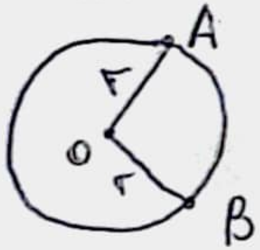
Задача 86. Върху окръжност $k(O, r)$ е фиксирана точка A, а точка B попада по случаен начин върху окръжността. Да се намери математическото очакване на лицето на $\triangle AOB$.

(86)



$$X_B \sim U(0, 2\pi r)$$

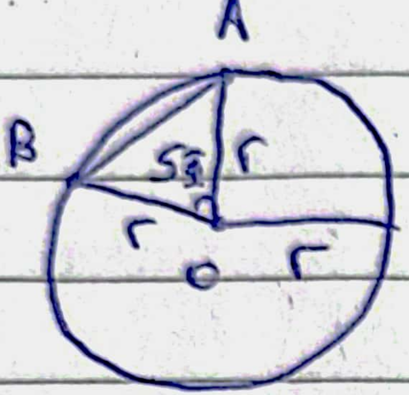
$$S = \frac{r^2 \cdot \sin(\phi)}{2} \qquad \phi = \begin{cases} \frac{X_B}{r} & ; X_B < \pi r \\ 2\pi - \frac{X_B}{r} & ; X_B > \pi r \end{cases}$$

$$\phi \sim U(0, \pi)$$

$$\mathbb{E}[S] = \int_0^{\pi} \frac{r^2 \cdot \sin(\phi)}{2} \cdot \frac{1}{\pi} d\phi = \frac{r^2}{2\pi}\left[-\cos(\phi)\right]_0^{\pi} = \frac{r^2}{2\pi}$$

(86)

Нека т. A има полярни координати $(r, 0)$, а т. B - (r, Φ), където $\Phi \sim Unif(0, 2\pi)$



$$S := g(x) = \frac{r^2 \cdot |\sin(x)|}{2}, \quad x \in [0, 2\pi]$$

$$\mathbb{E}[S] = \mathbb{E}[g(x)] = \int_0^{2\pi} g(x) \cdot f_\Phi(x) dx =$$

$$= \int_0^{2\pi} \frac{r^2 \cdot |\sin(x)|}{2} \cdot \frac{1}{2\pi} dx = \frac{r^2}{4\pi}\left[\int_0^{\pi} \sin(x) dx + \int_{\pi}^{2\pi} -\sin(x) dx\right] =$$

$$= \frac{r^2}{4\pi}\left[-\cos(x)\Big|_0^{\pi} + \cos(x)\Big|_{\pi}^{2\pi}\right] = \frac{r^2}{\pi}$$