#Решить краевую задачу для уравнения Лапласа в прямоугольнике.
#Сделать проверку.
#Построить график поверхности z=u(x, y)
#Постановка задача
Typesetting:-Settings(functionassign = false) :
$\Delta u := 0 :$ $0 < x < a, \quad 0 < y < b :$
$v0 := 3 :$
$a := 6 :$
$b := 2 :$

$$u(0, y) := v0 \cdot \left(1 - \frac{y}{b}\right);$$

$$3 - \frac{3}{2} y \tag{1}$$

$$u(a, y) := v0 \cdot \left(2 - \frac{y}{b}\right);$$

$$6 - \frac{3}{2} y \tag{2}$$

$$u(x, 0) := 2 \cdot v0 - v0 \cdot \left(\frac{x}{a} - 1\right)^2;$$

$$6 - 3\left(\frac{1}{6} x - 1\right)^2 \tag{3}$$

$$uy(x, b) := \frac{v0}{b} \cdot \left(\sin\left(\frac{\mathrm{Pi} \cdot x}{a}\right) - 1\right)$$

$$(x, b) \to \frac{v0\left(\sin\left(\frac{\pi x}{a}\right) - 1\right)}{b} \tag{4}$$

$$uy(x, b)$$

$$\frac{3}{2} \sin\left(\frac{1}{6} \pi x\right) - \frac{3}{2} \tag{5}$$

$$phi1(y) := 3 - \frac{3 y}{2} :$$

$$phi2(y) := 6 - \frac{3 y}{2} :$$

$$psi1(x) := 6 - 3\left(\frac{x}{6} - 1\right)^2$$

$$x \to 6 - 3\left(\frac{1}{6} x - 1\right)^2 \tag{6}$$

$$psi2(x) := \frac{3 \sin\left(\frac{x \pi}{6}\right)}{2} - \frac{3}{2}$$

$$x \to \frac{3}{2} \sin\left(\frac{1}{6} \pi x\right) - \frac{3}{2} \tag{7}$$

#Решение ищем в виде u(x,y)=w(x,y)+v(x,y), где w(x,y)-функция, удовлетворяющая граничным

условиям, например по переменной x

$w(x,y) := A\cdot x + B$:

$solve([subs(x=0, w(x,y)) = phi1(y), subs(x=a, w(x,y)) = phi2(y)], [A, B])$

$$\left[\left[A = \frac{1}{2}, B = 3 - \frac{3}{2}\,y\right]\right] \quad (8)$$

$w(x,y) := \frac{1\cdot x}{2} + 3 - \frac{3}{2}\cdot y$:

$\Delta w(x,y) := \frac{\partial^2}{\partial x^2} w(x,y) + \frac{\partial^2}{\partial y^2} w(x,y)$:

#Перейдем к новой задаче отосительно функции v(x,y)

#Для функции v(x,y) получаем краевую задача с однородными граничными условиями по переменной x

$\Delta v(x,y) := \Delta u - \Delta w(x,y)$: $0 < x < a, \quad 0 < y < b$:

$\Delta v(x,y)$

$$0 \quad (9)$$

$v(0,y) := phi1(y) - w(0,y)$

$$0 \quad (10)$$

$v(a,y) := phi2(y) - w(a,y)$

$$(a,y) \to \phi 2(y) - w(a,y) \quad (11)$$

$v(x,0) := psi1(x) - w(x,0)$

$$3 - 3\left(\frac{1}{6}\,x - 1\right)^2 - \frac{1}{2}\,x \quad (12)$$

$vy(x,b) := psi2(x) - \frac{\partial}{\partial y} w(x,y)\Big|_{y=b}$:

$vy(x,b)$

$$\frac{3}{2}\,\sin\left(\frac{1}{6}\,\pi x\right) \quad (13)$$

#$v(x,y) = X(x)\,Y(y)$

#Задача Штурма-Лиувилля

$X'' + \lambda X = 0$:

$X(0) = X(a) = 0$:

$\lambda n = \left(\frac{\pi n}{a}\right)^2$, $n = 1, 2 \ldots$:

$Xn(x) := \sin\left(\frac{\pi n}{a}\cdot x\right)$:

$v(x,y) = \sum_{n=1}^{\infty} Xn(x)\,Yn(y) = \sum_{n=1}^{\infty} \sin\left(\frac{\pi n}{a}\cdot x\right) Yn(y)$:

$\Delta v(x,y) = \sum_{n=1}^{\infty} \left(Y''n(y) - \left(\frac{\pi n}{a}\right) Yn(y)\right) \sin\left(\frac{\pi n}{a}\cdot x\right) = 0$:

$Y''n(y) - \left(\frac{\pi n}{a}\right) Yn(y) = 0$:

#Подставляем в граничные условия

$$v(x,0)=\sum_{n=1}^{\infty}\sin\left(\frac{\pi n}{a}\cdot x\right)Yn(0)=2\sin\left(\frac{2\pi x}{5}\right)=\text{phi}$$

$$vy(x,b)=\sum_{n=1}^{\infty}\sin\left(\frac{\pi n}{a}\cdot x\right)Y'n(b)=\frac{2x(x-5)}{25}=\text{psi}$$

$Yn(0)=phin$:

$Y'n(b)=psin$:

$$phin:=\frac{2}{a}\cdot\int_0^a\left(3-3\left(\frac{x}{6}-1\right)^2-\frac{x}{2}\right)\cdot\sin\left(\frac{x\cdot\pi\cdot n}{a}\right)dx$$

$$-\frac{6(\pi n\sin(\pi n)+2\cos(\pi n)-2)}{\pi^3 n^3} \tag{14}$$

$$psin:=\frac{2}{a}\cdot\int_0^a\left(\frac{3\sin\left(\frac{x\pi}{6}\right)}{2}\right)\cdot\sin\left(\frac{x\cdot\pi\cdot n}{a}\right)dx$$

$$-\frac{3\sin(\pi n)}{\pi(n^2-1)} \tag{15}$$

$Y''n-\lambda Yn=0$:

$$Yn(y):=an\cdot\sinh\left(\frac{\pi n}{a}\cdot y\right)+bn\cdot\cosh\left(\frac{\pi n}{a}\cdot(b-y)\right):$$

$$Yn(0)=bn\cdot\cosh\left(\frac{\pi n}{a}\cdot b\right)=phin$$

$$Y'n(b)=an\cdot\frac{\pi n}{a}\cosh\left(\frac{\pi n}{a}\cdot b\right)=psin$$

$$an:=\frac{psin}{\frac{\pi\cdot n}{a}\cdot\cosh\left(\frac{\pi\cdot n}{a}\cdot b\right)}$$

$$-\frac{18\sin(\pi n)}{\pi^2(n^2-1)\,n\cosh\left(\frac{1}{3}\pi n\right)} \tag{16}$$

$$bn:=\frac{phin}{\cosh\left(\frac{\pi\cdot n}{a}\cdot b\right)}$$

$$-\frac{6(\pi n\sin(\pi n)+2\cos(\pi n)-2)}{\pi^3 n^3\cosh\left(\frac{1}{3}\pi n\right)} \tag{17}$$

$$v(x,y):=\sum_{n=1}^{350}\left(an\cdot\sinh\left(\frac{\pi\cdot n}{a}\cdot y\right)+bn\cdot\cosh\left(\frac{\pi\cdot n}{a}\cdot(b-y)\right)\right)\cdot\sin\left(\frac{x\cdot\pi\cdot n}{a}\right):$$

$u\_new(x,y):=w(x,y)+v(x,y)$:

#Проверка

$$simplify\left(\frac{\partial^2}{\partial x^2} u\_new(x,y) + \frac{\partial^2}{\partial y^2} u\_new(x,y) \right)$$

$$0 \tag{18}$$

$$u\_new(x,y)\Big|_{x=0}$$

$$3 - \frac{3}{2} y \tag{19}$$

$$u\_new(x,y)\Big|_{x=a}$$

$$6 - \frac{3\,y}{2} \tag{20}$$

$$u\_new(x,y)\Big|_{y=0}$$

$$3 + \frac{1}{2} x + \frac{24}{29791} \frac{\sin\left(\frac{31}{6}\pi x\right)}{\pi^3} + \frac{8}{11979} \frac{\sin\left(\frac{11}{2}\pi x\right)}{\pi^3} + \frac{24}{42875} \frac{\sin\left(\frac{35}{6}\pi x\right)}{\pi^3} \tag{21}$$
$$+ \frac{24}{50653} \frac{\sin\left(\frac{37}{6}\pi x\right)}{\pi^3} + \frac{8}{19773} \frac{\sin\left(\frac{13}{2}\pi x\right)}{\pi^3} + \frac{24}{912673} \frac{\sin\left(\frac{97}{6}\pi x\right)}{\pi^3}$$
$$+ \frac{8}{323433} \frac{\sin\left(\frac{33}{2}\pi x\right)}{\pi^3} + \frac{24}{1030301} \frac{\sin\left(\frac{101}{6}\pi x\right)}{\pi^3} + \frac{24}{1092727} \frac{\sin\left(\frac{103}{6}\pi x\right)}{\pi^3}$$
$$+ \frac{8}{385875} \frac{\sin\left(\frac{35}{2}\pi x\right)}{\pi^3} + \frac{24}{1225043} \frac{\sin\left(\frac{107}{6}\pi x\right)}{\pi^3} + \frac{24}{1295029} \frac{\sin\left(\frac{109}{6}\pi x\right)}{\pi^3}$$
$$+ \frac{8}{455877} \frac{\sin\left(\frac{37}{2}\pi x\right)}{\pi^3} + \frac{24}{1442897} \frac{\sin\left(\frac{113}{6}\pi x\right)}{\pi^3} + \frac{24}{1520875} \frac{\sin\left(\frac{115}{6}\pi x\right)}{\pi^3}$$
$$+ \frac{8}{533871} \frac{\sin\left(\frac{39}{2}\pi x\right)}{\pi^3} + \frac{24}{1685159} \frac{\sin\left(\frac{119}{6}\pi x\right)}{\pi^3} + \frac{24}{1771561} \frac{\sin\left(\frac{121}{6}\pi x\right)}{\pi^3}$$
$$+ \frac{8}{620289} \frac{\sin\left(\frac{41}{2}\pi x\right)}{\pi^3} + \frac{24}{1953125} \frac{\sin\left(\frac{125}{6}\pi x\right)}{\pi^3} + \frac{24}{2048383} \frac{\sin\left(\frac{127}{6}\pi x\right)}{\pi^3}$$
$$+ \frac{8}{715563} \frac{\sin\left(\frac{43}{2}\pi x\right)}{\pi^3} + \frac{24}{2248091} \frac{\sin\left(\frac{131}{6}\pi x\right)}{\pi^3} + \frac{24}{2352637} \frac{\sin\left(\frac{133}{6}\pi x\right)}{\pi^3}$$

$$+\frac{8}{820125}\,\frac{\sin\left(\frac{45}{2}\,\pi x\right)}{\pi^3}+\frac{24}{2571353}\,\frac{\sin\left(\frac{137}{6}\,\pi x\right)}{\pi^3}+\frac{24}{2685619}\,\frac{\sin\left(\frac{139}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{8}{934407}\,\frac{\sin\left(\frac{47}{2}\,\pi x\right)}{\pi^3}+\frac{24}{2924207}\,\frac{\sin\left(\frac{143}{6}\,\pi x\right)}{\pi^3}+\frac{24}{3048625}\,\frac{\sin\left(\frac{145}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{8}{1058841}\,\frac{\sin\left(\frac{49}{2}\,\pi x\right)}{\pi^3}+\frac{24}{3307949}\,\frac{\sin\left(\frac{149}{6}\,\pi x\right)}{\pi^3}+\frac{24}{3442951}\,\frac{\sin\left(\frac{151}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{8}{9}\,\frac{\sin\left(\frac{1}{2}\,\pi x\right)}{\pi^3}+\frac{24}{125}\,\frac{\sin\left(\frac{5}{6}\,\pi x\right)}{\pi^3}+\frac{24}{343}\,\frac{\sin\left(\frac{7}{6}\,\pi x\right)}{\pi^3}+\frac{8}{243}\,\frac{\sin\left(\frac{3}{2}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{1331}\,\frac{\sin\left(\frac{11}{6}\,\pi x\right)}{\pi^3}+\frac{24}{2197}\,\frac{\sin\left(\frac{13}{6}\,\pi x\right)}{\pi^3}+\frac{8}{1125}\,\frac{\sin\left(\frac{5}{2}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{4913}\,\frac{\sin\left(\frac{17}{6}\,\pi x\right)}{\pi^3}+\frac{24}{6859}\,\frac{\sin\left(\frac{19}{6}\,\pi x\right)}{\pi^3}+\frac{8}{3087}\,\frac{\sin\left(\frac{7}{2}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{12167}\,\frac{\sin\left(\frac{23}{6}\,\pi x\right)}{\pi^3}+\frac{24}{15625}\,\frac{\sin\left(\frac{25}{6}\,\pi x\right)}{\pi^3}+\frac{8}{6561}\,\frac{\sin\left(\frac{9}{2}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{24389}\,\frac{\sin\left(\frac{29}{6}\,\pi x\right)}{\pi^3}+\frac{8}{4437351}\,\frac{\sin\left(\frac{79}{2}\,\pi x\right)}{\pi^3}+\frac{24}{13651919}\,\frac{\sin\left(\frac{239}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{13997521}\,\frac{\sin\left(\frac{241}{6}\,\pi x\right)}{\pi^3}+\frac{8}{4782969}\,\frac{\sin\left(\frac{81}{2}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{14706125}\,\frac{\sin\left(\frac{245}{6}\,\pi x\right)}{\pi^3}+\frac{24}{15069223}\,\frac{\sin\left(\frac{247}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{8}{5146083}\,\frac{\sin\left(\frac{83}{2}\,\pi x\right)}{\pi^3}+\frac{24}{15813251}\,\frac{\sin\left(\frac{251}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{16194277}\,\frac{\sin\left(\frac{253}{6}\,\pi x\right)}{\pi^3}+\frac{8}{5527125}\,\frac{\sin\left(\frac{85}{2}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{16974593}\,\frac{\sin\left(\frac{257}{6}\,\pi x\right)}{\pi^3}+\frac{24}{17373979}\,\frac{\sin\left(\frac{259}{6}\,\pi x\right)}{\pi^3}$$

$$+\frac{8}{5926527}\,\frac{\sin\left(\frac{87}{2}\,\pi x\right)}{\pi^3}+\frac{24}{18191447}\,\frac{\sin\left(\frac{263}{6}\,\pi x\right)}{\pi^3}+\frac{24}{68921}\,\frac{\sin\left(\frac{41}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{79507}\,\frac{\sin\left(\frac{43}{6}\,\pi x\right)}{\pi^3}+\frac{8}{30375}\,\frac{\sin\left(\frac{15}{2}\,\pi x\right)}{\pi^3}+\frac{24}{103823}\,\frac{\sin\left(\frac{47}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{117649}\,\frac{\sin\left(\frac{49}{6}\,\pi x\right)}{\pi^3}+\frac{8}{44217}\,\frac{\sin\left(\frac{17}{2}\,\pi x\right)}{\pi^3}+\frac{24}{148877}\,\frac{\sin\left(\frac{53}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{166375}\,\frac{\sin\left(\frac{55}{6}\,\pi x\right)}{\pi^3}+\frac{8}{61731}\,\frac{\sin\left(\frac{19}{2}\,\pi x\right)}{\pi^3}+\frac{24}{205379}\,\frac{\sin\left(\frac{59}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{226981}\,\frac{\sin\left(\frac{61}{6}\,\pi x\right)}{\pi^3}+\frac{8}{83349}\,\frac{\sin\left(\frac{21}{2}\,\pi x\right)}{\pi^3}+\frac{24}{274625}\,\frac{\sin\left(\frac{65}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{300763}\,\frac{\sin\left(\frac{67}{6}\,\pi x\right)}{\pi^3}+\frac{8}{109503}\,\frac{\sin\left(\frac{23}{2}\,\pi x\right)}{\pi^3}+\frac{24}{357911}\,\frac{\sin\left(\frac{71}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{389017}\,\frac{\sin\left(\frac{73}{6}\,\pi x\right)}{\pi^3}+\frac{8}{140625}\,\frac{\sin\left(\frac{25}{2}\,\pi x\right)}{\pi^3}+\frac{24}{456533}\,\frac{\sin\left(\frac{77}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{493039}\,\frac{\sin\left(\frac{79}{6}\,\pi x\right)}{\pi^3}+\frac{8}{177147}\,\frac{\sin\left(\frac{27}{2}\,\pi x\right)}{\pi^3}+\frac{24}{571787}\,\frac{\sin\left(\frac{83}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{614125}\,\frac{\sin\left(\frac{85}{6}\,\pi x\right)}{\pi^3}+\frac{8}{219501}\,\frac{\sin\left(\frac{29}{2}\,\pi x\right)}{\pi^3}+\frac{24}{704969}\,\frac{\sin\left(\frac{89}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{753571}\,\frac{\sin\left(\frac{91}{6}\,\pi x\right)}{\pi^3}+\frac{8}{268119}\,\frac{\sin\left(\frac{31}{2}\,\pi x\right)}{\pi^3}+\frac{24}{857375}\,\frac{\sin\left(\frac{95}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{10218313}\,\frac{\sin\left(\frac{217}{6}\,\pi x\right)}{\pi^3}+\frac{8}{3501153}\,\frac{\sin\left(\frac{73}{2}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{10793861}\,\frac{\sin\left(\frac{221}{6}\,\pi x\right)}{\pi^3}+\frac{24}{11089567}\,\frac{\sin\left(\frac{223}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{8}{3796875}\,\frac{\sin\left(\frac{75}{2}\,\pi x\right)}{\pi^3}+\frac{24}{11697083}\,\frac{\sin\left(\frac{227}{6}\,\pi x\right)}{\pi^3}$$

$$+\frac{24}{12008989}\frac{\sin\left(\frac{229}{6}\pi x\right)}{\pi^3}+\frac{8}{4108797}\frac{\sin\left(\frac{77}{2}\pi x\right)}{\pi^3}$$
$$+\frac{24}{12649337}\frac{\sin\left(\frac{233}{6}\pi x\right)}{\pi^3}+\frac{24}{12977875}\frac{\sin\left(\frac{235}{6}\pi x\right)}{\pi^3}$$
$$+\frac{24}{39651821}\frac{\sin\left(\frac{341}{6}\pi x\right)}{\pi^3}+\frac{24}{40353607}\frac{\sin\left(\frac{343}{6}\pi x\right)}{\pi^3}$$
$$+\frac{8}{13687875}\frac{\sin\left(\frac{115}{2}\pi x\right)}{\pi^3}+\frac{24}{41781923}\frac{\sin\left(\frac{347}{6}\pi x\right)}{\pi^3}$$
$$+\frac{24}{42508549}\frac{\sin\left(\frac{349}{6}\pi x\right)}{\pi^3}+\frac{8}{1193859}\frac{\sin\left(\frac{51}{2}\pi x\right)}{\pi^3}+\frac{24}{3723875}\frac{\sin\left(\frac{155}{6}\pi x\right)}{\pi^3}$$
$$+\frac{24}{3869893}\frac{\sin\left(\frac{157}{6}\pi x\right)}{\pi^3}+\frac{8}{1339893}\frac{\sin\left(\frac{53}{2}\pi x\right)}{\pi^3}+\frac{24}{4173281}\frac{\sin\left(\frac{161}{6}\pi x\right)}{\pi^3}$$
$$+\frac{24}{4330747}\frac{\sin\left(\frac{163}{6}\pi x\right)}{\pi^3}+\frac{8}{1497375}\frac{\sin\left(\frac{55}{2}\pi x\right)}{\pi^3}+\frac{24}{4657463}\frac{\sin\left(\frac{167}{6}\pi x\right)}{\pi^3}$$
$$+\frac{24}{4826809}\frac{\sin\left(\frac{169}{6}\pi x\right)}{\pi^3}+\frac{8}{1666737}\frac{\sin\left(\frac{57}{2}\pi x\right)}{\pi^3}+\frac{24}{5177717}\frac{\sin\left(\frac{173}{6}\pi x\right)}{\pi^3}$$
$$+\frac{24}{5359375}\frac{\sin\left(\frac{175}{6}\pi x\right)}{\pi^3}+\frac{8}{1848411}\frac{\sin\left(\frac{59}{2}\pi x\right)}{\pi^3}+\frac{24}{5735339}\frac{\sin\left(\frac{179}{6}\pi x\right)}{\pi^3}$$
$$+\frac{24}{5929741}\frac{\sin\left(\frac{181}{6}\pi x\right)}{\pi^3}+\frac{8}{2042829}\frac{\sin\left(\frac{61}{2}\pi x\right)}{\pi^3}+\frac{24}{6331625}\frac{\sin\left(\frac{185}{6}\pi x\right)}{\pi^3}$$
$$+\frac{24}{6539203}\frac{\sin\left(\frac{187}{6}\pi x\right)}{\pi^3}+\frac{8}{2250423}\frac{\sin\left(\frac{63}{2}\pi x\right)}{\pi^3}+\frac{24}{6967871}\frac{\sin\left(\frac{191}{6}\pi x\right)}{\pi^3}$$
$$+\frac{24}{7189057}\frac{\sin\left(\frac{193}{6}\pi x\right)}{\pi^3}+\frac{8}{2471625}\frac{\sin\left(\frac{65}{2}\pi x\right)}{\pi^3}+\frac{24}{7645373}\frac{\sin\left(\frac{197}{6}\pi x\right)}{\pi^3}$$
$$+\frac{24}{7880599}\frac{\sin\left(\frac{199}{6}\pi x\right)}{\pi^3}+\frac{8}{2706867}\frac{\sin\left(\frac{67}{2}\pi x\right)}{\pi^3}+\frac{24}{8365427}\frac{\sin\left(\frac{203}{6}\pi x\right)}{\pi^3}$$

$$+\frac{24}{8615125}\,\frac{\sin\left(\frac{205}{6}\,\pi x\right)}{\pi^3}+\frac{8}{2956581}\,\frac{\sin\left(\frac{69}{2}\,\pi x\right)}{\pi^3}+\frac{24}{9129329}\,\frac{\sin\left(\frac{209}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{9393931}\,\frac{\sin\left(\frac{211}{6}\,\pi x\right)}{\pi^3}+\frac{8}{3221199}\,\frac{\sin\left(\frac{71}{2}\,\pi x\right)}{\pi^3}+\frac{24}{9938375}\,\frac{\sin\left(\frac{215}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{18609625}\,\frac{\sin\left(\frac{265}{6}\,\pi x\right)}{\pi^3}+\frac{8}{6344721}\,\frac{\sin\left(\frac{89}{2}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{19465109}\,\frac{\sin\left(\frac{269}{6}\,\pi x\right)}{\pi^3}+\frac{24}{19902511}\,\frac{\sin\left(\frac{271}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{8}{6782139}\,\frac{\sin\left(\frac{91}{2}\,\pi x\right)}{\pi^3}+\frac{24}{20796875}\,\frac{\sin\left(\frac{275}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{21253933}\,\frac{\sin\left(\frac{277}{6}\,\pi x\right)}{\pi^3}+\frac{8}{7239213}\,\frac{\sin\left(\frac{93}{2}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{22188041}\,\frac{\sin\left(\frac{281}{6}\,\pi x\right)}{\pi^3}+\frac{24}{22665187}\,\frac{\sin\left(\frac{283}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{8}{7716375}\,\frac{\sin\left(\frac{95}{2}\,\pi x\right)}{\pi^3}+\frac{24}{23639903}\,\frac{\sin\left(\frac{287}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{24137569}\,\frac{\sin\left(\frac{289}{6}\,\pi x\right)}{\pi^3}+\frac{8}{8214057}\,\frac{\sin\left(\frac{97}{2}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{25153757}\,\frac{\sin\left(\frac{293}{6}\,\pi x\right)}{\pi^3}+\frac{24}{25672375}\,\frac{\sin\left(\frac{295}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{8}{8732691}\,\frac{\sin\left(\frac{99}{2}\,\pi x\right)}{\pi^3}+\frac{24}{26730899}\,\frac{\sin\left(\frac{299}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{27270901}\,\frac{\sin\left(\frac{301}{6}\,\pi x\right)}{\pi^3}+\frac{8}{9272709}\,\frac{\sin\left(\frac{101}{2}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{28372625}\,\frac{\sin\left(\frac{305}{6}\,\pi x\right)}{\pi^3}+\frac{24}{28934443}\,\frac{\sin\left(\frac{307}{6}\,\pi x\right)}{\pi^3}$$

$$+\frac{8}{9834543}\,\frac{\sin\left(\frac{103}{2}\,\pi x\right)}{\pi^3}+\frac{24}{30080231}\,\frac{\sin\left(\frac{311}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{30664297}\,\frac{\sin\left(\frac{313}{6}\,\pi x\right)}{\pi^3}+\frac{8}{10418625}\,\frac{\sin\left(\frac{105}{2}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{31855013}\,\frac{\sin\left(\frac{317}{6}\,\pi x\right)}{\pi^3}+\frac{24}{32461759}\,\frac{\sin\left(\frac{319}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{8}{11025387}\,\frac{\sin\left(\frac{107}{2}\,\pi x\right)}{\pi^3}+\frac{24}{33698267}\,\frac{\sin\left(\frac{323}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{34328125}\,\frac{\sin\left(\frac{325}{6}\,\pi x\right)}{\pi^3}+\frac{8}{11655261}\,\frac{\sin\left(\frac{109}{2}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{35611289}\,\frac{\sin\left(\frac{329}{6}\,\pi x\right)}{\pi^3}+\frac{24}{36264691}\,\frac{\sin\left(\frac{331}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{8}{12308679}\,\frac{\sin\left(\frac{111}{2}\,\pi x\right)}{\pi^3}+\frac{24}{37595375}\,\frac{\sin\left(\frac{335}{6}\,\pi x\right)}{\pi^3}$$
$$+\frac{24}{38272753}\,\frac{\sin\left(\frac{337}{6}\,\pi x\right)}{\pi^3}+\frac{8}{12986073}\,\frac{\sin\left(\frac{113}{2}\,\pi x\right)}{\pi^3}$$
$$+\frac{24\sin\left(\frac{1}{6}\,\pi x\right)\left(e^{\pi}\right)^{2/3}+24\sin\left(\frac{1}{6}\,\pi x\right)}{\pi^3\left(e^{\pi}\right)^{2/3}+\pi^3}$$

$$u\_new\_y(x,y) := \frac{\partial}{\partial y}u\_new(x,y):$$

$$u\_new\_y(x,y)\Big|_{y=b}$$
$$-\frac{3}{2}+\frac{1}{\pi^3\left(e^{\pi}\right)^{2/3}+\pi^3}\left(\frac{3}{2}\sin\left(\frac{1}{6}\,\pi x\right)\left(e^{\pi}\right)^{1/3}\pi^3 e^{\frac{1}{3}\pi}+\frac{3}{2}\sin\left(\frac{1}{6}\,\pi x\right)\left(e^{\pi}\right)^{1/3}\pi^3 e^{-\frac{1}{3}\pi}\right. \tag{22}$$
$$\left.-4\sin\left(\frac{1}{6}\,\pi x\right)\left(e^{\pi}\right)^{2/3}\pi e^{-\frac{1}{3}\pi}+4\sin\left(\frac{1}{6}\,\pi x\right)\pi e^{\frac{1}{3}\pi}\right)$$

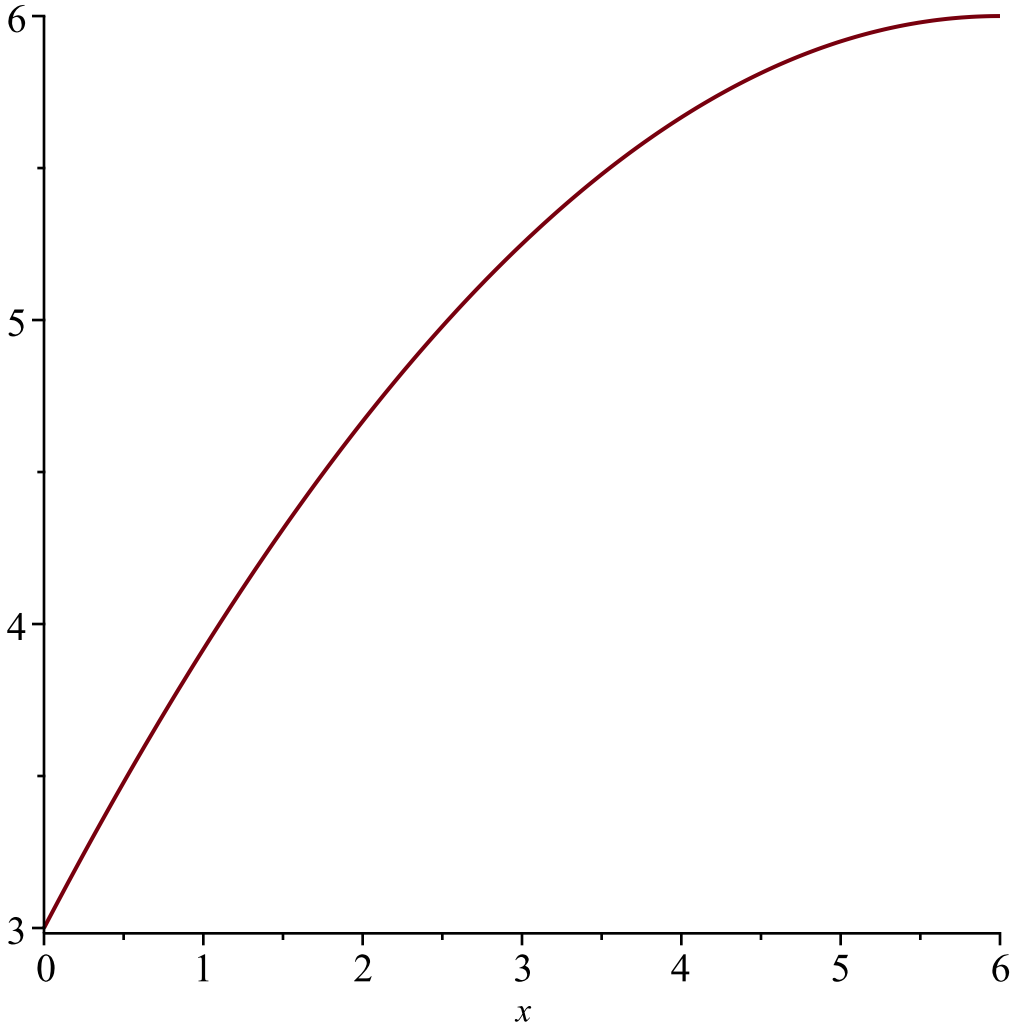
$simplify\left(u\_new\_y(x,y)\Big|_{y=b}\right)$

$$\frac{3}{2}\sin\left(\frac{1}{6}\pi x\right)-\frac{3}{2} \tag{23}$$

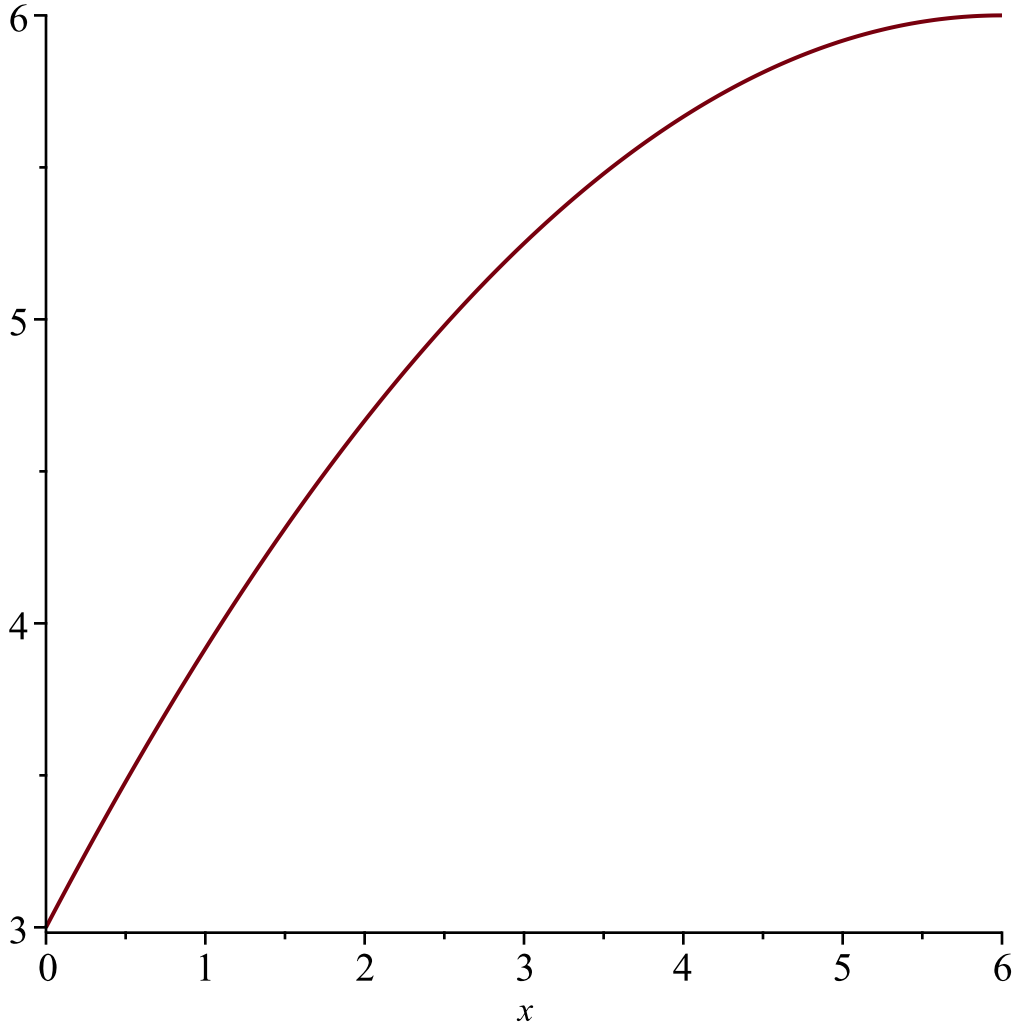
$with(plots):$

 # Так как анализ данных рядов не представляется возможным, построим совмещенные графики и убедимся, что они совпадают

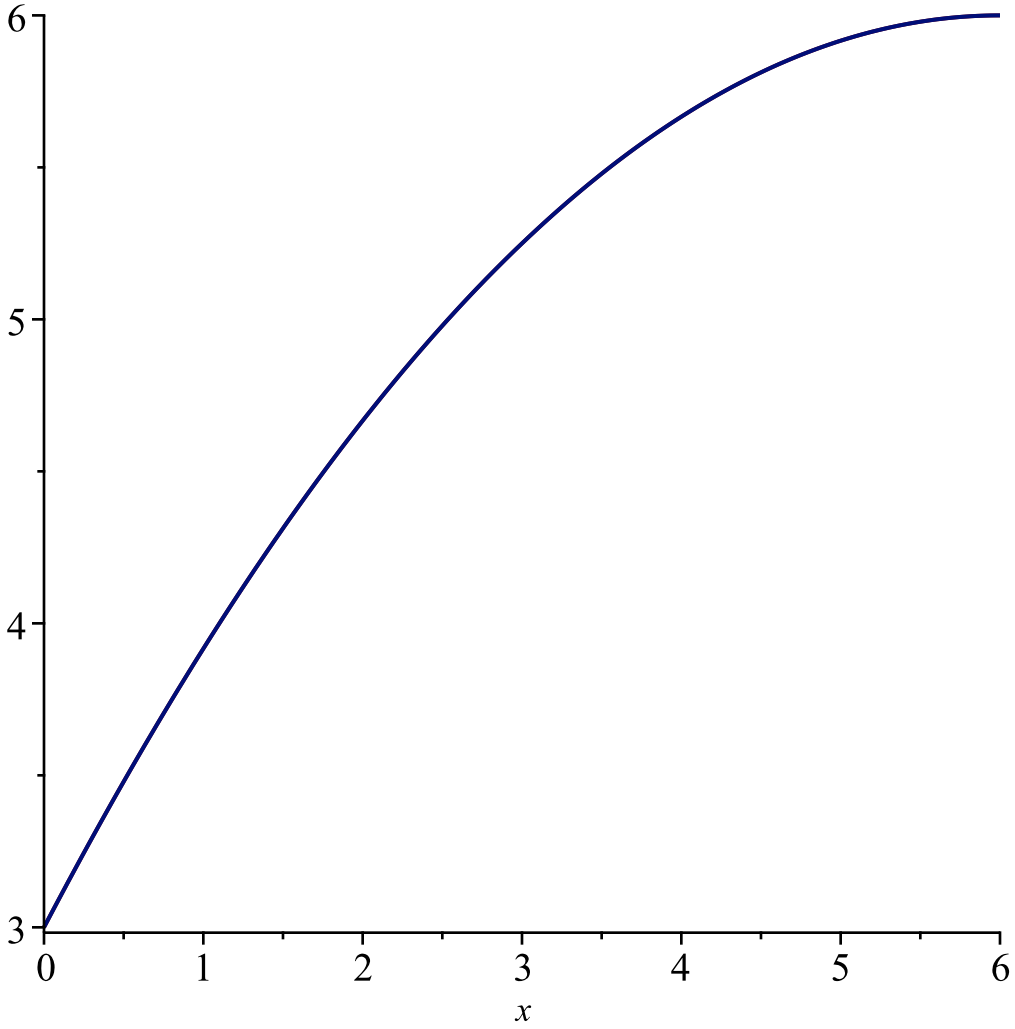
$plot(u(x,0),x=0..a)$



$plot\left(u\_new(x,y)\Big|_{y=0},x=0..a\right)$

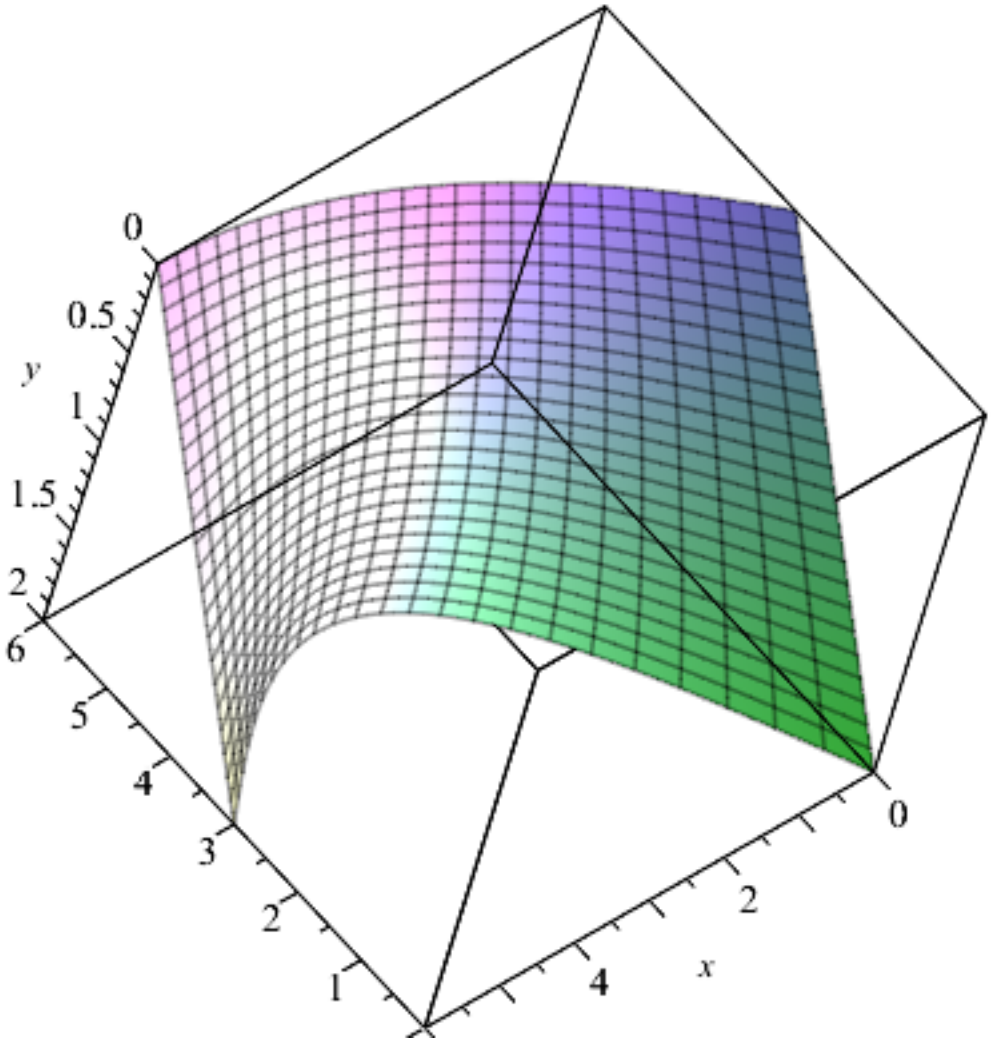


$$plot\left(\left[u(x,0),\ u\_new(x,y)\Big|_{y=0}\right],\ x=0..a\right)$$



#Видим, что графики наложились. Делаем вывод о верности нашего решения

$plot3d(u\_new(x, y), x = 0 ..a, y = 0 ..b)$


0
0.5
y
1
1.5
2
6
5
4
3
2
1
4
x
2
0