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PA 3

Part A: Solve for x in Ax = b; Solve for x in Ax=0

- If the area spanned by A DOESN'T = 0
  - To solve for x in Ax=b, I will use Cramer's Rule. **x1** = area(b,a2) / area(a1,a2) **x2** = area(a1,b) / area(a1,a2).
- If the area of A is = 0;
  - To solve for this scenario, I will use Gauss Elimination. Starting by finding the shear that will produce the upper triangle matrix, with 0 in the lower left corner.
  - After I have found the correct shear, I will multiply A by this shear, followed by back substitution to find the solution X.
- To solve for Ax = 0, I will choose an arbitrary value for x2 and use backwards substitution to solve for x1.
- To achieve this, I will use Python's LinearAlgebra library Solve() function.

Part B: Solve for EigenValues, R matrix, and Matrix Composition

<u>For part B, I will start by solving for the Eigenvalues.</u>

- I will use the quadratic formula to find the 2 eigenvalues associated with matrix A
- Python has a Linear Algebra library which aids in solving equations. I will use the Solve() function from this library to find the 2 corresponding eigenvalues.
- Once the values have been found, I will find which value has the greater absolute value, this will be Value1 with the other value being Value2.
- They will compose an eigen matrix, where Value1 == Matrix[11], and Value2 == Matrix[22]

<u>Next, I will need to solve for the R Matrix</u>

- This matrix is composed of 2 column vectors, r1 and r2
- To solve for these 2 vectors, I will use the associated eigenvalues. For r1 I will use the first eigenvalue, which will be used in a matrix as such [V1 0], [0 V1].
- After I have my first eigenvalue matrix I will subtract; Matrix A - Matrix eigenvalue1. I will call this resulting matrix the resultMatrix.
- The resultMatrix will be used to solve for the corresponding r vector of the homogeneous system. To achieve this, I will first need to use gauss elimination to transform the matrix into a system that can be solved.
- This involves first finding the shear matrix that will make the resultMatrix[21 == 0], followed by multiplying the shear matrix by the entire resultMatrix.
- As a result, I will use back substitution to solve for r1 vectors components.
- These steps will then be repeated to solve for r2, using the second eigenvalue.
- After finding both r1 and r2, we can combine them into the Matrix R = [r1 r2]

<u>Find Transpose Matrix</u>

- In order to find the Matrix composition, I will need to find the transpose matrix of R. To do this I will use python's Numpy library; .T function to transpose my R matrix.
    - rTranspose = rMatrix.T

Solve for Matrix Composition

- Solving for Matrix composition consists of multiplying the 3 components I previously solved for.
- I will start by multiplying the eigenvalue matrix by the Rtranspose matrix. Followed by multiplying this result by the R matrix.
- To achieve this, I will use the .matmul() function from the numpy library.

Testing

- To test the approach I have described, I will work alongside my program with a test input file.
- As I write the code to solve the problem, I will ensure the correct output by doing it by hand as well.
- This will help to avoid mathematical errors that could result in an incorrect answer.
- I will test with multiple different files with different starting matrices.
- As I test my code, I will output key values (value of my shear matrix, values of r1 & r2) and double check they are the expected values.

Part C: Solve for the area, distance to point, foot of point

To solve for the area of the triangle, I will take the cross product of the 3 points (determinant).

- Area = (1/2) |x1(y2 − y3) + x2(y3 − y1) + x3(y1− y2)|

To solve for the distance of p3 to the line produced by p1 & p2

- I plan to use the following function norm() from the numpy.Linalg library to solve for distance.
    - distance = norm(np.cross(p2-p1, p1-p3))/norm(p2-p1)

Solve for Foot of the point to line

- To solve for the foot point of the point to the line, I used a parametric version of a line. I needed to find q, which would be the foot of r (point).
- I needed to find t such that q = p + tv.

1. Find w. W = r(point 3) - p(point 1)
2. t = (v dot w) / normal(v) ^2
3. Plug t into q = p + tv and solve for q, the foot of r.