0\sin^2\theta}$, is the field line, the trajectory which particles move.

c) (i)

$$\boxed{V_{\nabla B} = \frac{1}{2} v_\perp r_L \left| \frac{\vec{B} \times \vec{\nabla} B}{B^2} \right| = \frac{1}{2} \frac{v_\perp^2}{\omega_c} \left| \frac{\nabla B}{B} \right|}$$

$$\bar{B} = \frac{\mu_0 M_E}{4\pi R^3} \left(\hat{r}\, 2\cos\theta + \hat{\theta} \sin\theta \right)$$

$$|\vec{B}| = \frac{\mu_0 M_E}{4\pi R^3} \left(4\cos^2\theta + \sin^2\theta \right)^{1/2} =$$

$$\left(R = R_0 \sin^2\theta , \quad \sin^2\theta = R/R_0 \right)$$

$$= \frac{\mu_0 M_E}{4\pi R^3} \left(4 - 3R/R_0 \right)^{1/2}$$

$$\nabla B = \frac{\partial}{\partial R} \left(\frac{\mu_0 M_E}{4\pi R^3} \left(4 - 3R/R_0 \right)^{1/2} \right) =$$

$$= \frac{\mu_0 M_E}{4\pi} \frac{\partial}{\partial R} \left(R^{-3} \cdot \left(4 - \frac{3R}{R_0} \right)^{\frac{1}{2}} \right)$$

$$= \frac{\mu_0 M_E}{4\pi} \left((-3) R^{-4} \cdot \left(4 - \frac{3R}{R_0} \right) + \right.$$

$$\left. + R^{-3} \cdot \frac{1}{2} \left(-\frac{3}{R_0} \right) \right)$$

$$\nabla B \Big|_{R=R_0} = -\frac{\mu_0 M_E}{4\pi} \frac{9/2}{R_0^4} = -\frac{9}{2} \frac{B(R=R_0)}{R_0}$$

$$\boxed{\nabla B = -\frac{9}{2} \frac{B}{R} \quad \text{near} \quad R = R_0}$$

$$|\vec{B}| = \frac{\mu_0 M_E}{4\pi R_0^3} = \frac{\mu_0 M_E}{4\pi (3 \cdot R_E)^3}$$

$$= \frac{1.25 \cdot 10^{-6} \cdot 8 \cdot 10^{22}}{4 \cdot 3.14 \cdot 3^3 \cdot 6.37^3 \cdot 10^{6 \cdot 3}} =$$

$= 1.11 \cdot 10^{-6}$ T – magnetic field near equator, when $\theta = 90^\circ$, $R = 3 R_E$, $\boxed{|\vec{B}| = 1.11 \cdot 10^{-6} \text{ T}}$

$$V_{\nabla B} = \frac{1}{2}\frac{v_\perp^2}{\omega_c}\left|\frac{\nabla B}{B}\right| = \frac{\frac{1}{2}v_\perp^2 \cdot \frac{9}{2}\frac{1}{R}}{\left(\frac{eB}{m}\right)} =$$

$$= \frac{\left(\frac{1}{2}v_\perp^2 m\right) 9/2}{eBR} = \frac{\frac{v_\perp^2}{v_0^2} E_p \, 9/2}{eBR} =$$

$$= \frac{\left(\frac{v_\perp^2}{v_0^2}\right)^{9/10}(E_p)_{ev} \cdot K}{eBR_0} = \frac{\frac{9}{10}\cdot\frac{9}{2}E_p \cdot k}{e \cdot B \cdot R_0}$$

$$V_{\nabla B} = \frac{\frac{9}{10}\cdot\frac{9}{2}\cdot 1{,}10^6 \cdot 1.6\cdot10^{-19}}{1.6\cdot10^{-19}\cdot 1{,}11\cdot10^{-6}\cdot(3\cdot 6{,}37\cdot10^6)} =$$

$$= 0.19\cdot10^6 \frac{M}{sec} = 190 \text{ KM/sec}$$

$$\boxed{V_{\nabla B} = 190 \frac{KM}{sec}}$$

This an inhomogenity drift of a possitive charged particle

<u>Westward</u>

(ii) Yes, the drift leads to a current. Protons move Westward, but electron move Eastward.

We can see this from the formula

$$V_{\nabla B} = \frac{1}{2}\frac{v_\perp^2}{\omega_c}\left|\frac{\nabla B}{B}\right|, \quad \omega = \frac{eB}{m}$$

ω has different dirrection for electrons and protons.

(iii) In a tokamak there is current coused by ∇B, but its dirrection is different, which up and down.

In the Earth magnetic field, current is along equator. lines around Earth.

d) (i) $ds = r_0 (\sin\theta)\sqrt{1+3\cos^2\theta}\, d\theta$

$v_{\parallel}^2 = v_0^2 \cos\alpha = v_0^2 (1-\sin^2\alpha) =$

$v_{\parallel} = v_0 \cos\alpha$, α = pinch angle

$\sin^2\alpha = \frac{v_{\perp}^2}{v_0^2}$

$$\tau_b = 4 \int_{90^\circ}^{\theta_b} \frac{ds}{v_{\parallel}} =$$

$$= 4 \int_{90^\circ}^{\theta_b} \frac{d\theta\, ds}{v_{\parallel}\, d\theta} =$$

$$= \frac{4 r_0}{v_0} \int_0^{\theta_b} \frac{(\sin\theta)(1+3\cos^2\theta)^{\frac{1}{2}} d\theta}{\cos\alpha(\theta)}$$

? ? ?

d) (I) Very very rough estimation of τ_b

Solving eqn.

$$\frac{v_\perp^2}{v_0^2} = \frac{9}{10} \frac{1}{\sin^6\theta} \sqrt{1+3\cos^2\theta}$$

gives $\theta \approx 80^\circ$

$$\Delta S = R_0 \sin(90^\circ - 80^\circ) =$$

$$= 3 \cdot 6.37 \cdot 10^6 M \cdot \sin(10^\circ) = 3.2 \cdot 10^6 M$$

$$v_{\parallel} = \frac{1}{3} v_0 = \frac{1}{3}\sqrt{\frac{2E}{m_p}} =$$

$$= \frac{1}{3}\sqrt{\frac{2 \cdot 10^6 \cdot 1.6 \cdot 10^{-19}}{1.67 \cdot 10^{-27}}} = 0.45 \cdot 10^7 \frac{m}{sec}$$

$$\tau_b = 4 \frac{\Delta S}{v_{\parallel}} = 4 \cdot \frac{3.2 \cdot 10^6}{4.5 \cdot 10^6} =$$

$$= 3 \text{ sec} \pm 1 \text{ sec}.$$

$$\boxed{\tau_b \approx 3 \text{ sec}}$$

e) Time travelling around Earth is $T = \frac{L}{V_{\nabla B}} = \frac{3 \cdot 2\pi \cdot 6 \cdot 10^6 \, m}{190 \cdot 10^3 \, m/c}$

≈ 590 sec.

$\frac{T}{\tau_b} \approx \frac{590}{3} \approx 200$ times.

$\Rightarrow T >> \tau_b \quad J = \oint p_{\parallel} ds \approx$ const.