Probability & Statistics. Assignment 5

1. The joint distribution of ξ and η is provided in the table below.

$\eta \setminus \xi$	-1	0	1
-2	$\frac{3}{17}$	$\frac{4}{17}$	$\frac{1}{17}$
2	$\frac{1}{17}$	$\frac{5}{17}$	$\frac{3}{17}$

(a) Find marginal distributions of ξ and η;

(b) find expected value and variance for ξ and η;

(c) determine if ξ and η are independent;

(d) find correlation coefficient of ξ and η;

(e) find conditional expected values $E(\xi|\eta)$ and $E(\eta|\xi)$.

Answer:

(a) $\xi \sim \begin{pmatrix} -1 & 0 & 1 \\ \frac{4}{17} & \frac{9}{17} & \frac{4}{17} \end{pmatrix}$, $\eta \sim \begin{pmatrix} -2 & 2 \\ \frac{8}{17} & \frac{9}{17} \end{pmatrix}$.

(b) $E\xi = 0$, $Var\xi = \frac{8}{17}$, $E\eta = \frac{2}{17}$, $Var\eta = \frac{1152}{289}$;

(c) are dependent. *Hint*: check the equality $P(\xi = 1, \eta = 2) = P(\xi = 1) \cdot P(\eta = 2)$;

(d) $\rho(\xi, \eta) = \frac{\sqrt{17}}{12}$;

(e) $E(\xi|\eta) \sim \begin{pmatrix} -\frac{2}{8} & \frac{2}{9} \\ \frac{8}{17} & \frac{9}{17} \end{pmatrix}$, $E(\eta|\xi) \sim \begin{pmatrix} -1 & \frac{2}{9} & 1 \\ \frac{4}{17} & \frac{9}{17} & \frac{4}{17} \end{pmatrix}$.

2. A fair coin is flipped thrice, and for every tails obtained we write a plus; for every heads obtained we write a minus. Random variable ξ is equal to the quantity of tails, and random variable η is equal to the quantity of sign changes in the sequence.

(a) Find marginal distributions of ξ and η;

(b) find expected value and variance for ξ and η;

(c) determine if ξ and η are independent;

(d) find correlation coefficient of ξ and η;

(e) find conditional expected values $E(\xi|\eta)$ and $E(\eta|\xi)$.

Answer:

(a) $\xi \sim \begin{pmatrix} 0 & 1 & 2 & 3 \\ \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \end{pmatrix}$, $\eta \sim \begin{pmatrix} 0 & 1 & 2 \\ \frac{1}{4} & \frac{2}{4} & \frac{1}{4} \end{pmatrix}$;

(b) $E\xi = \frac{3}{2}$, $Var\xi = \frac{3}{4}$, $E\eta = 1$, $Var\eta = \frac{1}{2}$;

(c) are dependent. *Hint*: check the equality $P(\xi = 0, \eta = 0) = P(\xi = 0) \cdot P(\eta = 0)$;

(d) $\rho(\xi, \eta) = 0$;

(e) $E(\xi|\eta) \sim \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{pmatrix}$, $E(\eta|\xi) \sim \begin{pmatrix} 0 & \frac{4}{3} & \frac{4}{3} & 0 \\ \frac{1}{8} & \frac{3}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$.

3. In some city the probability that it is sunny is equal to 0.4, and the probability that it is cloudy is equal to 0.6 (the weather each day is independent from the other days). How many days does one need (on average) to spend in the city in order to see it both in sunny and in cloudy weather?

   Answer: $\frac{19}{6}$.

4. Let Y be the number of times one needs to flip a coin in order to get tails and heads at least once. Find the expected value of Y.

   Answer: $E[Y] = 3$.

5. How many times does one have to flip a coin to get the results "heads", "heads" in succession? Is the result going to change if we replace the sequence with "tails", "heads"?

   Answer: $S_{HH} = 6$, $S_{TH} = 4$, where S_{HH} is an expected value of flips to get double "heads".

6. How many rolls does one need on average to get a sequence "6", "6" when rolling a symmetric six-sided die? And if we change this sequence to "6", "6", "6"?

   Answer: $S_{66} = 42$, $S_{666} = 258$.

7. A fair die is rolled until a four is obtained. Find the expected value of a sum obtained in all the rolls.

   Answer: $E[S] = 21$.

8. Find a correlation coefficient between the quantity of sixes and the quantity of fives obtained in K rolls of a fair die.

   Answer: $\rho(\xi, \eta) = -\frac{1}{5}$.

9. ζ is the quantity of threes and η is the quantity of odd digits obtained when rolling a fair die K times. Find correlation coefficient between η and ζ.

   Answer: $\rho(\zeta, \eta) = \frac{1}{\sqrt{5}}$.