Fusion: MCF - Homework #3

1 This problem describes the phenomenon of runaway electrons, a situation in which a small DC electric field can accelerate a certain population of plasma electrons to relativistic speeds. This occurs because – as discussed in lecture – the collision frequency decreases rapidly for large velocities: $\nu_{\rm ei} \propto 1/v^3$. Consequently, high energy electrons in the tail of the distribution function can gain a large amount of momentum from the electric field between collisions due to the fact that this time is so long. If the momentum gain is larger than the momentum loss in a collision, then an unstable, "runaway" situation exists.

The equation describing the parallel motion of an electron $v_{\rm e}(t)$ undergoing collisions in the presence of a DC electric field can be written as

$$m_{\rm e}\frac{{\rm d}v_{\rm e}}{{\rm d}t} = -eE_{||} - m_{\rm e}(\nu_{\rm ei}+\nu_{\rm ee})v_{\rm e}$$

where $E_{||} = -\left|E_{||}\right|$ is a negative constant and

$$\begin{aligned}\nu_{\rm ei}+\nu_{\rm ee} &= \frac{3\sqrt{2}}{16\pi}\frac{ne^4\ln\Lambda}{\epsilon_0^2 m_{\rm e}^{1/2}T_{\rm e}^{3/2}}\left[\frac{v_{T\rm e}}{v_{\rm e}}\right]^3 \\ &= 7.3\cdot 10^5\frac{n_{20}}{T_k^{3/2}}\left[\frac{v_{T\rm e}}{v_{\rm e}}\right]^3 \sec^{-1},\end{aligned}$$

where n_{20} is the particle density in units of $10^{20}\,{\rm m}^{-3}$, T_k is the temperature in keV and $v_{T\rm e} = (2T_{\rm e}/m_{\rm e})^{1/2}$.

(a) Derive a simplified form of the differential equation above by introducing a normalized velocity and electric field defined by

$$\begin{aligned} w_{\rm e} &= v_{\rm e}/v_{T\rm e} \\ E_0 &= \frac{e\left|E_{||}\right|}{(2m_{\rm e}T_{\rm e})^{1/2}} = 9.4\cdot 10^3\frac{\left|E_{||}\right|}{T_k^{1/2}}\sec^{-1}.\end{aligned}$$

In the second numerical formula above $\left|E_{||}\right|$ is meant in units of V/m. [4]

(b) Show that there is a critical value of the normalized initial velocity $w_0 \equiv w_{\rm e}(t=0)$ such that above $w_{\rm crit}(\nu, E_0)$, the electrons will run away. (This can be done directly by solving the equation, but I highly recommend that you just try to examine the curve of $\dot{w}_{\rm e}$ vs $w_{\rm e}$ and think a little.) [10]

(c) Consider a plasma with temperature $T_{\rm e} = 2\,{\rm keV}$ and an electric field $E_{||} = 0.5\,{\rm V/m}$. Calculate the critical densities for which $w_{\rm crit} = 1$ and $w_{\rm crit} = 10$.

Turn over

For a Maxwellian distribution function, the first case corresponds to almost the entire plasma consisting of runaways, while the second one corresponds to a negligible number of runaways. [6]