THE UNIVERSITY of York

DEPARTMENT OF PHYSICS

MSC IN FUSION ENERGY

and

FUSION CENTRE FOR DOCTORAL TRAINING

PROBLEM QUESTIONS

BOOKLET 2014/5

MSC IN FUSION ENERGY PROBLEM BOOK 2014/15

CONTENTS

FUSION TECHNOLOGY

1 (a) Calculate the thickness of Cd (A=112.4, ρ=8650 kg m^{-3}) that would be needed to attenuate the intensity of a collimated beam of thermal neutrons by a factor of 1000. The average cross section for thermal neutrons is 3000 b and you may assume that the scattering cross section is negligible.

(b) The flux of 14 MeV neutrons from a fusion reactor at a particular point is 10^{15} cm^{-2} s^{-1}. The cross sections for some of the key reaction processes from ^{56}Fe, which is a component of steel, at this energy are: σ(n,n') 820 mb, σ(n,p)=120 mb, σ(n,2n)=400 mb and $\sigma(n,\alpha)$=4 mb. Determine the reaction rate (in cm^{-2} s^{-1}) of neutrons from ^{56}Fe which lead to absorption processes, if the steel contains 80% ^{56}Fe and is 20 cm thick. (Density of iron = 7.8 g cm^{-3}).

(c) If the mean attenuation length of gold (A=197) for a collimated beam of neutrons of a certain energy is 2 cm, calculate the total removal cross section, σ_r for the removal of neutrons from the beam. (Density of gold = 19.3 g cm^{-3})

If the neutron scattering cross section is six times the absorption cross section what is the mean total distance travelled by the neutrons (i.e. the absorption mean free path) in gold?

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2 Consider a fusion power plant producing 3GW of fusion power. Suppose that the area of the confinement chamber is 1300m^2. Determine the resulting neutron wall load in MW/m^2.

Different material resilience to neutron damage can be expressed in terms of displacements per atom before the material (in the blanket) must be replaced. In steel, exposure to 1MW/m^2 for 1 year of full power operation induces 10 dpa.

Determine the lifetime of the blanket in full power years if the tolerable level of neutron damage is 20 dpa. If an improved material could be identified that could withstand 60dpa, what would the lifetime of this blanket be? If it takes 6 months to replace the blanket, calculate what fraction of the time the plant is shut down for blanket maintenance in both cases (ie for 20 and 60 dpa).

Use this number to calculate the plant availability, assuming a further 10% of the operation time is lost for other maintenance procedures. Hence estimate the cost of electricity in euro-cents per kWh for the two different materials. You should use an estimate of the cost of the fusion core to be 2M€m$^{\square 2}$, and double this to obtain the total site cost. Assume that you are required to repay 10% of the capital per year, and that the plant thermodynamic efficiency is 40%.

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3 A diagnostic beamline is required to deliver 1MW of neutral deuterium power at an energy of 100keV. A plan view schematic of the beamline is shown below together with the profile of the background gas pressure (in Pa) along it. Ions produced at the source are neutralised in the neutraliser with an efficiency

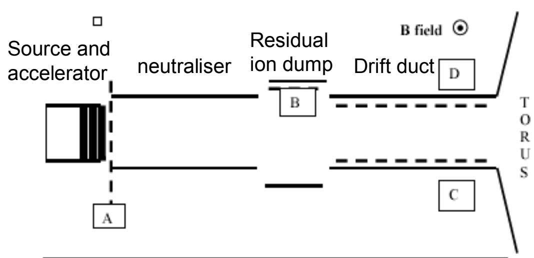
$$\frac{n^0}{n_0^+} = \frac{\sigma_{10}}{\sigma_{10}+\sigma_{01}}\{1-\exp[-N(\sigma_{10}+\sigma_{01})z]\}$$

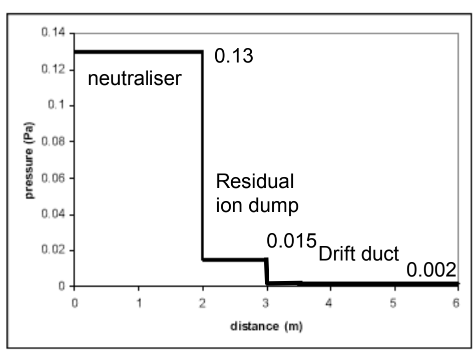
where N is the gas density (1Pa corresponds to N=2.65×10^{20}m^{-3}), z is the length of gas they pass through, n^0 is the neutral particle density at position z and $n^+{}_0$ is the ion density at z=0. In the residual ion dump, and drift duct neutral particles travelling in the z-direction are ionized at a rate:

$$\frac{dn^0}{dz} = -n^0 N\sigma_{01}$$

Calculate the power due to beam components (neutral and charged) at the planes A, B, C and D marked on the diagram in the case of a positive ion beam precursor and specify the extracted ion current required from the source Assume that there is no direct interception and that the residual ion dump is 100% effective. The stray field from the torus is vertical (ie out of the page) and zero at the residual ion dump, rising to 1T at the entry to the torus.
For the purposes of calculation only, you may take the beam as composed of 100% D^+ ions. Assume that all of the ions exiting the neutraliser are collected by the residual ion dump. The cross sections for 100keV deuterium are:

σ_{10} = 1.66x10^{-20}m^2
σ_{01} = 1.55x10^{-20}m^2





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4 Calculate the minimum injection velocity for fuel capsules needed to produce an electrical power output of 1 GW with a thermonuclear yield of 150 MJ per shot in a power plant if the wall of the reactor sits 10 metres from the centre of the reactor. You may assume the efficiency of the thermonuclear to electrical energy conversion in the reactor is 33%.

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5 Consider a single wire of radius a in a wire array Z-pinch. When the wire is vaporised and ionised by a current I, show that the plasma pressure p can be approximated by

$$p \approx \frac{\mu_0 I^2}{2\pi^2 a^2}.$$

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6 If N is the total number of tritium atoms in a reactor system, a simplified model of the evolution of N with time can be described by:

$$\frac{dN}{dt} = -\frac{P}{E_{DT}} + R_B \frac{P}{E_{DT}} - \lambda N$$

where P is the thermal power output of the reactor, E_{DT} is the energy released in a D-T fusion reaction, λ is the radioactive decay constant for tritium and R_B is the tritium breeding ratio, defined as the ratio of the production rate of tritium to the consumption rate of tritium.

Explain the physical meaning of each of the three terms on the right hand side of this equation. Solve this equation to get an expression for N as a function of time and estimate the time taken for a 1 GW(thermal) fusion reactor to double it’s initial total tritium inventory of 5kg.

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INERTIAL CONFINEMENT FUSION

1 Ignition and high gain burn of a compressed fuel blob of 50:50 DT, can only take place if two ρr criteria are satisfied.

i) State these two criteria.
ii) Calculate the thermonuclear energy yield, assuming 33% burn-up of the fuel, for fuel blobs just satisfying the high gain burn criteria given that the fuel is at: a. 50x solid density b. 2000x solid density

You may assume that, at the moment of ignition, the fuel takes the form of an Isochoric sphere.

In conventional ICF, the burn is ignited from a central hotspot. Assuming that this hotspot must be heated to 10keV in order to ignite, calculate the energy required to heat a hotspot which just satisfies the ρr requirement for ignition using the fact that the heat capacity of DT is ~100MJ/g/keV. You may assume an isochoric, spherical hotspot of density 100g/cc.

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2 An isochoric burning sphere of DT has a burn fraction of 60% and releases 1GJ of energy inside a reactor vessel. A mass of TNT high explosive is placed inside an identical reactor vessel and detonated. Given that the TNT releases the same energy as the burning DT sphere, compare the two scenarios by considering the momenta of the reaction products. TNT releases 4.6MJ/kg.

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3 If the ρr requirement for ignition in pure deuterium is approximately 100 times greater than for DT at densities of a few hundred g/cm^3 and temperatures of around 10keV, perform a calculation to illustrate why initiating burn in pure deuterium is impractical using a laser. The heat capacity of pure deuterium is approximately 100MJ/g/keV.

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4 Perform simple calculations to compare how each of the following affects ρr:
a) planar compression
b) cylindrical implosion
c) spherical implosion

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5 Assuming a fuel density of ρ=300g/cc , ignition pulse length t p $\approx$20ps and a required hotspot temperature of 12keV:

a) estimate the energy required to heat the hotspot in fast ignition
b) estimate the average power that must be deposited in the fast ignition hotspot by the ignitor pulse
c) estimate the required ignitor intensity incident on the surface of the hotspot in W/cm^2 assuming efficient coupling

For the heat capacity of DT fuel use 100MJ/g/keV. Loss mechanisms may be ignored. You may treat the hotspot as being approximately cylindrical with length=diameter=2r_{hs} where r_{hs} is given by the appropriate ρr criteria.

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6 In the context of nuclear fusion reactions, explain what we mean by a “secondary” reaction. Explain how such reactions can substantially increase the energy release when pure deuterium is burnt. Quantify how much energy is released per gram of deuterium if:

a) only the two primary reactions of DD occur
b) the fuel burns to the point where the only reaction products are He-4, neutrons and protons

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7 Using the equation that describes the Hugoniot curve of an ideal gas, show that the limiting compression for a strong shock in such a material is 4.

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8 By making comparison with the Fermi temperature, show that DT at 1100g/cc and 150eV may be considered cold (Fermi Degenerate).

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9(**) We have a spherical shell of DT ice, mass 2.2mg. A spherical implosion takes place which compresses the whole of this fuel to a spherical assembly with a uniform density of 1100g/cc

a) Calculate the average Fermi energy of the fuel ions in the imploded state and thereby calculate the minimum energy required to implode the fuel in Joules
b) In order to satisfy stability criteria, the initial fuel layer thickness is set to be exactly 1/12th the initial outer radius of the fuel. Find the initial capsule radius and thereby the Initial capsule volume in cm^3
c) Assuming that the pressure driving the implosion is constant and can be calculated from Energy= P ΔV, where ΔV can be approximated to be the initial capsule volume, calculate the pressure driving the implosion in Mbar
d) By setting the energy required to implode the fuel equal to the kinetic energy of the fuel calculate the implosion velocity in cm/s

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10 State the relative advantages and disadvantages of direct and indirect drive ICF. (Be concise: limit your answer to one side of A4, please!)

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11 For 1$\omega$and 3ω light from a Nd-glass laser system calculate:

i) The critical density as an electron number density, n_e in cm^{-3}
ii) The critical density as a mass density (ρ) in completely ionised C_nH_n plastic in gcm^{-3}. Note that by completely ionised we mean that all electrons are free, and there are no bound electrons remaining. Note that C n H n means that the plastic has an equal number of carbon and hydrogen atoms in it.

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12 a) In each case, explain briefly what the three parametric instabilities, SRS, SBS and $2\omega_{pe}$ are and why they are disadvantageous from the standpoint of ICF.
b) For 3ω laser light from an Nd-glass laser system, incident upon a Beryllium target (the ablation plume from which you may assume to be completely ionised) calculate the mass density (ρ) or density range in which each of the SRS, SBS, and $2\omega_{pe}$ parametric instabilities will grow in gcm^{-3}.

Note that by completely ionised we mean that all electrons are free, and there are no bound electrons remaining.

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13 a) Sketch a Planckian blackbody spectrum with kT=300eV
b) Explain how and why the emission from a real laser driven hohlraum differs from a true Planckian.

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14 Calculate the critical temperature for soft x-ray driven transonic ablation wave propagation in a 50mg/cc C_x H_x foam (you may assume: i) that the radiation wave propagates in the same way in a foam as it would in a uniform material, and ii) that C_x H_x will be completely ionized by the drive at the relevant temperatures). By C_x H_x we mean a material which has an equal number of carbon and hydrogen atoms in it.

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15 a) Briefly describe the 3 phases of RT instability growth
b) Mention a few of the difficulties of performing RT growth experiments of relevance to ICF

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16 a) Explain briefly what is meant by "ablative stabilisation" of the RT instability
b) Explain briefly what the "adiabat shaping" approach to combating RT instability growth is

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MAGNETIC CONFINEMENT FUSION

1 **a)** Consider a particle with charge q in a magnetic field $\vec{B} = B\hat{z}$ in the z direction. In addition to the Lorentz force, it is subjected to a force $\vec{F}_a$ which is perpendicular to $\vec{B}$ so that the velocity is determined by:

$$m\frac{d\vec{v}}{dt} = q\vec{v} \times \vec{B} + \vec{F}_a$$

By substitution, show that the solution is

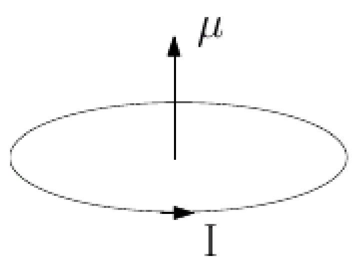
$$\vec{v} = v_\perp \sin(\Omega t)\hat{x} + v_\perp \cos(\Omega t)\hat{y} + v_{||}\hat{z} + \vec{D} \times \vec{B}$$

Find the gyro-frequency Ω, and constant vector $\vec{D}$. Hence show that the drift velocity $\vec{v}_D$ is given by

$$\vec{v}_D = \vec{D} \times \vec{B} = \frac{1}{q}\frac{\vec{F}_a \times \vec{B}}{B^2}$$

Hint $\hat{x} \times \hat{y} = \hat{z}$ and $\vec{A} \times \left(\vec{B} \times \vec{C}\right) = \vec{B}\left(\vec{A} \cdot \vec{C}\right) - \vec{C}\left(\vec{A} \cdot \vec{B}\right)$

b) The magnetic moment of a current loop is $\vec{\mu} = I\vec{S}$ where the direction follows the right hand convention.



Using your expression for Ω, and the Larmor radius $r_L = \dfrac{mv_\perp}{|q|B}$, show that the magnetic moment is given by

$$\vec{\mu} = -\frac{mv_\perp^2}{2B}\hat{b}$$

Where $\hat{b} = \vec{B}/B$ is the unit vector in the direction of $\vec{B}$. Show with a diagram why the sign is negative.

Continued on the next page

c) The potential energy of a magnetic dipole (like a current loop) in a magnetic field is $U = -\vec{\mu} \cdot \vec{B}$. Since the force on an object is always $\vec{F} = -\nabla U$, use your result from part (a) to show that in a non-uniform magnetic field a particle will drift with a velocity

$$\vec{v}_D = \frac{mv_\perp^2}{2qB} \frac{\vec{B} \times \nabla B}{B^2}$$

2 Considering a Maxwellian velocity distribution, and using the fact that for trapped particles,

$$\frac{v_{||}}{v_\perp} \le \sqrt{2\varepsilon}$$

where $v_{||}$ and $v_\perp$ are the components of velocity parallel and perpendicular to magnetic field lines, and □ is the inverse aspect ratio, show that the fraction of trapped particles, f_t, is

$$f_t = \frac{4}{\sqrt{\pi}} \int_0^\infty dt \int_0^{\sqrt{2\varepsilon}t} ds\, te^{-t^2} e^{-s^2}$$

where $t = v_\perp / v_{th}$, $s = v_{||} / v_{th}$ and $v_{th} = (2T/m)^{1/2}$ is the particle thermal speed. In the limit of small inverse aspect ratio, show that the trapped particle fraction is

$$f_t = \sqrt{2\varepsilon}$$

BD

3 Considering a large aspect ratio, circular cross-section tokamak, show that the ratios of the toroidal components of the bootstrap : Pfirsch-Schlüter : diamagnetic current densities is

$$1 : \sqrt{\varepsilon} : \frac{\varepsilon^{3/2}}{q^2}$$

where ε is the inverse aspect ratio and q is the safety factor. The relative contribution of the Pfirsch-Schlüter current to the total plasma current is actually smaller than this; explain why.

BD

4 i) Show that, for a large aspect ratio toroidal surface with major radius *R* and minor radius *a,* we may approximate

$$k_{||} = \frac{1}{R}\left(n - \frac{m}{q}\right)$$

where *m* and *n* are the wave's poloidal and toroidal wavenumbers, respectively, and the safety factor

$$q = \frac{a}{R}\frac{B_\phi}{B_\theta}\,.$$

(ii) Therefore show that the frequency of the centre of the TAE gap may be approximated by

$$\omega_{TAE} = \frac{B_0}{2qR\sqrt{\mu_0\rho_0}}$$

(iii) Give one reason why Alfvénic modes are bad for MCF and one reason why they may be good for MCF. BD

5 i) A deuterium neutral beam injection system has an energy of 40keV (i.e. each particle in the beam has an energy of 40keV). It is injected into a plasma with density 10^{19}m^{-3}. At what magnetic field strength does the speed of the incoming beam particles (neglecting any slowing-down) coincide with the Alfvén speed? (This is important because the coincidence of these speeds would be an indicator that Alfvénic modes might be excited in the plasma.)

(ii) You will recall from the lectures that the cold plasma dispersion relation is given by

$$\mathbf{M.E_1} \equiv \begin{pmatrix} S-N_{\parallel}^2 & -iD & N_{\parallel}N_{\perp} \\ iD & S-N^2 & 0 \\ N_{\parallel}N_{\perp} & 0 & P-N_{\perp}^2 \end{pmatrix}\begin{pmatrix} E_{1x} \\ E_{1y} \\ E_{1z} \end{pmatrix} = \mathbf{0}$$

$$S=\frac{R+L}{2},D=\frac{R-L}{2},=1-{}_e^2-P_i^2 \approx 1-XX_e^2,X$$

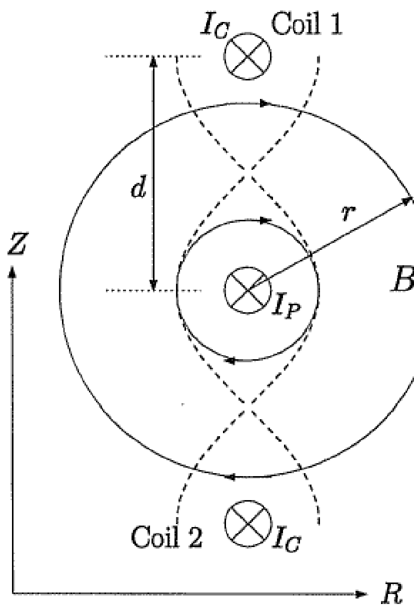
$$R=1-\frac{X_e^2}{1+Y_e}-\frac{X_i^2}{1+Y_i},L=1-\frac{X_e^2}{1-Y_e}-\frac{X_i^2}{1-Y_i}$$

$$X_s=\frac{\Pi_s}{\omega},Y_s=\frac{\Omega_s}{\omega},\Pi_s=\sqrt{\frac{n_0q_s^2}{\varepsilon_0 m_s}},\Omega_s=\frac{q_sB_0}{m_s}$$

Show that, if a wave's electric field **E** is perpendicular to both its direction of propagation **k** and the equilibrium magnetic field $\mathbf{B}_0$, then the wave's frequency $\omega^2 = 2\Omega_i^2$. [*Hint: Consider the structure of the matrix **M** for this particular orientation of **E**; then solve the resulting equation, which may require less ink if you write* $X_i = \delta X_e$ *and* $Y_i = \delta^2 Y_e$ *for some constant* δ *which you should specify).]*

BD

6 For improved performance, tokamak plasmas are almost always operated in x-point configurations with elongated plasmas. This problem sheet looks at the stability of a simplified version of this situation, shown in the figure below.



Here we approximate the plasma in a large aspect ratio tokamak (Rπr) as a toroidal wire at R = R_0, Z =0, carrying a current I_P. This produces a poloidal field $B_\theta = \mu_0 I_P/(2\pi r)$ (solid circles). Coils 1 & 2 are placed at $R = R_0$, $Z = \pm d$, each carrying a current I_C in the same direction as I_P. The forces on the plasma due to coils 1 & 2 balance so this is in vertical equilibrium, and the total magnetic field has an x-point configuration (dashed lines).

a) Moving the plasma in Z by a small amount Z, show that the total vertical force on the plasma is approximately given by

$$\partial F = 2\mu_0 I_C I_P \frac{R_0}{d^2}\delta Z$$

b) This force amplifies the original perturbation, and so the configuration is unstable. Show that the growth-rate of the instability is

$$\gamma_0 = \sqrt{2\mu_0 \frac{I_C I_P R_0}{d^2 M}}$$

where M is the mass of the plasma. For typical tokamaks, $\gamma_0 \sim 10^6 s^{-1}$, indicating that x-point plasmas should only exist for a few μ s.

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c) Now add two new coils at the same locations as coils 1 & 2, but which are not attached to any power supply - they are purely passive. Moving the plasma at a velocity $\upsilon = \partial \delta Z / \partial t$ induces a current in these coils. Using Faraday's and Ohm's laws, show that this current is

$$I_I = -\upsilon \frac{\mu_0 I_P A}{\eta 2\pi d}$$

where $I_I = -\upsilon \frac{\mu_0 I_P A}{\eta 2\pi d}$ is the resistance of a coil of area A and resistivity η.

Hint: Consider flux cut by coils. No (non-trivial) integration needed.

d) By considering the force exerted on the plasma by these currents, show that the growth-rate of the vertical instability is modified to

$$\gamma = \frac{\gamma_0^2}{2\gamma I}\left[-1 + \sqrt{1 + 4\gamma_I^2 / \gamma_0^2}\right] \qquad \gamma I = \frac{2\pi\eta}{\mu_0 A} \frac{I_C}{I_P}$$

Here γ_I is ~1/ a resistive timescale of the passive coils. Typical values (e.g. copper $\eta \sim 10^{-8}\,\Omega m$, steel $\eta \sim 10^{-6}\,\Omega m$) give $\gamma_I \sim 1 - 10^2\, s^{-1}$.

(i) Show that for small γ_I / γ_0, the instability growth-rate is limited to $\gamma \sim \gamma_I$.

(ii) Comment on the vertical stability of tokamak x-point configurations

BD

Two stream instability

7 Consider two beams of electrons, each with density n_0, travelling in opposite directions with velocity $\pm\upsilon_0$ through a background of cold (stationary) ions of density $2n_0$. In equilibrium there is no net charge, no current and no magnetic field. The beams are wide enough that the only important direction is parallel to the beams (x). This problem tests the stability of this situation to electrostatic perturbations by treating each electron beam as a fluid:

$$\frac{\partial n_{p,m}}{\partial t} + \mathbf{v}_{p,m} \cdot \nabla n_{p,m} = -n_{p,m} \nabla \cdot \mathbf{v}_{p,m}$$

$$m_e \left(\frac{\partial \mathbf{v}_{p,m}}{\partial t} + \mathbf{v}_{p,m} \cdot \nabla_{p,m} \mathbf{v}_{p,m} \right) = -e\mathbf{E}$$

where the p; m subscripts refer to the fluid with initial velocity plus and minus υ_0.

a) Linearise these equations with a perturbation of the form exp $(-iwt + ikx)$, and show that the velocity perturbation is given by

$$\tilde{\upsilon}_{p,m} = -\frac{e^2}{k m_e \varepsilon_0} (\tilde{n}_p + \tilde{n}_m) / (\pm k\upsilon_0 - w)$$

b) By substituting in expressions for $\tilde{n}_p$ and $\tilde{n}_m$, show that

$$1 = \frac{e^2 n_0}{m_e \varepsilon_0} \left[\frac{1}{(k\upsilon_0 - w)^2} + \frac{1}{(k\upsilon_0 + w)^2} \right]$$

Hint: write $\upsilon_m = -\upsilon_p (k\upsilon_0 - w)/(k\upsilon_0 + w)$

c) What happens when $\upsilon_0 = 0$?

d) Assuming w^2 is real, find the range of k which is unstable by considering when $w^2 = 0$.

BD

PLASMA DIAGNOSTIC TECHNIQUES

1 The JET tokamak at the Culham Science Centre has a major radius of 3 m and a minor radius (approx) of 1.5 m. Assuming that densities of 10^{20}m^{-3} and temperatures of 10 keV are produced with a plasma current of 3 MAmps in a pure hydrogen plasma, estimate the plasma conductivity and from this the power of ohmic heating in the plasma. Calculate a typical value for the loop voltage around the torus.

GJT

2 During the start-up phase of JET, assuming plasma temperatures of 100 eV, but other parameters as above, calculate the initial plasma conductivity, ohmic heating power and loop voltage around the torus.

GJT

3 Consider a cylindrical vacuum vessel of cross-sectional area 1 m^2 and length greater than 'v' where 'v' is the average velocity of gas particles in the vessel. Consider the particles between distance x and $x + dx$ from the cylinder end and determine an expression for the number of these particles striking the 1 m^2 cylinder end area per second. Integrate along the cylinder to show that the total number of particles striking the 1 m^2 area per second is $\frac{1}{4}nv$, where n is the density of particles. You can assume that particle trajectories are perfectly reflected on the cylinder walls. It may be useful to use the fact that a cone of angle 2θ subtends a solid angle of $2\pi(1 - \cos\theta)$ from the tip.

GJT

4 The velocity of electrons detected by a Langmuir probe set at voltage $V - V_p$ relative to the plasma potential V_p (for $V > V_p$) is determined by the number <v> of electrons able to overcome the potential drop $V - V_p$. We can write

$$< v > = \frac{\int_{v_0}^{\infty} f(v)\, v dv}{\int_0^{\infty} f(v)\, dv}$$

where $\frac{1}{2}mv_0^2 = e(V-V_p)$. For a Maxwellian velocity distribution in a particular direction $f(v) \alpha \exp(-mv^2/2kT)$, show that the Langmuir probe current $I \alpha e$<v> is given by

$$\ln\left(\frac{I}{I_{sat}}\right) \cong -\frac{e}{kT}\left(V - V_p\right)$$

where I_{sat} is a saturation current and e is the electron charge.

GJT

5 Assume a Gaussian ion density distribution is centred on the initial position of an ICF target such that the number density of ions is given by

$$n_i = A\exp\left(-\frac{r^2}{L_0^2}\right)$$

where r is the distance from the initial target position, L_0 is a scalelength and A is a normalisation factor such that the integral of n^i over all distances r is equal to the total number of ions N_T. Show that

$$A = \frac{N_T}{L_0^3 \pi^{3/2}}.$$

GJT

6 Assuming Faraday's Law $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ and electromagnetic wave propagation with both electric and magnetic fields varying as $\exp(i(kx - wt))$ show that the ratio of the magnitude of the electric to magnetic field is given $\frac{E}{B} = \frac{c}{N}$ where N is the refractive index of the medium.

GJT

7 A Mach-Zehnder interferometer operating at wavelength λ is used to measure the peak electron density n_0 of a plasma with a Gaussian shaped density profile such that

$$n_e = n_0 \exp\left(-\frac{x^2}{\Delta x^2}\right)$$

If the interferometer records one fringe shift with the probe arm passing through this density profile, show that

$$n_0 = \frac{\lambda}{\Delta x}\frac{2}{\sqrt{\pi}} n_c$$

where n_c is the critical density. You may assume $n_0 << n_c$.

GJT

8 If there is a constant magnetic field B_0 parallel to the direction of propagation of the probe beam in Q7, show that the Faraday rotation α of a plane polarised probing beam will be given by

$$\alpha = \frac{\omega_c \lambda}{c}$$

where ω_C is the electron cyclotron frequency for magnetic field strength B_0 and c is the speed of light.

GJT

9 Estimate the ratio of spectral width $\Delta\omega$ (full width at half maximum) to frequency ω of cyclotron emission for each harmonic caused by the thermal Doppler shift in frequency for a plasma of electron temperature 500 eV. For a tokamak of major radius 3 m, estimate the spatial resolution in temperature measurements that this ratio $\Delta\omega / \omega$ implies will be attainable from a cyclotron emission measurement.

GJT

10 In the non-relativistic limit, the power P emitted by an electron orbiting in a magnetic field in a particular direction varies with time t as $P \alpha \sin^2(\omega_C t)$ where ω_c is the cyclotron frequency. From the expression for the angular power radiated by an accelerating charge, explain why this temporal variation of radiation output occurs. Go on to explain why the temporal variation of output comprises radiation at frequency ω_c. Explain qualitatively how harmonics of ω_c are emitted.

GJT

11 The effect of plasma opacity on spectral lines emitted from a plasma is sometimes taken into account using the concept of 'escape factors'. The escape factor can be defined as the ratio between the intensity of optically thick emission and the intensity of emission that would occur if the plasma was optically thin. For a spectral line emitted from a uniform plasma with a total optical depth through the plasma of τ, show that the escape factor is given by

$$T = \frac{1 - \exp(-\tau)}{\tau} \ .$$

GJT

12 Show that recombination radiation into high lying quantum states (say, principal quantum number $n > 2$) for an ion of charge Z before recombination is less intense than free-free emission associated with the ion if

$$kT > \frac{2Z^2 R_d}{n^3}$$

where R_d is the Rydberg constant, k is Boltzmann's constant and T is the electron temperature.

GJT

13 Consider the rate coefficient K for collisional excitation between two levels of energy separation E_{21} for an ion in a plasma. The cross-section for the collisional excitation varies as follows

$$\sigma = \left(\frac{v_0^2}{v^2}\right)\gamma \quad (\text{if } v \geq v_0)$$

$$\sigma = 0 \qquad (\text{if } v < v_0)$$

where v_0 is the velocity of an electron of energy E_{21} and γ is a constant independent of electron velocity (in practice γ is *largely* independent of velocity). Show that

$$K = \frac{4E_{21}}{m}\left(\frac{m}{2\pi kT}\right)^{1/2} \gamma \exp\left(-\frac{E_{21}}{kT}\right)$$

where m is the electron mass, k is Boltzmann's constant and T is the electron temperature. Explain briefly why the cross-section σ is zero for $v < v_0$. Explain briefly the *perceived wisdom* in calculating collisional rate coefficients that 'the rate coefficient is proportional to the threshold value for the cross-section'.

GJT

14 The population density of the hydrogen ion excited to the first excited state (principal quantum number $n = 2$) in a tokamak is 10^{14} m^{-3} at the plasma edge. Given that the transition probability for the Lyman alpha line in hydrogen ($n = 2$ to 1) is 4.7×10^8 s^{-1}, calculate the emissivity (in Wm^{-3}) for the Lyman alpha line at the plasma edge. Given that the ground state hydrogen density ≈ electron density ≈ 10^{19} m^{-3}, estimate the rate coefficient for collisional excitation from the ground state $n = 1$ to the first excited state $n = 2$.

GJT

15 Consider a spectral line emitted from a uniform plasma of width L, with emissivity ε_0 and absorption coefficient K_0 at line centre. If the emissivity and absorption coefficient have line profiles of shape $\phi(\nu)$, show that the intensity profile $I(\nu)$ as a function of frequency ν from line centre can be written as

$$I(\upsilon) = \frac{\varepsilon_0}{K_0}\left[1 - \exp(-K_0 \phi L)\right].$$

Show that the ratio of the spectral line intensity in the line wings $I(\nu_{\text{wings}})$ to that at line centre $I(0)$ can be approximated to $K_0 L \phi(\nu_{\text{wings}})$. Briefly explain opacity broadening by referring to this ratio.

GJT

16 A small fraction $R = 10^{-9}\ \text{m}^{-1}$ of the laser light injected into a tokamak is absorbed and re-emitted into 4π steradian by an impurity ion in the plasma. If the aim of injecting the laser is to measure electron temperatures from Thomson scatter, estimate the minimum electron density that could be diagnosed for the Thomson scatter signal to exceed the re-emitted radiation signal.

GJT

STATISTICAL METHODS IN EXPERIMENTAL PHYSICS

The questions for this topic will be issued during the term.