Probability & Statistics. Assignment 2

1. There are 7 white balls and 3 black balls in the first urn, 8 white balls and 4 black balls in the second urn, 2 white balls and 13 black balls in the third urn. One of the urns is chosen at random, and a random ball is taken out of it.

   (a) Determine the probability that the ball is white.

   (b) It is known that the ball taken out of the urn is white. What is the probability that the first urn was chosen out of the three?

   4.2 from 1

   Answer: (a) 0.5; (b) $\frac{7}{15}$.

2. A hospital specializes in curing three types of diseases: A, B and C. On average, there are 50% of patients who suffer from disease A, 30% of patients with disease B, and 20% of patients with disease C (each of the patients has exactly one of these diseases). The probabilities to fully recover from the diseases are equal to 0.95, 0.9 and 0.85 respectively. A patient who came to the hospital recovered completely. What is the probability that he had disease B?

   Answer: $\frac{18}{61}$.

3. There are 5 white and 3 black balls in the first urn, 2 white and 6 black balls in the second urn. Two random balls are transferred from the first urn to the second, and after that some ball is taken from the second urn. This ball happens to be white. Find the probability that two balls of different colours were moved from the first urn to the second.

   Answer: $\frac{45}{91}$.

4. The quantity of passenger cars that pass along the gas station is on average four times greater than that of trucks. The probability that a car refuels is equal to 0.05 for trucks and 0.15 for passenger cars. A car has just left the gas station. What is the probability that it is a truck?

   Answer: $\frac{1}{13}$.

5. Three marksmen are shooting at a target. The probabilities for them to hit the target are equal to $\frac{4}{5}$, $\frac{3}{4}$, $\frac{2}{3}$ respectively. Each of the marksmen fires one shot, and the target is hit exactly two times. Find the probability that the third marksman has hit the target.

   4.13 from 1

   Answer: $\frac{7}{13}$.

6. Three marksmen are shooting at a target. The probabilities for them to hit the target are equal to $\frac{4}{5}$, $\frac{3}{4}$, $\frac{2}{3}$ respectively. At least two hits are needed for the target to be destroyed. The marksmen fire one shot each, and the target is destroyed. What is the probability that the third marksman has hit the target?

   Answer: 0.76.

7. A white ball is added into an urn that initially contained n balls. It is known that the probabilities of having $0, 1, 2, \ldots n$ white balls (at the start) in the urn are equal to each other. (a) One ball is taken at random from the urn. What is the probability that the ball is white? (b) The ball taken from the urn has turned out to be white. Find the most probable number of white balls that were in the urn from the start.

   4.15 from 1

   Answer: (a) $\frac{n+2}{2n+2}$; (b) n.

8. There are 7 white balls and 3 black balls in the first urn; 8 white balls and 4 black balls in the second; 2 white and 13 black balls in the third. One of the urns is chosen at random and a ball is taken out of this urn. The ball has happened to be white. It is returned into the urn, and another ball is randomly chosen from the urn. What is the probability that this ball is also white?

   4.16 from 1

   Answer: $\frac{857}{1350}$.

9. There were 12 white and 8 black balls in the first urn, 8 white and 4 black balls in the second urn, 10 white balls in the third urn. The fourth urn, that had been empty, was filled with the balls: 6 balls from the first urn, 5 balls from the second, and 4 balls from the third (all the balls were chosen at random). After that two balls were taken out of the fourth urn, and they happened to be white. What is the probability that these balls originate from different urns?

   Answer: ≈ 0.720.

10. Five people entered the lift at the ground floor of a nine-storey building. Find the probability that

    (a) none of them went out on the fifth floor;

    (b) (exactly) one of them went out at the fifth floor;

    (c) (exactly) three of them went out at the fifth floor.

    Answer: (a) ≈ 0.513; (b) ≈ 0.366; (c) ≈ 0.015.

11. What is more probable in case of two equally strong players: to win at least three games aut of four or to win at least six games out of eight (No draws are possible)?

    Answer: to win at least three games out of four.

12. There are ten devices installed on a workbench. During the trials each of them may break down with probability 0.15 independently from other devices. Find the probability that (exactly) three devices have broke down given that not all the devices went through the trials successfully.

    5.19 from 1

    Answer: $\frac{\binom{10}{3}\cdot 0.15^3\cdot 0.85^7}{1-0.85^{10}} \approx 0.15$.

13. On average 25% students subscribe to the newsletter. Determine the most probable number of subscribers out of (a) 100 students; (b) 103 students.

    5.23 from 1

    Answer: (a) 25; (b) 25 or 26.

14. A square is inscribed into a circle. Four points are chosen randomly inside the circle. What is the probability that there is only one point inside the square? What is the most probable number of points inside the circle?

    5.25 from 1

    Answer: $\frac{8}{\pi}\left(1-\frac{2}{\pi}\right)^3 \approx 0.122$; 3.

15. A marksman is shooting at a target until he misses three times. The probability to miss is equal to 0.2 for each of the shots he makes. What is the probability that (a) the marksman uses exactly seven bullets; (b) five bullets are sufficient for him?

    5.34 from 1

    Answer: (a) 0.049152; (b) 0.05792.

16. Two players are playing a match (that consists of several games), each of the games can finish in favour of the younger player with probability 0.6 and in favour of the older player with probability 0.4 The younger player has won exactly five games in the first eight games. What is the probability that he started the match with a defeat?

    Answer: $\frac{3}{8}$.

17. Three dice are rolled. What is the probability that the product of all numbers rolled is 24?

    5.40 from 1

    Answer: $\frac{5}{72}$.

18. Nine people enter the lift at the ground floor of a nine-storey building. What is the probability that the lift stops at each floor as it goes up?

    5.45 from 1

    Answer: $\frac{9!}{2^{25}} \approx 0.011$.

19. A woman arrives at the airport between 11 and 12 o'clock, waits for her husband for ten minutes, and if he does not arrive, she hails a taxi and leaves the airport alone (and the husband has a lot of trouble!) Her husband shows up at the airport at some time between 11 and 11 : 40. He waits for twenty minutes, and if he does not meet his wife, he goes home (and he is in a lot of trouble!) What is the probability that the husband ends up in having lots of trouble?

    Answer: .