MCF/Assignment 3

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a) $\partial_t \rho + \nabla(\rho v) = 0$

$$\partial_t(\rho_0 + \rho_1) + \nabla\cdot((\rho_0+\rho_1)(v_0+v_1)) = 0$$

$$\rho_1 = \rho_1 e^{-i\omega t + i\bar{k}\bar{r}}$$
$$v_1 = v_1 e^{-i\omega t + i\bar{k}\bar{r}}$$

$$\frac{\partial}{\partial t}\left(\rho_0 + \rho_1 e^{-i\omega t + i\bar{k}\bar{r}}\right) +$$
$$+ \nabla\left(\left(\rho_0 v_0 + \rho_0 v_1 + \rho_1 v_0 + \rho_1 v_1\right) = 0\right.$$

$$\frac{\partial}{\partial t}\left(\rho_0 + \rho_1 e^{-i\omega t + i\bar{k}\bar{r}}\right)$$
$$+ \nabla\left(\rho_0 v_0 + \rho_0 v_1 e^{-i\omega t + i\bar{k}\bar{r}} + \rho_1 e^{-i\omega t + i\bar{k}\bar{r}}\right.$$
$$\left. + \rho_1 e^{-i\omega t + i\bar{k}\bar{r}} + v_1 e^{-i\omega t + i\bar{k}\bar{r}}\right)$$

$$\boxed{-i\omega\rho_1 = -i\rho_0(\bar{k}\cdot\bar{v}_1)}$$

$$\bar{\nabla} \times \bar{B} = \mu_0 \bar{J}$$

$$\nabla \times (B_0 + B_1) = \mu_0 \bar{J}$$

$$\nabla \times \left(B_0 + B_1 e^{-i\omega t + i\bar{k}\bar{r}}\right) = \mu \bar{J}$$

$$\boxed{i\bar{k} \times B_1 = \mu_0 \bar{J}}$$

$$\partial_t \bar{B} - \nabla \times (\bar{V} \times \bar{B}) = 0$$

$$i\omega \bar{B} - (\bar{\nabla} \cdot \bar{B})\bar{V} - (\bar{\nabla} \cdot \bar{V})\bar{B}$$

$$\boxed{i\omega B = (i\bar{k} \cdot \bar{B})\bar{V} - (i\bar{k} \cdot \bar{V})\bar{B}}$$

$$\rho(\partial_t + \bar{V}\bar{\nabla})\bar{V} = \bar{J}\times B - \nabla\bar{P}$$

$$\partial_t \bar{V} = \frac{1}{\rho}\bar{J}\times\bar{B} - \frac{1}{\rho}\nabla\bar{P}$$

$$\boxed{-i\omega v_1 = \frac{1}{\rho}\bar{J}\times\bar{B} - \frac{1}{\rho}\, i\bar{k}\cdot\bar{p}_1}$$

6) Equation with cold dialectric tensor is

$$\begin{pmatrix} S-N_{\parallel}^2 & -iD & N_{\parallel}N_{\perp} \\ iD & S-N^2 & 0 \\ N_{\parallel}N_{\perp} & 0 & P-N_{\perp}^2 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0$$

$$\begin{cases} (S-N_{\parallel}^2)E_x - iDE_y + N_{\parallel}N_{\perp}E_z = 0 \\ iD\,E_x + (S-N^2)E_y + 0 = 0 \\ N_{\parallel}N_{\perp}E_x + 0 + (P-N_{\perp}^2)E_z = 0 \end{cases}$$

Let look the 2^d eqn.

$$iDE_x + (S-N^2)E_y = 0$$

$$i\left(\frac{R-L}{2}\right)E_x + \left(\frac{R+L}{2} - N\right)E_y = 0$$

Take into account that $N_{\perp} = 0$

$$N^2 = N_{\parallel}^2 + N_{\perp}^2 \Rightarrow N^2 = N_{\parallel}^2$$

$$\boxed{\frac{E_x}{E_y} = i\left(\frac{\frac{R}{2} - \frac{L}{2} - N_{\parallel}^2}{\frac{R}{2} + \frac{L}{2}}\right)}$$

if the wave is right hand circularly polarised

$$\frac{E_x}{E_y} = -i \Rightarrow i\,\frac{\frac{R}{2} - \frac{L}{2} - N_{\parallel}^2}{\frac{R}{2} + \frac{L}{2}} = -i$$

$$\Rightarrow \frac{R}{2} - \frac{L}{2} - N_{\parallel}^2 = -\frac{R}{2} - \frac{L}{2} \Rightarrow \boxed{N_{\parallel}^2 = R.}$$

if $\frac{E_x}{E_y} = i$ wave is left hand circularly polarised

$$\left(\frac{\frac{R}{2} - \frac{L}{2} - N_{\parallel}^2}{\frac{R}{2} + \frac{L}{2}}\right) i = +i$$

$$\frac{R}{2} - \frac{L}{2} - N_{\parallel}^2 = \frac{R}{2} + \frac{L}{2} \Rightarrow \boxed{N_{\parallel}^2 = L}$$

c) $n_j m_j \partial_t v_{j1} = n_j q_j (\bar{E}_1 + \bar{V}_{j1} \times \bar{B}_0)$

Let to drop some indexes

$$m \frac{\partial}{\partial t} v = q\,(\bar{E} + \bar{V} \times \bar{B})$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ 0 & 0 & B \end{vmatrix} = \hat{i}\, v_y B - \hat{j}\, v_x B$$

Also $\frac{\partial}{\partial t} v = \frac{\partial}{\partial t}\left(v e^{-i\omega t + ik}\right) = -i\omega v$

$$\begin{cases} -i\omega n m v_x = n q E_x + n q v_y B \\ -i\omega n m v_y = n q E_y - n q v_x B \end{cases}$$

Take into account $\omega_c = \frac{qB}{m}$

$$\begin{cases} -i\omega v_x = \frac{q}{m} E_x + \quad v_y & \mid \times i \\ -i\omega v_y = \frac{q}{m} E_y - \frac{qB}{m} v_x & \mid \times(-1) \end{cases}$$

$$* \boxed{\begin{cases} v_x = \frac{iq}{m\omega} E_x + \frac{\omega_c}{\omega} i v_y \\ i v_y = +\frac{iqm}{m\omega}(iE_y) + \frac{\omega_c}{\omega} v_x \end{cases}} \quad (i \times i = -1)$$

Now if we add 2 eqn in system $*$ we will get.

$$(v_x + i v_y) = \frac{iq}{m\omega}(E_x + E_y) + \frac{\omega_c}{\omega}(v_x + i v_y)$$

$$(v_x + i v_y)\left(1 - \frac{\omega_c}{\omega}\right) = iq(E_x + iE_y)$$

$$(v_x + i v_y) = \frac{iq(E_x + iE_y)}{m\omega\left(1 - \frac{\omega_c}{\omega}\right)}$$

$$v^+ = v_x + i v_y \quad \text{and} \quad E^+ = E_x + i E_y$$

$$v^+ = \frac{iq}{m\omega}\frac{E^+}{\left(1 - \frac{\omega_c}{\omega}\right)}$$

If we substruct eqn in the system$*$ and follow the similar manipylations. we will get

$$v^- = \frac{iq}{m\omega}\left(\frac{E^-}{1 - \frac{\omega_c}{\omega}}\right),$$

where
$v^- = v_x - i v_y$
$E^- = E_x - i E_y$

$$\boxed{v^{\pm} = \frac{iq}{m\omega}\frac{E^{\pm}}{\left(1 \mp \frac{\omega_c}{\omega}\right)}}$$