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4.4 Let $A\varepsilon CFG = \{\langle G \rangle \mid G$ is a CFG that generates ε

Show that $A\varepsilon CFG$ is decidable

Answer:

If a Context free Grammar $G=(V, \Sigma, R, S)$ includes the rule $S \to \varepsilon$, then G can generate ε.

There are still some CFGs that can result in ε without the rule $S \to \varepsilon$

ie. CFG H has rules: $S \to AB$, $A \to xB \mid \varepsilon$, $B \to xyA \mid \varepsilon$

H can be derived: $S \to AB \to \varepsilon\varepsilon \to \varepsilon$

therefore $\varepsilon \in L(H)$, and H doesn't have the rule $S \to \varepsilon$.

We cannot simply check if a CFG contains $S \to \varepsilon$.

Let $G' = (V', \Sigma, R', S')$ in Chomsky Normal Form then G' can generate ε iff G' contains the rule $S' \to \varepsilon$.

If we first convert G into an equivalent G'

If $S' \to \varepsilon$ is a rule of G' then G' generates ε. G must also generate ε since $L(G) = L(G')$

With G' in Chomsky Normal form the only possible ε-rule in G' is $S' \to \varepsilon$, so the only way $\varepsilon \in L(G')$ if G' has $S' \to \varepsilon$ in R'.

If G' doesn't include $S' \to \varepsilon$ then $\varepsilon \notin L(G')$ and then $\varepsilon \notin L(G)$

A machine decides $A_{\varepsilon CFG}$ if it follows:

$M =$ "On Input $\langle G \rangle$, where G is CFG:

1. Convert G to equivalent $G' = (V', \Sigma, R', S')$ in Chomsky
2. If G' includes rule $S' \to \varepsilon$, accept. Otherwise REJECT