Probability & Statistics. Final Examination. Problem 1

20 MINUTES

1. **PROBLEM SET 1.** It is known that random variable X has a binomial distribution, and $EX = 6$, $DX = 4$. Find $E\left(3^X\right)$.

2. **PROBLEM SET 2.** It is known that random variable X has a binomial distribution, and $EX = 10$, $DX = 6$. Find $E\left(5^X\right)$.

3. **PROBLEM SET 3.** It is known that random variable X has a binomial distribution, and $EX = 24$, $DX = 6$. Find $E\left(2^X\right)$.

Probability & Statistics. Final Examination. Problem 2

15 MINUTES

1. **PROBLEM SET 1.** Random variable ζ is uniformly distributed on interval $(2;7)$. Find expected value and variance of a volume of a sphere with radius ζ.

2. **PROBLEM SET 2.** Random variable ζ is uniformly distributed on interval $(1;6)$. Find expected value and variance of a volume of a sphere with radius ζ.

3. **PROBLEM SET 3.** Random variable ζ is uniformly distributed on interval $(6;8)$. Find expected value and variance of a volume of a sphere with radius ζ.

Probability & Statistics. Final Examination. Problem 3

20 MINUTES

1. **PROBLEM SET 1.** The probability of getting tails when tossing a crooked coin is equal to $p_0 = 0.42$. The coin was tossed 1000 times, and tails were obtained in 360 cases. What is the minimum number of times one has to toss the same coin in order that the percentage of tails differs from p_0 less than in the first thousand trials with probability $p_1 = 0.95$?

2. **PROBLEM SET 2.** The probability of getting tails when tossing a crooked coin is equal to $p_0 = 0.67$. The coin was tossed 1000 times, and tails were obtained in 690 cases. What is the minimum number of times one has to toss the same coin in order that the percentage of tails differs from p_0 less than in the first thousand trials with probability $p_1 = 0.98$?

3. **PROBLEM SET 3.** The probability of getting tails when tossing a crooked coin is equal to $p_0 = 0.54$. The coin was tossed 1000 times, and tails were obtained in 490 cases. What is the minimum number of times one has to toss the same coin in order that the percentage of tails differs from p_0 less than in the first thousand trials with probability $p_1 = 0.96$?

Probability & Statistics. Final Examination. Problem 4

25 MINUTES

1. **PROBLEM SET 1.** Random variables X and Z are independent and their probability densities are given by $f_X(x) = 5e^{-5x} \cdot I_{x>0}$, $f_Z(z) = 5e^{-5z} \cdot I_{z>0}$. Let us consider $U = \min(X; Z)$, $V = \max(X; Z)$. Find covariance of U and V.

2. **PROBLEM SET 2.** Random variables X and Z are independent and their probability densities are given by $f_X(x) = 3e^{-3x} \cdot I_{x>0}$, $f_Z(z) = 3e^{-3z} \cdot I_{z>0}$. Let us consider $U = \min(X; Z)$, $V = \max(X; Z)$. Find covariance of U and V.

3. **PROBLEM SET 3.** Random variables X and Z are independent and their probability densities are given by $f_X(x) = 6e^{-6x} \cdot I_{x>0}$, $f_Z(z) = 6e^{-6z} \cdot I_{z>0}$. Let us consider $U = \min(X; Z)$, $V = \max(X; Z)$. Find covariance of U and V.

Probability & Statistics. Final Examination. Problem 5

25 MINUTES

1. **PROBLEM SET 1.** Covariance matrix of random vector $\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$ is equal to $\begin{pmatrix} 2 & -1 & \lambda \\ -1 & 2 & 1 \\ \lambda & 1 & 3 \end{pmatrix}$.
   (a) Find all possible values of λ.
   (b) Find the value of λ that yields minimum to variance of $X + \lambda Y - 2Z$.

2. **PROBLEM SET 2.** Covariance matrix of random vector $\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$ is equal to $\begin{pmatrix} 3 & 1 & -2 \\ 1 & 1 & \lambda \\ -2 & \lambda & 6 \end{pmatrix}$.
   (a) Find all possible values of λ.
   (b) Find the value of λ that yields minimum to variance of $\lambda X - Y + 2Z$.

3. **PROBLEM SET 3.** Covariance matrix of random vector $\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$ is equal to $\begin{pmatrix} 1 & -1 & 1 \\ -1 & 3 & \lambda \\ 1 & \lambda & 2 \end{pmatrix}$.
   (a) Find all possible values of λ.
   (b) Find the value of λ that yields minimum to variance of $\lambda X + 2Y - Z$.

Probability & Statistics. Final Examination. Problems 6–8

60 MINUTES

1. **PROBLEM SET 1.**

   Problem #6. 14 white balls and 6 black balls are randomly placed in a row. Random variables η and ζ are equal (respectively) to quantities of white and black balls among the first 8 balls. Find the correlation coefficient of η and ζ.

   Problem #7. Probability density of random variable Y is given by $f_Y(y) = Ce^{-y^2+4y-10}$, $y \in \mathbb{R}$. Find the value of constant C, EY, $\operatorname{Var} Y$.

   Problem #8. Random variables $X_1, X_2, \ldots, X_{100}$ are independent and identically distributed, and they all have $N(0;4)$ distribution. Find the probability density of random vector $\begin{pmatrix} Y \\ Z \end{pmatrix}$ where $Y = X_{61} + X_{62} + \ldots + X_{100}$, $Z = X_1 + X_2 + \ldots + X_{80}$.

2. **PROBLEM SET 2.**

   Problem #6. 7 red balls and 13 yellow balls are randomly placed in a row. Random variables η and ζ are equal (respectively) to quantities of red and yellow balls among the first 6 balls. Find the correlation coefficient of η and ζ.

   Problem #7. Probability density of random variable Y is given by $f_Y(y) = Ce^{-y^2-6y+10}$, $y \in \mathbb{R}$. Find the value of constant C, EY, $\operatorname{Var} Y$.

   Problem #8. Random variables $X_1, X_2, \ldots, X_{100}$ are independent and identically distributed, and they all have $N(0;9)$ distribution. Find the probability density of random vector $\begin{pmatrix} Y \\ Z \end{pmatrix}$ where $Y = X_{41} + X_{42} + \ldots + X_{100}$, $Z = X_1 + X_2 + \ldots + X_{90}$.

3. **PROBLEM SET 3.**

   Problem #6. 5 green balls and 15 blue balls are randomly placed in a row. Random variables η and ζ are equal (respectively) to quantities of green and blue balls among the first 12 balls. Find the correlation coefficient of η and ζ.

   Problem #7. Probability density of random variable Y is given by $f_Y(y) = Ce^{-y^2-2y-6}$, $y \in \mathbb{R}$. Find the value of constant C, EY, $\operatorname{Var} Y$.

   Problem #8. Random variables $X_1, X_2, \ldots, X_{100}$ are independent and identically distributed, and they all have $N(0;4)$ distribution. Find the probability density of random vector $\begin{pmatrix} Y \\ Z \end{pmatrix}$ where $Y = X_{61} + X_{62} + \ldots + X_{100}$, $Z = X_1 + X_2 + \ldots + X_{80}$.