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Assignment 4

(i) From conservation of magnetic flux we can derive

$$B_0 r_0^2 = B_t r_t^2 ,$$

Where B_0 is an initial magnetic field $B_0 = 10T$,
but B_t is a magnetic field of compressed plasma.

Also Larmour radius

$$r_t = \frac{m v_t}{q B_t},$$ we can write system of equastions

$$\begin{cases} B_0 r_0^2 = B_t r_t^2 \\ r_t = \frac{m v_t}{q B_t} \end{cases} \Rightarrow \begin{cases} B_0 r_0^2 = \frac{m v_t}{q r_t} r_t^2 \\ B_t = \frac{m v_t}{q r_t} \end{cases}$$

$$\Rightarrow \boxed{r_t = \frac{q B_0 r_0^2}{m v_t}}$$

But $E_t = \frac{1}{2} m v_t^2 = 3.5$ MeV

energy of α particles.

$$v_t = \sqrt{2E_t/m}$$

$$\boxed{r_t = \frac{q B_0 r_0^2}{m\sqrt{2E_t/m}} = \frac{q B_0 r_0^2}{\sqrt{2mE_t}}}$$

$r_0 = 0.002$ m, $E_t = 3.5 \cdot 10^6$ eV

$B_0 = 10$T

$$r_t = \frac{2 \times 1.61 \times 10^{-19} \times 10^1 \times 2^2 \times 10^{-3\times2}}{\sqrt{2 \times 4 \times 1.67 \times 10^{-27} \times 3.5 \times 10^6 \times 1.61 \times 10^{-19}}}$$

$$= \frac{2 \times 1.6 \times 4}{\sqrt{2 \times 4 \times 1.67 \times 3.5 \times 1.61}} 10^{-19+1-6+20} =$$

$$= 1.48 \cdot 10^{-4} \text{ m}$$

Convergence ratio K

$$K = \frac{r_0}{r_t} = \frac{2 \times 10^{-3}}{1.48 \times 10^{-4}} \simeq 13.5 \text{ times.}$$

(ii) Magnetude of the axial B field can be calculated for the conversation of magnetic flux

$$B_0 r_0^2 = B_t r_t^2$$

$$B_t = B_0 \frac{r_0^2}{r_t^2} = B_0 k^2 = 10 \cdot 13.5^2$$

$$= \underline{1820 T}$$

(iii) The peak of azimutal B field

$$B_t = \frac{\mu_0 I}{2\pi r_t}$$

$$B_t = \frac{1.25 \times 10^{-6} \times 25 \times 10^6}{2 \times 3.14 \times 1.48 \times 10^{-4}} = 33,600 T$$

(iv) The azimutal B field is not used because it causes magnetic drift $v_{\nabla B}$ and the drift moves particles away from the centre.

(v) For adiabatically compressed gas $pV^{\gamma} = \text{const}$

$$P_0 V_0^{\gamma} = \text{const} = P_t V_t^{\gamma}$$

Where P_t and V_t are presure and volume in the compressed plasma (gas)

$$V_t = V_0/k = V_0 k^{-1}$$

k is the convergence ratio

$$P_t = P_0 k^{\gamma}; \quad V_t = V_0 k^{-1}$$

But $\frac{PV}{T} = \text{const}$ for ideal gas.

$$\frac{P_0 V_0}{T_0} = \text{const} = \frac{P_t V_t}{T_t}$$

$$\frac{P_0 V_0}{T_0} = \frac{P_0 k^{\gamma} V_0 k^{-1}}{T_t}$$

$$T_t = T_0 K^{\gamma - 1} \Rightarrow$$

$$\gamma = 5/3$$

$$T_t = T_0 K^{5/3 - 1} = T_0 K^{2/3}$$

$$\boxed{T_t = K^{2/3} T_0}$$

In our case $K = 13{,}5$

$$\boxed{T_t = (13{,}5)^{2/3} \cdot T_0 = 5{,}7 \times T_0}$$

For example if $T_0 = 300\ K$

$$T_t = 1{,}700\ K$$

T_t is temperature of compressed plasma.

(VI) We need to increase the plasma temperature in

$$\frac{4.3 \text{ keV}}{50 \text{ eV}} = 86 \text{ times.}$$

From a formula derived before

$$T_t = K^{2/3} T_0 ,$$

where $T_0 = 50$ eV, $T_t = 4.3$ keV

$$K = \left(\frac{T_t}{T_0} \right)^{3/2}$$

$$K = \left(\frac{4{,}300}{50} \right)^{3/2} = 86^{3/2} = \underline{800 \text{ times}}$$

So, convergence ratio has to be $\underline{K = 800}$ in order to increase the plasma temperature in 86 times, from 50 eV to 4,300 eV.

(vii) Preheat temperature or (Initial temperature)

$$T_t = T_0 K^{2/3} = T_0 \cdot (13,5)^{2/3}$$

$$\Rightarrow T_0 = \frac{T_t}{K^{2/3}} = \frac{4,300}{(13,5)^{2/3}} =$$

$$= \frac{4,300}{5.7} = 750 \text{ eV} \approx 830,000 \text{ K}$$

It can be achieved by heating a plasma by a laser.