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part d)

d) Corrections for part d)

MCF Assignment 5

$\boxed{r_{mod} \approx r_i}$ because, when $\frac{r_i}{r_0} \approx 0$

$$f(r_i) = \left(1 + \frac{4r_i^2}{r_0^2}\right)^{1/2} \approx f(r_i=0) + \frac{1}{1!} f'(r_i=0)\cdot r + \frac{1}{2!} f''(r_i=0)\, r_i^2$$

$$f(r_i=0) = \left(1 + \frac{4r_i^2}{r_0^2}\right)^{1/2}\Big|_{r_i=0} = 1$$

$$f'(r_i=0) = \left(\left(1 + \frac{4r_i^2}{4r_0^2}\right)^{1/2}\right)'\Big|_{r_i} = \left(1 + \frac{4r_i^2}{r_0^2}\right)^{-1/2} \cdot \frac{1}{2}\cdot 2\cdot 4\cdot \frac{r_i}{r_0^2}\Big|_{r_i=0} = 0$$

$$f''(r_i=0) = \left(\left(1 + \frac{4r_i^2}{r_0^2}\right)^{-1/2} \frac{4r_i}{r_0^2}\right)'\Big|_{r_i=0} =$$

$$= \left(1 + \frac{4r_i^2}{r_0^2}\right)^{-3/2} \frac{4r_i}{r_0^2}\left(\frac{-1}{2}\right) 4\cdot 2r_i + \left(1 + \frac{4r_i^2}{r_0^2}\right)\frac{4}{r_0^2}\Big|_{r_i=0} = \frac{4}{r_0^2}$$

$$\boxed{f(r_i) = 1 + \frac{0}{1!} r_i + \frac{1}{2!}\frac{4r_i^2}{r_0^2} = 1 + \frac{2r_i^2}{r_0^2}}$$

$$f(r_i) \approx \left(1 + \frac{4r_i^2}{r_0^2}\right)^{1/2} \approx 1 + \frac{2r_i^2}{r_0^2}$$

$$r_{mod} = \frac{r_0^2}{2r_i}\left[\left(1 + \frac{4r_i^2}{r_0^2}\right)^{1/2} - 1\right] = \frac{r_0^2}{2r_i}\left[f(r_i) - 1\right] =$$

$$= \frac{r_0^2}{2r_i}\left[1 + \frac{2r_i^2}{r_0^2} - 1\right] = \frac{r_0^2}{2r_i}\left[\frac{2r_i^2}{r_0^2}\right] = r_i; \quad \boxed{r_{mod} \approx r_i}$$