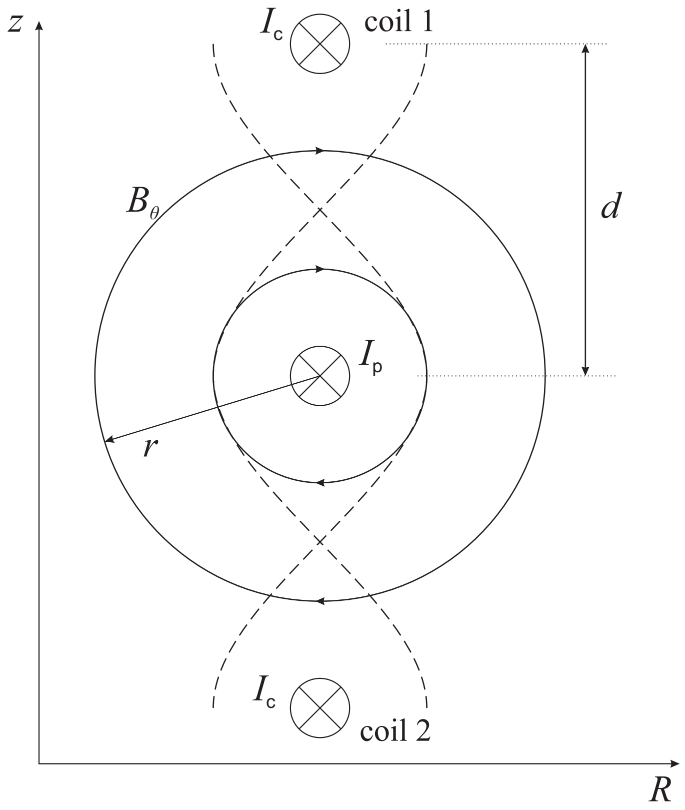
Fusion: MCF - Homework #6

1 For improved performance, tokamak plasmas are almost always operated in x-point configurations with elongated plasmas. In this problem we look at the stability of a simplified version of this situation, shown in the figure below.



Here we approximate the plasma in a large aspect ratio tokamak ($R \gg a$) as a toroidal wire at $R = R_0$, $z = 0$, carrying a current I_p. This produces a poloidal field $B_\theta = \mu_0 I_\mathrm{p}/(2\pi r)$, shown as solid circles in the figure. Coils 1 and 2 are placed at $R = R_0$, $z = \pm d$, each carrying a current I_C in the same direction as I_p. The forces on the plasma due to these coils balance so it is in vertical equilibrium, and the net magnetic field has an x-point configuration (dashed lines).

(a) By considering a motion of the plasma in the z direction by a small amount δz, show that the total vertical force on the plasma is approximately given by

$$\delta F \simeq 2\mu_0 I_\mathrm{p} I_\mathrm{C} \frac{R_0}{d^2} \delta z$$

[5]

(b) This force amplifies the original perturbation, and so the configuration is unstable. Show that the growth rate of the instability is

$$\gamma_0 = \left(2\mu_0 I_\mathrm{p} I_\mathrm{C} \frac{R_0}{d^2 M}\right)^{1/2}$$

continued

where M is the mass of the plasma. For a typical tokamak $\gamma_0 \sim 10^6\mathrm{s}^{-1}$, so x-point plasma should only exist for a few µs – which is clearly not the case in real experiments. [5]

(c) So let's add two new coils at the same locations as coils 1 and 2, but let these new coils not be attached to a power supply. Moving the plasma at a velocity $v = \partial_t \delta z$ will induce a current in these coils. Use Faraday's and Ohm's laws to show that this current is

$$I_{\mathrm{i}} = -v\frac{\mu_0 I_{\mathrm{p}} A}{2\pi d\eta}$$

where the total resistance of a coil of cross sectional area A and resistivity η is $2\pi R_0\eta/A$. [Hint: consider the flux cut by the coils. No non-trivial integration should be needed.] [5]

(d) By considering the force exerted on the plasma by these currents, show that the growth rate of the vertical instability is now modified to

$$\gamma_{\mathrm{mod}} = \frac{\gamma_0^2}{2\gamma_{\mathrm{i}}}\left[\left(1+\frac{4\gamma_{\mathrm{i}}^2}{\gamma_0^2}\right)^{1/2} - 1\right]; \qquad \gamma_{\mathrm{i}} = \frac{2\pi\eta}{\mu_0 A}\frac{I_{\mathrm{C}}}{I_{\mathrm{p}}}$$

This γ_{i} is a characteristic rate of resistive changes in the passive coils. Typical values (e.g. copper $\eta \sim 10^{-8}\Omega\mathrm{m}$, steel $\eta \sim 10^{-6}\Omega\mathrm{m}$) give $\gamma_{\mathrm{i}} \sim 1 - 10^2\mathrm{s}^{-1}$.

(i) Show that for small $\gamma_{\mathrm{i}}/\gamma_0$, the growth rate is $\gamma \simeq \gamma_{\mathrm{i}}$.

(ii) Comment on the vertical stability of tokamak x-point configurations. [5]

End of paper