a) Fourier transform

$$r(t) = \begin{cases} k, & |t-c| \le a/2 \\ 0, & |t-c| > a/2 \end{cases}$$

Property of Fourier transform:

$F(t) \to F(\omega)$, for $f(t-T) \to F(\omega) \cdot e^{-i\omega T}$

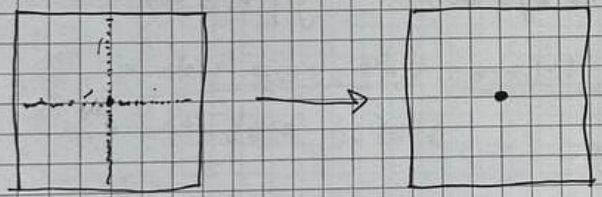
$$f(t) = \begin{cases} k, & |t| \le a/2 \\ 0, & |t| > a/2. \end{cases}$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt = \int_{-a/2}^{a/2} k e^{-i\omega t}\, dt = \frac{k}{i\omega}\left[e^{\frac{i\omega a}{2}} - e^{-\frac{i\omega a}{2}}\right] =$$

$$= ka\,\mathrm{sinc}\left(\frac{\omega a}{2}\right)$$

$$f(t) = \begin{cases} k, & |t-c| \le a/2 \\ 0, & |t-c| > a/2 \end{cases} \Rightarrow \underline{F(\omega) = ka\,\mathrm{sinc}\left(\frac{\omega a}{2}\right) \cdot e^{-i\omega c}}$$

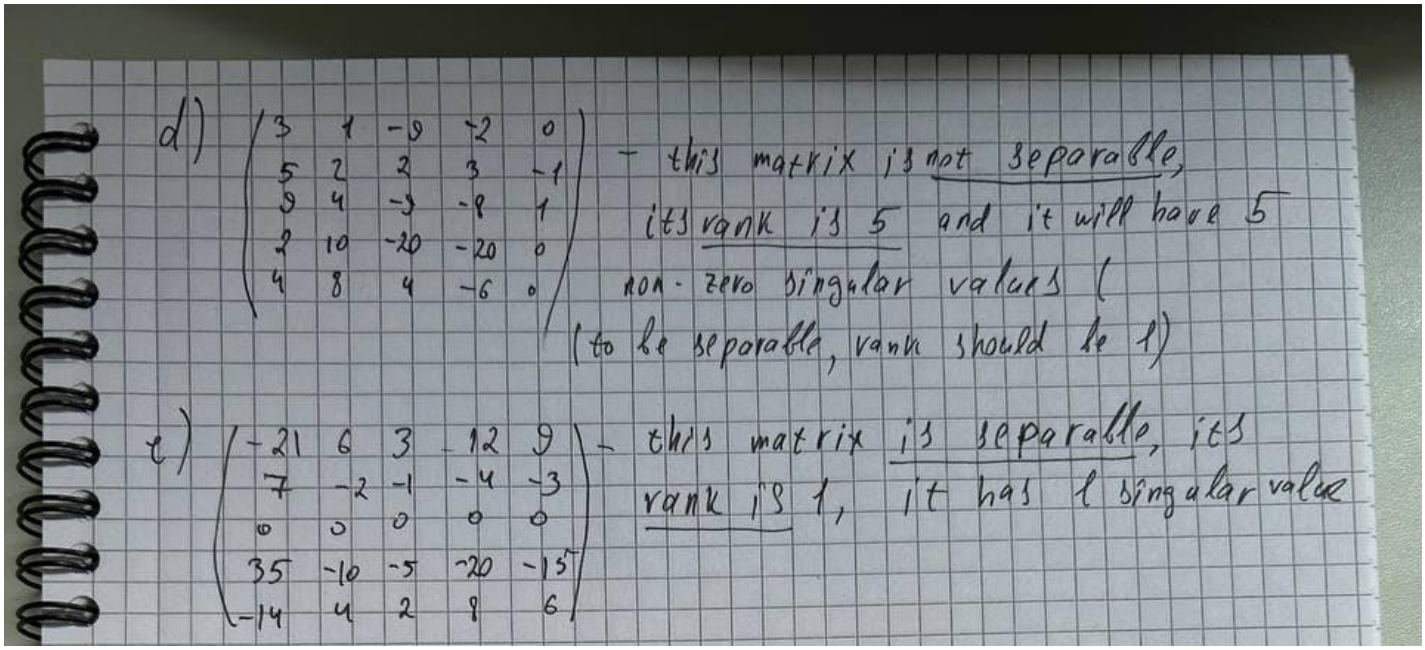
b)



the frequency spectrum looks like a sinc function in both x and y directions. The inverse transform of a sinc is a rectangle. Since the lines in the frequency spectrum are thin, the original function should be a rectangular dot (or a very small square)

c) Convolution theorem: the fourier transform of a convolution of two functions is the pointwise product of their fourier transform

The fourier transform of a delta function $\delta(x) = \begin{cases} 1, & x=0 \\ 0, & x \neq 0 \end{cases}$ is 1, so its convolution with a function or image will not change the frequency of a signal or image




```
    matrix = [
[-21, 6, 3, 12, 9],
[7, -2, -1, -4, -3],
[0, 0, 0, 0, 0],
[35, -10, -5, -20, -15],
[-14, 4, 2, 8, 6]
    ]
    a, b, c = np.linalg.svd(matrix)

    value = b[0]
    v1 = a[:, 0] * np.sqrt(value)
    v2 = c[0] * np.sqrt(value)

    print('v1', v1)
    print('v2', v2)
    print('v1 x v2\n', np.round(np.outer(v1, v2), 2))
```

```
v1 [-3.57900146  1.19300049  0.          5.96500244 -2.38600097]
v2 [ 5.86755837 -1.67644525 -0.83822262 -3.3528905  -2.51466787]
v1 x v2
 [[-21.   6.   3.  12.   9.]
 [  7.  -2.  -1.  -4.  -3.]
 [  0.  -0.  -0.  -0.  -0.]
 [ 35. -10.  -5. -20. -15.]
 [-14.   4.   2.   8.   6.]]
```

1) Distance transform 1D:

1) initialize $D[j] \to 0$ if j in P, otherwise ∞

2) Forward: for j from 1 to $n-1$

$$D[j] = \min(D[j], D[j-1]+1)$$

3) Backward: for j from $n-2$ to 0

$$D[j] = \min(D[j], D[j+1]+1)$$

2D case: Fwd pass finds closest above and to left

Bwd finds closest below and to right

init → Fwd → Bwd