5.21 Let $AMBIG_{CFG} = \{\langle G \rangle \mid G \text{ is ambiguous CFG}\}$
Show $AMBIG$ is undecidable
Reduce from PCP

$P = \frac{t_1}{b_1}, \frac{t_2}{b_2}, \ldots \frac{t_k}{b_k}$

CFG with rules $S \to T \mid B$
G $T \to t_1 T a_1 \mid \ldots \mid t_k T a_k \mid t_1 a_1 \mid \ldots \mid t_k a_k$
$B \to b_1 B a_1 \mid \ldots \mid b_k B a_k \mid b_1 a_1 \mid \ldots \mid b_k a_k$

P matches if $t_{i_1} \ldots t_{i_n} = b_{i_1} \ldots b_{i_n}$

$t_{i_1} \ldots t_{i_n} a_{i_n} \ldots a_{i_1}$ can be derived from T or B

If a CFG G is ambiguous, some string x can be derived in multiple ways.
x can be written as $y a_{j_1} \ldots a_{j_m}$ for some y that doesn't contain a_is.
with G's grammar one string in the form x can be derived by T and B
The Derivations are as follows
$S \to T \overset{*}{\Rightarrow} s = t_{j_m} \ldots t_{j_1} a_{j_1} \ldots a_{j_m}$
$S \to B \overset{*}{\Rightarrow} s = b_{j_m} \ldots b_{j_1} a_{j_1} \ldots a_{j_m}$

Therefore, $t_{j_m} \ldots t_{j_1} = b_{j_m} \ldots b_{j_1}$ we get a match for P

We have found:
P has a match $\Leftrightarrow$ G is ambiguous
This reduction works, therefore AMBIG is undecidable.