Alexei Kosykhin ①

Assignment 5

I part

a) We assume $n = n_D + n_T$ - number of ions in the plasma.

and $n_T \approx n_D$

T - tritium, D - Deterium

$n_T = n/2$

For fusion power we can write

$$P = \frac{1}{4} n^2 <\sigma v> \mathcal{E}_{fus} \left(\frac{T}{10}\right)^2 V$$

$$n = \sqrt{\frac{4P}{<\sigma v>(T/10)^2 \mathcal{E}_{fus} V}}$$

$$n = \sqrt{\frac{4 \times 5 \times 10^8 \ J/s}{10^{-22} \times \left(\frac{10\,keV}{10}\right)^2 \cdot 17.6 \times 10^6 \times 1.61 \times 10^{-19} \times 680}}$$

$$= \sqrt{\frac{4 \times 5 \times 10 \times 10^{7+22-6+19-2}}{17.6 \times 1.61 \times 6.8}} = \underline{1.02 \times 10^{20} \ par.}$$

n - is density

Total number of tritium ions

$$N_T = \frac{n}{2} V = 1.02 \times 10^{20} \times 6.8 \times 10^{2} =$$
$$= 3.5 \times 10^{22}$$

mass of tritium

$$m_T = N_T \cdot m_T = 3.5 \times 10^{22} \times 3 \times 9.1 \times 10^{-27} =$$
$$= \underline{0.94 \text{ g.}}$$

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b) Local neutrality condition:

$$-n_e + n_T + n_D + Z n_{He} = 0$$

We assume $n_T = n_D = n$

$$f_{He} = \frac{n_{He}}{n_e} \Rightarrow n_{He} = f_{He} n_e$$

$$\Rightarrow -n_e + 2n + Z_{He} n_{He} = 0$$

$$n = \frac{1}{2}(n_e - Z_{He} n_{He}) = \frac{1}{2}(n_e - Z_{He} f_{He} n_e) =$$

$$= \frac{n_e}{2}(1 - Z_{He} f_{He})$$

Fusion power

$$P = n_T \, n_D \langle \sigma v \rangle \left(\frac{T}{10}\right)^2 \varepsilon_{fus} V =$$

$$= \frac{n_e^2}{4} \left(1 - Z_{He} f_{He}\right)^2 \langle \sigma v \rangle \left(\frac{T}{10}\right)^2 \varepsilon_{fus} V$$

$$\boxed{P = \frac{n_e^4}{4} \left(1 - Z_{He} f_{He}\right)^2 \langle \sigma v \rangle \left(\frac{T}{10}\right)^2 \varepsilon_{fus} V}$$

If $f_{He} = 0$ Power $P = P_0$

$$(*) \quad P = \frac{n_e^2}{4}\left(1 - Z_{He} \cdot 0\right)^2 \langle \sigma v \rangle \left(\frac{T}{10}\right)^2 \varepsilon_{fus} V = P_0$$

If Power drops by 20% $P = \frac{8}{10} P_0$ and $f_{He} \neq 0$.

$$(**) \quad P' = \frac{n_e^2}{4}\left(1 - Z_{He} f_{He}\right)^2 \langle \sigma v \rangle \left(\frac{T}{10}\right)^2 \varepsilon_{fus} V = \frac{8}{10} P_0$$

We devide equ $(**)$ by $(*)$ and get

$$\frac{\frac{n_e^2}{4}\left(1 - Z_{He} f_{He}\right)^2 \langle \sigma v \rangle \left(\frac{T}{10}\right)^2 \cdot \varepsilon_{fus} V}{\frac{n_e^2}{4}\left(1 - Z_{He} \cdot 0\right)^2 \langle \sigma v \rangle \left(\frac{T}{10}\right)^2 \varepsilon_{fus} V} = \frac{\frac{8}{10} P_0}{P_0}$$

$$\Rightarrow \left(1 - Z_{He} f_{He}\right)^2 = \frac{8}{10} \Rightarrow$$

$$\Rightarrow f_{He} = \frac{1 - \sqrt{8/10}}{Z_{He}} = \frac{1 - \sqrt{8/10}}{2} = \underline{0.05}$$

$Z_{He} = 2$

c) In case of DEMO

$$\Gamma = \frac{15 \times 10 \times 10^{6}\,W}{7 \times 5 \times 1.61 \times 10^{-19}} = 2.6 \times 10^{25}\ \frac{\text{par.}}{\text{sec}}$$

Number of particles in 5 years

$$N = 2.6 \times 10^{25} \times 75\% \times 5 \times 365 \times 24 \times 3600 =$$
$$= 3 \times 10^{33} \text{ particles}$$

$$m_T = N \cdot m_T = 3 \times 10^{33} \times 3 \times 9.1 \times 10^{-27} =$$
$$= 8.2 \times 10^{7}\,kg$$

$$\frac{T}{M} = \frac{0.640}{8.2 \times 10^{7}} = 7.8 \times 10^{-9}$$

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d) ?

Part II

a) $q_\perp = \Gamma \gamma k T_e \Rightarrow$

$$\Rightarrow \Gamma = \frac{q_\perp}{\gamma k T_e}$$

$$\Gamma = \frac{5 \times 10 \times 10^6 \text{ W}}{7 \times 5 \times 1.61 \times 10^{-19}} = 8{,}8 \times 10^{24} \frac{\text{ions}}{\text{sec}}$$

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b) Number of particles in 5 years

$$N = 8.8 \times 10^{24} \times \frac{400}{3600} \times \frac{6}{12} \times 5 \times$$

$$\times 365 \times 24 \times 3600 = 7.6 \times 10^{31} \text{ particles}$$

Mass of tritium

$$M = N \times m_T = 3 \cdot 9{,}1 \times 10^{-27} \times 7{,}6 \times 10^{31} =$$

$$= 2.1 \times 10^6 \text{ kg}$$

$$\frac{T}{M} = \frac{0.640 \text{ kg}}{2.1 \times 10^6 \text{ kg}} = 3 \times 10^{-7}$$

Part III

a) In case of ITER

$$N_T = \frac{500 \times 10^6\ W}{17.6 \times 10^6\ eV} \times 1\%$$

$$= \frac{5 \times 10^8\ W \times 0.01}{17.6 \times 10^6 \times 1.61 \times 10^{-19}} = 1.76 \times 10^{24} \frac{part.}{sec}$$

$$\dot{M}_T = N_T \times m_T = 1.76 \times 10^{24} \times 3 \times 9.1 \times 10^{-27} =$$

$$= 0.048\ kg/sec$$

Time $T = \frac{3\ kg}{0.048} \approx 60\ sec \approx \underline{1\ min}$

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b) In case of DEMO

$$N_T = 5 \times 1.76 \times 10^{24} = 9 \times 10^{24}$$

$$\dot{M}_T = 5 \times 0.048\ \frac{kg}{s} \approx 0.25\ kg/s$$

Time $= \frac{1}{5}\ 60\ s = \underline{12\ sec}$