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5.16 $\Gamma = \{0, 1, \sqcup\}$ Busy Beaver function

$BB: \mathcal{N} \to \mathcal{N}$. For each value of k, consider all k-state TM's that halt when started with blank tape. Let $BB(k)$ be the max number of 1's that remain on tape among all of these machines. Show BB is not computable.

Suppose BB is computable and find a contradiction:

Let C be a TM that, on input 1^n, halts with $1^{BB(n)}$ on tape $\forall n$, since BB is computable.

Now we build TM M that halts when started with blank tape

TM M:

1. write n 1's on tape
2. double the 1s on the tape
3. Simulate F with input 1^{2n}

TM M will always halt on $BB(2n)$ 1s when started with blank tape

For implementation M needs at most n states for step 1. Step 2 and 3 needs c states for constant c.

$BB(n+c)$ = max # of 1's that $(n+c)$ state TMs will halt with, which is the min # of 1s M halts with

This implies $BB(n+c) \geq BB(2n)$ $\forall n$.

But $BB(k)$ is strictly increasing, therefore $BB(n+c) < BB(2n)$ when $n > c$

This is a contradiction, therefore $BB(k)$ is not computable