Probability & Statistics. Midterm Examination. Problem 1

25 MINUTES

1. **PROBLEM SET 1.** How many times (on average) does one need to roll a (symmetric **twenty-faced**) die in order to get a five and a six one after another? Does anything change if one needs to get two fives in a row? Justify your answer.

2. **PROBLEM SET 2.** How many times (on average) does one need to roll a (symmetric **twelve-faced**) die in order to get a five and a six one after another? Does anything change if one needs to get two fives in a row? Justify your answer.

3. **PROBLEM SET 3.** How many times (on average) does one need to roll a (symmetric **six-faced**) die in order to get a five and a six one after another? Does anything change if one needs to get two fives in a row? Justify your answer.

Probability & Statistics. Midterm Examination. Problem 2

25 MINUTES

1. **PROBLEM SET 1.** Probability density of random vector $(X, Y)^T$ is given by

$$f_{X,Y}(x, y) = \frac{25}{14\pi} \cdot \exp\left(-\frac{625}{98}\left(x^2 - \frac{48}{25}xy + y^2\right)\right).$$

   Find marginal probability density of X and determine the distribution law.

2. **PROBLEM SET 2.** Probability density of random vector $(X, Y)^T$ is given by

$$f_{X,Y}(x, y) = \frac{13}{24\pi} \cdot \exp\left(-\frac{169}{288}\left(x^2 - \frac{10}{13}xy + y^2\right)\right).$$

   Find marginal probability density of X and determine the distribution law.

3. **PROBLEM SET 3.** Probability density of random vector $(X, Y)^T$ is given by

$$f_{X,Y}(x, y) = \frac{17}{16\pi} \cdot \exp\left(-\frac{289}{128}\left(x^2 - \frac{30}{17}xy + y^2\right)\right).$$

   Find marginal probability density of X and determine the distribution law.

Probability & Statistics. Midterm Examination. Problem 3

20 MINUTES

1. **PROBLEM SET 1.** Two fair dice are rolled. Let us consider events

$$A_i = \{\text{the sum rolled is equal to } i\}$$

and

$$B_j = \{\text{there are } j \text{ points on the second die}\}.$$

Find all possible pairs of (i, j) such that events A_i and B_j are independent.

2. **PROBLEM SET 2.** Two fair dice are rolled. Let us consider events

$$A_i = \{\text{the sum rolled is equal to } i\}$$

and

$$B_j = \{\text{there are } j \text{ points on the second die}\}.$$

Find all possible pairs of (i, j) such that events A_i and B_j are independent.

3. **PROBLEM SET 3.** Two fair dice are rolled. Let us consider events

$$A_i = \{\text{the sum rolled is equal to } i\}$$

and

$$B_j = \{\text{there are } j \text{ points on the second die}\}.$$

Find all possible pairs of (i, j) such that events A_i and B_j are independent.

Probability & Statistics. Midterm Examination. Problem 4

20 MINUTES

1. **PROBLEM SET 1.** There are 7 black balls and 16 white balls in the first urn; 6 black balls and 3 white in the second urn; 7 black balls and 1 white in the third urn. A random ball is moved from the first urn to the second urn, the balls are thoroughly mixed, and after that a random ball is moved from the second urn to the third urn. What is the probability of getting a white ball out of the third urn?

2. **PROBLEM SET 2.** There are 8 black balls and 5 white balls in the first urn; 8 black balls and 7 white in the second urn; 1 black ball and 5 white in the third urn. A random ball is moved from the first urn to the second urn, the balls are thoroughly mixed, and after that a random ball is moved from the second urn to the third urn. What is the probability of getting a white ball out of the third urn?

3. **PROBLEM SET 3.** There are 2 black balls and 9 white balls in the first urn; 5 black balls and 13 white in the second urn; 2 black balls and 2 white in the third urn. A random ball is moved from the first urn to the second urn, the balls are thoroughly mixed, and after that a random ball is moved from the second urn to the third urn. What is the probability of getting a white ball out of the third urn?

Probability & Statistics. Midterm Examination. Problem 5

20 MINUTES

1. **PROBLEM SET 1.** Ten people enter the lift at the ground floor of a 13-storey house. Each of them can get off the lift at all the floors starting with the first one with equal probabilities and independently of each other. Let ξ be equal to the quantity of floors the lift does not reach at all. (For example, if the last person exits the lift at the seventh floor, it implies that the lift does not reach floors 8, 9, etc.) Find the expected value of ξ.

2. **PROBLEM SET 2.** Eleven people enter the lift at the ground floor of a 12-storey house. Each of them can get off the lift at all the floors starting with the first one with equal probabilities and independently of each other. Let ξ be equal to the quantity of floors the lift does not reach at all. (For example, if the last person exits the lift at the seventh floor, it implies that the lift does not reach floors 8, 9, etc.) Find the expected value of ξ.

3. **PROBLEM SET 3.** Twelve people enter the lift at the ground floor of an 11-storey house. Each of them can get off the lift at all the floors starting with the first one with equal probabilities and independently of each other. Let ξ be equal to the quantity of floors the lift does not reach at all. (For example, if the last person exits the lift at the seventh floor, it implies that the lift does not reach floors 8, 9, etc.) Find the expected value of ξ.

Probability & Statistics. Midterm Examination. Problems 6–8

55 MINUTES

1. **PROBLEM SET 1.**

   6. A taxi driver moves between four settlements W, X, Y, Z located at the vertices of a rectangle. Four highways connecting the settlements run along the sides of the rectangle, and $WX = 6$ kilometers, $WZ = 10$ kilometers. Having delivered a customer the driver waits for the next taxi call. Then he goes to the settlement the call came from, collects a customer, and goes to a new destination. A new call can emerge from any settlement with probability 0.25, and the customer chooses the settlement to travel to with probability $\frac{1}{3}$. Travelling between adjacent settlements, the driver always chooses the shortest route. The distances inside each of the settlements are very small (i.e. they may be neglected). Let ζ be the distance the driver needs to travel in order to collect and deliver a customer. Find the distribution of random variable ζ. Determine $E\zeta$ and $\operatorname{Var}\zeta$.

   7. Random variable X is uniformly distributed on $[0; 2]$. Find the probability density of $Y = \frac{3-X}{1+2X}$.

   8. Variables $y_1, y_2, \ldots, y_7$ can take only positive integer values. One of the solutions of the equation $y_1 + y_2 + \ldots + y_7 = 19$ is chosen at random. Determine the probability that $y_5 = 5$.

2. **PROBLEM SET 2.**

   6. A taxi driver moves between four settlements W, X, Y, Z located at the vertices of a rectangle. Four highways connecting the settlements run along the sides of the rectangle, and $WX = 11$ kilometers, $WZ = 3$ kilometers. Having delivered a customer the driver waits for the next taxi call. Then he goes to the settlement the call came from, collects a customer, and goes to a new destination. A new call can emerge from any settlement with probability 0.25, and the customer chooses the settlement to travel to with probability $\frac{1}{3}$. Travelling between adjacent settlements, the driver always chooses the shortest route. The distances inside each of the settlements are very small (i.e. they may be neglected). Let ζ be the distance the driver needs to travel in order to collect and deliver a customer. Find the distribution of random variable ζ. Determine $E\zeta$ and $\operatorname{Var}\zeta$.

   7. Random variable X is uniformly distributed on $[0; 3]$. Find the probability density of $Y = \frac{4-X}{1+3X}$.

   8. Variables $y_1, y_2, \ldots, y_6$ can take only positive integer values. One of the solutions of the equation $y_1 + y_2 + \ldots + y_7 = 23$ is chosen at random. Determine the probability that $y_5 = 3$.

3. **PROBLEM SET 3.**

   6. A taxi driver moves between four settlements W, X, Y, Z located at the vertices of a rectangle. Four highways connecting the settlements run along the sides of the rectangle, and $WX = 7$ kilometers, $WZ = 9$ kilometers. Having delivered a customer the driver waits for the next taxi call. Then he goes to the settlement the call came from, collects a customer, and goes to a new destination. A new call can emerge from any settlement with probability 0.25, and the customer chooses the settlement to travel to with probability $\frac{1}{3}$. Travelling between adjacent settlements, the driver always chooses the shortest route. The distances inside each of the settlements are very small (i.e. they may be neglected). Let ζ be the distance the driver needs to travel in order to collect and deliver a customer. Find the distribution of random variable ζ. Determine $E\zeta$ and $\operatorname{Var}\zeta$.

   7. Random variable X is uniformly distributed on $[0; 4]$. Find the probability density of $Y = \frac{9-2X}{2+X}$.

   8. Variables $y_1, y_2, \ldots, y_8$ can take only positive integer values. One of the solutions of the equation $y_1 + y_2 + \ldots + y_7 = 18$ is chosen at random. Determine the probability that $y_5 = 2$.