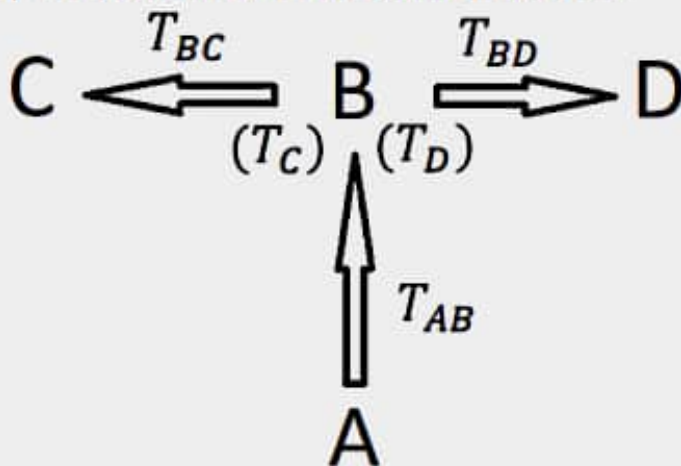


Задача 4. X и Y пътуват заедно от град A до B . След пристигането си, изчакват съответно автобуси до C и D . Предполагаме, че пътуванията траят съответно $T_{AB} \sim \text{Exp}(3)$, $T_{BC} \sim \text{Exp}(4)$ и $T_{BD} \sim \text{Exp}(5)$, а изчакванията в B са $T_C \sim \text{Exp}(1)$ и $T_D \sim \text{Exp}(2)$, като така дефинираните времена са независими. Нека ξ и η са времената на пътуване на X и Y .



1. (0.25 т.) Намерете $\mathbb{P}(T_C + \ln(\mathbb{E}T_D) > 0)$.
2. (0.75 т.) Намерете $\text{Cor}(\xi, \eta)$.

$$④ 1. P(\bar{I}_c + \ln(E[\bar{I}_p]) > 0)$$

$$E[\bar{I}_p] = \frac{1}{2}$$

$$P(\bar{I}_c + \ln(\frac{1}{2}) > 0) = P(\bar{I}_c > \ln 2) = e^{-\ln 2} = \frac{1}{2}$$

$$2. \text{ Here } Z := \xi \text{ u } W := \eta$$

$$E[Z] = E[\bar{I}_{AB} + \bar{I}_c + \bar{I}_{BC}] = \frac{1}{3} + \frac{1}{4} + 1 = \frac{19}{12}$$

$$E[W] = E[\bar{I}_{AB} + \bar{I}_D + \bar{I}_{BD}] = \frac{1}{3} + \frac{1}{2} + \frac{1}{5} = \frac{31}{30}$$

$$D[Z] = D[\bar{I}_{AB} + \bar{I}_c + \bar{I}_{BC}] \stackrel{iid}{=} \frac{1}{9} + \frac{1}{16} + 1 = \frac{169}{144} = \left(\frac{13}{12}\right)^2$$

$$D[W] = D[\bar{I}_{AB} + \bar{I}_D + \bar{I}_{BD}] \stackrel{iid}{=} \frac{1}{9} + \frac{1}{4} + \frac{1}{25} = \frac{361}{900} = \left(\frac{19}{30}\right)^2$$

$$E[ZW] = E[(\bar{I}_{AB} + \bar{I}_c + \bar{I}_{BC})(\bar{I}_{AB} + \bar{I}_D + \bar{I}_{BD})] = E[\bar{I}_{AB} + \bar{I}_c + \bar{I}_{BC}]E[\bar{I}_{AB} + \bar{I}_D + \bar{I}_{BD}] =$$

$$= E[\bar{I}_{AB}^2] + E[\bar{I}_{AB}\bar{I}_c] + E[\bar{I}_{AB}\bar{I}_{BD}] + \dots - E[\bar{I}_{AB}]E[\bar{I}_c] - E[\bar{I}_{AB}\bar{I}_D] - E[\bar{I}_{AB}\bar{I}_{BD}] - \dots$$

$$= \text{Cov}(\bar{I}_{AB}, \bar{I}_{AB}) + \underbrace{\text{Cov}(\bar{I}_{AB}, \bar{I}_c)}_{0 \text{ u } 14302.} + \underbrace{\text{Cov}(\bar{I}_{AB}, \bar{I}_D)}_{0} + \dots = \text{Cov}(\bar{I}_{AB}, \bar{I}_{AB}) = D[\bar{I}_{AB}] = \frac{1}{9} \Rightarrow \text{Cor}(Z, W) = \frac{\frac{1}{9}}{\frac{13}{12} \cdot \frac{19}{30}} \approx 0.1619$$

$$\text{Cov}(Z, W) = E[ZW] - E[Z]E[W]$$

$$\text{Cor}(Z, W) = \frac{\text{Cov}(Z, W)}{\sqrt{D(Z)D(W)}}$$