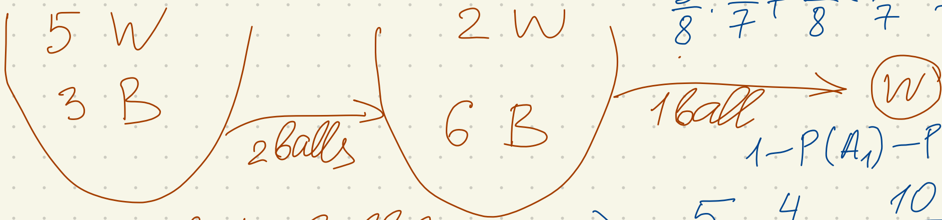
# August 27, lab

(31)




$$\frac{5}{8}\cdot\frac{3}{7}+\frac{3}{8}\cdot\frac{5}{7}$$

$$1-P(A_1)-P(A_2)$$

$A_1 = \{2 \text{ white balls}\}$ $\quad P(A_1) = \frac{5}{8}\cdot\frac{4}{7} = \frac{10}{28}$

$A_2 = \{2 \text{ black balls}\}$ $\quad P(A_2) = \frac{3}{8}\cdot\frac{2}{7} = \frac{3}{28}$

$A_3 = \{1 \text{ white } \& \text{ 1 black ball}\}$ $\quad P(A_3) = \frac{15}{28}$

$B = \{\text{a white ball is taken out of the } 2^{nd} \text{ urn}\}$

$$P(A_3|B) = \frac{P(B|A_3)P(A_3)}{P(B|A_1)P(A_1)+P(B|A_2)P(A_2)+P(B|A_3)P(A_3)} =$$

$$= \frac{0{,}3\cdot 15/28}{0{,}4\cdot 10/28 + 0{,}2\cdot 3/28 + 0{,}3\cdot 15/28} = \frac{45}{40+6+45} = \frac{45}{91}$$

6 $p_1 = \frac{4}{5}$, $p_2 = \frac{3}{4}$, $p_3 = \frac{2}{3}$

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7 W, 3 B | 8 W, 4 B | 2 W, 13 B

$A_i = \{\text{urn } i \text{ is chosen}\}$

$B = \{\text{a white ball is taken out of this urn}\}$

$P(A_i \mid B)$

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3) =$$

$$= \frac{7}{10}\cdot\frac{1}{3} + \frac{8}{12}\cdot\frac{1}{3} + \frac{2}{15}\cdot\frac{1}{3} = \frac{42+40+8}{60\cdot 3} = \frac{1}{2}$$

$$P(A_1|B) = \frac{P(B|A_1)P(A_1)}{P(B)} = \frac{7/30}{1/2} = \frac{7}{15} = \frac{21}{45}$$

$$P(A_2|B) = \frac{8/36}{1/2} = \frac{4}{9} = \frac{20}{45}$$

$$P(A_3|B) = \frac{2/45}{1/2} = \frac{4}{45}$$

$$\frac{21}{45}\cdot\frac{7}{10}^{(3} + \frac{20}{45}\cdot\frac{2}{3}^{(10} + \frac{4}{45}\cdot\frac{2}{15}^{(2} = \frac{441+400+16}{45\cdot 30} =$$

$$= \frac{857}{1350}$$

(12) $p = \frac{3}{20} \qquad q = \frac{17}{20}$

$A = \{$ exactly 3 devices have broken down $\}$

$B = \{$ at least one device does not work $\}$

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(A)}{1 - P(\overline{B})} = \frac{\binom{10}{3}\cdot\left(\frac{3}{20}\right)^3\left(\frac{17}{20}\right)^7}{1 - \left(\frac{17}{20}\right)^{10}}$$

(16) $p = 0{,}6;\ q = 0{,}4 \quad ; \quad 8$ games

$A = \{$ the younger wins 5 out of 8 $\}$

$B = \{$ the younger loses the $1^{st}$ game $\}$

$$P(B|A) = \frac{P(AB)}{P(A)} = \frac{\binom{7}{5}\cancel{p^5}\cancel{q^3}}{\binom{8}{5}\cancel{p^5}\cancel{q^3}} = \frac{7\cdot 6/2}{8\cdot 7\cdot 6/3!} =$$

$$= \frac{7\cdot 3}{7\cdot 8} = \frac{3}{8}$$

$$0\ \underbrace{\_\ \_\ \_\ \_\ \_\ \_\ \_}$$

5 "1"

2 "0"

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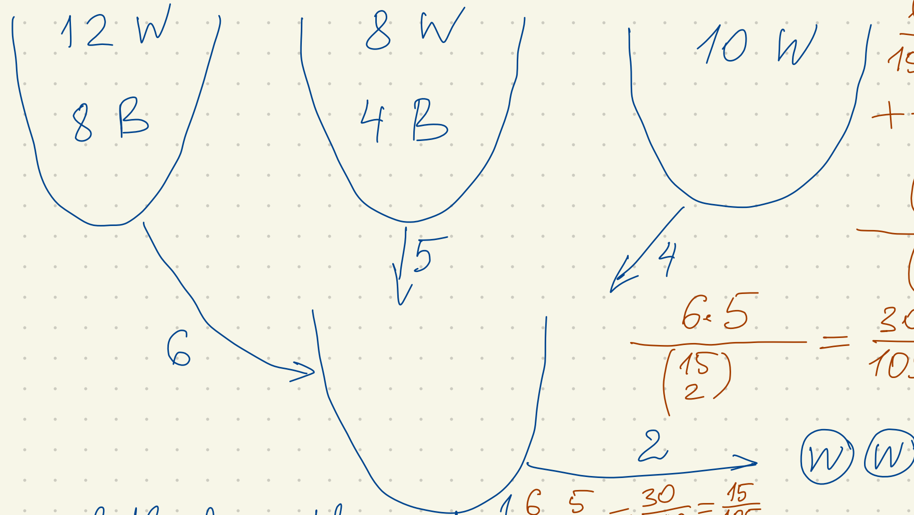


$$\frac{6}{15}\cdot\frac{5}{14}+\frac{5}{15}\cdot\frac{6}{14}$$

$$\frac{\binom{6}{2}}{\binom{15}{2}}$$

$$\frac{6\cdot 5}{\binom{15}{2}}=\frac{30}{105}$$

$A_1=\{\text{both from the } 1^{st}\}$ $\frac{6}{15}\cdot\frac{5}{14}=\frac{30}{210}=\frac{15}{105}$

$A_2=\{\text{both from the } 2^{nd}\}$ $\frac{20}{210}=\frac{10}{105}$

$A_3=\{\text{both from the } 3^{rd}\}$ $\frac{12}{210}=\frac{6}{105}$

$A_4=\{1^{st}\ \&\ 2^{nd}\}$ $\frac{60}{210}=\frac{30}{105}$

$A_5=\{1^{st}\ \&\ 3^{rd}\}$ $\frac{48}{210}=\frac{24}{105}$

$A_6=\{2^{nd}\ \&\ 3^{rd}\}$ $\frac{40}{210}=\frac{20}{105}$

$B=\{\text{2 white balls are taken}\}$

$$P(A_4+A_5+A_6|B)=?$$

$$P(A_4|B)+P(A_5|B)+P(A_6|B)=$$

$$=\frac{P(A_4B)+P(A_5B)+P(A_6B)}{P(B)}$$

$$P(B)=P(B|A_1)P(A_1)+P(B|A_2)P(A_2)+\ldots=\frac{12}{20}\cdot\frac{11}{19}\cdot\frac{15}{105}+$$

$$+\frac{8}{12}\cdot\frac{7}{11}\cdot\frac{10}{105}+1\cdot\frac{6}{105}+\underbrace{\frac{12}{20}\cdot\frac{8}{12}\cdot\frac{30}{105}}_{P(A_4B)}+\underbrace{\frac{12}{20}\cdot 1\cdot\frac{24}{105}}_{P(A_5B)}+\underbrace{\frac{8}{12}\cdot 1\cdot\frac{20}{105}}_{P(A_6B)}$$

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$$24 = 1\cdot 4\cdot 6 = 2\cdot 2\cdot 6 = 2\cdot 3\cdot 4$$

$$2^3\cdot 3$$

$$P(1,0,0,1,0,1) + P(0,2,0,0,0,1) +$$

$$+ P(0,1,1,1,0,0) = \frac{3!}{(1!)^3(0!)^3}\cdot\left(\frac{1}{6}\right)^3 + \frac{3!}{(0!)^4 2! 1!}\cdot\left(\frac{1}{6}\right)^3 +$$

$$+ \frac{3!}{(1!)^3(0!)^3}\cdot\left(\frac{1}{6}\right)^3 = \frac{15}{6^3} = \frac{5}{72}$$

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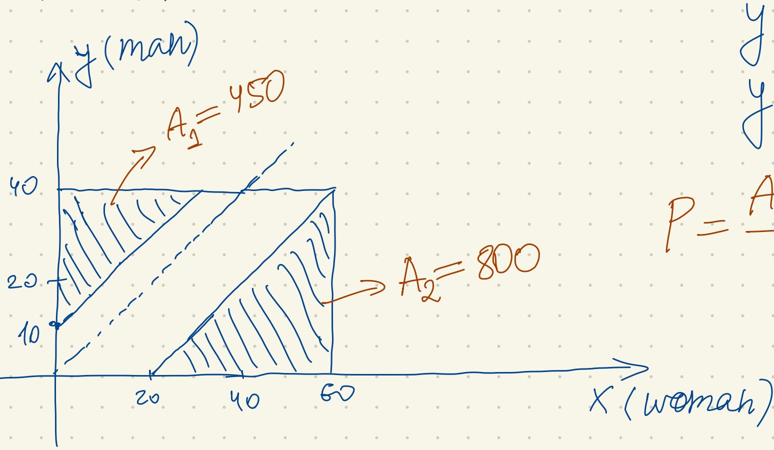


$$y > x \Rightarrow y > x + 10$$

$$y < x \Rightarrow y < x - 20$$

$$P = \frac{A_1 + A_2}{A} = \frac{1250}{2400} =$$

$$= \frac{25}{48}$$