Задача 85. Дадена е случайна величина X с плътност $f_X(x) = \begin{cases} c(x^2+2x), & x \in [0,1] \\ 0, & x \notin [0,1] \end{cases}$. Намерете

1. константата c;
2. $\mathbb{E}X$ и DX;
3. вероятността X да е по-малка от математическото си очакване;
4. очакването на случайната величина $X^2 + 3X$.

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(85) $f_X(x) = \begin{cases} c(x^2+2x) & , x \in [0,1] \\ 0 & , \text{иначе} \end{cases}$

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1. Намерете c

$$1 = \int_{-\infty}^{\infty} f_X(x)dx = \int_{-\infty}^{\infty} c(x^2+2x)\, I_{\{x \in [0,1]\}} dx =$$

$$= c \cdot \int_0^1 x^2+2x\,dx = c \cdot \left[\frac{x^3}{3} + x^2\right]_0^1 = \frac{4}{3} \cdot c = 1$$

$$\Rightarrow c = \frac{3}{4}$$

2. $$\mathbb{E}[X] = \int_{-\infty}^{\infty} x \cdot f_X(x)dx = \int_0^1 x \cdot c\,(x^2+2x)dx =$$

$$= c \cdot \int_0^1 x^3+2x^2 dx = c\left[\frac{x^4}{4} + \frac{2x^3}{3}\right]_0^1 = \frac{11}{12} \cdot c$$

$$\mathbb{D}[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = c \cdot \frac{7}{10} + \frac{121}{144} \cdot c^2$$

$$\mathbb{E}[g(x)] = \int_{-\infty}^{\infty} g(x) f_X(x) dx$$

$$\mathbb{E}[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)dx = \int_0^1 x^2 \cdot c(x^2+2x)dx = c \cdot \int_0^1 x^4+2x^3 dx = c\left[\frac{x^5}{5} + \frac{x^4}{2}\right]_0^1 = c \cdot \frac{7}{10}$$

3. $$\mathbb{P}(X < \overset{a}{\overset{\shortparallel}{\mathbb{E}[X]}}) = F_X(a) \overset{a \in (0,1)}{=} \int_{-\infty}^{a} f_X(t)dt$$

$$f_X(t) = \frac{d}{dt} F_X(t)$$

$$F_X(a) = \int_0^a c(x^2+2x)dx = c\left[\frac{x^3}{3} + x^2\right]_0^{a = \mathbb{E}[X]} =$$

4. $$\mathbb{E}[X^2+3X] = \mathbb{E}[X^2] + \mathbb{E}[3X] = c \cdot \frac{7}{10} + 3 \cdot \frac{11}{12} \cdot c$$

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