Ec141, Spring 2019

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Review Sheet 3

This review sheet is designed to assist you in your exam preparations. I suggest preparing written answers to each question. You may find it useful to study with your classmates (indeed I encourage you to do so and also to be generous with one another as you prepare). In the exam you may bring in a single 8.5 x 11 sheet of notes. No calculators or other aides will be permitted. Please bring blue books to the exam. The midterm exam will occur in class on Thursday, May 2nd.

[1] This question is inspired by the Moving to Opportunity (MTO) social mobility experiment. Consider a population of public housing residents living in high poverty neighborhoods at baseline. As part of a social experiment, each resident is randomly assigned to receive a restricted housing voucher ($Z = 1$) or not ($Z = 0$). The voucher subsidizes rent for any housing unit located in a low poverty neighborhood; the voucher cannot be used in a high poverty neighborhood. At follow-up respondents either live in a low poverty neighborhood ($W = 1$) or not ($W = 0$). Let Y be an outcome of interest, for example earnings or a measure of academic achievement. We will call respondents in low poverty neighborhoods "treated" and all others "controls".

[a] Let $W(z)$ for $z \in \{0, 1\}$ denote each a respondent's treatment assignment given "encouragement" $Z = z$. Consider the following table:

	$W(0)$	$W(1)$
Complier	0	1
Defier	1	0
Always-taker	1	1
Never-taker	0	0

Explain how this table divides the population into four subpopulations. Describe these subpopulations in words.

[b] Let $Y(w, z)$ denote a unit's potential outcome given treatment $W = w$ and encouragement $Z = z$. Explain, in words, the restriction that $Y(w, z) = Y(w)$ for $w \in \{0, 1\}$ and $z \in \{0, 1\}$. Is this restriction reasonable in the present context?

[c] Assume that $W(1) \geq W(0)$. What type of behavior does this restriction rule out. It is reasonable in the present context?

[d] Explain how random assignment ensures that

$$(Y(0), Y(1), W(0), W(1)) \perp Z = z \text{ for } z \in \{0, 1\}.$$

What if, instead of random assignment, vouchers were allocated to households by a case-worker? How might the above restriction be violated?

[e] Show that, under the restrictions outlined above that

$$\mathbb{E}[W|Z = 1] - \mathbb{E}[W|Z = 0] = \Pr(W(0) = 0, W(1) = 1).$$

[f] Show that, under the restrictions outlined above that

$$\begin{aligned}\beta_{\mathrm{WALD}} &= \frac{\mathbb{E}\left[\left.Y\right|Z=1\right]-\mathbb{E}\left[\left.Y\right|Z=0\right]}{\mathbb{E}\left[\left.W\right|Z=1\right]-\mathbb{E}\left[\left.W\right|Z=0\right]} \\ &= \mathbb{E}\left[\left.Y\left(1\right)-Y\left(0\right)\right|W\left(0\right)=0,W\left(1\right)=1\right].\end{aligned}$$

Comment on your result? How does your interpretation depend on the magnitude of $\Pr\left(W\left(0\right)=0,W\left(1\right)=1\right)$?

[f] Show that if $\Pr\left(W\left(0\right)=1,W\left(1\right)=1\right)=0$ that

$$\beta_{\mathrm{WALD}}=\mathbb{E}\left[\left.Y\left(1\right)-Y\left(0\right)\right|W=1\right].$$

Is this likely to be true in the present context? Is a public housing recipient likely to be able to move to a low poverty neighborhood without voucher support?