Fusion: MCF - Homework #4

1 (a) Starting from the equations of ideal MHD, i.e.

$$\begin{aligned}
\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) &= 0 \\
\rho(\partial_t + \mathbf{v} \cdot \nabla)\mathbf{v} &= \mathbf{J} \times \mathbf{B} - \nabla p \\
(\partial_t + \mathbf{v} \cdot \nabla)\left[\frac{p}{\rho^\gamma}\right] &= 0 \\
\partial_t \mathbf{B} - \nabla \times (\mathbf{v} \times \mathbf{B}) &= 0 \\
\nabla \times \mathbf{B} &= \mu_0 \mathbf{J}
\end{aligned}$$

derive the linearised set of equations we used in lecture to derive the basic MHD (or Alfvén) waves. [4]

(b) Using the cold plasma dielectric tensor, prove that

$$N_{||}^2 = L = 1 - \sum_j \frac{\omega_{\mathrm{p}j}^2}{\omega(\omega - \omega_{\mathrm{c}j})}$$

corresponds to a left-hand circularly polarized wave, and

$$N_{||}^2 = R = 1 - \sum_j \frac{\omega_{\mathrm{p}j}^2}{\omega(\omega + \omega_{\mathrm{c}j})}$$

corresponds to a right-hand circularly polarized wave. [8]

(c) Use the linearised cold plasma momentum equation in the absence of collisions, i.e.

$$n_j m_j \partial_t v_{j1} = n_j q_j \left(\mathbf{E}_1 + \mathbf{v}_{j1} \times \mathbf{B}_0\right)$$

to show that the perturbed fluid velocity is given by

$$\begin{aligned}
v^+ &= \frac{iq}{m\omega} \frac{E^+}{\left(1 - \frac{\omega_{\mathrm{c}j}}{\omega}\right)} \\
v^- &= \frac{iq}{m\omega} \frac{E^-}{\left(1 + \frac{\omega_{\mathrm{c}j}}{\omega}\right)},
\end{aligned}$$

where $v^\pm = v_x \pm i v_y$, and $E^\pm = E_x \pm i E_y$; thereby showing that if $E^- \neq 0$, then $v^- \to \infty$ at ω_{ce}; and $E^+ \neq 0$, then $v^+ \to \infty$ at $\omega_{\mathrm{c}i}$.

End of paper [8]