Question 1.60

$C_k = \Sigma^* a \Sigma^{k-1}$ $\quad \Sigma = \{a, b\}$

Create an NFA with k+1 states that recognizes C_k

Let NFA M have k+1 states $Q = \{0 \text{ to } K\}$

starting state $q_0 = 0$, Final state

A state in Q refers to how many of the last k bits M has read.

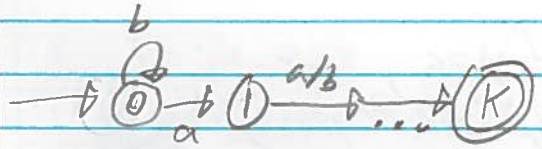
State functions are as follows

While in 0, stay in 0 until an a appears $\delta(0, b) = 0$

When an a appears, assume it is k away from the end $\delta(0, a) = \{0, 1\}$

This continues until state K $\delta(i-1, a \cup b) = i$ for $2 \leq i \leq k$

It reaches state k and accepts iff there are k-1 inputs from the transition input from 0 to 1



1.61 Prove C_k cannot be accepted by any DFA with $Q = 2^k$

let there be two different inputs c and d which are numbered $c_1 \ldots c_k$ and $d_1 \ldots d_k$. since they are different for some i, (one must be a and the other b.

let e be the the following inputs that distinguish ce and de as the kth member of the string.

These rules make all inputs of length K fully distinguishable. The DFA must have 2^k states or more.