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Q6. Use complementary slackness

$$\begin{aligned} \max \quad & 3.1x_1 + 10x_2 + 8x_3 - 45.2x_4 + 18x_5 \\ \text{s.t.} \quad & x_1 + x_2 + x_3 - x_4 + 2x_5 \leq 5 \\ & 2x_1 - 4x_2 + 1.2x_3 + 2x_4 + 7x_5 \leq 16 \\ & x_1 + x_2 - 3x_3 - x_4 - 10x_5 \leq -20 \\ & 3x_1 + x_2 + 3x_3 + \frac{7}{2}x_4 + \frac{7}{3}x_5 \leq 10 \\ & x_2 + x_3 + 6x_4 + 2x_5 \leq 4.5 \\ & 2x_3 - x_4 + x_5 \leq 2 \\ & x_1, x_5 \geq 0 \end{aligned}$$

Dual is

$$\begin{aligned} \min \quad & 5y_1 + 16y_2 - 20y_3 + 10y_4 + 4.5y_5 + 2y_6 \\ \text{s.t.} \quad & y_1 + 2y_2 + y_3 + 3y_4 \geq 3.1 \\ & y_1 - 4y_2 + y_3 + 1y_4 + y_5 + 2y_6 \geq 10 \\ & y_1 + 1.2y_2 - 3y_3 + 3y_4 + y_5 \geq 8 \\ & -y_1 + 2y_2 - y_3 + 1.5y_4 + 6y_5 \geq -45.2 \\ & 2y_1 + 7y_2 - 10y_3 + \frac{7}{3}y_4 + 2y_5 + y_6 \geq 18 \end{aligned}$$

Using the thm

If the primal is feasible and the cost is bounded, the dual is as well and its cost is also bounded. Additionally, their optimum are the same.

Since both the primal and dual are feasible and bound they share optimum solns.

Therefore,  $x_1 = x_2 = 0.5$ ,  $x_3 = x_4 = 0$ ,  $x_5 = 2$  is optimal