Descriptive Statistics

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Plan

1. Overview of Last Week
2. Measurement
3. Descriptive Statistics
4. Readings
5. Wrap up

Measurement

- Social science is about developing and testing causal theories:
  - Does minimum wage change levels of employment?
  - Does outgroup contact influence views on immigration?
- Theories are made up of concepts:
  - Minimum wage, level of employment, outgroup contact, views on immigration.
  - We took these for granted when talking about causality.
- Important to consider how we measure these concepts.
  - Some more straightforward: what is your age?
  - Others more complicated: what does it mean to "be liberal"?
  - Have to create an operational definition of a concept to make it into a variable in our dataset.

Example

- Concept: presidential approval.
- Conceptual definition:
  - Extent to which US adults support the actions and policies of the current US president.
  - Operational definition:
- "On a scale from 1 to 5, where 1 is least supportive and 5 is more supportive, how much would you say you support the job that Donald Trump is doing as president?"

Measurement Error

- **Measurement error**: chance variation in our measurements.
  - individual measurement = exact value + chance error
  - chance errors tend to cancel out when we take averages.
- No matter how careful we are, a measurement could have always come out differently.
  - Panel study of 19,000 respondents: 20 reported being a citizen in 2010 and then a non-citizen in 2012.
  - Data entry errors.
- **Bias**: systematic errors for all units in the same direction.
  - individual measurement = exact value + bias + chance error.
  - "Did you vote?" $\sim$ overreporting

Definitions

- A **variable** is a series of measurements about some concept.
- **Descriptive statistics** are numerical summaries of those measurements.
  - If we smart enough, we wouldn't need them: just look at the list of numbers and completely understand.
- Two salient features of a variable that we want to know:
  - **Central tendency**: where is the middle/typical/average value.
  - **Spread** around the center: are all the data close to the center or spread out?

Center of the Data

- "Center" of the data: Typical/average value
- **Mean**: sum of the values divided by the number od observations
- **Median**: the "middle" of a sorted list of numbers.
- Median more robust to outliers
  - Example 1: data $= \{0, 1, 2, 3, 5\}$, mean $= 2.2$, median $= 2$
  - Example 2: data $= \{0, 1, 2, 3, 100\}$, mean $= 21.2$, median $= 2$

Spread of the data

- Are the data close to the center?
- **Range**: [min(x), max(x]
- **Quantile** (quartile, quintile, percentile, etc):
  - 25th percentile = lower quartile (25% of the data below this value)
  - 50th percentile = median (50% of the data below this value)
  - 75th percentile = upper quartile (75% of the data below this value)
- **Interquartile range (IQR)**: a measure of variability
  - How spread out is the middle half of the data?
  - Is most of the data really close to the median or are the values spread out?
- One definition of outliers: over $1.5 \times$ IQR above the upper quartile or below lower quartile.

Standard deviation

- **Standard deviation**: On average, how far away are data points from the mean?

$$\sqrt{\frac{\sum((x-\bar{x})^2)}{n-1}}$$

- Steps:

1. Subtract each data point by the mean
2. Square each resulting difference
3. Take the sum of these values
4. Divide by n-1
5. Take the square root

Variance

- **Variance** = standard deviation (squared)

$$\frac{\sum((x - \bar{x})^2)}{n - 1}$$

How large is large?

- Need a way to put any variable on common units.
- **z-score**:

$$\frac{x - \bar{x}}{\sigma}$$

- Interpretation:
  - Positive values above the mean, negative values below the mean
  - Units now on the scale of standard deviations away from the mean
  - Intuition: data more than 3 SDs away from mean are rare.

Shape

- *Skewness*
  - Positive/right skew
  - Symmetric
  - Negative/left skew
- *Kurtosis*: peakedness of a distribution

Skewness

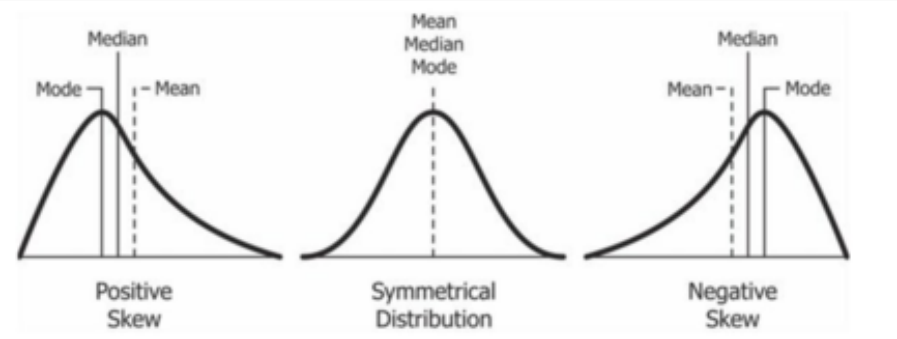


Figure 1: Skewness

Kurtosis

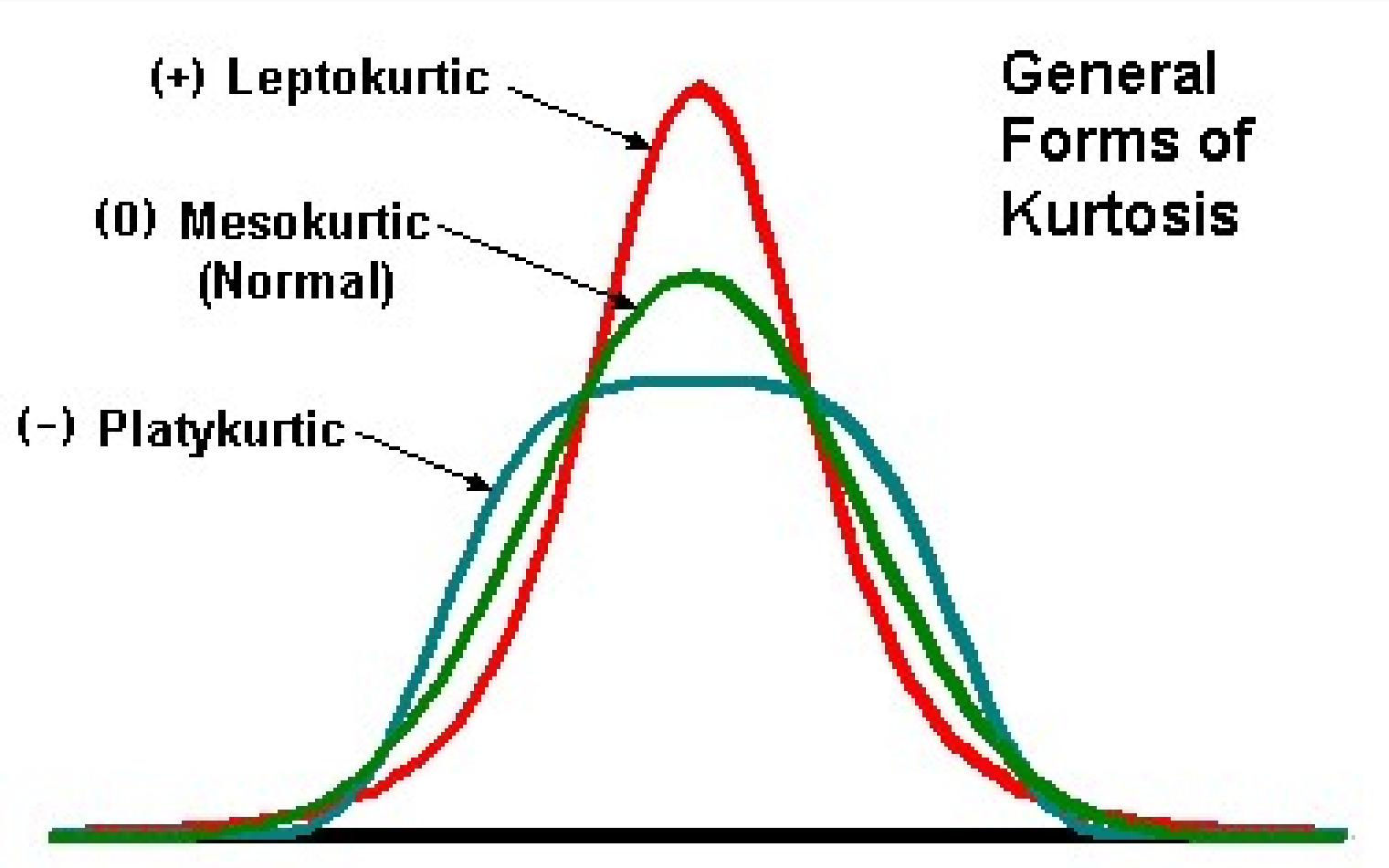


Figure 2: Kurtosis

Correlation

- How do variables move together on average?
- If I know one variable is big, does that tell me anything about how big the other variable is?
  - Positive correlation: when X is big, Y is also big
  - Negative correlation: when X is big, Y is small
  - High correlation: data cluster tightly around a line.

Relationship

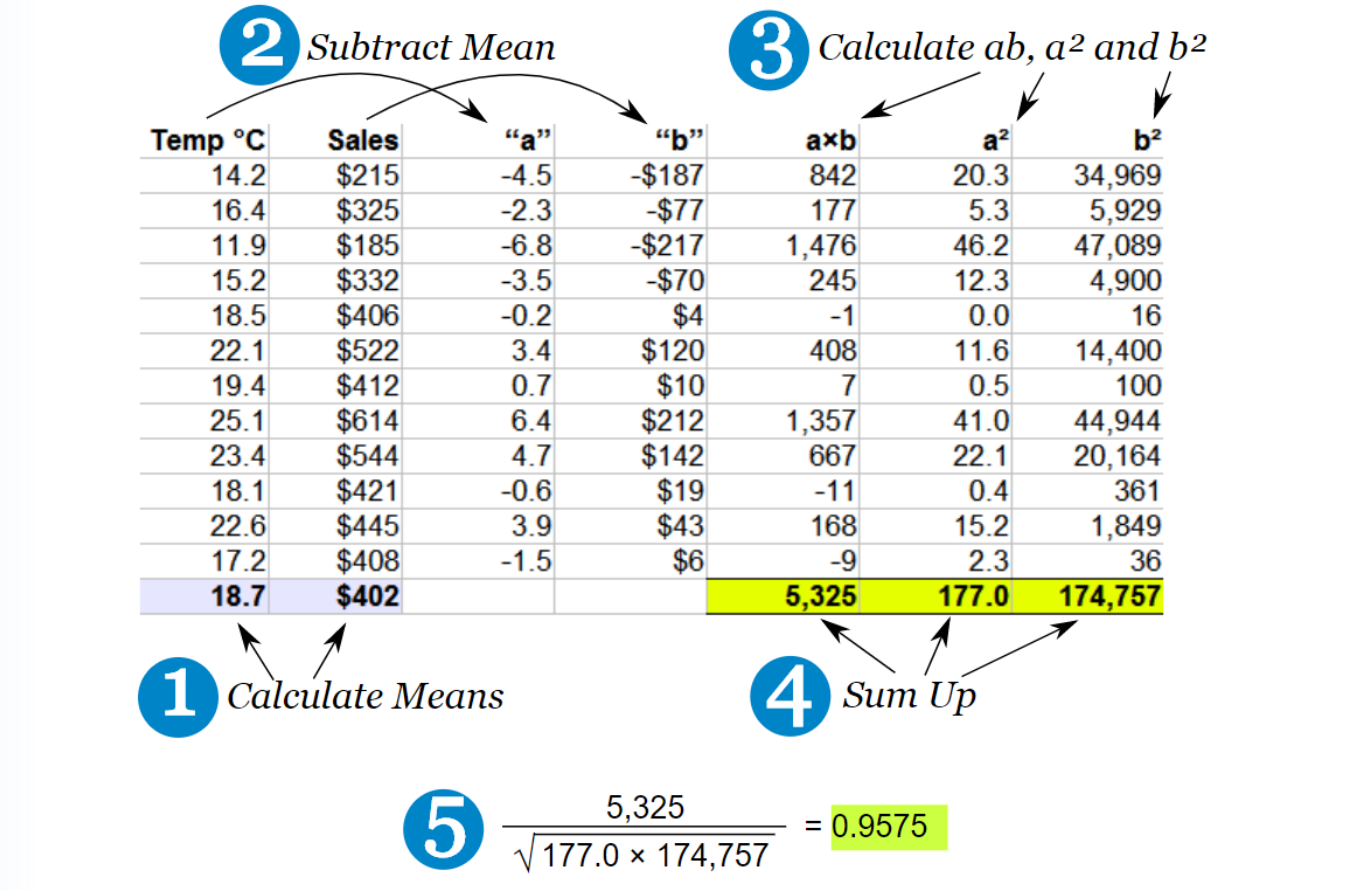
- **Covariance**: provides a measure of the strength of the correlation between two or more sets of random variates.

$$cov_{x,y} = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{N-1}$$

- **Correlation** is the degree to which two or more quantities are linearly associated

$$r = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \overline{x})^2(y_i - \overline{y})^2}}$$

Correlation



Temp °C	Sales	"a"	"b"	a×b	a²	b²
14.2	$215	-4.5	-$187	842	20.3	34,969
16.4	$325	-2.3	-$77	177	5.3	5,929
11.9	$185	-6.8	-$217	1,476	46.2	47,089
15.2	$332	-3.5	-$70	245	12.3	4,900
18.5	$406	-0.2	$4	-1	0.0	16
22.1	$522	3.4	$120	408	11.6	14,400
19.4	$412	0.7	$10	7	0.5	100
25.1	$614	6.4	$212	1,357	41.0	44,944
23.4	$544	4.7	$142	667	22.1	20,164
18.1	$421	-0.6	$19	-11	0.4	361
22.6	$445	3.9	$43	168	15.2	1,849
17.2	$408	-1.5	$6	-9	2.3	36
18.7	**$402**			**5,325**	**177.0**	**174,757**

Figure 3: Correlation by hand

Correlation

- Correlation is **Positive** when the values **increase** together
- Correlation is **Negative** when one value **decreases** as the other increases
- Correlation can have a value:
  - 1 is a perfect **positive** correlation
  - 0 is no correlation (the values don't seem linked at all)
  - -1 is a perfect **negative** correlation

Correlation

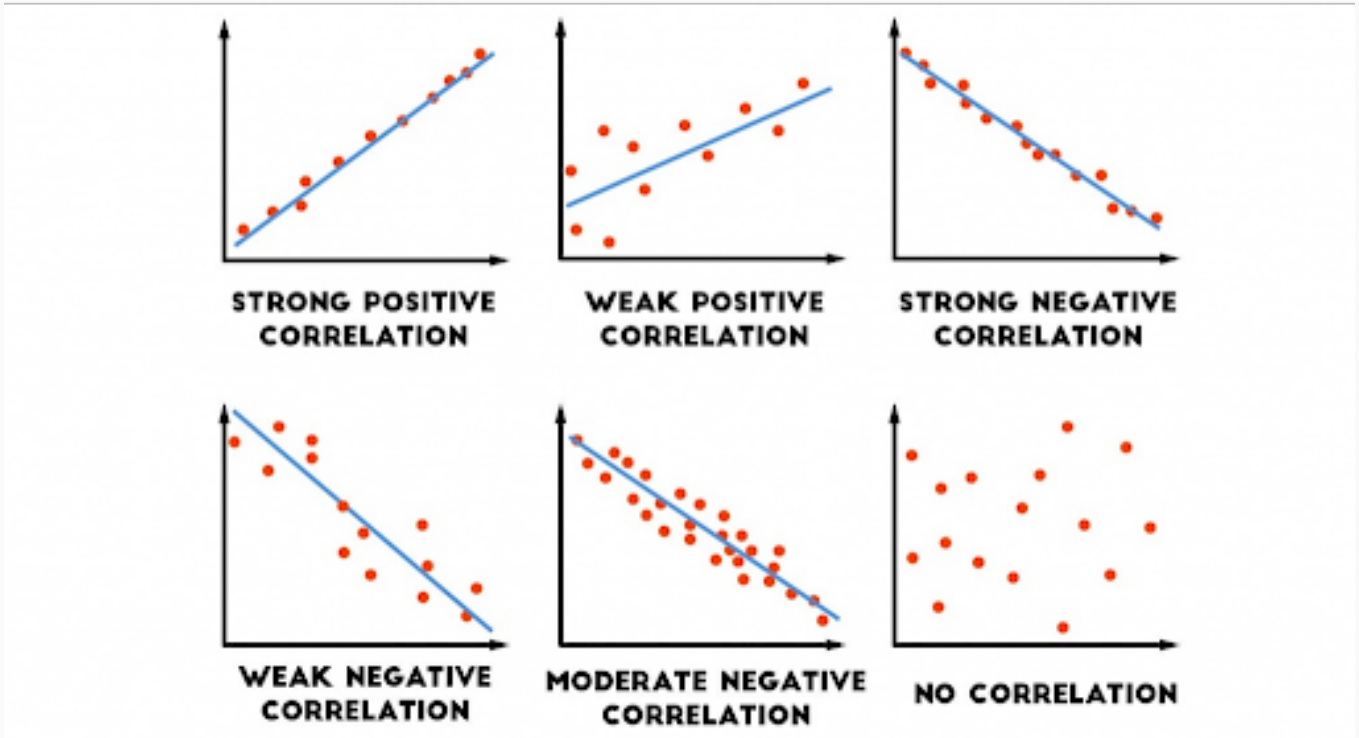


Figure 4: Correlation

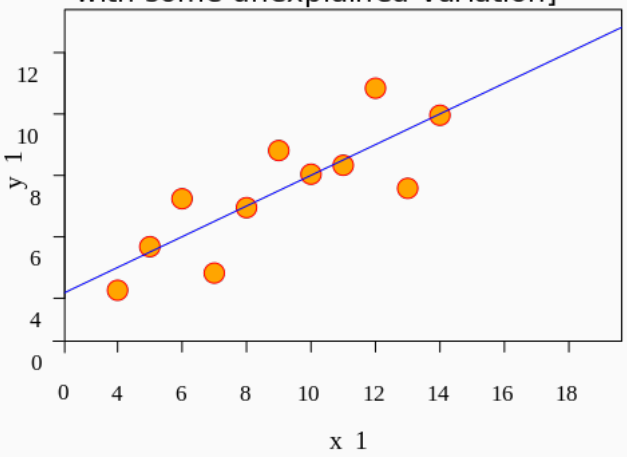
Properties of correlation coefficient

- Correlation measures **linear** association.
- Interpretation:
  - Correlation is between -1 and 1
  - Correlation of 0 means no linear association.
  - Positive correlations $\sim$ positive associations.
  - Negative correlations $\sim$ negative associations.
  - Closer to -1 or 1 means stronger association.
- Order doesn't matter: cor(x,y) = cor(y,x)
- Not affected by changes of scale:
  - Celsius vs. Fahreneheit; dollars vs. pesos; cm vs. in.

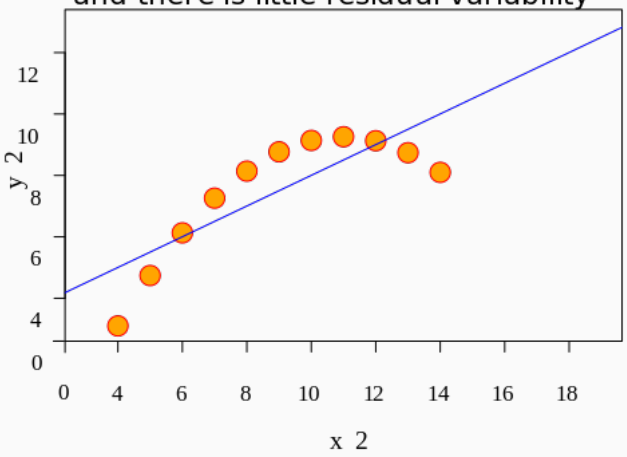
Correlation is not good at curves

Anscombe's Quartet

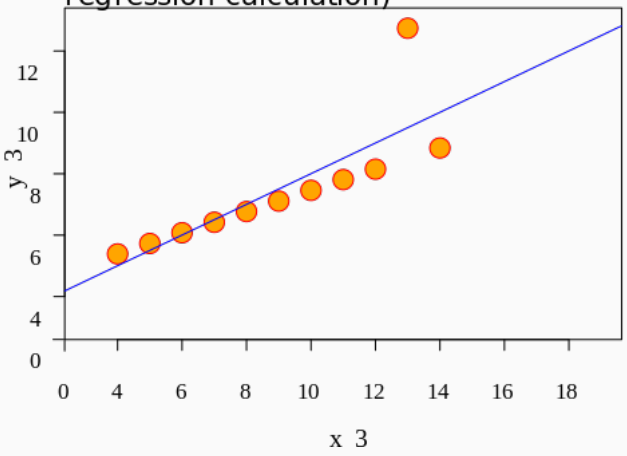
"what most people would see in their mind's eye [for a linear relationship with some unexplained variation]"



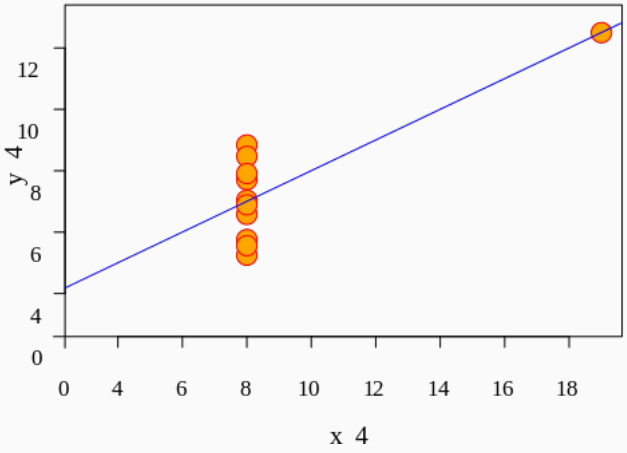
"y has a smooth curved relation with x, possibly quadratic, and there is little residual variability"



"all but one of the observations lie close to a straight line (not the one yielded by the standard regression calculation)"



"all the information about the slope of the regression line resides in one observation"



Correlation is not Causation

- What it really means is that a correlation does not prove one thing causes the other:
  - One thing might cause the other
  - The other might cause the first to happen
  - They may be linked by a different thing
  - Or it could be random chance!

Correlation is not causation

- Any correlation is potentially causal
  - X might cause Y
  - Y might cause X
  - X and Y might be caused by Z
  - X and Y might cause Z
  - There may be no causal relationship

Spurious Correlations

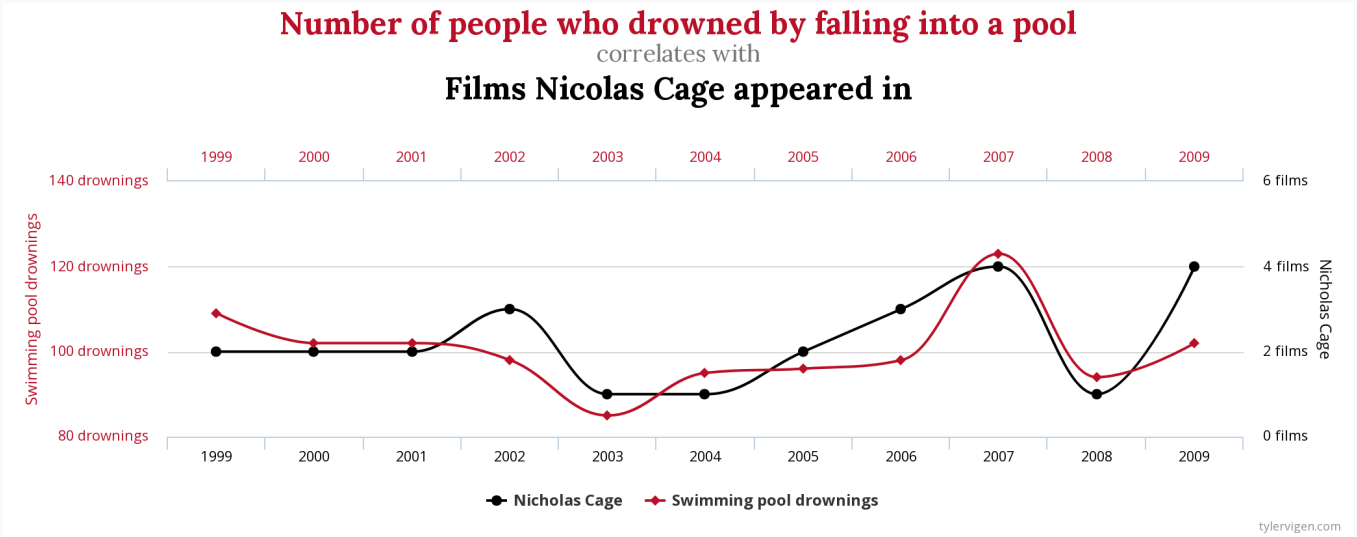


Figure 6: Correlation is NOT Causation

Naive Causal Inference

- Correlations are not necessarily causal
- Our mind thinks they are because humans are not very good at the kind of causal inference problems that social scientists care about
- Instead, we're good at understanding **physical causality**

Physical Causality

- Action and reaction
- Example:
  - Picture a ball resting on top of a hill
  - What happens if I push the ball?
- Features:
  - Observable
  - Single-case
  - Deterministic
  - Monocausal

Pre-Post Change Heuristic

- Our intuition about causation relies too heavily on simple comparisons of pre-post change in outcomes before and after something happens
- Why can this be wrong?

Flaws in causal inference from pre-post comparisons

- Maturation or trends
- Regression to the mean
- Selection
- Simultaneous historical changes
- Instrumentation changes
- Monitoring changes behaviour

Maturation or trends

- Is a shift in an outcome before and after a policy change the impact of the policy or a small part of a longer time trend?
- Example:

Regression to the mean

- Is a shift in an outcome before and after a policy change the impact of the policy or a function of statistical variation?
- Example:

Selection

- Is a shift in an outcome before and after a policy the impact of the policy or the result of the policy being implemented when outcomes are extreme?

Simultaneous changes

- Is the shift in an outcome before and after a policy the impact of the policy or the result of a simultaneous historical shift?

Instrumentation changes

- Is the shift in an outcome before and after a policy the impact of the policy or a change in how the outcome is measured?

Monitoring changes behaviour

- Is the shift in an outcome before and after a policy the impact of the policy or a change in response to measuring the outcome per se?

Examples

- Age and conservatism
- GDP and democracy
- Personality traits and political ideologies
- Healthcare spending and happiness

In R...

- `mean()`
- `median()`, `min()`, `max()`, `quantile()`
- `var()`
- `sd()`
- `cov()`
- `cor()`

Studying Feelings Toward Democracy

- 2018 Pew Study
- 27 countries, $\sim$ 30,000 respondents
- **Question**: How satisfied are you with the way democracy is working in our country – very satisfied, somewhat satisfied, not too satisfied, or not at all satisfied?
  1. very satisfied
  2. somewhat satisfied
  3. not too satisfied
  4. not at all satisfied
  5. don't know
  6. refused

Data

- Load the data:

```
library(haven) # package to read the data
library(dplyr) # package for data manipulation
library(tidyverse) # package for 'tidy' data
pew2018 <- read_sav("Pew 2018.sav",
                    user_na=TRUE) %>%
                    as_factor()
pew2018 = pew2018 %>% select(COUNTRY, satisfied_democracy, age, sex, d_ptyid_us)
pew2018$partyid2 <- fct_collapse(pew2018$d_ptyid_us,
                         DK = c("No preference (DO NOT READ)",
                                "Other party (DO NOT READ)",
                                "Don't know (DO NOT READ)",
                                "Refused (DO NOT READ)"),
                         Rep = "Republican",
                         Ind = "Independent",
                         Dem = "Democrat")
```

Glimpsing at data

- `dim()`: Retrieve the dimension
- `names()`: Get the names
- `str()`: Display compactly the internal structure
- `glimpse()`: is the dplyr-version of str() showing values of each variable the whole sceen width, but does not display the number of levels and names of factor variables. But this feature of str() cannot be displayed completly with either many or long levels names.
- `View()`: With RStudio you can see and inspect the data set comfortably. The View() function invokes a spreadsheet-style data viewer.

Glimpsing at data

```
dim(pew2018) # Dimensions
```

```
## [1] 30109     6
```

```
names(pew2018) # Column names
```

```
## [1] "COUNTRY"             "satisfied_democracy" "age"
## [4] "sex"                 "d_ptyid_us"          "partyid2"
```

```
glimpse(pew2018) # Structure of data
```

```
## Rows: 30,109
## Columns: 6
## $ COUNTRY             <fct> United States, United States, United States, Un...
## $ satisfied_democracy <fct> Not at all satisfied, Not too satisfied, Not to...
## $ age                 <fct> 60, 69, 71, 82, 46, 57, 64, 81, 84, Refused (DO...
## $ sex                 <fct> Male, Male, Male, Female, Male, Female, Female,...
## $ d_ptyid_us          <fct> Democrat, Independent, Democrat, Republican, De...
## $ partyid2            <fct> Dem, Ind, Dem, Rep, Dem, Dem, Dem, Dem, DK, DK,...
```

Contingency table

- The _table()_ function shows us how many respondents are in each category of a categorical variable:

```
table(pew2018$satisfied_democracy)
```

```
##
##           Very satisfied       Somewhat satisfied        Not too satisfied
##                     3403                    10402                     8801
##     Not at all satisfied Don't know (DO NOT READ)    Refused (DO NOT READ)
##                     6785                      662                       56
```

- We can use prop.table() to show what proportions of the data each response represents:

```
prop.table(table(pew2018$satisfied_democracy))
```

```
##
##           Very satisfied       Somewhat satisfied        Not too satisfied
##              0.113022684              0.345478096              0.292304627
##     Not at all satisfied Don't know (DO NOT READ)    Refused (DO NOT READ)
##              0.225347903              0.021986781              0.001859909
```

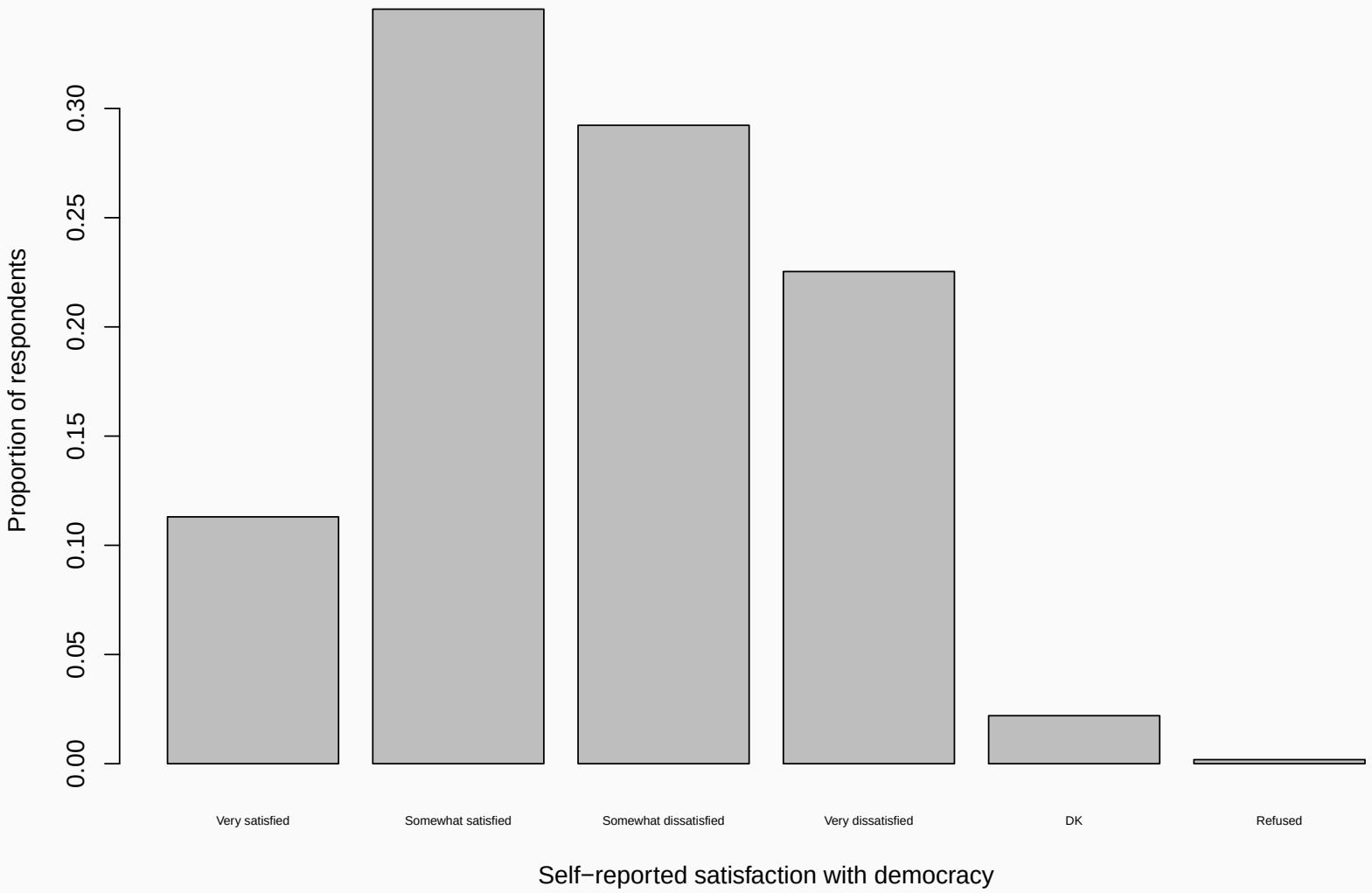
Barplot

- The `barplot()` function can help us visualize a contingency table:

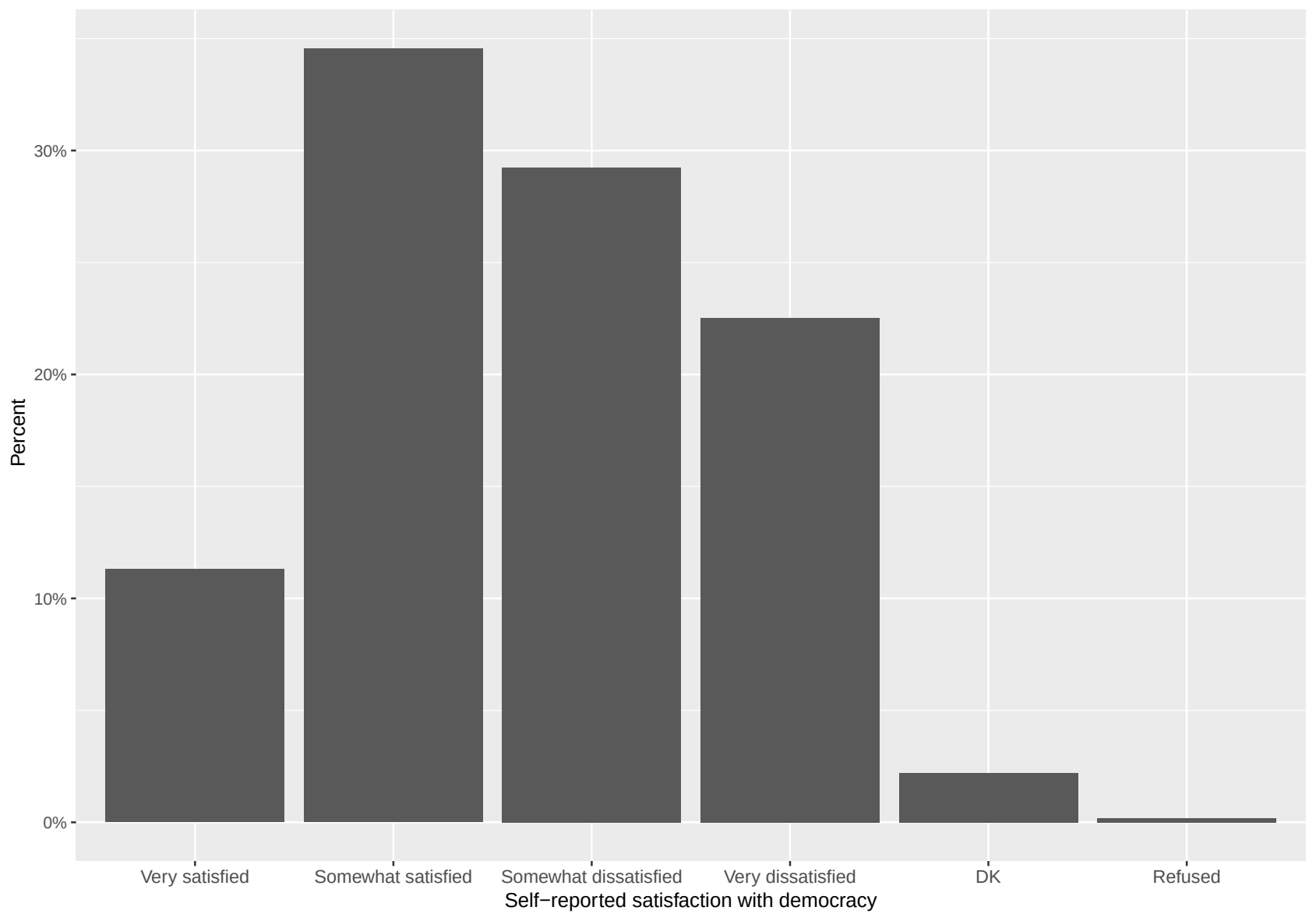
```
barplot(prop.table(table(pew2018$satisfied_democracy)),
xlab = "Self-reported satisfaction with democracy",
ylab = "Proportion of respondents")
```

- Arguments:
  - First is the height each bar should take (we're using proportions in this case)
  - names are the labels for the each category
  - xlab, ylab are axis labels

Barplot


Proportion of respondents
0.00
0.05
0.10
0.15
0.20
0.25
0.30
Very satisfied
Somewhat satisfied
Somewhat dissatisfied
Very dissatisfied
DK
Refused
Self−reported satisfaction with democracy

Bar plot (Using ggplot)



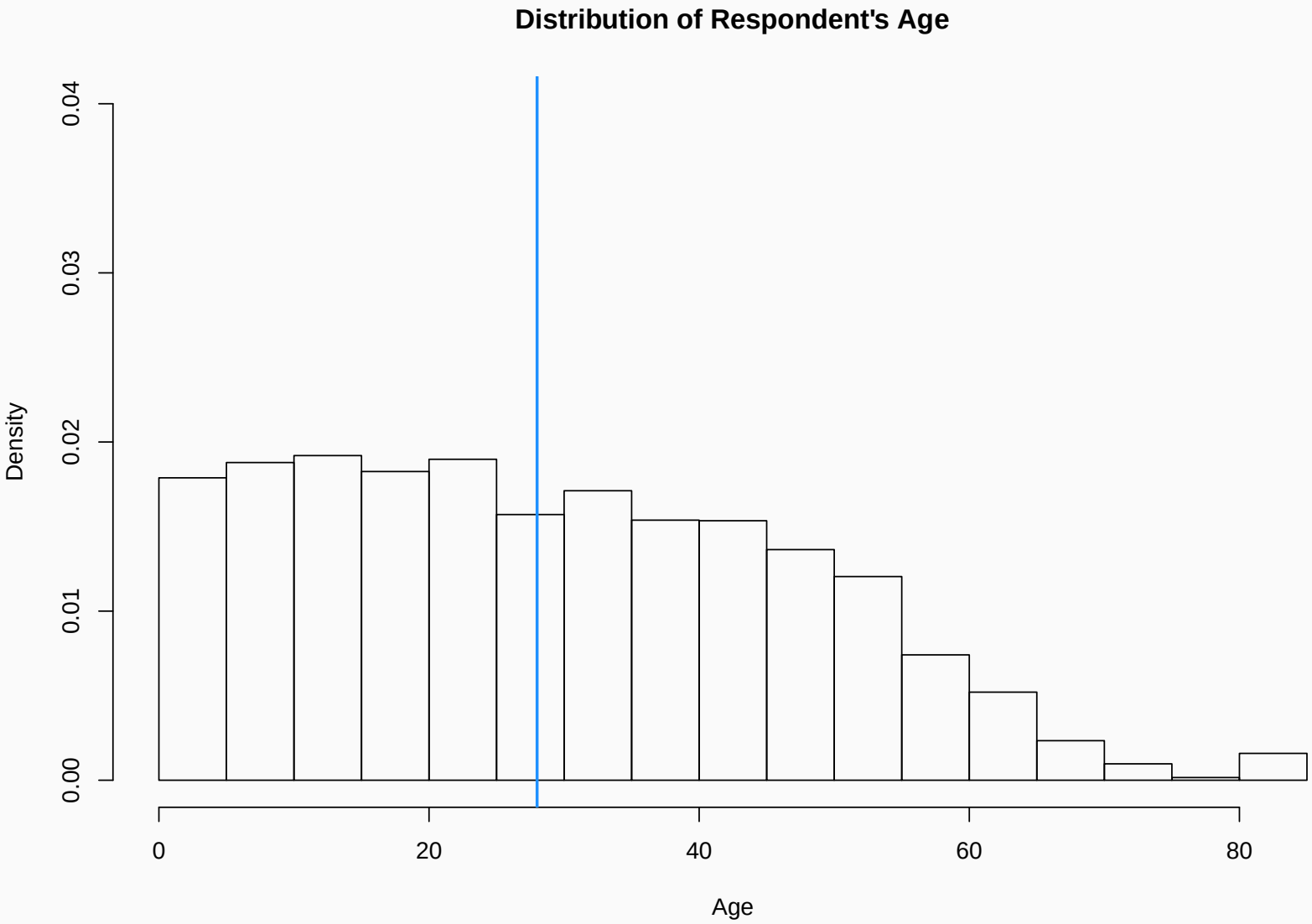
Histogram

- Visualize density of continuous/numeric variable.
- How to create a histogram by hand:
  1. create bins along the variable of interest
  2. count number of observations in each bin
  3. **density** = bin height
- In R, we use `hist()` with `freq = FALSE`:

```
hist(as.numeric(pew2018$age), freq = FALSE, ylim = c(0, 0.04),
xlab = "Age", main = "Distribution of Respondent's Age")
```

- Other arguments:
  - `ylim` sets the range of the y-axis to show (if you don't set it, uses the range of the data).
  - `main` sets the title for the figure.

Histogram


Distribution of Respondent's Age
Density
0.00
0.01
0.02
0.03
0.04
0
20
40
60
80
Age

What is Density

- The areas of the blocks = proportion of observations in those blocks.
- area of the blocks sum to 1 (100%)
- Can lead to confusion: height of block can go above 1!

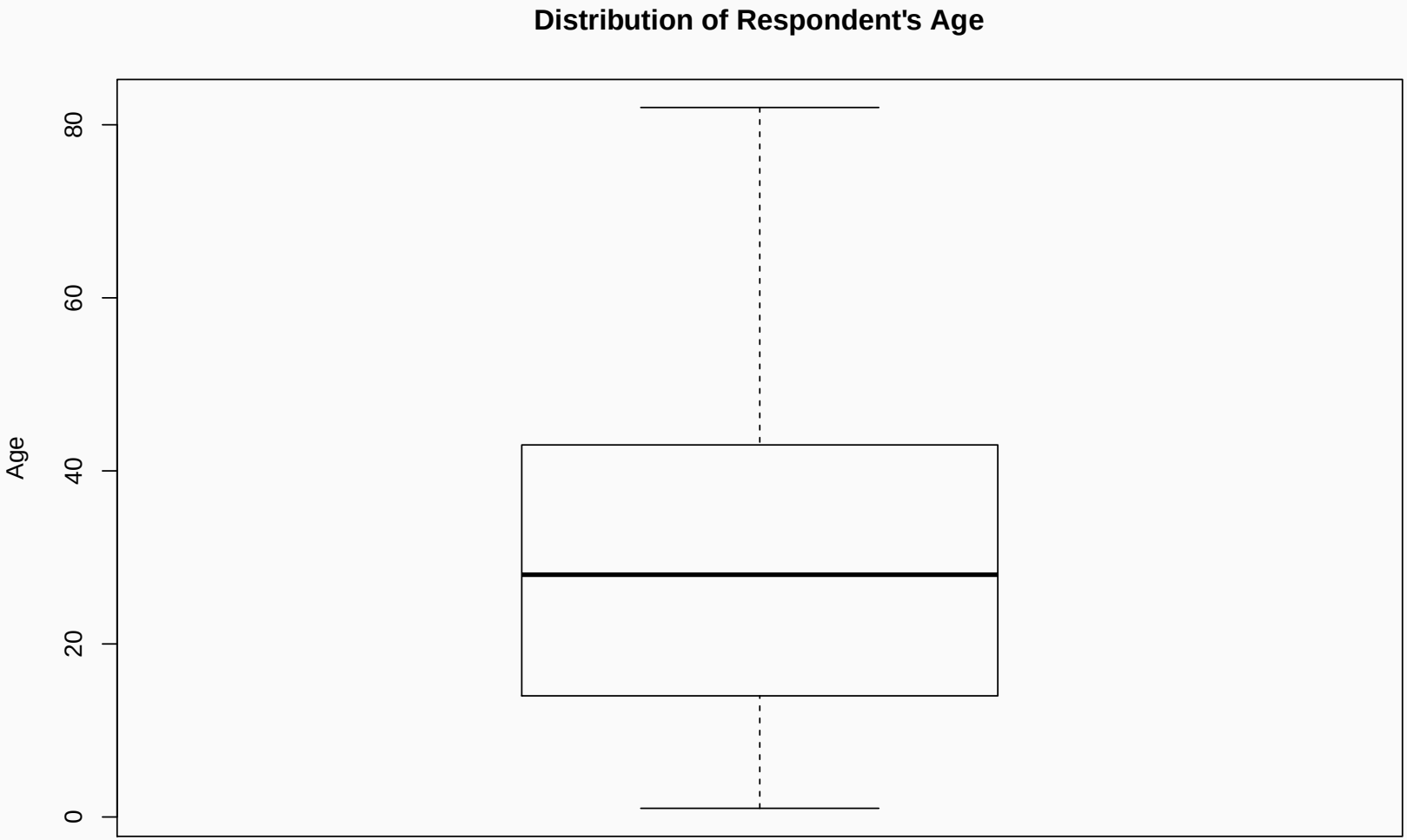
Boxplot

- A boxplot can characterize the distribution of continuous variables
- Use `boxplot()`:

```
boxplot(as.numeric(pew2018$age),
ylab = "Age",
main = "Distribution of Respondent's Age")
```

- "Box" represents range between lower and upper quartile.
- "Whiskers" represents either:
  - 1.5 $\times$ IQR or max/min of the data, whichever is tinyer.
  - Points beyond whiskers are outliers

Boxplot



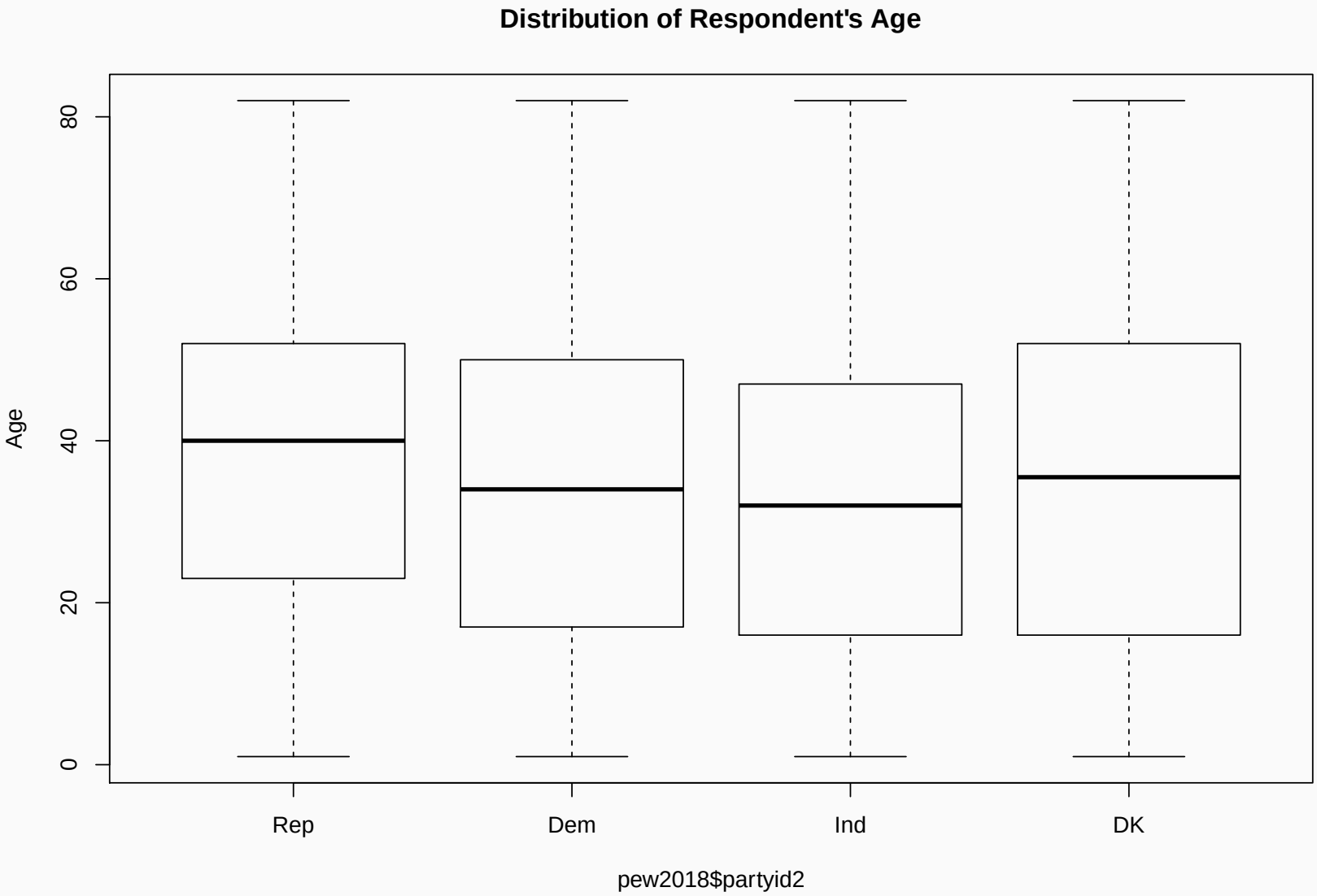
Comparing Distributions with the boxplot

- Useful for comparing a variable across groups:

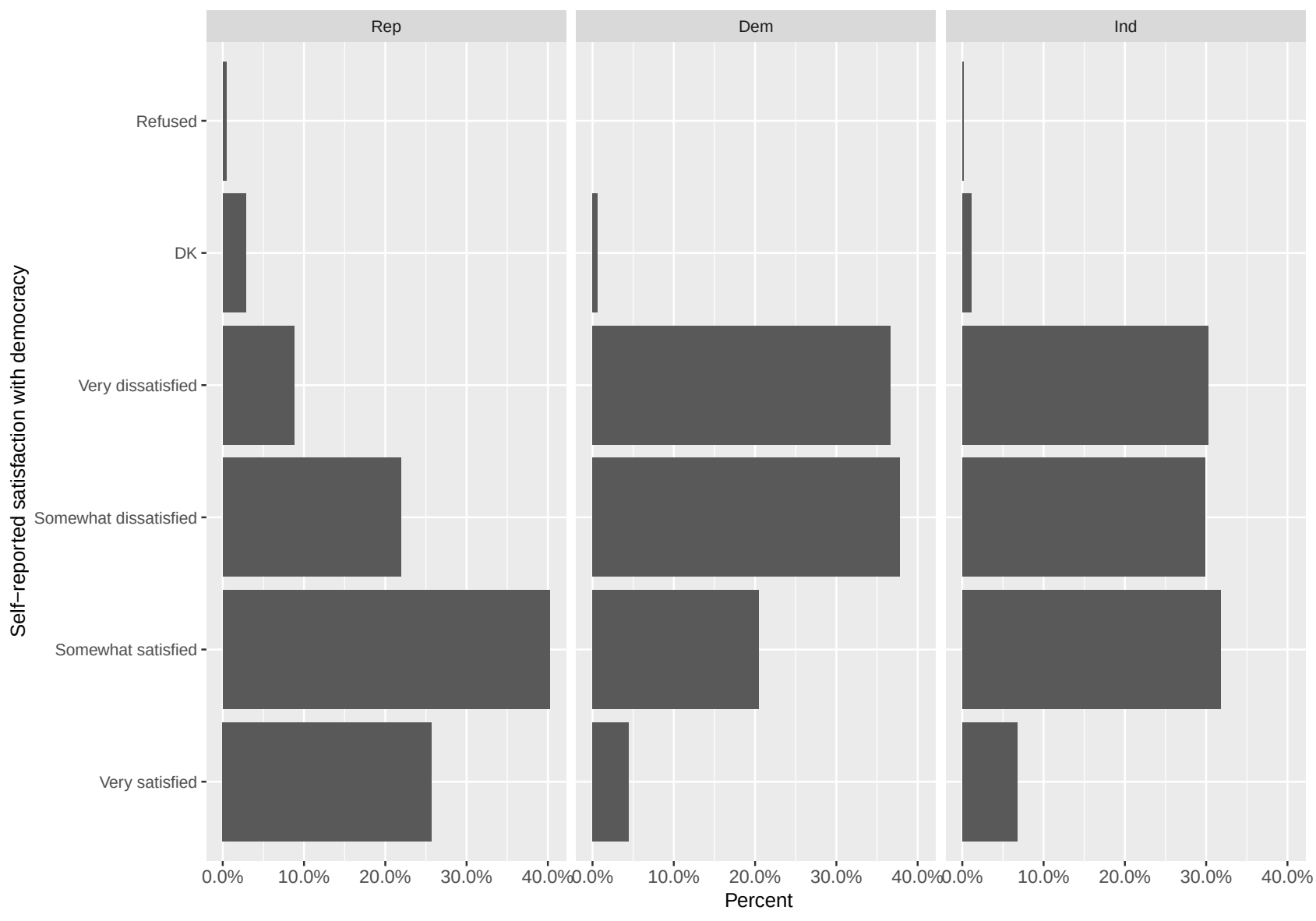
```
boxplot(as.numeric(pew2018$age) ~ pew2018$sex,
ylab = "Age",
main = "Distribution of Respondent's Age")
```

- First argument is called a formula, `y ~ x`:
  - `y` is the continuous variable whose distribution we want to explore.
  - `x` is the grouping variable.
  - When using a formula, we need to add a `data` argument

Comparing Distributions with the boxplot



Satisfaction with democracy by party



Satisfaction with democracy by party

- Why are Republicans more satified with democracy?

Correlations in R

- Use the `cor()` function
- Missing values: set the use = "pairwise" ~ available case analysis

```
# Read Happiness Data
happ2019 = read.csv("C:/Users/afisher/Documents/R Code/Resources/Data/Happiness/2019.csv")

# Structure of dataset
str(happ2019)
```

```
## 'data.frame':    156 obs. of  9 variables:
##  $ Overall.rank                : int  1 2 3 4 5 6 7 8 9 10 ...
##  $ Country.or.region           : Factor w/ 156 levels "Afghanistan",..: 44 37 106 58 99 134 133 100 24
##  $ Score                       : num  7.77 7.6 7.55 7.49 7.49 ...
##  $ GDP.per.capita              : num  1.34 1.38 1.49 1.38 1.4 ...
##  $ Social.support              : num  1.59 1.57 1.58 1.62 1.52 ...
##  $ Healthy.life.expectancy     : num  0.986 0.996 1.028 1.026 0.999 ...
##  $ Freedom.to.make.life.choices: num  0.596 0.592 0.603 0.591 0.557 0.572 0.574 0.585 0.584 0.532 ...
##  $ Generosity                  : num  0.153 0.252 0.271 0.354 0.322 0.263 0.267 0.33 0.285 0.244 ...
##  $ Perceptions.of.corruption   : num  0.393 0.41 0.341 0.118 0.298 0.343 0.373 0.38 0.308 0.226 ...
```

```
# Correlation
cor(happ2019$Score, happ2019$GDP.per.capita)
```

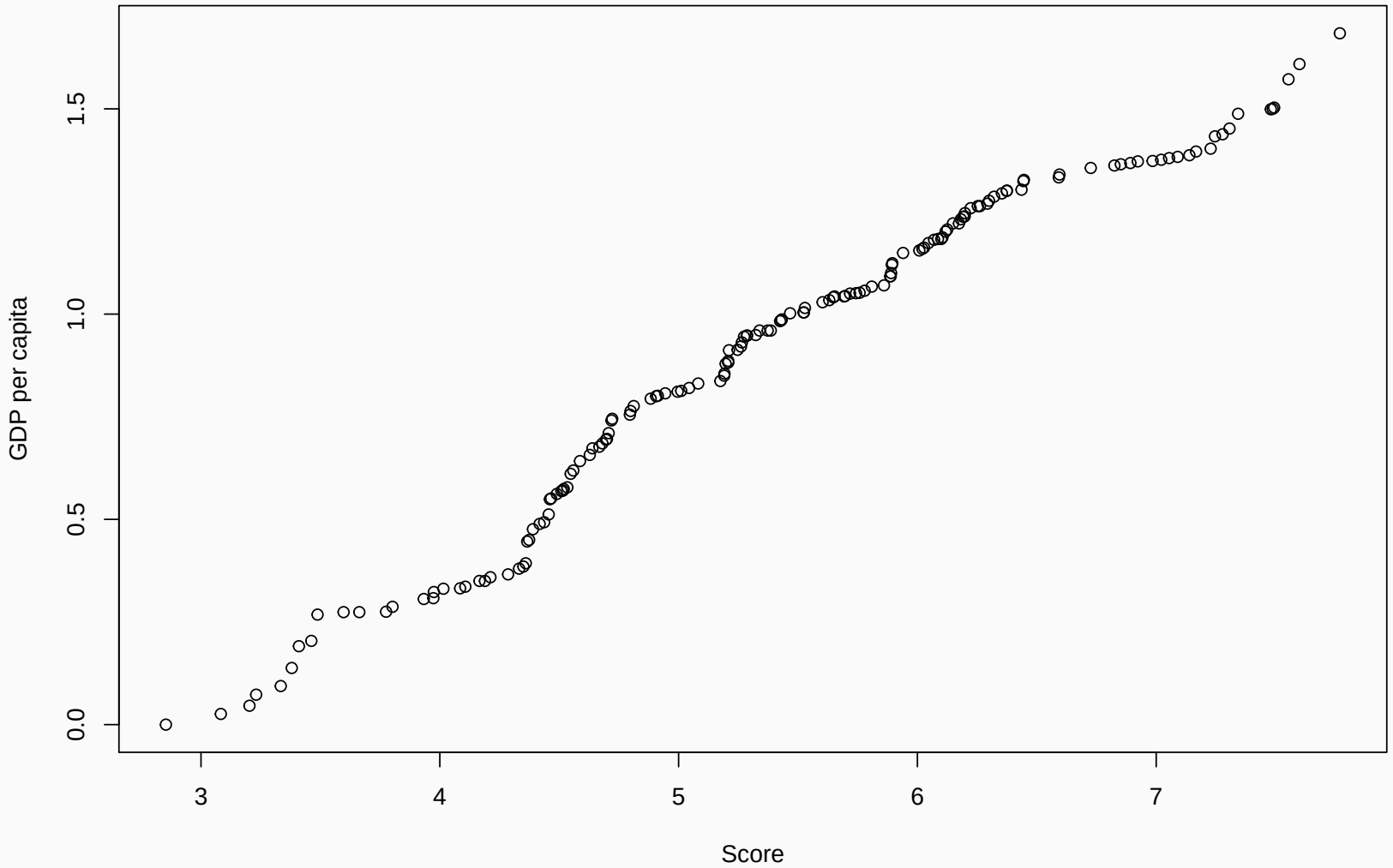
```
## [1] 0.7938829
```

QQ-plot example

- **Quantile-quantile plot (qq-plot)**: Plot the **quantiles** of each distribution against each other.
- Example points:
  - (min of X, min of Y)
  - (median of X, median of Y)
  - (25th percentile of X, 25th percentile of Y)
- 45 degree line indicates quality of the two distributions

QQ-plot example

```
qqplot(happ2019$Score, happ2019$GDP.per.capita, xlab = 'Score', ylab="GDP per capita")
```

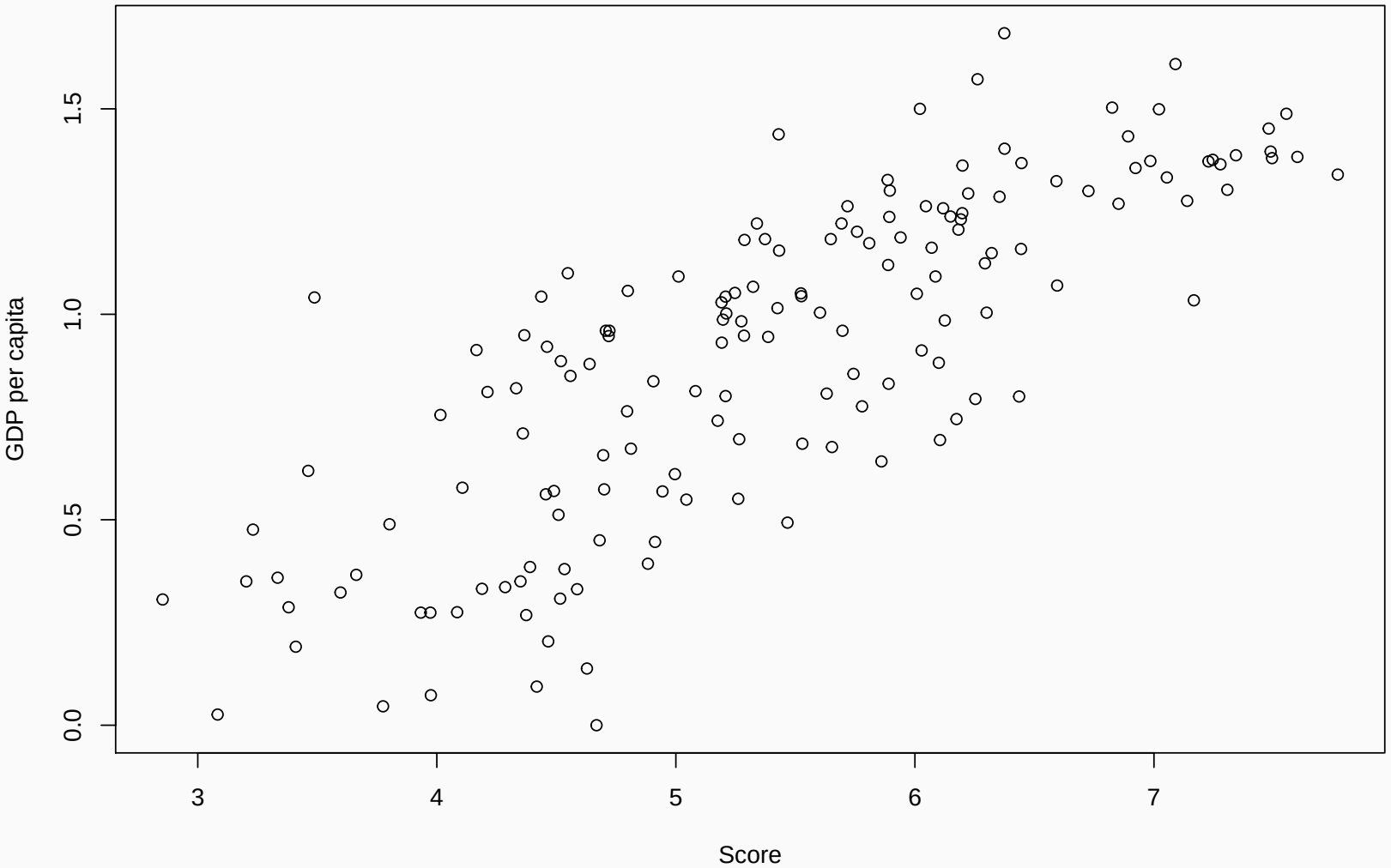


Scatterplot

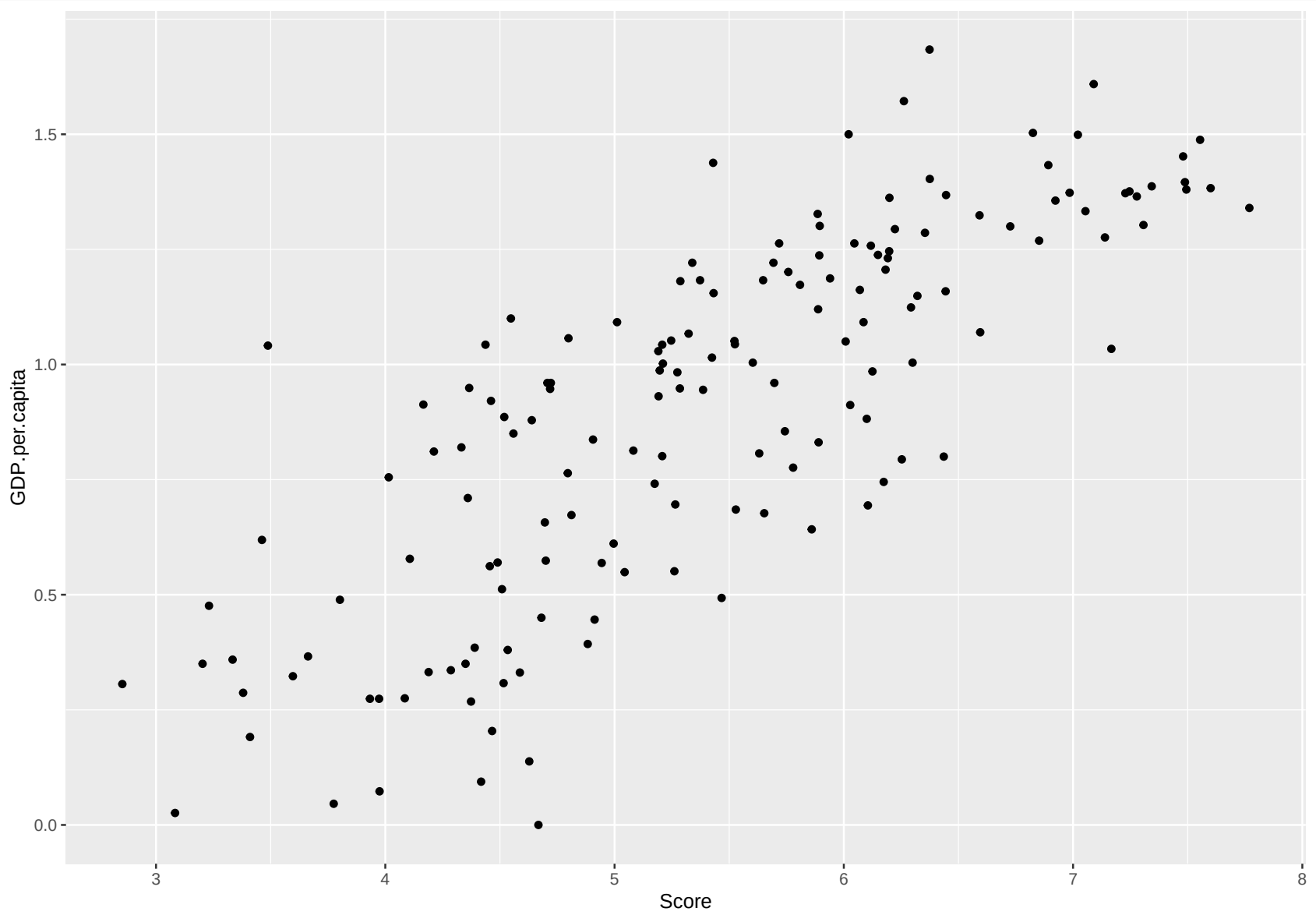
```r
## Base R
plot(happ2019$Score, happ2019$GDP.per.capita, xlab = 'Score', ylab="GDP

## ggplot
ggplot(happ2019) +
  aes(x=Score, y=GDP.per.capita)+
  geom_point()
```

Base R - Scatterplot



ggplot - Scatterplot



Confidence Intervals

- What if you'd like to know what percentage of people in the U.S. are night owls (people who stay up late at night).
- In order to obtain a completely right answer, you'd have to ask each person in the country this question, but polling over 300 million people isn't very practical.

Confidence Intervals

- Get a much smaller random sample of people and then find the percentage of night owls in that sample.
- Problem:
  - Not confident that this percentage is correct or how far off this number is from the right answer for the entire population.
- So we'll try to find an "interval" that provides the assertion:
  - "I am 95% confident that the percentage of people in the U.S. are night owls is between 12% and %16."
  - This declaration is based on what's called a "confidence interval," in this case 14 +/- 2 and the confidence is 95%.

Confidence Intervals in Polls

- When a pollster reports an estimate and a margin of error, in a way they're reporting a 95% confidence interval.
- This means confidence intervals are a way of quantifying the uncertainty of an estimate.
- Further, if we take many different random samples, compute confidence intervals for each of those samples, 95% of those confidence intervals will be such that the population average would lie between those limits.

Polling Example

- Candidate Gobermouch is leading in the polls over Candidate Fopdoodle, 48% to 43%, a difference of 5 percentage points. The poll's margin of error is 3%.
- Does Gobermouch have a lead over Fopdoodle that is outside the margin of error?

Confidence Intervals and Margin of Error

- A margin of error of $\pm$ 3% means that Gobermouch's support could be as high as 51% but as low as 45%.
- Similarly, Fopdoodle's support could be as high as 46% but as low as 40%.
- Those ranges, more appropriately called "confidence intervals," overlap.
- Gobermouch's support is not "outside the margin of error."

Confidence Interval

- A confidence interval (which is most often a "95% confidence interval") means that the "real answer" will fall within the calculated range 95% of the time
- In other words, if the pollsters repeated their survey 100 times, 95 of the ranges they calculate would contain the "real answer" and 5 would not. That's right.
- Even the best pollsters will get it wrong 5% of the time. (And this does not take into consideration systematic bias in sampling.)

Confidence Intervals and Statistical Significance

- In our example, Gobermouch is at 48% (with a confidence interval of 45% to 51%) and Fopdoodle is at 43% (with a confidence interval of 40% to 46%).
- However, "Is Gobermouch's lead over Fopdoodle *statistically significant*?"
- Different question and requires a different statistical approach.

Confidence Intervals and Statistical Significance

- If we subtracted Fopdoodle's support from Gobermouch's (or vice versa), would the result be zero?"
- If the result is zero (or if the confidence interval contains zero), then there is no statistically significant difference
- *You can't know this by simply determining if the confidence intervals for each candidate's support overlap*

Why Overlapping Confidence Intervals mean Nothing about Statistical Significance

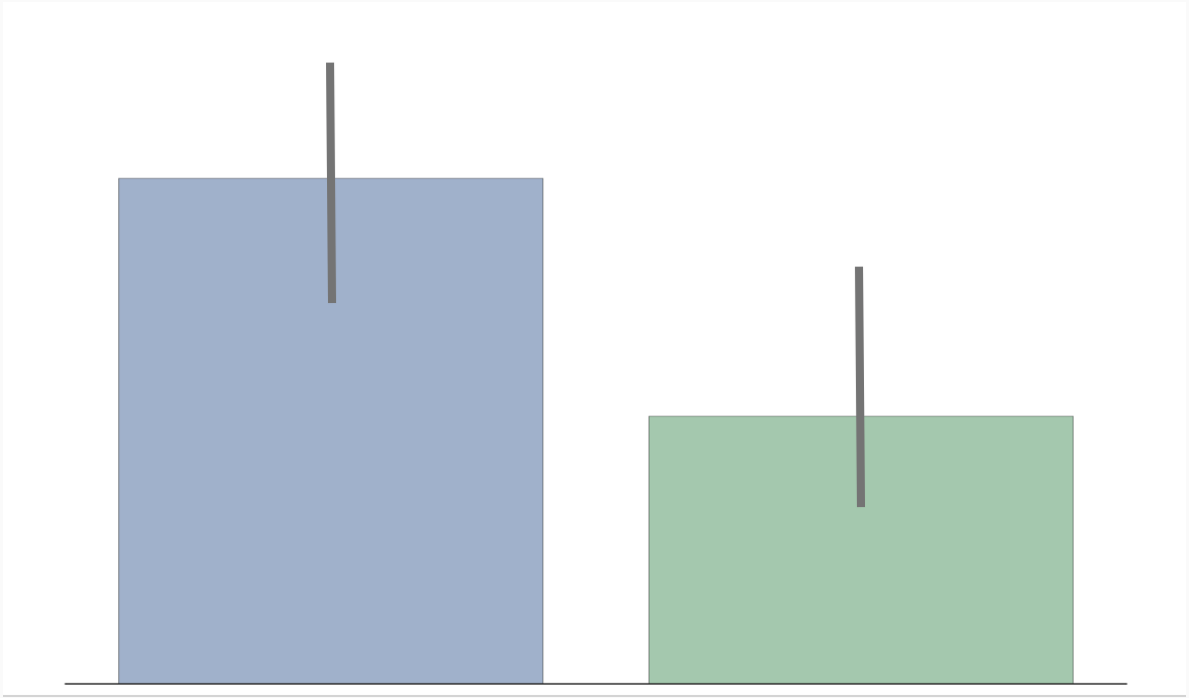


Figure 7: Correlation is NOT Causation

- "The confidence intervals of the two groups overlap, hence the difference is not statistically significant" — A lot of People

Overlapping confidence intervals/error bars say nothing about statistical significance.

- When 95% confidence intervals for the means of two independent populations don't overlap, there will indeed be a statistically significant difference between the means (at the 0.05 level of significance).
- However, the opposite is not necessarily true. CI's may overlap, yet there may be a statistically significant difference between the means.

An example

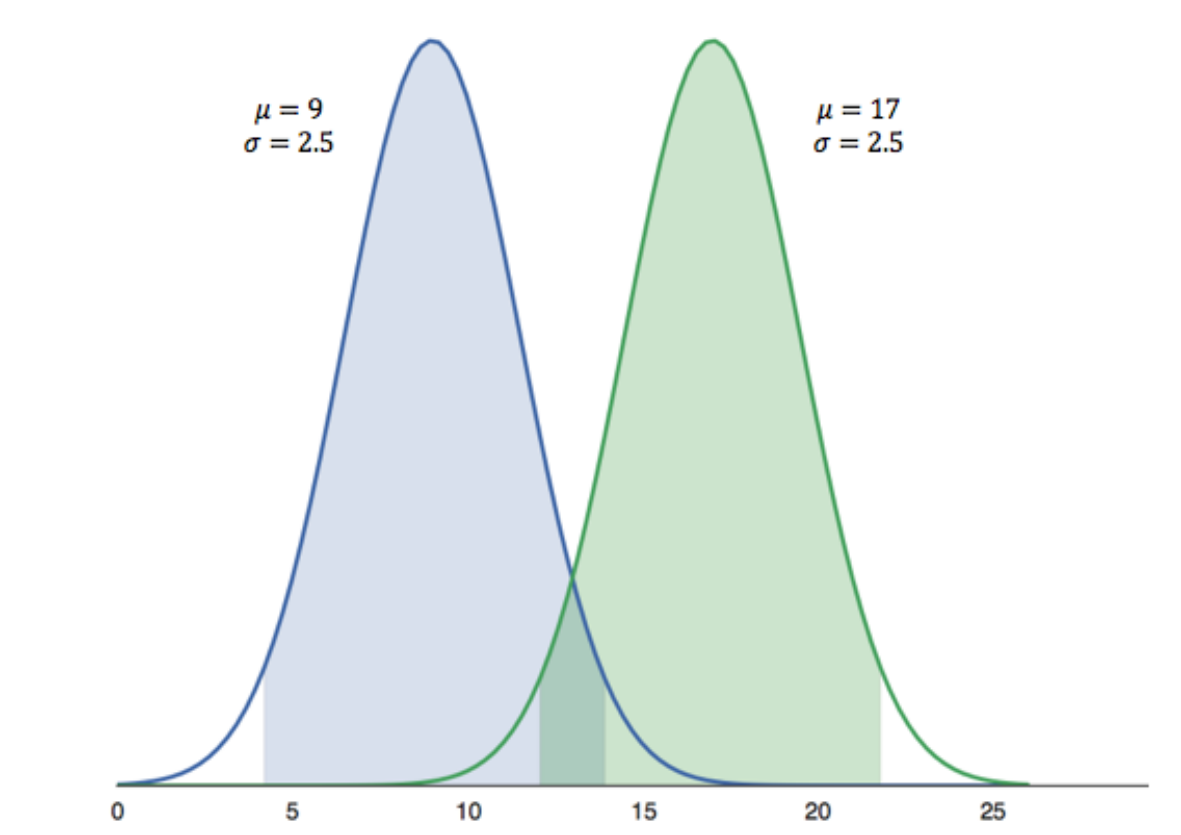


Figure 8: Correlation is NOT Causation

- Group Blue's average age is 9 years with an error of 2.5 years. Group Green's average age is 17, also with an error of 2.5 years.

- The shaded regions show the 95% confidence intervals (CI).

Difference between groups

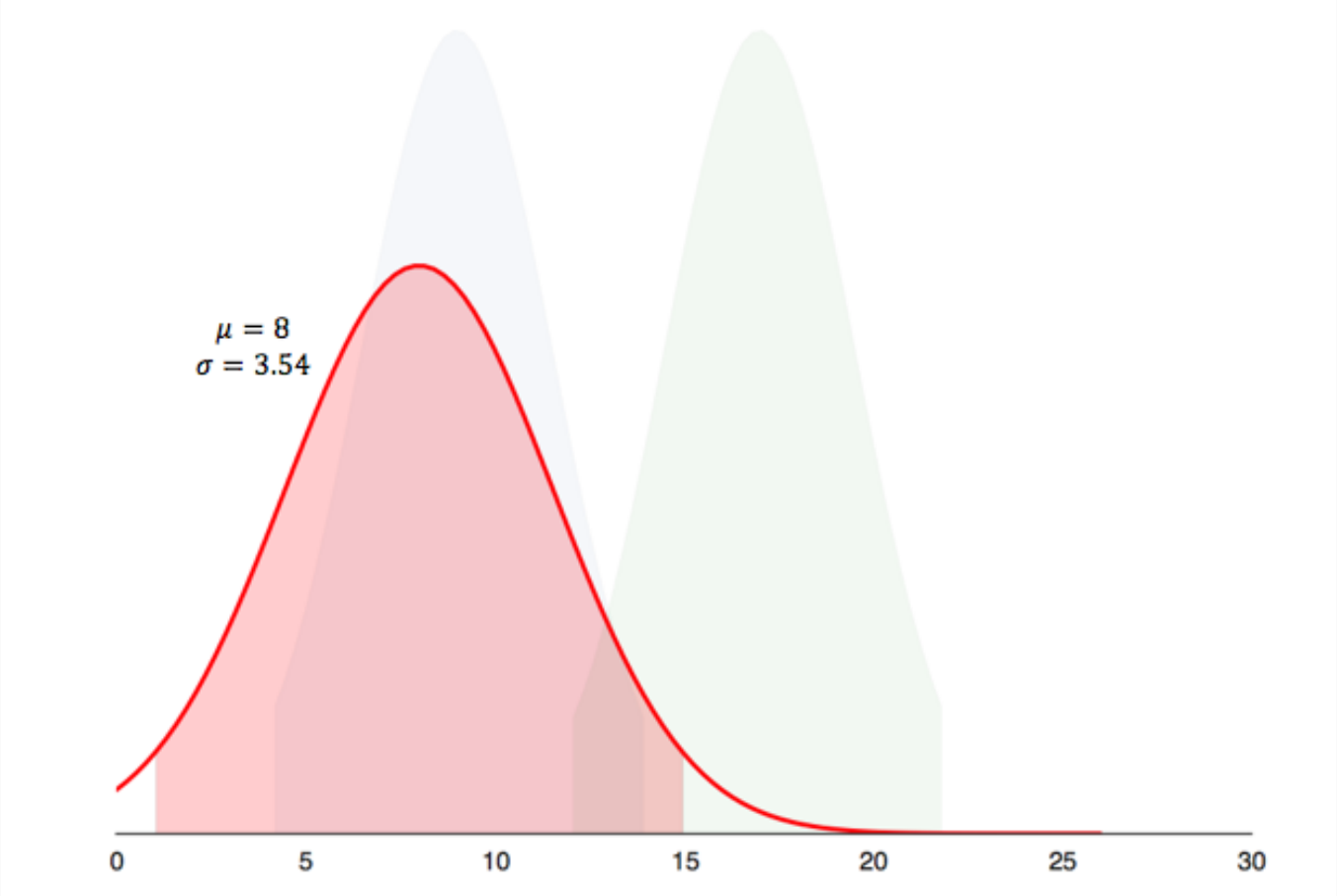


Figure 9: Correlation is NOT Causation

Difference between groups

- Instead of building a distribution for each group, we build one distribution for the difference in mean age between groups.
- If the 95% CI of the difference contains 0, then there is no difference in age between groups. If it doesn't contain 0, then there is a statistically significant difference between groups.
- As it turns out the difference is statistically significant, since the 95% CI (shaded region) doesn't contain 0.

See also

- https://towardsdatascience.com/why-overlapping-confidence-intervals-mean-nothing-about-statistical-significance-48360559900a
- https://www.acsh.org/news/2019/12/14/election-polls-should-report-confidence-intervals-not-just-margins-error-14452