Probability & Statistics. Assignment 3

1. Distribution of random variable ξ is given by $\xi \sim \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ 0.1 & 0.3 & 0.2 & \cdot & 0.2 \end{pmatrix}$. Determine the missing probability, draw the graph of a cumulative distribution function $F_\xi(t)$. Find expected value and variance of ξ.

   6.1 from 1

2. Random variable ζ can take values -1, 0 and 1. Find its probability mass function given that $E\zeta = 0$, $\operatorname{Var}\zeta = 0.5$.

   6.10 from 1

   Answer: $\zeta \sim \begin{pmatrix} -1 & 0 & 1 \\ 0.25 & 0.5 & 0.25 \end{pmatrix}$.

3. Find the range of variance for random variable η if its cumulative distribution function is given by $F_\eta(x) = \begin{cases} 0, & x \leqslant 0, \\ 0.3, & 0 < x \leqslant 2, \\ b, & 2 < x \leqslant 6, \\ 1, & x > 6, \end{cases}$ if b is a parameter that belongs to $(0.3; 1)$.

   Answer: $0.84 \leqslant \operatorname{Var}\eta \leqslant 7.6$.

4. The cards are drawn at random one after another from a deck of 52 cards (and they are not returned back to the deck). One continues to draw until he meets the ace of diamonds. What is the average number of cards drawn from the deck? Find the probability that one does not need to take more than half of all the cards.

   Answer: $\frac{53}{2}$; 0.5.

5. The probabilities to pass the exam for three students are equal to 0.9, 0.8, 0.7 respectively. (They pass or fail independently from each other.) Let ξ be equal to the number of students who have passed the exam. Find $E\xi$ and $\operatorname{Var}\xi$.

   Answer: $E\xi = 2.4$, $\operatorname{Var}\xi = 0.46$.

6. Eight random balls are taken out of the urn that contains 10 white balls and 15 black balls. What is the average number of white balls among the ones taken?

   6.23 from 1

   Answer: 3.2.

7. n letters have been written and n envelopes have been inscribed for these letters. An absent minded secretary places the letters into envelopes at random and sends them with the evening post. (a) What is the probability that at least one letter reaches its destination? (b) Find the average quantity of letter that reach their destination.

   Answer: (a) $1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \ldots + \frac{(-1)^{n+1}}{n!}$; (b) 1.

8. Ten people entered the lift at the ground floor of a nine-storey building. How many stops is the lift going to make on average as it goes up?

   6.26 from 1

   Answer: $8\left(1 - \left(\frac{7}{8}\right)^{10}\right) \approx 5.895$.

9. One of three-digit numbers is chosen at random. Determine the expected value of the quantity of different even digits used to write down this number.

   Answer: 1.31.

10. Forty three equally strong sportsmen take part in a ski race; 18 of them belong to club A, 10 to club B and 15 to club C. What is the average place for (a) the best participant from club B; (b) the worst participant from club B?

    Answer: (a) fourth place; (b) fortieth place.

11. Is it possible for random variable X to have a binomial distribution if (a) $EX = 6$ and $\operatorname{Var} X = 3$; (b) $EX = 7$ and $\operatorname{Var} X = 4$?

    Answer: (a) yes; $Bin(12; 0.5)$; (b) no.

12. Let Y be number of sixes and Z be number of fours one gets when rolling six dice. Find the expected value and variance of $Y + Z$.

    Answer: $E(Y+Z) = 2$, $\operatorname{Var}(Y+Z) = \frac{4}{3}$.

13. Indicate such value of p such that variance of $\theta \sim \operatorname{Bin}(n,p)$ reaches its maximum value.

    Answer: $p = \frac{1}{2}$.

14. Each game between the two players ends up with the victory of the first or with the victory of the second with equal probabilities (no draws are possible). Each victory yields one point. Find the expected value of games played until at least one player scores 3 points.

    Answer: $\frac{33}{8}$.

15. Six people entered the lift at the ground floor of a nine-storied house. Find the expected value for (a) the number of stops where exactly one person gets off the lift; (b) the number of stops where exactly two persons leave the lift.

    Answer: (a) $6\left(\frac{7}{8}\right)^5 \approx 3.077$, (b) $8\left(1-\left(\frac{7}{8}\right)^6-\frac{6}{8}\left(\frac{7}{8}\right)^5\right) \approx 1.332$.

16. Six people enter the lift at the ground floor of a nine-storey building. It is known that the lift made its first stop on the fourth floor. What is the average quantity of people who got off the lift at this floor?

    Answer: $\frac{6}{5}\left(1-\left(\frac{4}{5}\right)^6\right)^{-1} \approx$.

17. Find the expected value and variance of a^ξ given that $\xi \sim Bin(n,p)$.

    6.50 from 1

    Answer: $E\left(a^\xi\right) = (pa+q)^n$, $\operatorname{Var}\left(a^\xi\right) = \left(pa^2+q\right)^n - (pa+q)^{2n}$.

18. The probability to detect a small object in a region is equal to $\frac{1}{3}$ for each of the flights. (a) How many flights does one need on average to detect the object? (b) What is the probability that one needs more than two flights to detect the object?

    Answer: (a) 3; (b) $\frac{4}{9}$.

19. How many times (on average) do we need to roll a die in order to get each of the results 1, 2, 3, 4, 5, 6 at least once?

    6.59 from 1

    Answer: 14.7.

20. How many times (on average) does one need to roll a die in order to get each of the odd numbers (1, 3, 5) at least once?

    Answer: 11.

21. Let Z be a random variable with geometric distribution. Prove that $P(Z = n+k | z > n) = P(Z = k)$ (*lack of memory property of geometric distribution*).