Probability & Statistics Final Examination Retake, July 24, 2020

Problem 1 (20 minutes)

Two players take part in a match that consists of three games. The results of all the games are independent of each other, and the probabilities for the first and the second player to win a game are equal to 0,2 and 0,3 respectively (thus draws are also possible).

Each victory yields 1 point, and each draw yields 0,5.

a) Find the probability that the first player won the match given that he lost his first game.

b) Find the probability that the first player lost the first game given that he won the match.

c) What is the probability that the third game is not needed (i.e. the winner is determined after two games).

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Problem 2 (20 minutes)

Random variables $\xi_1, \xi_2, \ldots, \xi_{100}$ are independent and have $N(1; 9)$ distribution.

$$X = \sum_{i=1}^{90} \xi_i, \qquad Y = \sum_{i=71}^{100} \xi_i.$$

Find the joint probability density $f_{X,Y}(x, y)$.

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Problem 3 (25 minutes)

A (six-sided) die is rolled once. If number ς is rolled then a coin is tossed ς times; ξ and η are quantities of heads and tails obtained (respectively). Find the correlation between ξ and η.

/Both the die and the coin are fair./

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Problem 4 (25 minutes)

On the election one votes for candidate X with probability 0,65 in the first electoral district (2000 electors) and with probability 0,4 in the second electoral district (3000 voters).

It is known that candidate X got 1250 and 1280 votes in these districts. Estimate the probability that these numbers refer to the first and the second districts respectively. Round your answer up to three decimal points.

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Problems 5–7 (45 minutes)

(5) It is known that random variables ξ and $\frac{1}{\xi}$ have the same distribution, and

$$\xi \sim \begin{pmatrix} 1 & 2 & \cdot \\ \cdot & 1/4 & \cdot \end{pmatrix}.$$

(dots designate unknown numbers).
Find $E\xi$ and $Var\,\xi$.

(6) Find the minimum value of variance of ξ given that ξ has a uniform distribution on some interval, $E\xi \geq 4$, $P(\xi > 1) = 5/7$.

(7) Independent random variables ξ and η are exponentially distributed with parameters λ and μ respectively. Find the probability density of $X = \min(\xi, \eta)$.