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Assignment 2

a)

$$m_e \frac{dv_e}{dt} = -eE_{\parallel} - m_e(\nu_{ei}+\nu_{ee})v_e$$

$$\left.\begin{cases} w_e = v_e/v_{T_e} \Rightarrow v_e = w_e \cdot v_{T_e} \\ dv_e/dt = (dw_e/dt)\cdot v_{T_e} \\ v_{T_e} = (2T_e/m_e)^{1/2} \\ (\nu_{ei}+\nu_{ee}) = \frac{7.3\cdot 10^5 n_{20}}{T_K^{3/2}}\left[\frac{v_{T_e}}{v_e}\right]^3 \\ E_0 = \frac{eE_{\parallel}}{(2mT_e)^{1/2}} \end{cases}\right\}$$

$$m_e \frac{dw_e}{dt}\cdot v_{T_e} = -eE_{\parallel} - \frac{m_e \cdot 7.3\cdot 10^5 n_{20}}{T_K^{3/2}}\left[\frac{v_{T_e}}{v_e}\right]^3 v_e$$

$$\frac{dw_e}{dt} = \frac{-eE_{\parallel}}{m_e v_{T_e}} - \frac{7.3\cdot 10^5 n_{20}}{T_K^{3/2}}\frac{1}{w_e^2}$$

$$\frac{dw_e}{dt} = -\frac{eE_{\parallel}}{m_e(2T_e/m_e)^{1/2}} - \frac{7.3\cdot 10^5 n_{20}}{T_K^{3/2}}\frac{1}{w_e^2}$$

$$\boxed{\frac{dw_e}{dt} = E_0 - \frac{7.3\cdot 10^5 n_{20}}{T_K^{3/2}}\frac{1}{w_e^2}}$$

b)

From the physical problem, electron velocity w_e has to be increasing. It can be possibly if only w_e satisfies some initial criteria.

If we look at the formula

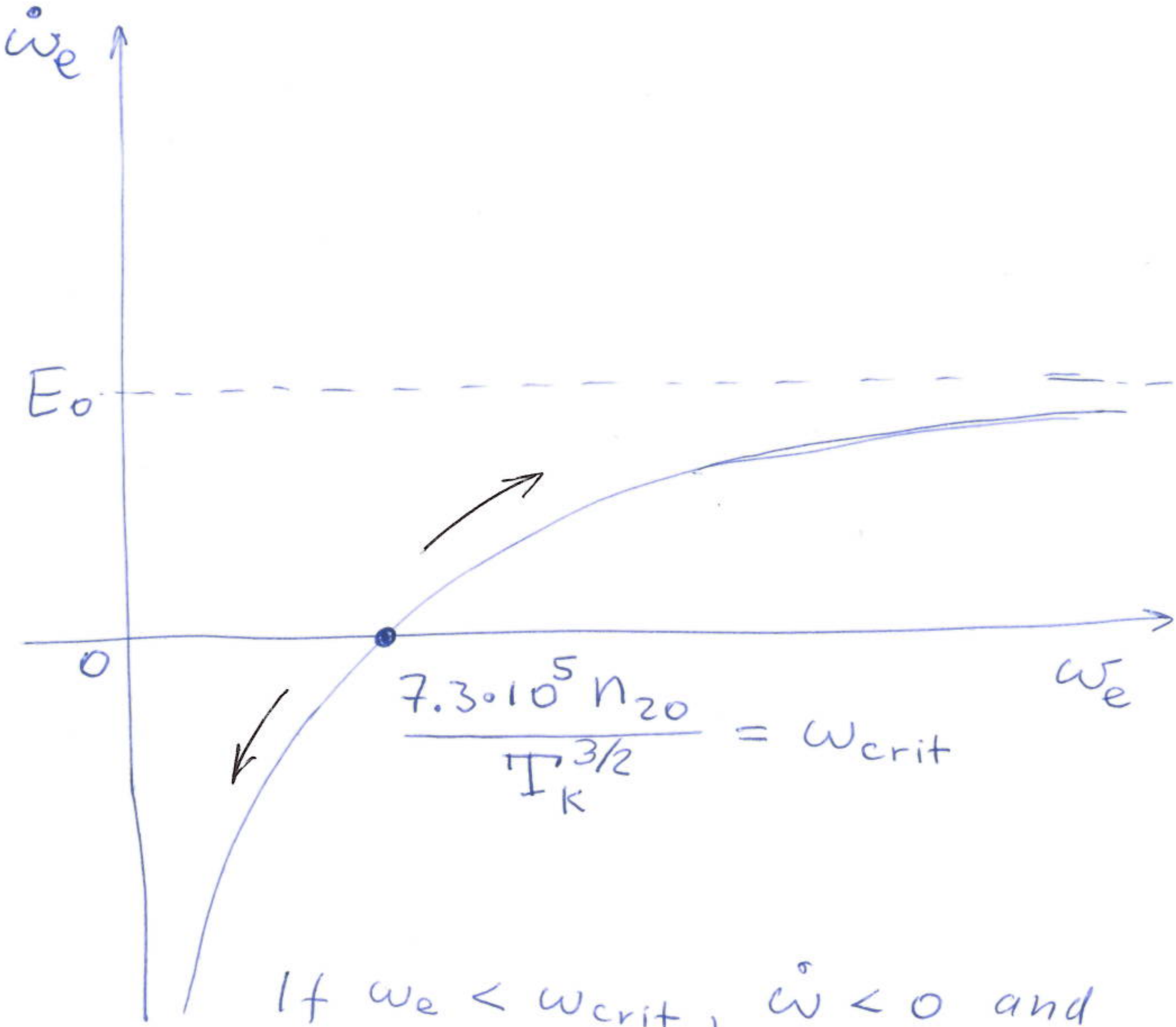
$$\frac{dw_e}{dt} = E_0 - \frac{7.3 \cdot 10^5 n_{20}}{T_K^{3/2}} \frac{1}{w_e^2}$$

the acceleration dw_e/dt has to be possitive figure from the beginning otherwise velocity w_e will never be increasing.

$$\frac{dw_e}{dt} = E_0 - \frac{7{,}3 \cdot 10^5 n_{20}}{T_K^{3/2} w_e^2} > 0 \Rightarrow$$

$$\Rightarrow \boxed{w_e > \sqrt{\frac{7{,}3 \cdot 10^5 n_{20}}{T_K^{3/2}} \frac{1}{E_0}}}$$

Alternativelly, we can examine a graph $\dot{\omega}_e(\omega_e) = \sqrt{E_0 - \frac{7.3 \cdot 10^5 n_{20}}{T_K^{3/2}} \frac{1}{\omega_e^2}}$



If $\omega_e < \omega_{crit}$, $\dot{\omega} < 0$ and $\omega(t)$ will be decreasing, what is not possible from the physical problem.

In addition if we solve the equ,

$$\frac{dw}{dt} = E_0 - \frac{7,3\cdot10^5 n_{20}}{T_K^{3/2}} \frac{1}{w_e^2},$$

$$\frac{dw}{dt} = a - b\frac{1}{w_e^2}, \quad a = E_0, \; b = \frac{7,3\cdot10^5 n_{20}}{T_K^{3/2}}$$

we will get a solution

$$\frac{1}{2a^2}\left(\sqrt{ab}\,\log\left(a\cdot w(t) - \sqrt{a\cdot b}\right) - \sqrt{ab}\,\log\left(a\cdot w(t) + \sqrt{a\cdot b}\right) + 2a\,w(t)\right)$$

$$= t + c_1,$$

$$\left(a\cdot w(t) - \sqrt{a\cdot b}\right) > 0 \quad \text{or}$$

$$w(t) > \sqrt{b/a},$$

or $$\boxed{w(t) > \frac{7,3\cdot10^5 n_{20}}{T_K^{3/2}} \frac{1}{E_0}}$$

Simplified solution of the eq

$$\frac{dw}{dt} = 1 - 1/w^2 \,|: (1 - 1/w^2)$$

$$\frac{dw}{1 - 1/w^2} = dt$$

$$\int\left(\frac{1}{w-1} + \frac{1}{w+1} + 1\right)dw = \int 1\, dt$$

$$\log(w-1) + \log(w+1) + w = t + c$$

Function $\log(x)$ is defined if only $x > 0$, so $\log(a\,w(t) - \sqrt{ab})$, is defined when $a\,w(t) - \sqrt{ab} > 0$, or $w > \sqrt{b/a}$

c)

From the eqn. $\frac{dw_e}{dt} = E_0 - \frac{7{,}3 \cdot 10^5 n_{20}}{T_k^{3/2}} \frac{1}{w_e^2}$

we can find critical values for density n_{20}, when $w_{e\,crit} = 1$ or 10.

$$n_{20} < \frac{E_0 T_k^{3/2} w_e^2}{7{,}3 \cdot 10^5}$$

i) When $w_{e\,crit} = 1$

$$E_0 = \frac{9.4 \cdot 10^3 \cdot E_{\parallel}}{T_k^{1/2}} = \frac{9.4 \cdot 10^3 \cdot 0.5}{2^{1/2}} =$$

$$n = \frac{9.4 \cdot 10^3 \cdot 0.5 \cdot 2^{3/2} \cdot 1^2 \, n_{20}}{7.3 \cdot 10^5 \cdot 2^{1/2}} =$$

$$= 1.28 \cdot 10^{-2} n_{20} = 1{,}28 \cdot 10^{-18} \text{ part.}/M^3$$

ii) When $w_{e\,crit} = 10$

$$n = \frac{9.4 \cdot 10^3 \cdot 0.5 \cdot 2^{3/2} \cdot (10)^2 n_{20}}{7.3 \cdot 10^5 (2)^{1/2}} =$$

$$= 1.28 \cdot n_{20} = 1{,}28 \cdot 10^{20} \text{ particles}/M^3$$