Statistical Significance

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What Does Probability Distribution Mean?

- If we want to predict a variable accurately then the first task we need to perform is to understand the underlying behaviour of our target variable.
- Determine the possible outcomes of the target variable
- Start assigning probabilities to the events (values)

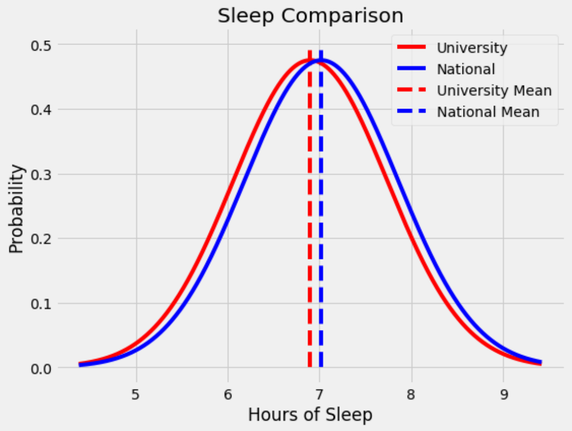
Probability Distribution

- We can throw a dice 10000 times and as there are 6 possible values that a dice can take, we can create 6 buckets.
- Start recording the number of occurrences for each value.
- We can plot the chart and it will form a curve. This curve is known as probability distribution curve and the likelihood of the target variable getting a value is the probability distribution of the variable.

You are getting very sleepy...

- Students at Haverford average 6.80 hours of sleep per night
- National college average of 7.02 hours
- You have to decide if this is a serious issue

Sleep Comparison



What it mean to prove something with data?

- Statistical Significance is built on a few simple ideas:
  - hypothesis testing,
  - the normal distribution
  - p values.

Hypothesis testing

- The "hypothesis" refers to initial belief about the situation before the study
  - **Alternative Hypothesis:** The average amount of sleep by students at our university is below the national average for college student.
  - **Null Hypothesis:** The average amount of sleep by students at our university is not below the national average for college students.
- This is an example of a one-sided hypothesis test because we are concerned with a change in only one direction

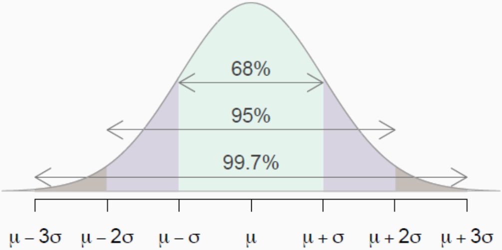
Normal Distribution

- The normal distribution is used to represent how data from a process is distributed defined by
  - the mean, given the Greek letter μ (mu)
  - the standard deviation, given the letter σ (sigma)

Normal Distribution - Background

- Firstly, the most important point to note is that the normal distribution is also known as the **Gaussian distribution**
- It is named after *Carl Friedrich Gauss*
- Normal distribution is simple and hence its simplicity makes it extremely popular.

Normal Distribution



Normal Distribution

- We can determine how anomalous a data point is based on how many standard deviations it is from the mean
  - 68% of data is within $\pm$ 1 standard deviations from the mean
  - 95% of data is within $\pm$ 2 standard deviations from the mean
  - 99.7% of data is within $\pm$ 3 standard deviations from the mean

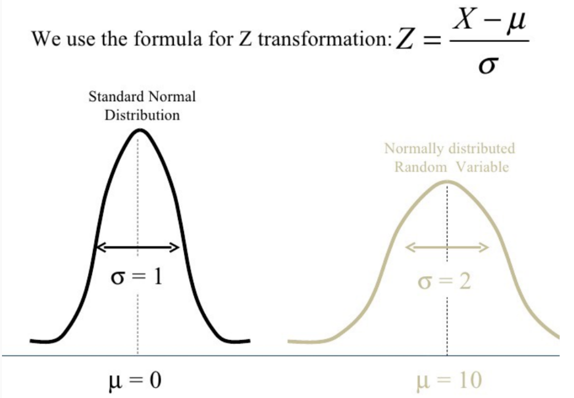
Example

- Average female height in the US is 65 inches with a standard deviation of 4 inches.
- If we meet a new acquaintance who is 73 inches tall, we can say she is two standard deviations above the mean and is in the tallest 2.5% of females
    - 2.5% of females will be shorter than $\mu - 2\sigma$ (57 in) and 2.5% will be taller than $\mu + 2\sigma$.
- Instead of saying our data is two standard deviations from the mean, we assess it in terms of a z-score

Z-Score

- Conversion to a z-score is done by subtracting the mean of the distribution from the data point and dividing by the standard deviation
- The higher or lower the z-score, the more unlikely the result is to happen by chance and the more likely the result is meaningful

Z-Score


We use the formula for Z transformation: $Z = \dfrac{X - \mu}{\sigma}$
Standard Normal Distribution
Normally distributed Random Variable
σ = 1
σ = 2
μ = 0
μ = 10

P-Values

- A p-value is the probability of observing results at least as extreme as those measured when the null hypothesis is true...
  - The p-value is NOT the probability the claim is true.
  - The p-value is NOT the probability the null hypothesis is true.

P-Values

- The p-value is actually the probability of getting a sample like ours, or more extreme than ours IF the null hypothesis is true.
- So, we assume the null hypothesis is true and then determine how "strange" our sample really is.
- If it is not that strange (a large p-value) then we don't change our mind about the null hypothesis.
- As the p-value gets smaller, we start wondering if the null really is true and well maybe we should change our minds (and reject the null hypothesis).

P-Values

- Whether or not the result can be called statistically significant depends on the p-value (known as alpha) we establish for significance before we begin the experiment
- If the observed p-value is less than alpha, then the results are statistically significant.
- We need to choose alpha before the experiment because if we waited until after, we could just select a number that proves our results are significant no matter what the data shows

THE 0.05

- Most commonly used value is 0.05, corresponding to a 5% chance the results occurred at random
- If you ran the experiment 100 times — again, assuming the null hypothesis is true — you'd see these same numbers (or more extreme results) five times.
  - R.A. Fischer, the father of modern statistics, choose a p-value of 0.05 for indeterminate reasons and it stuck)!

But Really Why 0.05?

- https://www.openintro.org/book/stat/why05/

Summary

- As a summary so far, we have covered three ideas:
- **Hypothesis Testing:** A technique used to test a theory
- **Normal Distribution:** An approximate representation of the data in a hypothesis test.
- **p-value:** The probability a result at least as extreme at that observed would have occurred if the null hypothesis is true

Back to Haverford Students' late nights

- Students across the country average 7.02 hours of sleep per night according to the National Sleep Foundation
- In a poll of 200 students at Haverford the average hours of sleep per night was 6.90 hours with a standard deviation of 0.84 hours.
- Our alternative hypothesis is the average sleep of students at Haverford is below the national average for college students.
- We will use an alpha value of 0.05 which means the results are significant f the p-value is below 0.05.

First we need a z-score

- subtracting the population mean (the national average) from our measured value and dividing by the standard deviation over the square root of the number of samples.

$$\frac{x - \bar{x}}{\sigma/\sqrt{n}}$$

- When you are estimating the standard error, SE, for the mean (the SE is the standard deviation of the means of samples), the larger your sample size, the smaller the standard deviation. for example, if you took a sample of 200, you would be much more likely to get close to the true mean than if you took a sample of 2. In other words, the larger your "n", the smaller the standard deviation.

First we need a z-score

$$\frac{6.90 - .7.02}{0.84/\sqrt{200}} = -2.03$$

- The z-score is called our test-statistic. Once we have a test-statistic, we can use a table or a programming language such as R to calculate the p-value.

```
# Calculate the results
z_score = (6.90 - 7.02) / (0.84 / sqrt(200))
p_value = pnorm(z_score)
# Print our results
sprintf('The p-value is %f for a z-score of %f.', p_value, z_score)
```

```
## [1] "The p-value is 0.021676 for a z-score of -2.020305."
```

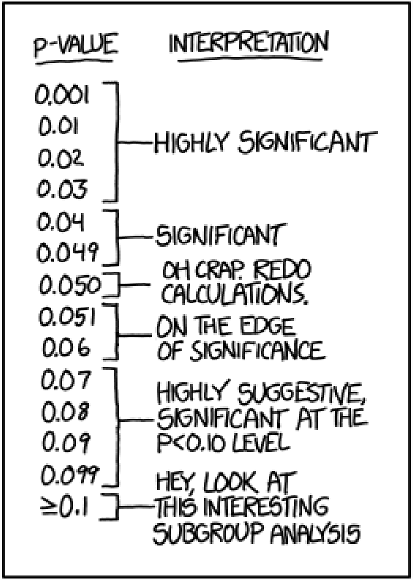
So what do we know?

- Based on the p-value of 0.02116, we can reject the null hypothesis. (Statisticians like us to say reject the null rather than accept the alternative.)
- There is statistically significant evidence our students get less sleep on average than college students in the US at a significance level of 0.05. The p-value shows there is a 2.12% chance that our results occurred because of random noise.
- Notice that our p-value, 0.02116, would not be significant if we had used a threshold of 0.01.

Some thoughts on p-values

- We should think about the p-value and the sample size in addition to the conclusion.
- might have statistical significance, but that does mean it is practically meaningful.
- This was an observational study, which means there is only evidence for correlation and not causation.

P-hacking


P-VALUE
INTERPRETATION
0.001
0.01
0.02
0.03
HIGHLY SIGNIFICANT
0.04
0.049
SIGNIFICANT
0.050
OH CRAP. REDO CALCULATIONS.
0.051
0.06
ON THE EDGE OF SIGNIFICANCE
0.07
0.08
0.09
0.099
HIGHLY SUGGESTIVE, SIGNIFICANT AT THE P<0.10 LEVEL
≥0.1
HEY, LOOK AT THIS INTERESTING SUBGROUP ANALYSIS

BIG Debates on p-values

- Replication crises
- Publication bias
- Propose a change to $P < 0.005$
  - fewer false positive
- Rejecting the null doesn't tell you anything about the mechanism
- It doesn't tell you if the experiment is well designed, or well controlled for, or if the results have been cherry-picked.

The case against p-values

- A famous 2015 paper in Science attempted to replicate 100 findings published in a prominent psychological journal. Only 39 percent passed
- Studies that yielded highly significant results (less than p=.01) are more likely to reproduce than those that are just barely significant at the .05 level.
- The increased burden of proof — the proposal authors hope — would nudge labs into adopting other practices science reformers have been calling for, such as data sharing and thinking more long-term about their work.

The case against p$<$.005

- High standards could impede scholars with low budgets.
- It keeps scientific communities fixated on p-values

How else to evaluate good social science

- Concentrating on effect sizes (how big of a difference does an intervention make, and is it practically meaningful?)
- Confidence intervals (what's the range of doubt built into any given answer?)
- Whether a result is novel study or a replication (put some more weight into a theory many labs have looked into)
- Whether a study's design was preregistered (so that authors can't manipulate their results post-test), and that the underlying data is freely accessible (so anyone can check the math)
- There are also alternative statistical techniques — like Bayesian analysis — that, in some ways, more directly evaluate a study's results

Confidence Intervals

- What if you'd like to know what percentage of people in the U.S. are night owls (people who stay up late at night).
- In order to obtain a completely right answer, you'd have to ask each person in the country this question, but polling over 300 million people isn't very practical.

Confidence Intervals

- Get a much smaller random sample of people and then find the percentage of night owls in that sample.
- Problem:
  - Not confident that this percentage is correct or how far off this number is from the right answer for the entire population.
- So we'll try to find an "interval" that provides the assertion:
  - "I am 95% confident that the percentage of people in the U.S. are night owls is between 12% and %16."
  - This declaration is based on what's called a "confidence interval," in this case 14 +/- 2 and the confidence is 95%.

Confidence Intervals in Polls

- When a pollster reports an estimate and a margin of error, in a way they're reporting a 95% confidence interval.
- This means confidence intervals are a way of quantifying the uncertainty of an estimate.
- Further, if we take many different random samples, compute confidence intervals for each of those samples, 95% of those confidence intervals will be such that the population average would lie between those limits.

Polling Example

- Candidate Gobermouch is leading in the polls over Candidate Fopdoodle, 48% to 43%, a difference of 5 percentage points. The poll's margin of error is 3%.
- Does Gobermouch have a lead over Fopdoodle that is outside the margin of error?

Confidence Intervals and Margin of Error

- A margin of error of $\pm$ 3% means that Gobermouch's support could be as high as 51% but as low as 45%.
- Similarly, Fopdoodle's support could be as high as 46% but as low as 40%.
- Those ranges, more appropriately called "confidence intervals," overlap.
- Gobermouch's support is not "outside the margin of error."

Confidence Interval

- A confidence interval (which is most often a “95% confidence interval”) means that the “real answer” will fall within the calculated range 95% of the time
- In other words, if the pollsters repeated their survey 100 times, 95 of the ranges they calculate would contain the “real answer” and 5 would not. That’s right.
- Even the best pollsters will get it wrong 5% of the time. (And this does not take into consideration systematic bias in sampling.)

Confidence Intervals and Statistical Significance

- In our example, Gobermouch is at 48% (with a confidence interval of 45% to 51%) and Fopdoodle is at 43% (with a confidence interval of 40% to 46%).
- However, “Is Gobermouch’s lead over Fopdoodle *statistically significant*?”
- Different question and requires a different statistical approach.

Confidence Intervals and Statistical Significance

- If we subtracted Fopdoodle's support from Gobermouch's (or vice versa), would the result be zero?"
- If the result is zero (or if the confidence interval contains zero), then there is no statistically significant difference
- *You can't know this by simply determining if the confidence intervals for each candidate's support overlap*

Why Overlapping Confidence Intervals mean Nothing about Statistical Significance

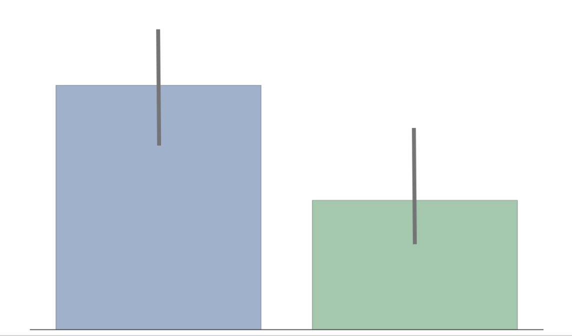


Figure 1: Correlation is NOT Causation

- "The confidence intervals of the two groups overlap, hence the difference is not statistically significant" — A lot of People

Overlapping confidence intervals/error bars say nothing about statistical significance.

- When 95% confidence intervals for the means of two independent populations don't overlap, there will indeed be a statistically significant difference between the means (at the 0.05 level of significance).
- However, the opposite is not necessarily true. CI's may overlap, yet there may be a statistically significant difference between the means.

An example

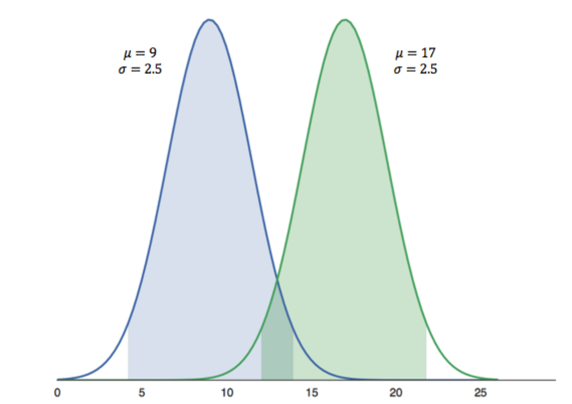


Figure 2: Correlation is NOT Causation

- Group Blue's average age is 9 years with an error of 2.5 years. Group Green's average age is 17, also with an error of 2.5 years.
- The shaded regions show the 95% confidence intervals (CI)

Difference between groups

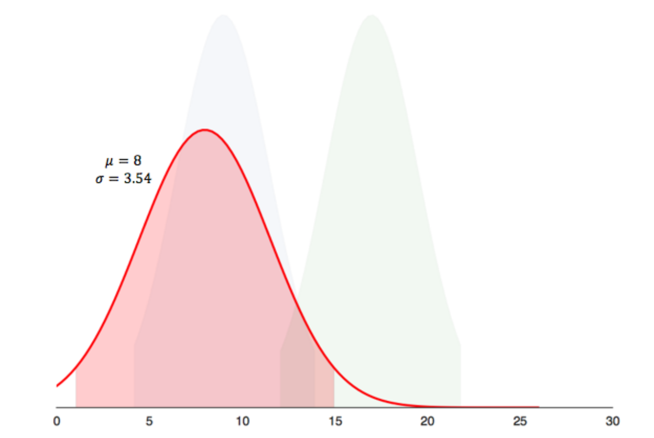


Figure 3: Correlation is NOT Causation

Difference between groups

- Instead of building a distribution for each group, we build one distribution for the difference in mean age between groups.
- If the 95% CI of the difference contains 0, then there is no difference in age between groups. If it doesn't contain 0, then there is a statistically significant difference between groups.
- As it turns out the difference is statistically significant, since the 95% CI (shaded region) doesn't contain 0.

For more see...

- Slides based on:
  - https://towardsdatascience.com/statistical-significance-hypothesis-testing-the-normal-curve-and-p-values-93274fa32687
- P-values:
  - https://www.vox.com/science-and-health/2017/7/31/16021654/p-values-statistical-significance-redefine-
  - https://www.vox.com/latest-news/2019/3/22/18275913/statistical-significance-p-values-explained
  - https://warwick.ac.uk/fac/soc/economics/staff/vetroeger/publications/pvaluedebate_vt1.pdf
- Confidence Intervals
  - https://towardsdatascience.com/why-overlapping-confidence-intervals-mean-nothing-about-statistical-significance-