5.31

$f(x)\begin{cases} 3x+1 & \text{for odd } x \\ x/2 & \text{for even } x \end{cases}$ for $\forall x \in N$

Stop when 1 is reached.

the question of whether all positive starting points end up at 1; it is called 3x+1 prob.

Suppose A_{TM} were decidable by a TM H. use H to describe a TM that is guaranteed to state an answer

Turing machine for terminate on 1

TM Q: Input <x>

1. If x=1, ACCEPT
2. if x is even, x is now $x/2$. if x is odd, x is now 3x+1
3. Loop to step 1

If Q finds what it's looking for, it will never terminate.

We now build another Turing machine that tests inputs through Q. TM L will loop through all natural numbers.

TM L: Input <Doesnt matter>

1. For each $x \in N$ [1, 2, 3 ...]
2. Run H on <Q, x>
3. If H rejects, ACCEPT. Other, Loop.

If we simulate L, if there is no counterexample it will never terminate. Another TM is needed

TM F: Input <Doesn't matter>

1. Run H on <L, ε>
2. If H accepts, then ACCEPT, else REJECT

If TM F accepts a counterexample has been found.