October 22, 2020

Assignment 8, #4.

$p = 0{,}7$, $q = 0{,}3$, $n = 5000$

a) $P(3450 \text{ for } A) = ?$ $\qquad 3450 - 1550$

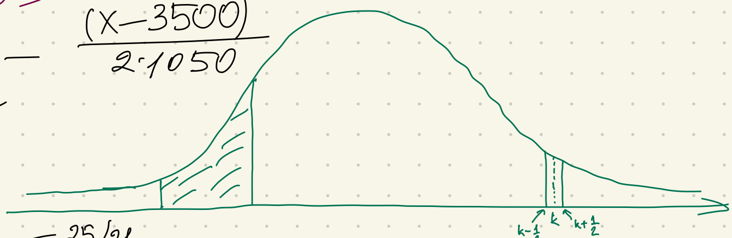
$E\xi = np = 3500$, $\quad Var\,\xi = npq = 1050$

$\xi \sim N(3500;\ 1050)$

local Moivre–Laplace theorem

$$P(\xi = 3450) \approx f_\xi(3450)$$

$$f_\xi(x) = \frac{1}{\sqrt{2\pi}\sqrt{1050}}\, e^{-\frac{(x-3500)^2}{2\cdot 1050}}$$



$$f_\xi(3450) = \frac{1}{\sqrt{2100\pi}}\, e^{-25/21} \approx$$

$$t = \frac{x-3500}{\sqrt{1050}}$$

$$P(\xi > 3450) \approx \int_{3450}^{+\infty\ 5000} \frac{1}{\sqrt{2\pi\cdot 1050}} \cdot e^{-\frac{(x-3500)^2}{2\cdot 1050}}\, dx =$$

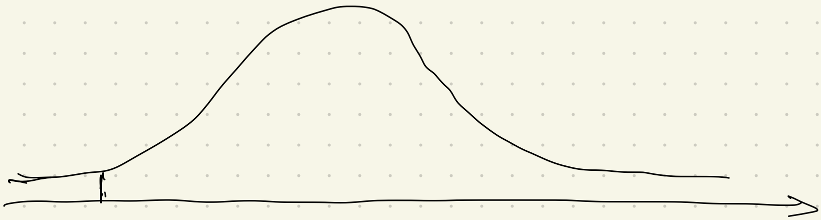
$$= \int_{-\frac{50}{\sqrt{1050}}}^{+\infty\ \frac{1500}{\sqrt{1050}} \approx 46{,}29} \frac{1}{\sqrt{2\pi}}\, e^{-t^2/2}\, dt = \Phi\left(\frac{50}{\sqrt{1050}}\right) \approx 0{,}5 + \Phi_0(1{,}543) \approx$$

$$\approx 0{,}5 + 0{,}4386 = 0{,}9386$$

$\Phi(1{,}54) = 0{,}4382$

$\Phi(1{,}55) = 0{,}4394$

$0{,}4382 + 0{,}3(0{,}012)$



$$\frac{-50}{\sqrt{1050}}$$

$$\Phi_0\left(\frac{50}{\sqrt{1050}}\right) + \Phi_0(46{,}29)$$

$$\Phi_0(3) = 0{,}4986$$

$$\Phi_0(4) = 0{,}499968$$

$$\Phi_0(5) = 0{,}4999997$$

5) $P = 95\%$ 　　　　　　　　　　　　　　　　　Xi

"success" = choosing the first cloakroom

$p = 0,5$ 　　　 $q = 0,5$

$500 + k$ places in the cloakroom

ξ = number of pairs who choose the first cloakroom

$$P(500 - k \le 2\xi \le 500 + k) \geq 0,95$$

$\xi \sim Bin(500;\ 0,5)$ 　　　 $E\xi = 250$, $Var\,\xi = 125$

$$\xi \sim N(250;\ 125)$$

$$P\left(-k/2 \le \xi - 250 \le k/2\right) \geq 0,95$$

$$P\left(-\frac{k}{10\sqrt{5}} \le \frac{\xi - 250}{\sqrt{125}} \le \frac{k}{10\sqrt{5}}\right) \geq 0,95$$

$$2\Phi_0\left(\frac{k}{10\sqrt{5}}\right) \geq 0,95$$

$$\Phi_0\left(\frac{k}{10\sqrt{5}}\right) \geq 0,475, \quad \frac{k}{10\sqrt{5}} \geq 1,96$$

$$k \geq 43,83 \qquad k = 44$$

544

$$P(500-k \le \xi \le 500+k) =$$

$$\xi \sim Bin(1000\,;\,0{,}5)$$

$$E\xi = 500,\quad Var\xi = 250$$

$$= P(-k < \xi - 500 < k) = P\left(-\frac{k}{\sqrt{250}} < \frac{\xi - 500}{\sqrt{250}} < \frac{k}{\sqrt{250}}\right)$$

$$= 2\Phi_0\left(\frac{k}{\sqrt{250}}\right) \ge 0{,}95$$

$$\frac{k}{\sqrt{250}} \ge 1{,}96,\quad k \ge 30{,}99$$

531

$\xi \sim Bin(n, p)$

$n \to \infty$, $p \to 0$, $np \to \lambda = const > 0$

$p = \frac{\lambda}{n} + o\left(\frac{1}{n}\right)$, $n \to \infty$.

Then $P(\xi = k) \xrightarrow{n\to\infty} e^{-\lambda} \cdot \frac{\lambda^k}{k!}$

$\binom{n}{k} p^k (1-p)^{n-k} = \frac{n!}{k!(n-k)!} \left(\frac{\lambda}{n} + o\left(\frac{1}{n}\right)\right)^k \cdot$

$\cdot \left(1 - \frac{\lambda}{n} + o\left(\frac{1}{n}\right)\right)^{n-k} = \frac{1}{k!} \cdot n(n-1) \ldots (n-k+1) \cdot \frac{(\lambda + o(1))^k}{n^k} \cdot$

$\cdot \left(1 - \frac{\lambda}{n} + o\left(\frac{1}{n}\right)\right)^n \cdot \left(1 - \frac{\lambda}{n} + o\left(\frac{1}{n}\right)\right)^{-k} \xrightarrow{n\to\infty}$

$\frac{n}{n} \cdot \frac{n-1}{n} \cdot \ldots \cdot \frac{n-k+1}{n}$

$\frac{1}{k!} \lambda^k e^{-\lambda}$

#1) 600 pages 600 typos

$p = P(\text{typo is on page } 13) = \frac{1}{600}$

$n = 600$

$\lambda = np = 1$ $\xi \sim \text{Poisson}(1)$ /number of typos on page 13/

$P(\text{no typos on p. } 13) = P(\xi = 0) \approx e^{-1} \approx 0{,}3679$

$\eta \sim \text{Bin}\left(600; \frac{1}{600}\right)$

$P(\text{no typos on p. } 13) = P(\eta = 0) = \left(\frac{599}{600}\right)^{600} \approx 0{,}3676$

$P(\xi = 2) \approx e^{-1} \cdot \frac{1^2}{2!} = \frac{e^{-1}}{2}$

$P(\xi \leq 3) \approx e^{-1} + e^{-1} + \frac{e^{-1}}{2} + \frac{e^{-1}}{6}$