Linear models with random and fixed effects

Consider

- A dataset

y	x
4	A
3	A
2	B
1	B

- A model M_1 of the form (with the use of lm notation) `y~x`.
- A model M_2 of the form (with the use of lmer notation) `y~(1|x)`.

Solve

1. Find the design matrix X for model M_1,
2. Find the joint distribution of $(\hat{\mu}, \hat{\beta}_X)$ for model M_1 and data from Table 1. Draw contours of this distribution. Here you may assume that μ and β_X are known,
3. Find the distribution of prediction $\hat{y}$ for $x = A$ under model M_1 and data from Table 1,
4. Find the subspace K orthogonal to subspace spanned by design matrix X derived for model M_2,
5. Write down the REML likelihood for model M_2 and data presented in Table 1,
6. Write down Henderson equations for model M_2 and data presented in Table 1. Here you may use R to find numerical estimate for σ_x^2.

Hint

If some additional assumptions are **really** needed to solve any of following task you can introduce them. But justify it.

Literature

Feel free to use any source. Everything related to our course is in

- *Analiza danych z programem R, Modele liniowe z efektami stałymi, losowymi i mieszanymi*, Biecek Przemysław, PWN, 2011.
- *Linear Mixed-Effects Models Using R*, Gałecki Andrzej, Burzykowski Tomasz, Springer, 2013