Question 1.54

$F = \{a^i b^j c^k \mid i,j,k \geq 0 \text{ and if } i = 1, \text{ then } j = k\}$

(a) Show F is not Regular

Myhill Nerode

We claim all strings of b^i must be distinct for $i \geq 0$

Two strings ab^{i_1} and ab^{i_2} can be distinguished by c^i

$ab^{i_1}c^{i_1} \in F$ but $ab^{i_2}c^{i_1} \notin F$

Because all strings have distinct equivalences classes, there are infinitely many, therefore no DFA can accept it.

(b) Show that the pumping Lemma would accept F

Pumping Lemma: with pumping length p $s = xyz$

$|xy| \leq p$ s.t. $xy^i z$ is in the language for $i \geq 0$

let $p = 2$

if $i = 1$ then x is empty and $y = a$, since $i = 1$ then $j = k$, adding more a's by pumping it is always within language.

Let $i = 2$, $x = \varepsilon$, $y = aa$ there is always an even number of a's, therefore within the language

if $i = 0$ all strings of $b^j c^k$ are within the language. pumping still applies