

Q2. Alg VI

$$\min \sum_{u \in V} x_u$$

$$\text{s.t. } x_u + x_v + x_w \geq 1 \quad \forall \Delta(u, v, w) \in G$$

$$0 \leq x_u \leq 1$$

$$\forall x_u \in V$$

Σ_x



$$\text{opt(LP)} = 7/3$$

$$\text{opt(ILP)} = 1$$

set every x_u 0 or 1

Let x^* be opt soln of linear prog

$$\text{let } \hat{x}_u = \begin{cases} 1 & \text{if } x_u^* \geq 1/3 \\ 0 & \text{if } x_u^* < 1/3 \end{cases}$$

$$\text{by } x_u + x_v + x_w \geq 1 \quad x_u, x_v \geq 1/3 \text{ or } x_w, x_v \geq 1/3$$

$$\text{or } x_u, x_w \geq 1/3$$

At least one or two of every \hat{x}_u or \hat{x}_v or \hat{x}_w is 1

$$\text{cost} = \sum_{u \in V} \hat{x}_u \text{ comp. to cost of } x^* \leq 3 \sum_{u \in V} x_u^*$$

$$\text{because } \hat{x}_u \leq 3x_u^*$$

$$\text{Output} \leq 3 \cdot \text{opt(LP)} \leq 3 \text{ (Min } V \text{ removed for no } \Delta)$$

$$7/3 \leq 3 \cdot 1 \leq 3$$

Therefore 3 factor