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Question 1.47

Let  $\Sigma = \{1, \#\}$

$Y = \{w \mid w = x_1 \# x_2 \# \dots \# x_k, k \geq 0, \text{ each } x_i \in 1^*, x_i \neq x_j, (i \neq j)\}$

Prove  $Y$  is irregular

Pumping Lemma

$|s| \geq p$   $s$  can be written as  $s = xyz$

1.  $|xy| \leq p$

2.  $|y| > 0$

3.  $xy^i z \in Y$  for all  $i \geq 0$

If  $Y$  were regular then its complement is regular

Let  $X = \{1^n \# 1^n \mid n \geq 0\}$

let  $w = xyz$   $|w| \geq k$

$|xy| \leq k$   $|y| \geq 1$

$x = 1^r, y = 1^s, z = 1^{k-r-s} \# 1^k$

$\Leftrightarrow$  long as  $r+s \leq k$

By 3rd rule  $i=0$   $xy^0 z = 1^{k-r} \# 1^k$  this does not equal  $1^n \# 1^n$  since  $r$  must be greater than 1.

Since  $X$  is not regular,  $Y$  is not regular