Alexei Kosykhin

Assignment 2

a) Number particles entering the torus per second.

$$n_{torus} = \frac{1\ MW}{100\ KeV} = \frac{1 \cdot 10^6\ J/sec}{1 \cdot 10^5 \cdot 1.602 \cdot 10^{-19} J}$$

$$= \underline{6.242 \times 10^{19}} \text{ particles/second}$$

---

b) Number ionised particles equastion

$$\frac{d\dot{n}_0(z)}{dz} = -N\dot{n}_0(z)\sigma_{01}$$

Solving the equastion

$$\int_{z_1}^{z_2} \frac{d\dot{n}_0(z)}{dz} = -\int_{z_1}^{z_2} N\sigma_{01}\, dz$$

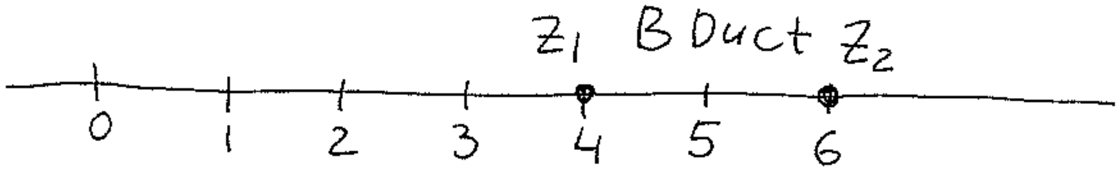
$$\ln \dot{n}(z_2)/\dot{n}(z_1) = -N\sigma_{01}(z_2 - z_1)$$

$$(1) \quad \boxed{\dot{n}_0(z_2) = \dot{n}_0(z_1) \exp\{-N\sigma_{01}(z_2 - z_1)\}}$$

z_1 is a point where a particle starts being ionised, z_2 - finish.

b) i) From eqn. (1) we can find $\dot{n}_o(z_1)$

$$\dot{n}_o(z_1) = \dot{n}_o(z_2) \Big/ \left\{ exp\left(-N\sigma_{01}(z_2 - z_1)\right) \right\}$$



We will calculate number particles entering B Duct

$$\dot{n}_{o(B\,Duct)} = \dot{n}_{o(torus)} / exp\left\{-N\sigma_{01}(z_2 - z_1)\right\}$$

$$= 6.242\times10^{19} / exp\left(-5.3\times10^{17} \times 1.55\times10^{-20} \times (6-3)\right)$$

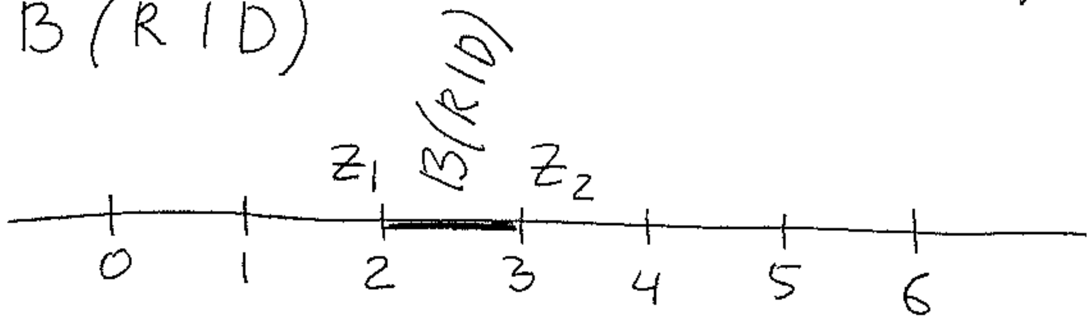
$$\left\{ \text{density } N = 2.65\times10^{20} \times 0.002 = 5.3\times10^{17} \frac{p}{M^3} \right\}$$

$$= 6.242\times10^{19} / exp\left(-2.465\times10^{-2}\right) =$$

$$= 6.242\cdot10^{19} / (0.976) =$$

$= \underline{6.395\times10^{19}}$ particles / second entering Drift Duct.

b) ii) Now as long as we know number of particles entering B(Duct) we can calculate number of particles entering B(RID)



$$\dot{n}_0(\text{BRID}) = \dot{n}_0(\text{BDuct}) / \exp(-N \cdot \sigma (z_2 - z_1))$$

$$= 6.395 \times 10^{19} / \exp(-2.65 \times 10^{20} \times 0.015 \times 1)$$

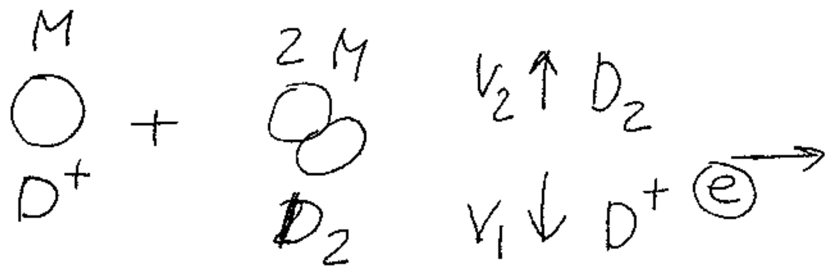
1.55×10^{-20}

$$= 6.395 \times 10^{19} / \exp(-0.0616) =$$

$$= \underline{6.801 \times 10^{19}}$$ particles entering B(RID) per second.

b) iii)

$$D^0 + D_2 \rightarrow D^+ + e + D_2$$



$$\begin{cases} M v_1 = 2 M v_2 & \text{momentum} \\ \dfrac{M v_1^2}{2} + \dfrac{(2M) v_2^2}{2} = \dfrac{M v_0^2}{2} & \text{energy conservation} \end{cases}$$

$$\begin{cases} v_1 = 2 v_2 \\ \dfrac{M (2 v_2)^2}{2} + \dfrac{(2M) v_2^2}{2} = \dfrac{M v_0^2}{2} \end{cases}$$

$$\begin{cases} \dfrac{M}{2} 4 v_2^2 + \dfrac{M}{2} 2 v_2^2 = \dfrac{M}{2} v_0^2 \\ v_1 = 2 v_2 \end{cases}$$

$$6 v_2^2 = v_0^2 \Rightarrow \begin{cases} v_2 = \dfrac{v_0}{\sqrt{6}} \\ v_1 = 2 v_2 \end{cases}$$

$$\begin{cases} V_1 = \frac{2}{\sqrt{6}} V_0 \\ V_2 = \frac{1}{\sqrt{6}} V_0 \end{cases}$$

$$E_{D_2} = (2M)\,\frac{1}{2}\left(\frac{1}{\sqrt{6}} V_0\right)^2 = \frac{1}{3} E_0$$

$$E_{D^+} = (M)\,\frac{1}{2}\left(\frac{1}{\sqrt{6}}\right)^2 = \frac{1}{2}\,\frac{2}{3} M V_0^2$$

$$= \frac{2}{3} E_0$$

where $E_0 = \frac{M}{2} V_0^2$ – energy of D^+

Total energy of D^+ entering B(RID)

$$= (6.801 - 6.242)\cdot 10^{19}\cdot 10^5 \,\text{eV} =$$

$$= 0.558\cdot 10^{19}\cdot 10^5\cdot 1.602\cdot 10^{-19} =$$

$$= 0.089\cdot 10^6 \,\text{W} = 89 \text{ KW}$$

Power loading on the plates =

$$= \frac{2}{3}\cdot 89 \text{ KW} \approx \underline{29.3 \text{ KW}}$$

c) ii) $\dot{n}^+$ ions,

$\dot{n}^0$ neutralised ions

$\dot{n}^x$ not neutralised ions.

$$\dot{n}^+ = \dot{n}^0 + \dot{n}^x \Rightarrow$$

$$\Rightarrow \dot{n}^x = \dot{n}^+ - \dot{n}^0 = \frac{\dot{n}^0}{\left(\frac{\dot{n}^0}{\dot{n}^+}\right)} - \dot{n}^0 =$$

$$= \dot{n}^0\left(\frac{1}{\left(\frac{\dot{n}^0}{\dot{n}^+}\right)} - 1\right)$$

$$\boxed{\begin{array}{l}\text{Not neutralised ions}\\ \dot{n}^x = \dot{n}^0\left(\frac{1}{(\dot{n}^0/\dot{n}^+)} - 1\right)\end{array}}$$

$$\dot{n}^x = 6.801\times10^{15}\left(\frac{1}{0.4604} - 1\right) =$$

$$= \underline{7.983\times10^{19}} \text{ ions are not neutralised per second}$$

Check

ions neutralised	$6.80\cdot10^{19}$
ions not neutralised	$+\ 7.98\cdot10^{19}$
total ions	$14.77\cdot10^{19}$

c) iii)

Total energy of ions =

$= 14.77 * 10^{19} \cdot 100\,KeV =$

$= 14.77 \times 10^{19} \times 10^{5}\,eV \times 1.602 \times 10^{-19} =$

$= 2.36\,MW$ – input

$1\,MW$ – output

$$\text{Efficiency} = \frac{1\,MW}{2.36\,MW} = \underline{0.423 \text{ times}}$$

$\approx 42\%$