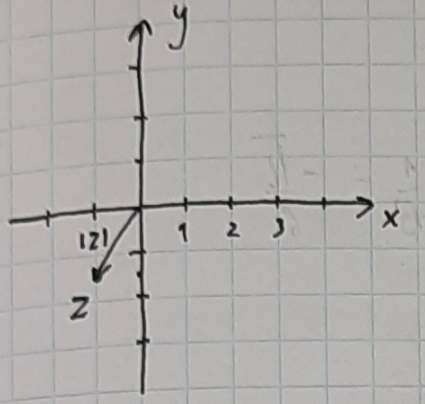
ТФКЗ №1 Жуковін ФІ-21

① а) $z = -1 - i\sqrt{3}$



$\mathrm{Re}\, z = -1$

$\mathrm{Im}\, z = -\sqrt{3}$

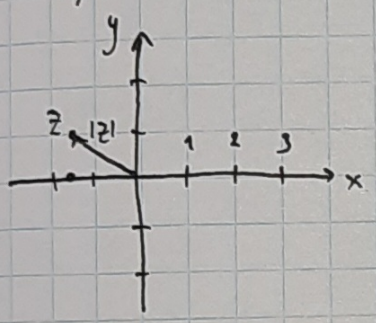
$|z| = \sqrt{1+3} = 2$

$\arg z = \operatorname{arctg}\sqrt{3} - \pi = \frac{\pi}{3} - \pi = -\frac{2\pi}{3}$

$z = 2\left(\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}\right)$

$z = 2e^{i2\pi/3}$

② б) $z = -\sqrt{3} + i$



$\mathrm{Re}\, z = -\sqrt{3}$

$\mathrm{Im}\, z = 1$

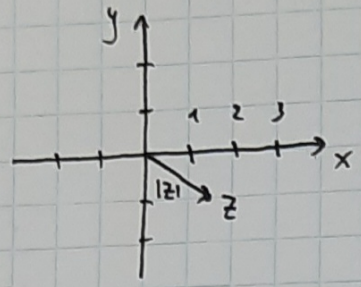
$|z| = \sqrt{3+1} = 2$

$\arg z = \operatorname{arctg}\frac{1}{-\sqrt{3}} + \pi = -\frac{\pi}{6} + \pi = \frac{5\pi}{6}$

$z = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

$z = 2e^{i5\pi/6}$

b) $z = \sqrt{3} - i$



$\operatorname{Re} z = \sqrt{3}$

$\operatorname{Im} z = -1$

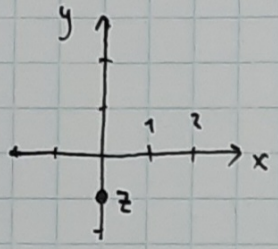
$|z| = 2$

$\arg z = \operatorname{arctg} -\frac{1}{\sqrt{3}} = -\frac{\pi}{6}$

$z = 2\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$

$z = 2e^{i\pi/6}$

2) $z = -i$



$\operatorname{Re} z = 0$

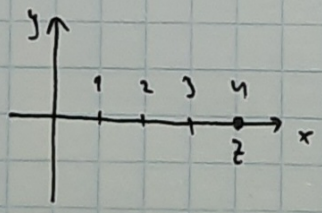
$\operatorname{Im} z = -1$

$|z| = 1$

$\arg z = -\frac{\pi}{2}$

$z = \cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$ $\quad z = e^{i\pi/2}$

g) $z = 4$



$\operatorname{Re} z = 4$ $\quad \operatorname{Im} z = 0$ $\quad |z| = 4$

$\arg z = 0$

$z = 4(\cos 0 + i\sin 0)$

$z = 4e^{0i}$

② a) $z = 2 \pm 5i$

$|z| = \sqrt{4+25} = \sqrt{29}$

$\arg z = \operatorname{arctg} \pm \frac{5}{2}$

b) $z = -2 \pm 5i$

$|z| = \sqrt{29}$

$\arg z = \begin{cases} \operatorname{arctg} \pm \frac{5}{2} \\ \operatorname{arctg} -\frac{5}{2} + \pi \\ \operatorname{arctg} \frac{5}{2} - \pi \end{cases}$

c) $z = ib, \quad b \in \mathbb{R}, \quad b \neq 0$

$|z| = |b|$

$\arg z = \pm \frac{\pi}{2}$

d) $z = e^{-5i}$

$|z| = 1, \quad \arg z = -5$

③ $z = \frac{5+i}{1+2i} = \frac{(5+i)(1-2i)}{1-4} = \frac{5-10i+i+2}{-3} = -\frac{7}{3} + 3i$

$\operatorname{Re} z = -\frac{7}{3} \qquad \operatorname{Im} z = 3$

④ $x(1-2i) + y(2i-3) = 4-8i$

$x - 2ix + 2iy - 3y = 4 - 8i$

$\begin{cases} x - 3y = 4 \\ -2ix + 2iy = -8i \end{cases}$ $\begin{cases} x = 4 + 3y \\ y - x = -4 \end{cases}$ $y - 4 - 3y = -4$

$-2y = 0$ $\begin{cases} y = 0 \\ x = 4 \end{cases}$

⑤ $\begin{cases} z_1 = 1 - 2i \\ z_2 = -2 + i \end{cases}$

$z_1 + z_2 = (1-2) + i(-2+1) = -1 - i$

$z_1 - z_2 = (1+2) + i(-2-1) = 3 - 3i$

$z_1 \cdot z_2 = -2 + 2 + i(1+4) = 5i$

$\frac{z_1}{z_2} = \frac{(1-2i)}{(-2+i)} = \frac{(1-2i)(-2-i)}{4+1} =$

$= \frac{-2 - i + 4i - 2}{5} = -\frac{4}{5} + \frac{3i}{5}$

⑥ $z_1 = 3 + 3i$

$|z_1| = \sqrt{18} = 3\sqrt{2}$

$\arg z_1 = \operatorname{arctg} 1 = \frac{\pi}{4}$

$z_1 = 3\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = 3\sqrt{2}\, e^{i\frac{\pi}{4}}$

$z_2 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

$|z_2| = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$

$\arg z_2 = \operatorname{arctg}\sqrt{3} - \pi = \frac{\pi}{3} - \pi = -\frac{2\pi}{3}$

$z_2 = \cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3} = e^{-i2\pi/3}$

$z_3 = 2\sqrt{3} - 2i$

$|z_3| = \sqrt{12+4} = 4$

$\arg z_3 = \operatorname{arctg} -\frac{1}{\sqrt{3}} + \pi = -\frac{\pi}{6} + \pi = \frac{5\pi}{6}$

$z_3 = 4\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) = 4e^{i5\pi/6}$

$$z = \frac{-\frac{3}{2} + \frac{3\sqrt{3}}{2} + i\left(-\frac{3\sqrt{3}}{2} - \frac{3}{2}\right)}{2\sqrt{3} - 2i} = \frac{\frac{3}{2}(\sqrt{3}-1) - i\frac{3}{2}(\sqrt{3}+1)}{2\sqrt{3} - 2i} =$$

$$= \frac{\left(\frac{3}{2}(\sqrt{3}-1) - i\frac{3}{2}(\sqrt{3}+1)\right)(2\sqrt{3} + 2i)}{12 + 4} =$$

$$= \left(3 - 3 - 3\sqrt{3} + 3\sqrt{3}i - 6i - 9i - 3\sqrt{3}i + 3\sqrt{3} + 3\right)\frac{1}{16} =$$

$$= \frac{12}{16} - \frac{15}{16}i$$