$$A^{i'}_{k}\tilde{A}^{j}_{i'} = \frac{\partial x_{i'}}{\partial x_k}\frac{\partial x_j}{\partial x_{i'}} = 1$$

Si $k=j$, $$\frac{\partial x_{i'}}{\partial x_k}\frac{\partial x_k}{\partial x_{i'}} = 1$$

$A^{i'}_{k} \Rightarrow \cos(\theta_{i'k})$, siendo $\theta_{i'k}$ el ángulo entre i' y k

$$\Rightarrow A^{i'}_{k}\tilde{A}^{k}_{i'} = \cos(\theta_{i'k})\cos(\theta_{ki'}) = \cos^2(\theta_{i'k})$$

$$= \cos^2(\theta_{1'1}) + \cos^2(\theta_{2'2}) + \cos^2(\theta_{3'3}) = \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

a) $(x,y) \to (-y,x)$

$$\underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{A}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ x \end{pmatrix}, |\det(A)| = 1$$

b) $(x,y) \to (x,-y)$

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{B}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}, |\det(B)| = |-1| = 1$$

c) $(x,y) \to (x-y, x+y)$

$$\underbrace{\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}}_{C}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x-y \\ x+y \end{pmatrix}, |\det(C)| = |1-(-1)| = 2$$

d) $(x,y) \to (x+y, x-y)$

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}}_{D}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ x-y \end{pmatrix}, |\det(D)| = |-1-1| = |-2| = 2$$

1.5.7

2a) $\nabla(\phi\psi) = \partial_i(\phi\psi)^i = ((\partial_i\phi)\psi) + (\phi(\partial_i\psi))$

2d) $\nabla\cdot(\nabla\times a) = \nabla\cdot(\varepsilon_{ijk}\partial^j a^k) = \partial_i\varepsilon_{ijk}\partial^j a^k = \varepsilon_{ijk}\partial^i\partial^j a^k$

$\doteq -\varepsilon_{jik}\partial^j\partial^i a^k = 0$

$\nabla\times(\nabla\cdot a)$ no está definido puesto que no se hace producto vectorial con un escalar $(\nabla\cdot a)$.

$2f)\ \nabla\times(\nabla\times a)=\varepsilon_{ijk}\,\partial^j(\nabla\times a)^k=\varepsilon_{ijk}\varepsilon^{kmn}\partial^j\partial^n a^n=\varepsilon_{ijk}\varepsilon^{mnk}\partial_m a_n\partial^j$

$\Rightarrow(\delta^i_m\delta^j_n-\delta^i_n\delta^j_m)\partial^j\partial_m a_n=\delta^i_m\delta^j_n\partial^j\partial_m a_n-\delta^i_n\delta^j_m\partial^j\partial_m a_n$

$=\partial_m(\partial^j a_j)-a_n(\partial^j\partial_j)=\nabla(\nabla\cdot a)-a(\nabla\cdot\nabla)=\nabla(\nabla\cdot a)-\nabla^2 a$

1.65

2ab $e^{i3\theta}=(e^{i\theta})^3=\cos(3\theta)+i\,\mathrm{Sen}(3\theta)=(\cos\theta+i\,\mathrm{Sen}\theta)^3$

$\Rightarrow\cos^3(\theta)+3\cos^2\theta\, i\,\mathrm{Sen}\theta+3\cos\theta(i\,\mathrm{Sen}\theta)^2+(i\,\mathrm{Sen}\theta)^3$

$=\cos^3\theta+3i\cos^2\theta\,\mathrm{Sen}\theta-3\cos\theta\,\mathrm{Sen}^2\theta-i\,\mathrm{Sen}^3\theta$

$R_e\Rightarrow\cos(3\theta)=\cos^3(\theta)-3\cos\theta\,\mathrm{Sen}^2\theta$

$I_m\Rightarrow\mathrm{Sen}(3\theta)=3\cos^2\theta\,\mathrm{Sen}\theta-\mathrm{Sen}^3\theta$

5. a) $\sqrt{2i}\Rightarrow 2e^{i\frac{\pi}{2}}$

raíces $\rightarrow\sqrt{2}\,e^{i\left(\frac{\pi/2+2\pi k}{n}\right)},\ k=(0,1)$

$k=0\Rightarrow\sqrt{2}\,e^{i\frac{\pi}{4}}=\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right)+i\,\mathrm{Sen}\left(\frac{\pi}{4}\right)\right)=1+i$

$k=1\Rightarrow\sqrt{2}\,e^{i\left(\frac{\pi}{4}+\pi\right)}=\sqrt{2}\left(\cos\left(\frac{\pi}{4}+\pi\right)+i\,\mathrm{Sen}\left(\frac{\pi}{4}+\pi\right)\right)=-1-i$

b) $\sqrt{1-\sqrt{3}i}\Rightarrow\sqrt{4}\,e^{i\frac{\pi}{3}}=2e^{i\frac{\pi}{3}}$

$k=0\Rightarrow\sqrt{2}\,e^{i\frac{\pi}{6}}=\sqrt{2}\left(\cos\left(\frac{\pi}{6}\right)+i\,\mathrm{Sen}\left(\frac{\pi}{6}\right)\right)=\sqrt{2}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)$

$k=1\Rightarrow\sqrt{2}\,e^{i\left(\frac{\pi}{6}+\pi\right)}=\sqrt{2}\left(\cos\left(\frac{\pi}{6}+\pi\right)+i\,\mathrm{Sen}\left(\frac{\pi}{6}+\pi\right)\right)=\sqrt{2}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)$

c) $(-1)^{1/3}\Rightarrow e^{i\pi}$

$k=0\Rightarrow e^{i\frac{\pi}{3}}=\cos\left(\frac{\pi}{3}\right)+i\,\mathrm{Sen}\left(\frac{\pi}{3}\right)=\frac{1}{2}+\frac{\sqrt{3}}{2}i$

$k=1\Rightarrow e^{i\left(\frac{\pi}{3}+\frac{2\pi}{3}\right)}=e^{i\pi}=-1$

$k=2\Rightarrow e^{i\left(\frac{\pi}{3}+\frac{4\pi}{3}\right)}=e^{i\frac{5\pi}{3}}=\cos\left(\frac{5\pi}{3}\right)+i\,\mathrm{Sen}\left(\frac{5\pi}{3}\right)=\frac{1}{2}-\frac{\sqrt{3}}{2}i$

d) $\sqrt[6]{8}=2^{3/6}=\sqrt{2}=\sqrt{2}\,e^{i0}$

$k=0\Rightarrow\sqrt{2}\,e^{i0}=\sqrt{2}$

$k=1\Rightarrow\sqrt{2}\,e^{i\frac{\pi}{3}}=\frac{\sqrt{2}}{2}+i\frac{\sqrt{6}}{2}$

$k=2 \Rightarrow \sqrt{2}\, e^{i\frac{2\pi}{3}} = -\frac{\sqrt{2}}{2} + i\frac{\sqrt{6}}{2}$

$k=3 \Rightarrow \sqrt{2}\, e^{i\pi} = -\sqrt{2}$

$k=4 \Rightarrow \sqrt{2}\, e^{i\frac{4}{3}\pi} = -\sqrt{2}\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$k=5 \Rightarrow \sqrt{2}\, e^{i\frac{5\pi}{3}} = \sqrt{2}\left(\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)$

e) $\sqrt[4]{-8-8\sqrt{3}i} \Rightarrow 16 e^{i\frac{4\pi}{3}}$

$k=0 \Rightarrow 2e^{\frac{\pi}{3}i} = 1+\sqrt{3}i$

$k=1 \Rightarrow 2e^{\left(\frac{\pi}{3}+\frac{\pi}{2}\right)i} = -\sqrt{3}+i$

$k=2 \Rightarrow 2e^{\left(\frac{\pi}{3}+\pi\right)i} = -1-\sqrt{3}i$

$k=3 \Rightarrow 2e^{\left(\frac{\pi}{3}+\frac{3\pi}{2}\right)i} = \sqrt{3}-i$

6) a) $\ln(-ie) = \ln(e) + \ln(-i)$

$e^{v} = -i \;,\; v = a+bi$

$e^{a}e^{bi} = -i \rightarrow e^{a}(\cos b + i \operatorname{Sen} b) = -i.$

$R_e \Rightarrow e^{a}\cos b = 0, \; I_m \Rightarrow e^{a}\operatorname{Sen} b = -1$

$b = \left(\frac{\pi}{2}, 3\frac{\pi}{2}, \ldots, n\frac{\pi}{2}\right)$ n impar

$b = \left(-\frac{\pi}{2}, \frac{3\pi}{2}, -\frac{\pi}{2}+2\pi n\right)$, n número natural

y $a=0$

Si tomamos una solución como lo es $b=-\frac{\pi}{2}$:

$-i = e^{-\frac{\pi}{2}i}$.

$\therefore \ln(-ie) = \ln(e) + \ln\left(e^{-\frac{\pi}{2}i}\right) = 1 - \frac{\pi}{2}i$

b) $\ln(1-i) = \frac{1}{2}\ln(2) - \frac{\pi}{4}i$

$e^{v} = (1-i)$

$e^{a}(\cos b + i\operatorname{Sen} b) = e^{a}e^{bi} = \sqrt{2}\, e^{-\frac{\pi}{4}i}$

$e^{a} = \sqrt{2} \Rightarrow a = \frac{1}{2}\ln(2)$

$e^{b} = e^{-\pi/4} \Rightarrow b = -\frac{\pi}{4}$

$\ln(1-i) = \ln\left(e^{\frac{1}{2}\ln(2) - \frac{\pi}{4}i}\right) = \frac{1}{2}\ln(2) - \frac{\pi}{4}i$

c) $\mathrm{Ln}(e) = 1 + 2n\pi i$

$e^a e^{bi} = e^a(\mathrm{Cos}\,b + i\,\mathrm{Sen}\,b) = e$

$\mathrm{Re} \Rightarrow e^a \mathrm{Cos}\,b = e$

$\mathrm{Im} \Rightarrow e^a \mathrm{Sen}\,b = 0 \Rightarrow b = 2\pi n$, n número entero

Y en $2\pi n$, $\mathrm{Cos}\,b = 1$, entonces $e^a = e$, $a = 1$

$\ln(e) = \ln(e^{1+2\pi n i}) = 1 + 2\pi n i$

d) $\ln(i) = (2n + \frac{1}{2})\pi i$

$e^a e^{bi} = e^{(\frac{\pi}{2} + 2\pi n)i}$

$a = 0$, $bi = (\frac{\pi}{2} + 2\pi n)i$

$\ln(e^{(\pi/2 + 2\pi n)i}) = \pi/2 + 2n\pi i = (2n + \frac{1}{2})\pi i$