Модуль 1, задача 2

Решить краевую задачу для уравнения Лапласа в прямоугольнике. Сделать проверку. Построить график поверхности $z = u(x, y)$.

Вариант 1

$$\begin{cases} u_{xx} + u_{yy} = 0, \qquad 0 < x < a, \quad 0 < y < b; \\ u\Big|_{x=0} = \dfrac{v_0 y(b-y)}{ab}, \qquad u_x\Big|_{x=a} = \dfrac{v_0}{b}; \\ u_y\Big|_{y=0} = \dfrac{v_0}{a}\big(1 + \sin\dfrac{3\pi x}{2a}\big), \quad u\Big|_{y=b} = \dfrac{v_0 x}{b} \end{cases}$$

Вариант 2

$$\begin{cases} u_{xx} + u_{yy} = 0, \qquad 0 < x < a, \quad 0 < y < b; \\ u_x\Big|_{x=0} = \dfrac{v_0}{a}, \quad u\Big|_{x=a} = \dfrac{v_0 y}{b}; \\ u_y\Big|_{y=0} = \dfrac{v_0 x^2}{a^2 b}, \quad u_y\Big|_{y=b} = \dfrac{v_0}{b}\big(1 + \cos\dfrac{\pi x}{2a}\big) \end{cases}$$

Вариант 3

$$\begin{cases} u_{xx} + u_{yy} = 0, \qquad 0 < x < a, \quad 0 < y < b; \\ u\Big|_{x=0} = v_0\big(1 + \dfrac{y}{b}\big), \quad u\Big|_{x=a} = \dfrac{v_0 y}{b}; \\ u_y\Big|_{y=0} = \dfrac{v_0}{b}\Big(1 + \sin\dfrac{2\pi x}{a}\Big), \quad u\Big|_{y=b} = v_0 + v_0\big(\dfrac{x}{a} - 1\big)^2 \end{cases}$$

Вариант 4

$$\begin{cases} u_{xx} + u_{yy} = 0, \qquad 0 < x < a, \quad 0 < y < b; \\ u_x\Big|_{x=0} = \dfrac{v_0 y}{ab} + \dfrac{v_0}{a}\big(\sin\dfrac{3\pi y}{2b} - 1\big), \quad u\Big|_{x=a} = \dfrac{v_0 y^2}{b^2}; \\ u\Big|_{y=0} = v_0\big(1 - \dfrac{x}{a}\big), \quad u_y\Big|_{y=b} = \dfrac{v_0(a+x)}{ab} \end{cases}$$

Вариант 5

$$\begin{cases} u_{xx} + u_{yy} = 0, \qquad 0 < x < a, \quad 0 < y < b; \\ u_x\Big|_{x=0} = \dfrac{v_0}{a} + \dfrac{v_0 y^2}{2ab^2}, \qquad u_x\Big|_{x=a} = \dfrac{v_0}{a}\Big(1 + \sin\dfrac{3\pi y}{2b}\Big) + \dfrac{v_0 y}{ab}; \\ u\Big|_{y=0} = \dfrac{v_0 x}{a}, \quad u_y\Big|_{y=b} = \dfrac{v_0(x-a)}{ab} \end{cases}$$

Вариант 6

$$\begin{cases} u_{xx} + u_{yy} = 0, \qquad 0 < x < a, \quad 0 < y < b; \\ u\Big|_{x=0} = v_0\big(1 - \dfrac{y}{b}\big), \qquad u\Big|_{x=a} = v_0\big(2 - \dfrac{y}{b}\big); \\ u\Big|_{y=0} = 2v_0 - v_0\big(\dfrac{x}{a} - 1\big)^2, \quad u_y\Big|_{y=b} = \dfrac{v_0}{b}\big(\sin\dfrac{\pi x}{a} - 1\big) \end{cases}$$

Вариант 7

$$\begin{cases} u_{xx} + u_{yy} = 0, \qquad 0 < x < a, \quad 0 < y < b; \\ u\Big|_{x=0} = v_0\big(1 - \dfrac{y}{b}\big), \qquad u\Big|_{x=a} = v_0\big(2 - \dfrac{y}{b}\big); \\ u\Big|_{y=0} = v_0\Big(1 + \sin\dfrac{2\pi x}{a}\Big) + \dfrac{v_0 x}{a}, \quad u_y\Big|_{y=b} = \dfrac{v_0 x(x-a)}{a^2} - \dfrac{v_0}{b} \end{cases}$$

Вариант 8

$$\begin{cases} u_{xx} + u_{yy} = 0, \qquad 0 < x < a, \quad 0 < y < b; \\ u\Big|_{x=0} = \dfrac{v_0(y-b)}{a}, \qquad u_x\Big|_{x=a} = \dfrac{v_0}{b}\Big(1 + \cos\dfrac{\pi y}{2b}\Big); \\ u_y\Big|_{y=0} = \dfrac{v_0}{a}, \quad u\Big|_{y=b} = \dfrac{v_0 x(x-a)}{ab} \end{cases}$$

Вариант 9

$$\begin{cases} u_{xx}+u_{yy}=0, \qquad 0<x<a, \quad 0<y<b; \\ u_x\Big|_{x=0}=\dfrac{v_0}{a}\Big(1+\cos\dfrac{3\pi y}{2b}\Big), \qquad u_x\Big|_{x=a}=\dfrac{v_0y^2}{ab^2}; \\ u_y\Big|_{y=0}=\dfrac{v_0}{b}, \quad u\Big|_{y=b}=\dfrac{v_0x}{a} \end{cases}$$

Вариант 10

$$\begin{cases} u_{xx}+u_{yy}=0, \qquad 0<x<a, \quad 0<y<b; \\ u\Big|_{x=0}=\dfrac{v_0y(y+b)}{b^2}, \qquad u\Big|_{x=a}=\dfrac{v_0y}{b}+v_0\cos\dfrac{\pi y}{2b}; \\ u_y\Big|_{y=0}=\dfrac{v_0}{b}, \quad u\Big|_{y=b}=2v_0-\dfrac{v_0x}{a} \end{cases}$$

Вариант 11

$$\begin{cases} u_{xx}+u_{yy}=0, \qquad 0<x<a, \quad 0<y<b; \\ u_x\Big|_{x=0}=\dfrac{v_0(y-b)}{ab}, \qquad u\Big|_{x=a}=\dfrac{2v_0y}{b}; \\ u\Big|_{y=0}=v_0\big(1-\dfrac{x}{a}\big)+v_0\cos\dfrac{3\pi x}{2a}, \quad u\Big|_{y=b}=\dfrac{v_0x(x+a)}{a^2b} \end{cases}$$

Вариант 12

$$\begin{cases} u_{xx}+u_{yy}=0, \qquad 0<x<a, \quad 0<y<b; \\ u\Big|_{x=0}=\dfrac{v_0y}{b}, \quad u_x\Big|_{x=a}=\dfrac{v_0(y-b)}{ab}; \\ u_y\Big|_{y=0}=\dfrac{v_0}{b}+\dfrac{v_0x^2}{2a^2b}, \quad u_y\Big|_{y=b}=\dfrac{v_0}{b}\Big(1+\sin\dfrac{\pi x}{2a}\Big)+\dfrac{v_0x}{ab} \end{cases}$$

Вариант 13

$$\begin{cases} u_{xx}+u_{yy}=0, \qquad 0<x<a, \quad 0<y<b; \\ u\Big|_{x=0}=2v_0-v_0\big(\dfrac{y}{b}-1\big)^2, \qquad u_x\Big|_{x=a}=\dfrac{v_0}{a}\big(\sin\dfrac{2\pi y}{b}-1\big); \\ u\Big|_{y=0}=v_0\big(1-\dfrac{x}{a}\big), \quad u\Big|_{y=b}=v_0\big(2-\dfrac{x}{a}\big) \end{cases}$$

Вариант 14

$$\begin{cases} u_{xx}+u_{yy}=0, \qquad 0<x<a, \quad 0<y<b; \\ u\Big|_{x=0}=v_0\big(1-\dfrac{y}{b}\big), \qquad u\Big|_{x=a}=v_0\big(2-\dfrac{y}{b}\big); \\ u\Big|_{y=0}=v_0\big(1+\sin\dfrac{3\pi x}{a}\big)+\dfrac{v_0x}{a}, \quad u_y\Big|_{y=b}=\dfrac{v_0x(x-a)}{a^2}-\dfrac{v_0}{b} \end{cases}$$

Вариант 15

$$\begin{cases} u_{xx}+u_{yy}=0, \qquad 0<x<a, \quad 0<y<b; \\ u_x\Big|_{x=0}=\dfrac{v_0}{b}\big(1+\sin\dfrac{\pi y}{2b}\big), \qquad u\Big|_{x=a}=\dfrac{v_0y}{a}; \\ u\Big|_{y=0}=\dfrac{v_0x(a-x)}{ab}, \quad u_y\Big|_{y=b}=\dfrac{v_0}{a} \end{cases}$$

Вариант 16

$$\begin{cases} u_{xx}+u_{yy}=0, \qquad 0<x<a, \quad 0<y<b; \\ u_x\Big|_{x=0}=\dfrac{v_0y^2}{ab^2}, \qquad u_x\Big|_{x=a}=\dfrac{v_0}{a}\big(1+\cos\dfrac{3\pi y}{2b}\big); \\ u_y\Big|_{y=0}=\dfrac{v_0}{b}, \quad u\Big|_{y=b}=\dfrac{v_0x}{a} \end{cases}$$

Вариант 17

$$\begin{cases} u_{xx}+u_{yy}=0, \qquad 0<x<a, \quad 0<y<b; \\ u_x\Big|_{x=0}=\dfrac{v_0}{a}\big(1+\sin\dfrac{\pi y}{b}\big), \qquad u\Big|_{x=a}=v_0+v_0\big(\dfrac{y}{b}-1\big)^2; \\ u\Big|_{y=0}=v_0\big(\dfrac{x}{a}+1\big), \quad u\Big|_{y=b}=\dfrac{v_0x}{a} \end{cases}$$

Вариант 18

$$\begin{cases} u_{xx}+u_{yy}=0, \qquad 0<x<a, \quad 0<y<b; \\ u\Big|_{x=0}=v_0\big(1-\dfrac{y}{b}\big), \qquad u_x\Big|_{x=a}=\dfrac{v_0(b+y)}{ab}; \\ u_y\Big|_{y=0}=\dfrac{v_0x}{ab}+\dfrac{v_0}{b}\big(\sin\dfrac{\pi x}{2a}-1\big), \quad u\Big|_{y=b}=\dfrac{v_0x^2}{a^2} \end{cases}$$

Вариант 19

$$\begin{cases} u_{xx}+u_{yy}=0, \qquad 0<x<a, \quad 0<y<b; \\ u_x\Big|_{x=0}=\dfrac{v_0}{a}\big(1+\sin\dfrac{3\pi y}{2b}\big)+\dfrac{v_0y}{ab}, \qquad u_x\Big|_{x=a}=\dfrac{v_0}{a}+\dfrac{v_0y^2}{2ab^2}; \\ u\Big|_{y=0}=\dfrac{v_0x}{a}, \quad u_y\Big|_{y=b}=\dfrac{v_0(x-a)}{ab} \end{cases}$$

Вариант 20

$$\begin{cases} u_{xx}+u_{yy}=0, \qquad 0<x<a, \quad 0<y<b; \\ u\Big|_{x=0}=v_0\big(1-\dfrac{y}{b}\big), \qquad u\Big|_{x=a}=v_0\big(2-\dfrac{y}{b}\big); \\ u\Big|_{y=0}=2v_0-v_0\big(\dfrac{x}{a}-1\big)^2, \quad u_y\Big|_{y=b}=\dfrac{v_0}{b}\big(\sin\dfrac{3\pi y}{a}-1\big) \end{cases}$$

Вариант 21

$$\begin{cases} u_{xx}+u_{yy}=0, \qquad 0<x<a, \quad 0<y<b; \\ u\Big|_{x=0}=v_0\big(1+\sin\dfrac{2\pi y}{b}\big)+\dfrac{v_0y}{b}, \qquad u_x\Big|_{x=a}=\dfrac{v_0y(y-b)}{b^2}-\dfrac{v_0}{a}; \\ u\Big|_{y=0}=v_0\big(1-\dfrac{x}{a}\big), \quad u\Big|_{y=b}=v_0\big(2-\dfrac{x}{a}\big) \end{cases}$$

Вариант 22

$$\begin{cases} u_{xx}+u_{yy}=0, \qquad 0<x<a, \quad 0<y<b; \\ u_x\Big|_{x=0}=\dfrac{v_0}{b}, \quad u\Big|_{x=a}=\dfrac{v_0y(y-b)}{ab}; \\ u\Big|_{y=0}=\dfrac{v_0(x-a)}{b}, \quad u_y\Big|_{y=b}=\dfrac{v_0}{a}\left(1+\cos\dfrac{3\pi x}{2a}\right) \end{cases}$$

Вариант 23

$$\begin{cases} u_{xx}+u_{yy}=0, \qquad 0<x<a, \quad 0<y<b; \\ u_x\Big|_{x=0}=\dfrac{v_0}{a}, \quad u\Big|_{x=a}=\dfrac{v_0y}{b}; \\ u_y\Big|_{y=0}=\dfrac{v_0}{b}\big(1+\cos\dfrac{\pi x}{2a}\big), \quad u_y\Big|_{y=b}=\dfrac{v_0x^2}{a^2b} \end{cases}$$

No	v_0	a	b
1	3	6	2
2	2	5	3
3	1	3	5
4	4	2	3
5	3	1	3
6	3	6	2
7	2	5	1
8	1	3	5
9	4	2	5
10	3	1	3
11	3	6	2
12	2	5	1
13	1	3	5
14	4	2	3
15	3	1	3
16	3	6	2
17	2	5	3
18	1	3	5
19	4	2	1
20	3	1	3
21	3	6	2
22	2	5	3
23	1	3	5