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Assignment 3

$$a) W_{\text{pellet-electric}} = 150 \text{ MJ} \times 33\% = \underline{49.5 \text{ MJ}}$$

Frequency at which pellets should be installed in the reactor

$$f_{\text{pellet}} = \frac{1 \text{ GW}}{49.5 \text{ MJ}} = 20.2 \approx 20 \frac{\text{times}}{\text{sec}}$$

As the reactor has an internal radius $R = 5 \text{ m}$, speed at which pellets should travel inside the reactor

$$v_{\text{pellet}} = R \cdot f_{\text{pellet}} = 5 \cdot 20 = \underline{100 \frac{\text{m}}{\text{sec}}}$$

b) Total fusion energy of the reactor

$$W_{\text{fusion}} = 20.2 \times 150 \text{ MW} = \underline{3 \text{ GW}}$$

$$W_{\alpha \text{ particles}} = \frac{1}{5} W_{\text{fusion}} = 600 \text{ MW}$$

α power per m^2 of the wall

$$w_{\alpha} = W_{\alpha \text{ part.}} / S$$

$$S = 4\pi R^2 = 4 \cdot 3.14 \cdot 5^2 = 314 \text{ m}^2$$

S - area of internal reactor wall

$$w_{\alpha} = 600 / 314 = 1.91 \text{ MW/m}^2$$

Inner wall surface temperature

$$T = \frac{w_{\alpha} \cdot d}{k} + T_{\text{cool}}$$

d is thickness of the wall

k is thermal conductivity

c) According Stefan-Boltzman law heat flux emitted from the hot surface is

$$Q_{IR} = \epsilon \sigma A (T_2^4 - T_1^4),$$

where A is area of the surface

ϵ - is emissivity

σ is Stefan-Boltzman constant

T_2 - surface temperature

T_1 - background temperature.

For steel with $\epsilon = 0.6$

$$\begin{aligned} Q_{IR} &= 0.6 \times 5.67 \times 10^{-8} (1300^4 - 300^4) = \\ &= 0.6 \times 5.67 \times 10^{-8} \times 3.0 \times 10^{12} = 100 \frac{\text{KW}}{\text{m}^2} \end{aligned}$$

For tungsten with $\epsilon = 0.2$

and temperature $260^\circ\text{C} = 530 \text{ K}$

$$\begin{aligned} Q_{IR} &= 0.2 \times 5.67 \times 10^{-8} (530^4 - 300^4) = \\ &= 0.2 \times 5.67 \times 10^{-8} \times 0.071 = 801 \text{ W/m}^2 \end{aligned}$$

d) The pellet surface

$$\begin{aligned} S_{\text{pellet}} &= 4\pi r^2 = 4\pi (0.001)^2 = \\ &= 12.56 \times 10^{-6} = \underline{1.256 \times 10^{-5} \text{ m}^2} \end{aligned}$$

Heat absorbed by a pellet in case of steel during being in a reactor for $\tau \approx 0.05$ sec.

$$\begin{aligned} Q_{\text{IR pellet}} &= Q_{\text{IR}} \cdot S_{\text{pellet}} \cdot \tau = \\ &= 100 \text{ kW} \cdot 1.256 \cdot 10^{-5} \times 0.05 = \\ &= 10^5 \times 1.256 \times 10^{-5} \times 5 \times 10^{-2} = \\ &= 10^{5-5-2} \times 6.3 = \underline{0.063 \text{ J}} \end{aligned}$$

In case of tungsten, heat absorbed by the pellet will be

$$q_{\text{IR}} = 801 \times 1.26 \times 10^{-5} \times 5 \times 10^{-2} = \underline{5 \cdot 10^{-4} \text{ J}}$$

Volume of D-T ice

$$V_{DTice} = \frac{4}{3} \pi (r_2^3 - r_1^3) =$$
$$= \frac{4}{3} 3.14 (1.3^3 - 0.8^3) = 2.04 \text{ mm}^3 \approx$$
$$\approx 2 \cdot 10^{-9} \text{ m}^3$$

pellet mass

$$m_{\text{pellet}} = \rho \cdot V = 2 \cdot 10^5 \times 2 \cdot 10^{-9} =$$
$$= \underline{4.4 \cdot 10^{-4} \text{ g}}$$

In case of steel the pellet temperature will increase by

$$\Delta T = \frac{q_{IR \text{ pellet}}}{C m_{\text{pellet}}} = \frac{0.063}{8.8 \times 4.4 \times 10^{-4}} = \underline{16.3 \text{ K}}$$

In case of tungsten the pellet temperature will increase by

$$\Delta T = \frac{5 \cdot 10^{-4}}{8.8 \times 4.4 \times 10^{-4}} = \underline{0.13 \text{ K}}$$

e) The intensity at which a pellet absorbs heat from a hot surface is

$$P_{\text{pellet}} = \frac{q}{\tau} = \frac{0.063 \text{ J}}{0.05} = 1.26 \text{ W}$$

Where q is energy received from a hot surface during being in a reactor.

In the case of filled in a reactor gas Xe, molecules of the gas will move towards the pellet and heat the pellet. at energy flux

$$\Gamma_E = \frac{1}{4} \langle v \rangle n E_k S_{\text{pellet}},$$

where $\langle v \rangle$ average gas speed,
 E_k energy of ^a gas molecule,
 S area of the pellet

$$\epsilon_K = \frac{3}{2} K_B T$$

$$n = \frac{p}{K_B T}$$

$$\langle v \rangle = \left(\frac{3 K_B T}{m_{Xe}} \right)^{\frac{1}{2}}$$

$$\Rightarrow \frac{q}{L} = \Gamma_\epsilon = \frac{1}{4} \left(\frac{3 K_B T}{m_{Xe}} \right)^{\frac{1}{2}} p \frac{3/2 K_B T}{K_B T}$$

$$= \frac{1}{4} \frac{\sqrt{3}}{2} \left(\frac{K_B T}{m_{Xe}} \right)^{\frac{1}{2}} p s_{\text{pellet}}$$

$$p = \frac{q/L}{s_{\text{pellet}} \left(\frac{K_B T}{m_{Xe}} \right)^{1/2}} \frac{8}{3\sqrt{3}}$$

$$p = \frac{8}{3\sqrt{3}} \frac{q/L}{\left(\frac{K_B T}{m_{Xe}} \right)^{1/2} s_{\text{pellet}}}$$

$$S = 1.26 \cdot 10^{-5} \text{ W/m}^2$$

$$q/\tau = 1.26 \text{ W}$$

$$k_B = 1.38 \cdot 10^{-23}$$

$$m_{Xe} = 1.31 \times 1.61 \times 10^{-27} = 2.2 \cdot 10^{-25} \text{ kg}$$

$$T = 1300 \text{ K} = 1.3 \times 10^3 \text{ K}$$

$$p = \frac{8}{3\sqrt{3}} \frac{1.26}{\left(\frac{1.38 \cdot 10^{-23} \cdot 1.3 \cdot 10^3}{2.2 \times 10^{-25}} \right)^{1/2} \cdot 1.26 \cdot 10^{-5}} =$$

$$= \underline{\underline{550 \text{ Pa}}}$$

If we fill xenon with pressure of $p = 550 \text{ Pa}$ in a reactor temperature of the pellet will increase by 16 K , as in the case of the hot wall radiation.