Probability & Statistics. Final Examination. Problem Set 1. Part 1

Name ______________________________ **Group Number** ________

Solutions for each part of this examination have to be written on separate sheets! Do not write solutions of two different parts of this examination on one sheet of paper. You can use a single sheet for several problems from one part of the examination (i.e. you can write problems 1, 2, 3, 4 on one sheet). Do not forget to sign each of the sheets that you hand in. You can use this sheet for solving problems as well.

I am aware that using any electronic devices, books etc. at the exam as well as communicating with other students is strictly prohibited. Only two handwritten informational sheets are allowed at the examination. Any violation of these rules immediately leads to exam cancellation. Signature

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1. Two players take turns in rolling a 10-sided fair die. The first of them to get a result of one wins the game. Find the probability for each of the players to win the game.

2. A regular hexagon inscribed into a circle divides it into seven parts. Seven points are chosen at random inside the circle. Find the probability that there is exactly one point in each of the seven parts of the circle.

3. 10 000 people take part in the marathon. The probability to run all the way to the finish is equal to 0,64. (a) Find[1] the probability that at least 6 500 people get to the finish. (b) Let ξ be a number of people who got to the finish. Find the shortest possible interval that contains the value of ξ with probability of 95%.

4. There are three red balls and nine green balls in the first urn, and there are five red balls and seven yellow balls in the second urn. Two balls from the first urn and two balls from the second urn have been transferred into the third urn that had initially been empty. It has happened that the third urn contains balls of all three different colours. Find the probability that the balls transferred from the first urn were of different colours.

[1] You should provide an approximate numerical answer for both questions in this problem.

Probability & Statistics. Final Examination. Problem Set 1. Part 2

Name ______________________________ **Group Number** ________

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5. ξ is uniformly distributed over some interval, and it is known that $E\xi \leqslant 10$, $P(\xi \leqslant 14) = \frac{6}{7}$. Find the smallest possible value of the variance of ξ.

6. $\xi_1, \xi_2, \ldots, \xi_{50}$ are independent $N(0;1)$ random variables; $\eta_1 = \sum\limits_{i=1}^{50} \xi_i$; $\eta_2 = \sum\limits_{i=41}^{50} \xi_i$. Find the joint probability density of η_1 and η_2.

7. Using maximum likelihood method find the estimator for the unknown parameter θ of an exponential distribution. Is this estimator (a) unbiased; (b) consistent? Justify your answer.

8. Let ξ and η be continuously distributed random variables, their joint distribution density given by $f_{\xi,\eta}(x;y)$. Prove that their ratio $\zeta = \frac{\xi}{\eta}$ has a probability density of

$$f_\zeta(z) = -\int\limits_{-\infty}^{0} tf(zt,t)\,dt + \int\limits_{0}^{+\infty} tf(zt,t)\,dt.$$

Probability & Statistics. Final Examination. Problem Set 2. Part 1

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1. Two players take turns in rolling a 12-sided fair die. The first of them to get a result of one wins the game. Find the probability for each of the players to win the game.

2. A regular triangle inscribed into a circle divides it into four parts. Four points are chosen at random inside the circle. Find the probability that there is exactly one point in each of the four parts of the circle.

3. 1 900 people take part in the marathon. The probability to run all the way to the finish is equal to 0,81. (a) Find[2] the probability that no more than 1 500 people get to the finish. (b) Let ξ be a number of people who got to the finish. Find the shortest possible interval that contains the value of ξ with probability of 99%.

4. There are five red balls and seven green balls in the first urn, and there are ten red balls and two yellow balls in the second urn. Two balls from the first urn and two balls from the second urn have been transferred into the third urn that had initially been empty. It has happened that the third urn contains balls of all three different colours. Find the probability that the balls transferred from the first urn were of different colours.

[2] You should provide an approximate numerical answer for both questions in this problem.

Probability & Statistics. Final Examination. Problem Set 2. Part 2

Name ______________________________ **Group Number** ________

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5. ξ is uniformly distributed over some interval, and it is known that $E\xi \leqslant 7$, $P(\xi \leqslant 11) = \frac{5}{9}$. Find the smallest possible value of the variance of ξ.

6. $\xi_1, \xi_2, \dots, \xi_{40}$ are independent $N(0;1)$ random variables; $\eta_1 = \sum\limits_{i=1}^{40} \xi_i$; $\eta_2 = \sum\limits_{i=26}^{40} \xi_i$. Find the joint probability density of η_1 and η_2.

7. Using maximum likelihood method find the estimator for the unknown parameter θ of a Poisson distribution. Is this estimator (a) unbiased; (b) consistent? Justify your answer.

8. Let ξ and η be continuously distributed random variables, their joint distribution density given by $f_{\xi,\eta}(x;y)$. Prove that their product $\zeta = \xi\eta$ has a probability density of

$$f_\zeta(z) = -\int\limits_{-\infty}^{0} \frac{1}{t} f\left(\frac{z}{t}, t\right)\, dt + \int\limits_{0}^{+\infty} \frac{1}{t} f\left(\frac{z}{t}, t\right)\, dt.$$