Aleksas Murauskas 260718389

4.13 Let $A = \{\langle R, S\rangle \mid R$ & S are regular expressions and $L(R) \subseteq L(S)\}$. Show A is decidable

$L(R) \subseteq L(S)$ iff $\forall w \in L(R), w \in L(S)$
iff $\forall w \in L(R), w \notin \overline{L(S)}$
iff $L(R) \cap \overline{L(S)}$

Need DFA D s.t.
$L(D) = L(R) \cap \overline{L(S)}$

This can be constructed by converting the regular expression to NFA (Thm 1.54)
The closure of regular languages is true under complement and intersection.
Then convert NFA to DFA D

We can use Thm 4.4 to test if $L(D)$ is empty

Thm 4.4 = For DFA (B), L(B) denote language of B
$E_{DFA} = \{\langle B\rangle : B$ is a DFA and $L(B) = \emptyset\}$
is decidable

We can design turing machine s.TM X that decides A
Input: $\langle R, S\rangle$ when R & S are reg expressions
1. Construct DFA D as described
2. Run TM T to use Thm 4.4 on input $\langle D\rangle$
3. if T accepts, ACCEPT. Otherwise, REJECT