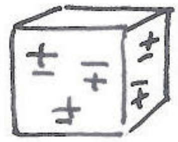
Assignment 2

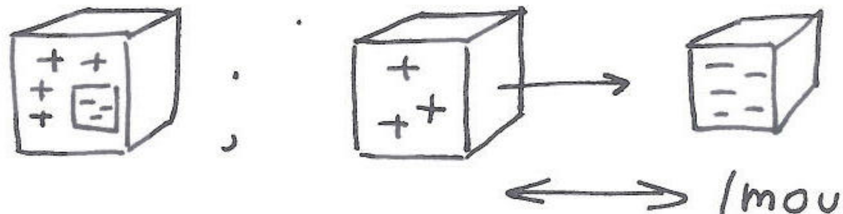
Date 04/12/2015

a) **Physics behind electron plasma waves.**

Here we should consider a movement of a group of electrons, not a single electron. For example, if we consider a box of quasi-neutral plasma, and there is no perturbance, nothing will happen.



But now we will collect all electrons from the box and pull them out.



(movement back and forward)

So, only possetivly charged ions will be left in the box. The box of electron will be attracted back to the box of ions. (Because negatively and positevely charges are attracted to each other).
Here we will neglect of ion's movement because they are much heavier electrons. So only the group of electrons will be moving.

As the electrons have inertia, they won't stop moving and be oscillating. As a result of this, there electron plasma wave will arise.

However, if the electrons have thermal energy, they will be able to escape from one box and move to another box, and "transfer" information, It means they will have non-zero group velocity.

b) <u>Electron force balance eqn.</u>

i) $$m_e n_e \left[\frac{\partial \underline{u}_e}{\partial t} + (\underline{u}_e \cdot \nabla) \underline{u}_e \right] = - n_e e \underline{E}$$

Where u_e is flow, n_e is density, $\underline{E}$ is electric field

If a magnetic field is present a term $[\underline{u}_e \times \underset{\sim}{B}]$ should be added to right side of the eqn.

This equation means retaining enertia.

In this equation we can clearly see a patern of convective derivative $\frac{D(\,)}{dt} = \frac{\partial (\,)}{\partial t} + u \cdot \nabla (\,)$

which is $\left(\frac{\partial \underline{u}_e}{\partial t} + (\underline{u}_e \cdot \nabla) \underline{u}_e \right)$.

<u>Electron continuity</u>

$$\frac{\partial n_e}{\partial t} + \underline{\nabla} \cdot (n_e \underset{\sim}{u}_e) = 0$$

Where n_e is density, u_e is flow.

Continuity equation means

the conservation of matter, which requires the total number of particles in valume V can change only if there is a net flux of particles across the surface S surounding that volume V.

In this equation we can see the patern of convective derivative $\left(\frac{D()}{\partial t} = \frac{\partial}{\partial t}(\ ) + u \cdot \nabla(\ )\right)$ whish is $\left(\frac{\partial}{\partial t} n_e + \nabla \cdot (n_e \underline{u}_e)\right)$

Poisson equation

$$\varepsilon_0 \underline{\nabla} \cdot \underline{E} = \rho = (n_i - n_e)e$$

This equation means that presence of charge imbalance causes flux of electric field.

I don't find any paterns of convective derivative in this equation.

(i)

c) (i) <u>Electron force balance eqn</u>

$$m_e n_e \left[\frac{\partial u_e}{\partial t} + (\underline{u_e} \cdot \nabla) u_e \right] = -n_e e \underline{E}$$

if $\underline{E}^{(1)} = 0$ and $u_e = 0 \Rightarrow \frac{\partial u}{\partial t} = 0$

<u>Electron continuity eqn.</u>

$$\frac{\partial n_e^{(1)}}{\partial t} + n_e^{(0)} \nabla \cdot u_e^{(1)} = 0 ,$$

if there is no flow $n_e^{(0)} \underline{\nabla} \cdot \underline{u_e} = 0$

$$\Rightarrow \frac{\partial n_e^{(1)}}{\partial t} = 0$$

<u>Poisson equation</u>

$$\varepsilon_0 \nabla \cdot \underline{E}^{(1)} = -e n_e^{(1)}$$

if there no electric field $E = 0$

$\Rightarrow e n_e^{(1)} = 0 \Rightarrow n_e^{(1)} = 0.$

But $n_e = n_i = n^{(0)}$ - uniform in space

<u>quasi-neutral.</u>

$\underline{u}_e = \underline{u}^{(0)} = 0$ zero flow

$\underline{E} = E^{(0)} = 0$ zero electric field

means the "equilibrium" state.

(ii) and (iii)

1) From electron force balance eqn.

$$m_e \frac{\partial \underline{u}_e^{(1)}}{\partial t} = \left(\underline{E}_e^{(1)} + [\underline{u}_e^{(1)} \times \underline{B}^{(1)}] \right)$$

$$\left\{ \begin{aligned} [\underline{u}_e^{(1)} \times \underline{B}^{(0)}] &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_{ex}^{(1)} & u_{ey}^{(1)} & u_{ez}^{(1)} \\ 0 & 0 & B_z^{(0)} \end{vmatrix} = \\ &= \hat{i}\, u_{ey}^{(1)} B_z^{(0)} - j\, u_{ex}^{(1)} B_z^{(0)} \end{aligned} \right\}$$

here $\underline{B^{(0)} = B}$

$$\begin{cases} -i\omega m_e \cdot u_{ex}^{(1)} = -e E_x^{(1)} - e B_z u_{ey}^{(1)} \\ -i\omega m_e \cdot u_{ey}^{(1)} = -e B_z u_{ex}^{(1)} \end{cases}$$

$$u_{ey}^{(1)} = i B_z e\, u_{ex}^{(1)} / \omega m_e$$

$$-i\omega m_e \cdot u_{ex}^{(1)} = -e E_x^{(1)} - \frac{e B_z\, i B_z \cdot e\, u_{ex}^{(1)}}{\omega m_e}$$

$$u_{ex}^{(1)} \left(1 - \frac{e^2 B_z^2}{\omega^2 m_e^2} \right) = - \frac{i e E_x^{(1)}}{\omega m_e}$$

$$\boxed{u_{ex}^{(1)} = \frac{-ie E_x^{(1)}}{\omega m_e \left(1 - \frac{e^2 B_z^2}{\omega^2 m_e^2} \right)}} \qquad (1)$$

2) <u>From continuity equation</u>

$$\frac{\partial n_e^{(1)}}{\partial t} + n_e^{(0)} \nabla \cdot u_e = 0$$

<u>and from Poisson equation</u>

$$\varepsilon_0 \nabla \cdot \underline{E}^{(1)} = -e\, n_e^{(1)}$$

$$-i\omega n^{(1)} + n^{(0)} i k u_{ex}^{(1)} = 0$$

$$n_e^{(1)} = \frac{n_e^{(0)} k u_{ex}^{(1)}}{\omega}$$

$$\varepsilon_0 i k E_x^{(1)} = -e\, n_e^{(1)} = \frac{n^{(0)} k u_{ex}^{(1)} e}{-\omega}$$

$$\boxed{u_{ex}^{(1)} = \frac{-i \varepsilon_0 E_x^{(1)} \omega}{e\, n_e^{(0)}}} \qquad (2)$$

3) From equs (1) and (2) $u_{ex}^{(1)} = e_{ex}^{(2)}$

$$\Rightarrow -\frac{i e E_x^{(1)}}{\omega m_e \left(1 - \dfrac{e^2 B_z^2}{\omega^2 m_e^2}\right)} = \frac{-i \varepsilon_0 E_x^{(1)} \omega}{e\, n_e^{(0)}}$$

$$\underbrace{\frac{e^2 n_e^{(0)}}{\varepsilon_0 m_e}}_{=\omega_{pe}^2} \frac{1}{\omega^2} \frac{1}{\left(1 - \underbrace{\dfrac{e^2 B_z^2}{m_e^2}}_{=\omega_{ce}^2} \dfrac{1}{\omega^2}\right)} = 1$$

Where $\omega_{pe} = \sqrt{\dfrac{e n_e^{(0)}}{\varepsilon_0 m_e}}$ is electron plasma frequency

$\omega_{ce} = \dfrac{e^2 B_z^2}{m_e^2}$ is cyclotron electron frequeney

$$\frac{\omega_{pe}^2}{\omega^2\left(1 - \dfrac{\omega_{ce}^2}{\omega^2}\right)} = 1$$

$$\omega_{pe}^2 = \omega^2\left(1 - \frac{\omega_{ce}^2}{\omega^2}\right) = \omega^2 - \omega_{ce}^2$$

$$\boxed{\omega^2 = \omega_{pe}^2 + \omega_{ce}^2}$$

Where ω_{pe} is electron plasma frequency, and ω_{ce} is electron cyclotron frequency.