MCF Assignment 5

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Date: 11/03/2016

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a) $B_\theta = \mu_0 I_p/(2\pi r)$

$$F = \int_l B_\theta j_c \, dl = \frac{\mu_0 j_c}{2\pi r} \int_0^{2\pi R_0} =$$

$$= \frac{\mu_0 I_p I_c R_0}{r} \qquad \left\{ \begin{array}{l} 2\pi R_0 j_c = I_c \\ r = d \end{array} \right\}$$

$$\boxed{F = \frac{\mu_0 I_p I_c R_0}{d}}$$

Let consider ~~a~~ tiny perturbations δz and $-\delta z$ which will couse δF

$$\delta F = F(d-\delta z) - F(d+\delta z) =$$

$$= \frac{\mu_0 I_p I_c R_0}{d-\delta z} - \frac{\mu_0 I_p I_c R_0}{d+\delta z} =$$

$$= \mu_0 I_p I_c R_0 \cdot \left(\frac{1}{d-\delta z} - \frac{1}{d+\delta z} \right)$$

$$\left\{ \begin{array}{l} \dfrac{1}{d-\delta z} - \dfrac{1}{d+\delta z} = \dfrac{d+\delta z - d + \delta z}{(d-\delta z)\cdot(d+\delta z)} \\ = 2\delta z/(d^2 - \delta^2 z) = \underline{2\delta z/d^2} \end{array} \right\}$$

$$\delta F = \mu_0 I_p I_c R_0 \frac{2\,\delta z}{d^2}$$

$$\boxed{\delta F = 2 \mu_0 I_p I_c \frac{R_0}{d^2} \delta z}$$

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b) $\delta F = M \ddot{z}$ M is the plasma mass.

$$M \ddot{z} = 2 \mu_0 I_p I_c \frac{R_0}{d^2} \delta z$$

$$\ddot{z} = \frac{2 \mu_0 I_p I_c \; R_0}{d^2 M} \delta z$$

Let find the general solution of the equation.

$$z(t) = c_1 e^{\left(2\mu_0 I_p I_c R_0/(d^2 M)\right)^{\frac{1}{2}} t} + c_2 e^{-\left(2\mu_0 I_p I_c R_0/(d^2 M)\right)^{\frac{1}{2}} t}$$

but 2^{nd} term when $t \to \infty$, $\to 0$

So $z(t) = c_1 \exp\left(-\left(\frac{2\mu_0 I_p I_c R_0}{d^2 M}\right)^{\frac{1}{2}}\right)$

Let call $\boxed{\gamma_0 = \left(2\mu_0 I_p I_c \frac{R_0}{d^2 M}\right)^{\frac{1}{2}}}$ as the growth rate.

d)(i) $Y_{mod} = \frac{r_0^2}{2r_1}\left[\left(1+\frac{4r_1^2}{r_0^2}\right)^{1/2} - 1\right]$

· Here we will use Taylor series

$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots$

$\left\{\left(\left(1+\frac{4r_1^2}{r_0^2}\right)^{1/2}\right)' = \left(1+\frac{4r_1^2}{r_0^2}\right)^{-\frac{1}{2}} \cdot \frac{1}{2} \cdot \frac{4 \cdot 2 r_1}{r_0^2} \right\} \neq$

$= \left(\underbrace{1}+\frac{4r_1^2}{r_0^2}\right)^{-1/2} \cdot \frac{4r_1}{r_0^2} \}$

$f(r_1) = 1 + \frac{4r_1}{r_0^2} r_1 + \dots$

$= 1 + \frac{4r_1^2}{r_0^2} + \dots$

$Y_{mod} = \frac{r_0^2}{2r_1}\left[\left(1+\frac{4r_1^2}{r_0^2}\right) - 1\right] =$

$= \frac{r_0^2}{2r_1} \frac{4r_1^2}{r_0^2} = \underline{2r_1}$ (not r_1 ?)

$\boxed{Y_{mod} = 2r_1}$ not $Y_{mod} = r_1$?

c) $\eta \, j_i = - \bar{v} \times \bar{B}$

$$\begin{cases} I_i = j_i A \Rightarrow j_i = I_i / A \\ B = B_\theta = \mu_0 I_p / (2\pi r) \end{cases}$$

$$\frac{\eta I_i}{A} = - \frac{v \cdot \mu_0 I_p}{2\pi r}$$

$$I_i = - \frac{v \cdot \mu_0 I_p A}{2\pi r \eta},$$

when $r = d$

$$\boxed{I_i = - \frac{v \mu_0 I_p A}{2\pi d \eta}}$$