

7.21 Let  $\text{Double-SAT} = \{ \langle \phi \rangle \mid \phi \text{ has at least 2 satisfying assignments} \}$ . Show NPComplete

Double SAT can be easily certified, since the truth assignment is the certificate, and clauses can be checked in poly time.

We can reduce 3sat to double sat. Given a 3cnf function  $\psi$ , create new CNF  $\psi'$   
 $\psi' = \psi \wedge (x_n \vee \bar{x}_n)$  ( $x_n$  is not used in  $\psi$ ).

This reduction is clearly poly time  
we now see

$\psi'$  has a satisfying assignment  $\leftrightarrow \psi$  has two satisfying assignments.

If  $\psi'$  has satisfying assignment  $\leftrightarrow$  it has two satisfying assignments.

in  $\psi'$ , if  $\psi$  has any solution, then the clause  $(x_n \vee \bar{x}_n)$  will always provide at least two satisfying assignments  $x_n = \text{true}$  and  $x_n = \text{false}$

Now the other direction

If there are two satisfying assignments  $\leftrightarrow \psi$  has a satisfying assignment.  
this is trivial.

In either case Double SAT and 3SAT are validated.