Lab 1

(2) $P = \dfrac{n}{n+m}$ $\quad n$ black, m white

k

$$\frac{\binom{n+m-1}{n-1}}{\binom{n+m}{n}}$$

(5) $1 - \dfrac{20 \cdot 18 \cdot 16 \cdot 14}{20 \cdot 19 \cdot 18 \cdot 17}$

(7)

(11) $9/19$ $\quad | \underbrace{\quad\quad}_{10} | \underbrace{\quad\quad}_{10} | \quad 20!$

$$\frac{10 \cdot 9 \cdot 18! \cdot 2}{20!}$$

(12) $(a;b)$ $\quad a \neq b \quad a > b \quad (a;b) \quad a < b$

$(4;2) \quad (2;4)$

$$\frac{100 \cdot 99}{2}$$

(14) BBB $\quad$ 1/4 $\quad$ GBG

BBG $\quad$ GGB

BGB $\quad$ BGG

GBB $\quad$ GGG

(16) $P(A+B)=P(A)+P(B)-P(AB)=\frac{4}{36}+\frac{18}{36}-\frac{2}{36}$

$=\frac{5}{9}$

(17) MASTE$_1$RPIE$_2$CE$_3$

$X_1=\{$REST can be made$\}$

$X_2=\{$STRIP...$\}$ $\quad X_3=\{$PEST...$\}$

$P(X_1X_2X_3)=\frac{3}{\binom{11}{6}}=\frac{3}{462}$ $\quad \frac{11\cdot\cancel{10}^{2}\cdot\cancel{9}^{3}\cdot\cancel{8}\cdot 7}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}}=462$

RESTIP

$P(X_1+X_2+X_3)=P(X_1)+P(X_2)+P(X_3)-P(X_1X_2)-$

$-P(X_1X_3)-P(X_2X_3)+P(X_1X_2X_3)=$

$P(X_1)=\frac{46}{462}=P(X_3)$ $\quad P(X_2)=\frac{6}{462}$

$\left.\begin{array}{l}\text{RST EEE} \to 1\\ \text{RST EE}\cancel{\text{E}} \to 15\\ \text{RST E}\cancel{\text{E}}\cancel{\text{E}} \to 30\end{array}\right\}46$ $\quad \binom{3}{1}\cdot\binom{5}{2}$

$P(X_1X_3)=\frac{15}{462}$ $\quad P(X_1X_2)=P(X_2X_3)=\frac{3}{462}$

RSTPEE $\to 3$

RSTPE$\cancel{\text{E}}$ $\to 12$

$=\frac{46}{462}+\frac{6}{462}+\frac{46}{462}-\frac{15}{462}-\frac{3}{462}-\frac{3}{462}+\frac{3}{462}=$

19) 1) $P(AB) \geq P(A) + P(B) - 1$

$$P(A+B) = P(A) + P(B) - P(AB)$$

$$\underbrace{P(AB) = P(A) + P(B) - P(A+B)} \geq P(A) + P(B) - 1$$

2) $P(A_1 A_2 \ldots A_{n-1}) \geq P(A_1) + P(A_2) + \ldots + P(A_{n-1}) - (n-2)$

$$\Downarrow$$

$$P((A_1 A_2 \ldots A_{n-1}) A_n) \geq P(A_1) + P(A_2) + \ldots + P(A_n) - (n-1)$$

$$= P(A_1 A_2 \ldots A_{n-1}) + P(A_n) - P(A_1 A_2 \ldots A_{n-1} + A_n) \geq$$

$$\geq P(A_1) + P(A_2) + \ldots + P(A_{n-1}) - (n-2) + P(A_n) -$$

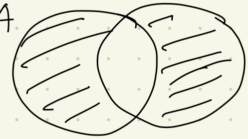
$$- P(A_1 A_2 \ldots A_{n-1} + A_n) \geq P(A_1) + \ldots + P(A_n) - (n-1)$$

(24) $P(AB) = P(A)\cdot P(B)$ independent

$P(AB) = 0$ disjoint

$A \triangle B = A + B - AB$

A B



(29)

HT

HHHT

HHHHHT

...

$$\frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \dots = \frac{1/4}{1 - 1/4} = \frac{1}{3}$$

$A = \{H, H, H, \dots, H \dots\}$

$P(A) \le \frac{1}{2^n}$

(31)

$* * H T T T$ 6 turns $\frac{4}{64} = \frac{1}{16}$

$* * * H T T T$ $\frac{7}{128}$

$B + \overline{B} = \Omega$

$P(B) + P(\overline{B}) = 1$

(25) A and B are independent $\Rightarrow$

A and $\overline{B}$ are independent

$$P(AB) = P(A)P(B) \Longrightarrow P(A\overline{B}) = P(A)\cdot P(\overline{B})$$

$\Updownarrow$

$$P(A\overline{B}) = P(A)\cdot(1 - P(B))$$

$$P(A\overline{B}) = P(A) - P(A)\cdot P(B)$$

$\Updownarrow$

$$P(A\overline{B}) = P(A) - P(AB)$$

$$P(AB) + P(A\overline{B}) = P(A)$$

$$P(AB + A\overline{B}) = P(A)$$

$$P(A) = P(A\Omega) \longleftarrow P(A(B + \overline{B})) = P(A)$$

B and C are disjoint

$\Downarrow$

AB and CD are disjoint for $\forall A, D$.