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Assignment 1

Q 1 a)

From definition of β

$$\beta = \frac{p}{B^2/2\mu_0}$$, where p is pressure, B is magnetic field, $\mu_0 = 1.25 \cdot 10^{-6}$

$$p = \frac{\beta B^2}{2\mu_0}$$

Denote w as fusion power density

$$w = \frac{p^2}{4\times10^5} = \frac{\beta^2 B^4}{4\mu_0^2 \cdot 4\times10^5} = \frac{\beta^2 B^4}{1.6 \cdot 10^6 \mu_0^2}$$

$$w = \frac{(0.05)^2 \cdot 6^4}{1.6 \cdot 10^6 (1.25 \cdot 10^{-6})^2} = 1{,}296{,}000 \frac{W}{M^3}$$

$$\underline{w \approx 1.3 \ MW/M^3}$$

Tokamak volume $V = \frac{4\pi^2}{9} R^3$

Total tokamak power $W = w \cdot V$

$$W = w \cdot \frac{4\pi^2}{g} R^3$$

$$\Rightarrow \quad R = \left(\frac{g}{4\pi^2} \frac{W}{w} \right)^{\frac{1}{3}}$$

$$R = \left(\frac{9}{4 \cdot 3.14^2} \frac{3 \cdot 10^9}{1.3 \cdot 10^6} \right)^{\frac{1}{3}} \approx 8.08 \text{ m}$$

Tokamak radius $R \approx 8.08$ m

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b) Tokamak cost $=$

$= 2 \cdot S \cdot 4 \cdot M€$

where $S = 2\pi^2 R^2$ tokamak area

$$\text{Cost} = 2 \cdot 2\pi^2 R^2 \cdot 4 \text{ M€} =$$

$$= 2 \cdot 2 \cdot (3.14)^2 (8.08)^2 \cdot 4 \text{ M€} =$$

$$= 10{,}300 \text{ M€} = 10.3 \text{ B€}$$

c) We assume the power of 3 GW consists of energy realised by neutrons – 2,400 MW and energy realised by α particles – 600 MW. Multiplication by 1.3 times applies only for neutrons energy 2,400 x 1.3 and doesn't apply for α particles energy of 600 MW

Electric power produced

$W_E = (600 + 2{,}400 \times 1.3) \times 40\% =$

$= 1{,}488$ MW

However we need substruct energy for heating plasma by the beams. – 200 MW/(50%) = 400 MW

Net $W_{E\,Net} = 1{,}488 - 400 = \underline{1{,}088 \text{ MW}}$

Annual electricity production at 75% availability

$W_{Annual} = 1{,}088 \cdot \text{MW} \cdot 365 \cdot 24 \cdot 75\% = \underline{7.15 \cdot 10^9 \text{ KWh}}$

Annual cost of running a tokamak

$$\text{Cost}_{\text{annual}} = 10.3\ \text{B€} \times 10\% = 1.03\ \frac{\text{B€}}{\text{year}}$$

$$\text{Cost of 1 kW}\cdot\text{hour} = \frac{\text{Annual cost}}{W_{\text{annual}}} =$$

$$= \frac{1.03 \cdot 10^9\ €}{7.15 \cdot 10^9\ \text{kWhr}} = \underline{0.144\ \frac{€}{\text{kW}\cdot\text{hr}}}$$

d) Let's assume that annual cost of running a tokamak is 1.03 B€ ≈ ≈ 1. B€ consists of a fixed cost of 0.5 B€ and reapering components of 0.5 B€

$$C_{cost} = C_{fixed} + C_0 = 0.5\ B€ + 0.5\ B€$$

Cost of components will be reducing every each generation of tokamaks.

Number of tokamaks N	Number of generations i	Cost of Components C_{comp}
$n = 1 = 2^0$	$i = 0$	$C_0 (0.85)^0$
$n = 2 = 2^1$	$i = 1$	$C_c \cdot (0.85)^1$
$n = 4 = 2^2$	$i = 2$	$C_c \cdot (0.85)^2$
. . . . .	. . . . .	. . . . . .
$n = 2^i$	i	$C_{comp}(0.85)^i$

$$n = 2^i \Rightarrow i = \log_2 n$$

$$C_{comp} = C_0 \cdot (0.85)^{\log_2 n}$$

Cost reduction formula

$$C_{cost}(n) = 0.5\,B€ + 0.5\,B€ (0.85)^{\log_2 n} \quad \text{annually}$$

$$\boxed{C_{cost}(n) = 0.5\left(1 + (0.85)^{\log_2 n}\right) \frac{B€}{year}}$$

Where n is number of tokamaks

Cost of capital cost

$$\boxed{C_{capital}(n) = 5\left(1 + (0.85)^{\log_2 n}\right) B€}$$

Let's assume in 5 year there will be built 32 tokamaks

$$C_{capital}(32) = 5\left(1 + (0.85)^{\log_2 32}\right) B€ =$$

$$= \underline{7.2\ B€}$$

Let's assume in 10 years there will be built 1,024 tokamaks.

$$C_{capital}(1{,}024) = 5\left(1 + (0.85)^{\log_2 1024}\right) \text{B€}$$

$$C_{capital}(1{,}024) = \underline{6.0 \text{ B€}}$$

Q2 Lifetime (years) =

$$= \frac{(\text{dpa limit})(1\ MW/M^2)}{(10\ \text{dpa})(\text{power loaded})\left(\frac{1\ MW}{M^2}\right)}$$

$$\left(\begin{matrix}\text{Power loaded} \\ \text{on } M^2\end{matrix}\right) \doteq \frac{2.400\ MW}{S}$$

$$= \frac{2.400\ MW}{1.289\ M^2} = 1.86\ MW/M^2$$

$$\left\{ S = 2\pi^2 R^2 = 2 \cdot \pi^2 (8.08)^2 = 1.289\ M^2 \right\}$$

lifetime at 20 dpa

$$\underline{\text{lifetime}} = \frac{20\ \text{dpa}}{10\ \text{dpa}} \; \frac{1\ MW/M^2}{1.86\ MW/M^2} =$$

$$= \underline{1.075\ \text{years}}$$

$$\left(\begin{matrix}\text{Shut down} \\ \text{fraction}\end{matrix}\right) = \frac{0.5 + 10\% \cdot 1.075}{1.075 + 0.5 + 10\% \cdot 1.075} = 36\%$$

$$\text{Availability} = 100\% - 36\% = 64\%$$

dpa Limit	Life time (years)	Shutdown period (years)	Availability	Shutdown fruction
20	1.075	0.5	64%	36%
40	2.15	0.5	75%	25%
60	3.23	0.5	80%	20%
80	4.30	0.5	82%	18%
120	6.45	0.5	85%	15%

Q3

At 100% availability cost of electricity will be €cents 19.2 kW·hr

dpa limit	Availability	CoE [€cents/kW·hr]
20	64%	16.9
40	75%	14.4
60	80%	13.5
80	82%	13.2
120	85%	12.7
unlimited (theoretically)	100%	10.8

In order to reduce a blanket time of replacement and installation and to increase availability and reduce cost of electricity per KW·hr., I would like to suggest:

1) To design a blanket in modules rather small numerous parts / details. Because removing and installing big modules will take less time and efforts.

2) To automise the process of removing and installing a blanket and its components. For excample to use robotics.

3) To make access to tokamaks details and parts more easer.