3D. Propagation model

$$\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}_t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}_{t-1} + \begin{bmatrix} \cos(\theta)\Delta t & 0 \\ \sin(\theta)\Delta t & 0 \\ 0 & \Delta t \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix}_t + \begin{bmatrix} \eta_x \\ \eta_y \\ \eta_\theta \end{bmatrix}_t$$

$X = \begin{bmatrix} x \\ y \\ t \end{bmatrix}$ - state vector

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ - dynamics matrix

$B = \begin{bmatrix} \cos(\theta)\Delta t & 0 \\ \sin(\theta)\Delta t & 0 \\ 0 & \Delta t \end{bmatrix}$ - input gain matrix

$u = \begin{bmatrix} v \\ w \end{bmatrix}$ - control sequence

$\eta = \begin{bmatrix} \eta_x \\ \eta_y \\ \eta_\theta \end{bmatrix}$ - noise vector $\sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0{,}2 & 0 & 0 \\ 0 & 0{,}2 & 0 \\ 0 & 0 & 0{,}1 \end{bmatrix} \right)$

$$X_t = AX_{t-1} + \underbrace{B(\theta)u}_{\text{nonlinear}} + \eta$$

$$f = B(\theta)u$$

$$B(\theta)u = \begin{bmatrix} v\cos\theta\Delta t \\ v\sin\theta\Delta t \\ w\Delta t \end{bmatrix}$$

$$J(B(\theta)u, X) = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial \theta} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial \theta} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial \theta} \end{pmatrix}$$

$$J = \begin{pmatrix} 0 & 0 & -v\Delta t\sin\theta \\ 0 & 0 & v\cos\theta\Delta t \\ 0 & 0 & 0 \end{pmatrix} \Bigg|_{x_0, y_0, \theta_0}$$