1) $A_1, A_2, \ldots, A_n$ is exhaustive

$$A_1 + A_2 + \ldots + A_n = \Omega$$

2) $A_1, A_2, \ldots, A_n$ are mutually exclusive

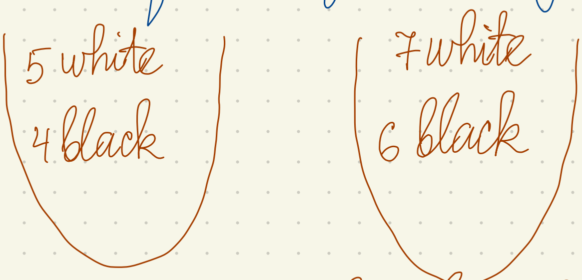
$$i \neq j \Rightarrow A_i A_j = \emptyset$$

$$(A_1 \cup A_2 \cup \ldots \cup A_n) \cap B =$$
$$= (A_1 \cap B) \cup (A_2 \cap B) \cup \ldots \cup (A_n \cap B)$$

B is some event

$$P(B) = P(B\Omega) = P(B(A_1 + A_2 + \ldots + A_n)) =$$
$$= P(BA_1 + BA_2 + \ldots + BA_n) = P(BA_1) + P(BA_2) + \ldots +$$
$$+ P(BA_n) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \ldots + P(B|A_n)P(A_n)$$

Law of total probability



$A_i = \{$ urn i has been chosen $\}$

$B = \{$ a black ball is taken out of the urn $\}$

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) =$$
$$= \frac{4}{9} \cdot \frac{1}{2} + \frac{6}{13} \cdot \frac{1}{2} = \frac{26+27}{117} = \frac{53}{117}$$

$$P(A_i|B) = \frac{P(A_i B)}{P(B)} = \frac{P(B|A_i)P(A_i)}{P(B|A_1)P(A_1)+\dots+P(B|A_n)P(A_n)}$$

Bayes rule

5 white
4 black
$\frac{4}{9} \approx 44\%$

7 white
6 black
$\frac{6}{13} \approx 46\%$

$$\underbrace{P(A_1|B)} = \frac{P(B|A_1)P(A_1)}{P(B)} = \frac{4/9 \cdot 1/2}{53/117} = \underbrace{\frac{26}{53}}_{\text{a posteriori}}$$

$P(A_1) = \frac{1}{2}$ — a priori

---

10 white 10 black

5 W
5 B

5 W
5 B

1 B

10 W
9 B

$$\frac{1}{2} + \frac{1}{2} \cdot \frac{9}{19}$$

$$P_1 = \frac{1}{5}, \quad P_2 = \frac{1}{6}, \quad P_3 = \frac{1}{7}$$

$A_1 = \{$the second shooter has hit the target$\}$

$A_2 = \{$the second shooter has missed$\}$

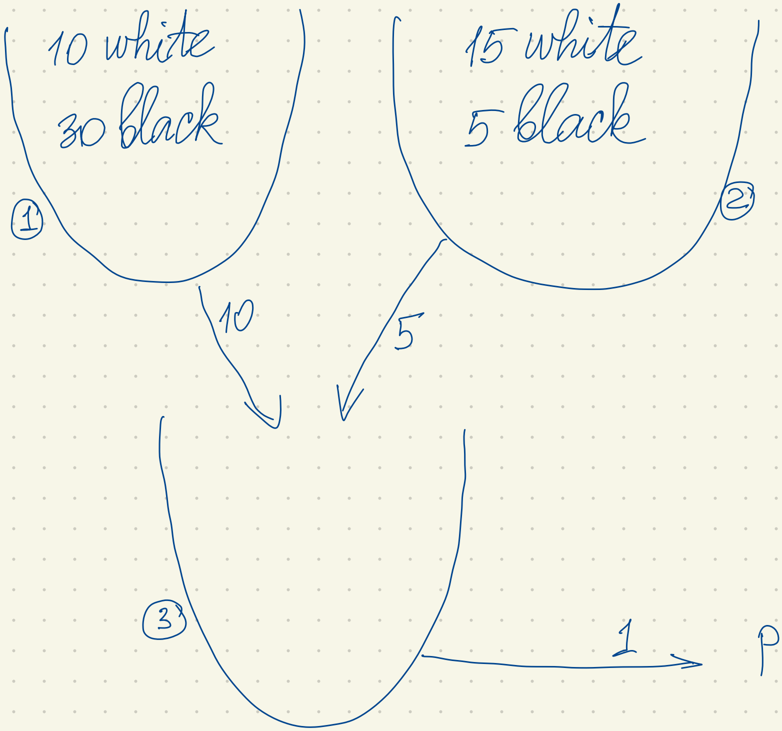
$B = \{$the target has been hit exactly once$\}$

~~? $A_i = \{$shooter i has hit the target$\}$~~

$$P(A_1 | B) = \frac{P(B|A_1)P(A_1)}{P(B|A_1)P(A_1) + P(B|A_2)P(A_2)} =$$

$$P(B|A_1) = \frac{4}{5} \cdot \frac{6}{7}; \quad P(B|A_2) = \frac{1}{5} \cdot \frac{6}{7} + \frac{4}{5} \cdot \frac{1}{7} = \frac{10}{35}$$

$$= \frac{24/35 \cdot 1/6}{24/35 \cdot 1/6 + 10/35 \cdot 5/6} = \frac{24}{24+50} = \frac{12}{37}$$



$P(\text{black}) = ?$

$A_i = \{$the ball taken from the third urn was initially situated in urn $i\}$

$B = \{$the ball is black$\}$

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) =$$

$$= \frac{3}{4}\cdot\frac{2}{3} + \frac{1}{4}\cdot\frac{1}{3} = \frac{7}{12}$$

Bernoulli's trials

p – a probability of success

$q = 1-p$ – a probability of failure

n trials

μ_n = number of successes in n trials

$$\underbrace{P(\mu_n = k) = \binom{n}{k} p^k q^{n-k}}$$

$$\underbrace{11\ldots1}_{k}\underbrace{00\ldots0}_{n-k} \qquad \underbrace{p \cdot p \cdot \ldots \cdot p}_{k} \cdot \underbrace{q \cdot q \cdot \ldots \cdot q}_{n-k} = p^k \cdot q^{n-k}$$

P(the first wins at least 4 games out of 5) $\quad p = q = \frac{1}{2}$

P(— " — 8 games out of 10)

$$P(\mu_5 = 4) + P(\mu_5 = 5) = \binom{5}{4} \cdot p^4 \cdot q + \binom{5}{5} p^5 =$$

$$= \frac{1}{32} \cdot 5 + \frac{1}{32} \cdot 1 = \frac{3}{16} = \frac{24}{128}$$

$$P(\mu_{10} = 8) + P(\mu_{10} = 9) + P(\mu_{10} = 10) =$$

$$= \left(\binom{10}{8} + \binom{10}{9} + \binom{10}{10}\right) \cdot \left(\frac{1}{2}\right)^{10} = (45 + 10 + 1) \cdot \left(\frac{1}{2}\right)^{10} =$$

$$= \frac{7}{128}$$

70 dice are rolled. What is the most probable number of "6" obtained?

$$\frac{P(\mu_{70}=n)}{P(\mu_{70}=n-1)} = \frac{\binom{70}{n}\left(\frac{1}{6}\right)^{n}\cdot\left(\frac{5}{6}\right)^{70-n}}{\binom{70}{n-1}\cdot\left(\frac{1}{6}\right)^{n-1}\cdot\left(\frac{5}{6}\right)^{71-n}} =$$

$$= \frac{\frac{\cancel{70!}}{n!(70-n)!}\cdot\frac{1}{6}}{\frac{\cancel{70!}}{(n-1)!(71-n)!}\cdot\frac{5}{6}} = \frac{71-n}{5n} > 1$$

$71-n > 5n$

$n < \frac{71}{6}$

$n \leq 11$

$$P(\mu_{70}=0) < P(\mu_{70}=1) < \ldots < P(\mu_{70}=11) > P(\mu_{70}=12) > \ldots > P(\mu_{70}=70)$$

71

$$\frac{72-n}{5n} > 1 \qquad n < 12$$

$$P(\mu_{71}=0) < P(\mu_{71}=1) < \ldots < P(\mu_{71}=11) = P(\mu_{71}=12) > P(\mu_{71}=13) > \ldots > P(\mu_{71}=71)$$

k different outcomes; their probabilities are

$p_1, p_2, \ldots, p_k$ $\qquad \sum_{i=1}^{k} p_i = 1$

n experiments

$P(n_1, n_2, n_3, \ldots, n_k) =$ where $n_1 + n_2 + \ldots + n_k = n$

$$= p_1^{n_1} p_2^{n_2} \ldots p_k^{n_k} \cdot \frac{n!}{n_1! \, n_2! \ldots n_k!}$$

$$\underbrace{\underbrace{1\,1 \ldots 1}_{n_1} \ \underbrace{2\,2 \ldots 2}_{n_2} \ \ldots \ \underbrace{k\,k \ldots k}_{n_k}}_{n} \qquad n - n_1 - n_2$$

$$\binom{n}{n_1} \cdot \binom{n-n_1}{n_2} \cdot \binom{n-n_1-n_2}{n_3} \cdot \ldots \cdot \cancel{\binom{n-n_1-\ldots-n_{k-1}}{n_k}}^{1} =$$

$$= \frac{n!}{n_1! \, \cancel{(n-n_1)!}} \cdot \frac{\cancel{(n-n_1)!}}{n_2! \, \cancel{(n-n_1-n_2)!}} \cdot \frac{\cancel{(n-n_1-n_2)!}}{n_3! \, (n-n_1-n_2-n_3)!} \cdot \ldots \cdot$$

$$\cdot \frac{(n-n_1-\ldots-n_{k-1})!}{n_k! \, (n-n_1-\ldots-n_k)!}$$

10 people $\qquad$ 10-storey house $\qquad$ ground

$1, 2, \ldots, 8, 9$ (9 outcomes)

$$p_1 = p_2 = \ldots = p_9 = \frac{1}{9}$$

$$P(2,1,1,\ldots,1) + P(1,2,1,1,\ldots,1) + \ldots +$$

$$+ P(1,1,\ldots,1,2) = 9 \cdot P(2,1,1,\ldots,1) =$$

$$= 9 \cdot \frac{10!}{2!1!\ldots1!} \cdot \left(\frac{1}{9}\right)^{10} = \frac{10!}{2 \cdot 9^9}$$

$p = 0,3$; $q = 0,7$

a match until 6 victories

probability that a winner scores exactly one more point than a looser = ?

$\cancel{P(\mu_{11} = 5) + P(\mu_{11} = 6)}$

6 : 5 5 : 6

5 : 5

1 1 0 0 0 0 0 0 1 1 1

$$P(\mu_{10} = 5) = \frac{10!}{5!\,5!}\, 0,3^5 \cdot 0,7^5$$

4 : 4

6 : 4

5 : 4 & the first wins

4 : 5 & the second wins