h) $\langle a|b\rangle = \frac{1}{2}\left[\langle a|b\rangle - |q_1\rangle \odot \langle a|b\rangle \odot |q_1\rangle\right]$

$= \frac{1}{2}\left[a^0 b^0 + \vec{a}\cdot\vec{b} + a^0\vec{b} - b^0\vec{a} + (\vec{b}\times\vec{a})\right] -$

$\Big[|q_1\rangle \odot \big(a^0 b^0 + a^i b^i + a^0 b^i |q_i\rangle - b^0 a^i |q_i\rangle +$

$(b^2 a^3 - b^3 a^2)|q_1\rangle - (b^1 a^3 - b^3 a^1)|q_2\rangle + (b^1 a^2 - b^2 a^1)|q_3\rangle \big) \odot |q_1\rangle$

$$\det \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix} \Rightarrow (\vec{b}\times\vec{a})$$

$= \frac{1}{2}\left[a^0 b^0 + \vec{a}\cdot\vec{b} + a^0\vec{b} - b^0\vec{a} + (\vec{b}\times\vec{a})\right] -$

$\Big[a^0 b^0 |q_1\rangle + a^i b^i |q_1\rangle + a^0 b^1(-1) + a^0 b^2 |q_3\rangle + a^0 b^3(-|q_2\rangle)$

$- b^0 a^1(-1) - b^0 a^2 |q_3\rangle - b^0 a^3(-|q_2\rangle) +$

$-(b^2 a^3 - b^3 a^2) - (b^1 a^3 - b^3 a^1)|q_3\rangle + (b^1 a^2 - b^2 a^1)|-q_2\rangle \Big]$

$\odot |q_1\rangle$

$\Big[-a^0 b^0 - \vec{a}\cdot\vec{b} - a^0 b^1 |q_1\rangle + a^0 b^2 |q_2\rangle + a^0 b^3 |q_3\rangle$

$+ b^0 a^1 |q_1\rangle - b^0 a^2 |q_2\rangle - b^0 a^3 |q_3\rangle - (b^2 a^3 - b^3 a^2)|q_1\rangle$

$- (b^1 a^3 - b^3 a^1)|q_2\rangle + (b^1 a^2 - b^2 a^1)|q_3\rangle \Big]$

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$$= \frac{1}{2}\left[a^0 b^0 + \vec{a}\cdot\vec{b} + a^0\vec{b} - b^0\vec{a} + (\vec{b}\times\vec{a}) + \right.$$

$$\left[+ a^0 b^0 + \vec{a}\cdot\vec{b} + a^0 b^1 |q_1\rangle - a^0 b^2 |q_2\rangle - a^0 b^3 |q_3\rangle \right.$$

$$- b^0 a^1 |q_1\rangle + b^0 a^2 |q_2\rangle + b^0 a^3 |q_3\rangle + (b^2 a^3 - b^3 a^2)|q_1\rangle$$

$$\left. + (b^1 a^3 - b^3 a^1)|q_2\rangle - (b^1 a^2 - b^2 a^1)|q_3\rangle \right]$$

$$= \frac{1}{2}\left[2 a^0 b^0 + 2\vec{a}\cdot\vec{b} + 2 a^0 b^1 |q_1\rangle - 2 b^0 a^1 |q_1\rangle \right.$$

$$\left. + 2(b^2 a^3 - b^3 a^2)|q_1\rangle \right]$$

$$= \underbrace{a^0 b^0 + \vec{a}\cdot\vec{b}}_{F^0} + |q_1\rangle \underbrace{(a^0 b^1 - b^0 a^1 + b^2 a^3 - b^3 a^2)}_{F^1}$$

$\Rightarrow$ asocia un número complejo al par de vectores

1) $\langle a | a \rangle = a^0 a^0 + \vec{a}\cdot\vec{a} + |q_1\rangle (a^0 a^1 - a^0 a^1 + a^2 a^3 - a^3 a^2)$

$$= (a^0)^2 + (\vec{a}\cdot\vec{a}) = (a^0)^2 + (a^1)^2 + (a^2)^2 + (a^3)^2$$

$$= \|a\|^2 \geq 0$$

vectores

(1) $\langle a|a\rangle = a^0 a^0 + \vec{a}\cdot\vec{a} + |q_1\rangle(a^0 a^1 - a^0 a^1 + a^2 a^3 - a^3 a^2)$

$= (a^0)^2 + (\vec{a}\cdot\vec{a}) = (a^0)^2 + (a^1)^2 + (a^2)^2 + (a^3)^2$

$= \|a\|^2 \geq 0$

Si $\langle a|a\rangle = 0$ $|a\rangle = |0\rangle$?

$(a^0)^2 + (a^1)^2 + (a^2)^2 + (a^3)^2 = 0 \rightarrow a^i = 0$

entonces $|a\rangle = |0\rangle$

(2) $\langle b|a\rangle = b^0 a^0 + \vec{b}\cdot\vec{a} + |q_1\rangle(b^0 a^1 - a^0 b^1 +$

$a^2 b^3 - a^3 b^2) = a^0 b^0 + \vec{a}\cdot\vec{b} - |q_1\rangle(a^0 b^1 - b^0 a^1$

$+ b^2 a^3 - b^3 a^2) = \langle a|b\rangle^*$

(3) $\langle a|\alpha b + \gamma c\rangle = a^0(\alpha b^0 + \gamma c^0) + \vec{a}\cdot(\alpha\vec{b} + \gamma\vec{c})$

$+ |q_1\rangle(a^0(\alpha b^1 + \gamma c^1) - (\alpha b^0 + \gamma c^0)a^1 + a^3(\alpha b^2 + \gamma c^2)$

$- a^2(\alpha b^3 + \gamma c^3))$

$= \alpha a^0 b^0 + \gamma a^0 c^0 + \alpha\,\vec{a}\cdot\vec{b} + \gamma\,\vec{a}\cdot\vec{c} + |q_1\rangle(\alpha a^0 b^1 +$

$\gamma a^0 c^1 - \alpha b^0 a^1 - \gamma c^0 a^1 + \alpha b^2 a^3 + \gamma c^2 a^3$
$- \alpha a^2 b^3 - \gamma a^2 c^3) = \alpha \langle a|b\rangle + \gamma \langle a|c\rangle$

4) $\langle \alpha a + \beta b | c \rangle$
$= (\alpha a^0 + \beta b^0)(c^0) + (\alpha \vec{a} + \beta \vec{b}).(\vec{c})$
$+ |q_1\rangle ((\alpha a^0 + \beta b^0)(c^1) - (c^0)(\alpha a^1 + \beta b^1)$
$+ (c^2)(\alpha a^3 + \beta b^3) - (c^3)(\alpha a^2 + \beta b^2))$
$= \alpha \langle a|c\rangle + \beta \langle b|c\rangle$

5) $\langle a|0\rangle = a^0.0 + \vec{a}.\vec{0} + |q_1\rangle (a^0.0 - 0.a^1$
$+ 0a^3 - 0.a^2)$
$= 0 + |q_1\rangle(0) = 0 = \langle 0|a\rangle$

Si cumple todas las propiedades del producto Interno