Problem Sheet: Advanced Plasmas

January 22, 2015

1. The N-particle distribution function and electromagnetic fields $\mathbf{E}$ & $\mathbf{B}$ in the plasma can be written $N = f + \delta N$, $\mathbf{E} = \mathbf{E}_s + \delta\mathbf{E}$, $\mathbf{B} = \mathbf{B}_s + \delta\mathbf{B}$. Derive the equation for f from the Klimontovich equation. Identify the collision operator.

2. Consider a plasma which, in equilibrium, consists of two well defined populations of electrons: a beam (in the x-direction) and a background. The equilibrium distribution functions of the beam and background are, respectively

$$f_B = \frac{n_B}{\pi^{3/2} v_{TB}^3} \exp\left[-\frac{|\mathbf{v} - v_B \hat{\mathbf{e}}_x|^2}{v_{TB}^2}\right] \qquad f_M = \frac{n_M}{\pi^{3/2} v_{TM}^3} \exp\left[-\frac{v^2}{v_{TM}^2}\right]$$

The beam density is low, i.e. $n_B \ll n_M$.

(i) Determine $F_0(v_x)$ where $F_0(v_x) = \int dv_y dv_z f_0$ (f_0 is the equilibrium distribution function of the electrons in the plasma). You may need the following standard integral

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

(ii) Sketch F_0 in the case where $v_B \gg v_{TM}$ and $v_B \gg v_{TB}$.

(ii) Using the equation derived in the lectures, determine the rate of Landau damping for this distribution for a wave with frequency ω and wavenumber k where $\mathbf{k} = k\hat{\mathbf{e}}_x$. Let $\omega/k = v_B \pm \Delta v$ and assume that ω/k is much greater that v_{TM} so that the contribution from the background Maxwellian can be ignored. Assume also that although ω/k is close enough to v_B that the beam is the only part of the distribution that contributes, it is far enough from v_B that the expression given in the lecture notes is valid.

3. Consider a plasma in equilibrium with a magnetic field in the z-direction (assume no electric fields)and a density gradient in the x, z-plane (assume no temperature gradients). Let the distribution function for a species with mass m and charge Ze be equal to $f_M + \delta f$ with $\delta f \ll f_M$

(i) Using a Krook collision operator show that the equation satisfied by δf is

$$\frac{f_M}{n}\mathbf{v}\cdot\nabla_{\mathbf{x}}n + \frac{Ze}{m}(\mathbf{v}\times\mathbf{B})\cdot\nabla_{\mathbf{v}}\delta f = -\nu_c\delta f$$

ν_c is the collision frequency
(ii) Show that the particle flux is given by $\mathbf{\Gamma} = \int \mathbf{v}\delta f d^3\mathbf{v}$
(iii) Show that the components of the particle flux are

$$\Gamma_x = -\frac{\tau_c T}{m[1+(\omega\tau_c)^2]}\frac{\partial n}{\partial x}$$
$$\Gamma_y = \omega\tau_c\frac{\tau_c T}{m[1+(\omega\tau_c)^2]}\frac{\partial n}{\partial x}$$
$$\Gamma_z = -\frac{\tau_c T}{m}\frac{\partial n}{\partial z}$$

Here T is the temperature of the plasma species, $\tau_c = 1/\nu_c$

4. Consider a plasma with density and temperature gradients in the z-direction, in steady-state with no macroscopic electric and magnetic fields.

(i) If the initally Maxwellian background is perturbed by δf write down and using the BGK collision operator, write down a linearised equation for the distribution function of a species with charge Ze and mass m.

(ii) Relate the density and temperature gradients in the plasma if it is at constant pressure and has an 'ideal gas' equation of state, i.e. $P = nT$.

(iii) Using the expression for the heat-flow in the z-direction, $q_z = \int d^3\mathbf{v}(mv^2/2)v_z\delta f$, show that the thermal conductivity is $-(5nT)/(2m\nu_c)$. You may need the following integral

$$\int_0^\infty x^n e^{-ax^2}dx = 1.3.5.\ldots.(n-1)(2a)^{-(n+1)/2}\sqrt{\frac{\pi}{2}}$$