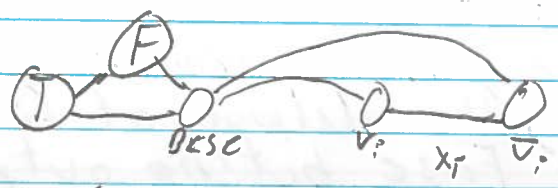
7.27 Show 3 colour is NP complete

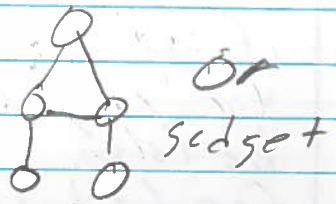
Prove NP:
Certifier: for each node a colour from 1-3
Check for each edge $e(u,v)$ if $u_{color} \neq v_{color}$
This is polynomial time

To prove completeness
$3\text{-SAT} \leq_p 3\text{-colour}$

CNF ϕ has n variables x_n and m clauses, C_m
We can create a graph s.t G_ϕ is 3colourable iff
ϕ is satisfiable

Parts:





Given $C = (a \vee b \vee c)$
if one of abc is colored true then OR
gadget can be 3 colored s.t. output node
of OR-gadget is true.

if a, b, c are colored false in a 3 color then output
of OR gadget is True

Example
$\phi = (a \vee \bar{b} \vee c)$