Lab 5

19(a) $E\xi = 1$, $Var\,\xi = 4$ $\qquad \frac{x-1}{2} = t$

$$P(-3<\xi<1) = \int_{-3}^{1} \frac{1}{\sqrt{2\pi}\,2} e^{-\frac{(x-1)^2}{8}} dx =$$

$$= \int_{-2}^{0} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = \int_{0}^{2} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = \Phi_0(2) \approx$$

$\approx 0,4772$

20(b) $\xi \sim N(-1;1)$ $\qquad P(0<\xi<x) = 0,8$

$$P(0<\xi<x) < P(-1<\xi<+\infty) = \frac{1}{2}$$

20(c) $P(-1-x<\xi<-1+x) = 0,8$

$$\int_{-1-x}^{-1+x} \frac{1}{\sqrt{2\pi}} e^{-\frac{(y+1)^2}{2}} dy = 0,8 \qquad y+1=t$$

$$\int_{-x}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = 0,8$$

$2\Phi_0(x) = 0,8$, $\quad \Phi_0(x) = 0,4$

$x \approx 1,28$

22) $P(|\xi - E\xi| < 1) = 0,3 \qquad P(|\xi - E\xi| < 2) = ?$

$\xi \sim N(\mu; \sigma^2)$

$|\xi - \mu| < 1 \Leftrightarrow \mu - 1 < \xi < \mu + 1$

$$\int_{\mu-1}^{\mu+1} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \qquad \frac{x-\mu}{\sigma} = t$$

$$= \int_{-1/\sigma}^{1/\sigma} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = \underbrace{2\Phi_0\left(\frac{1}{\sigma}\right) = 0,3}_{\Phi_0\left(\frac{1}{\sigma}\right) = 0,15}$$

$$P(|\xi - E\xi| < 2) = 2\Phi_0\left(\frac{2}{\sigma}\right) =$$

$\Phi_0(0,38) \approx 0,1480$

$\Phi_0(0,39) \approx 0,1517$

$\underbrace{\frac{1}{\sigma} = 0,385}$

$$= 2\Phi_0(0,77) \approx 0,2793 \cdot 2 =$$

$$= 0,5586$$

(25) $\eta \sim N(1; \sigma^2),\ \sigma > 0 \qquad P(2 < \eta < 4) \longrightarrow \max$

$$\boxed{\frac{x-1}{\sigma} = t}$$

$$P(2<\eta<4) = \int_2^4 \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-1)^2}{2\sigma^2}}\,dx =$$

$$= \int_{1/\sigma}^{3/\sigma} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\,dt = \int_{-\infty}^{3/\sigma} \ldots - \int_{-\infty}^{1/\sigma} \ldots = \Phi\left(\frac{3}{\sigma}\right) - \Phi\left(\frac{1}{\sigma}\right) = h(\sigma)$$

$$h'(\sigma) = f\left(\frac{3}{\sigma}\right)\cdot\left(-\frac{3}{\sigma^2}\right) - f\left(\frac{1}{\sigma}\right)\cdot\left(-\frac{1}{\sigma^2}\right) =$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{9}{2\sigma^2}}\left(-\frac{3}{\sigma^2}\right) + \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}} \cdot \frac{1}{\sigma^2} =$$

$$= \frac{1}{\sigma^2\sqrt{2\pi}} e^{-\frac{9}{2\sigma^2}}\left(-3 + e^{\frac{4}{\sigma^2}}\right)$$

$$h': \quad + \nearrow \quad \frac{2}{\sqrt{\ln 3}} \quad - \searrow$$

$$\frac{4}{\sigma^2} = \ln 3, \qquad \sigma = \frac{2}{\sqrt{\ln 3}}$$

(26) $\xi \sim N(-2; 9)$

$$\int_{-\infty}^{+\infty} (3-x)(x+5) f_\xi(x)\,dx$$

$$E\big((3-\xi)(\xi+5)\big) =$$

$$= E(15 - 2\xi - \xi^2) = 15 - 2E\xi - E\xi^2 =$$

$$= 15 - 2(-2) - \big(Var\,\xi + (E\xi)^2\big) = 15 + 4 - 13 = 6$$

(28) $\xi \sim N(0;\sigma^2)$ $\qquad E|\xi|,\ Var|\xi|$

$\frac{x}{\sigma}=t$

$d(-e^{-t^2/2})$

$$E|\xi| = \int_{-\infty}^{+\infty} |x| \cdot \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{x^2}{2\sigma^2}} dx =$$

$$= \int_{-\infty}^{+\infty} \sigma |t| \cdot \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = 2\sigma \int_0^{+\infty} \frac{t e^{-t^2/2} dt}{\sqrt{2\pi}} =$$

$$= 2\sigma \cdot \frac{1}{\sqrt{2\pi}} \cdot \left(-e^{-t^2/2}\right)\Big|_{t=0}^{+\infty} = \frac{2\sigma}{\sqrt{2\pi}} = \sqrt{\frac{2}{\pi}}\,\sigma$$

$$E|\xi|^2 = E\xi^2 = \sigma^2$$

$$Var|\xi| = \sigma^2 - \frac{2}{\pi}\sigma^2$$

$$F_{|\xi|}(t) = P(|\xi|<t) = P(-t<\xi<t) \cdot I_{t>0} =$$

$$= 2P(0<\xi<t) \cdot I_{t>0} = 2\Phi_0\left(\frac{t}{\sigma}\right) \cdot I_{t>0}$$

$$P(0<\xi<t) = \int_0^t \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{x^2}{2\sigma^2}} dx = \left/\!\!\left/ \frac{x}{\sigma} = y \right/\!\!\right/ =$$

$$= \int_0^{t/\sigma} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy = \Phi_0\left(\frac{t}{\sigma}\right)$$

$$f_{|\xi|}(t) = \frac{2}{\sigma} f\left(\frac{t}{\sigma}\right) \cdot I_{t>0} = \frac{2}{\sigma} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma^2}} \cdot I_{t>0}$$

31 Cauchy distribution

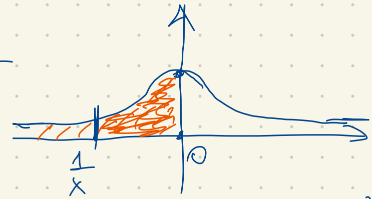
$\tau > 0$

$$f_\xi(x) = \frac{\tau}{\pi(\tau^2 + (x-a)^2)}, \quad x \in \mathbb{R}$$

$$\int_{-\infty}^{+\infty} \frac{dx}{\tau^2 + (x-a)^2} = \frac{1}{\tau} \arctan \frac{x-a}{\tau} \Big|_{x=-\infty}^{+\infty} = \frac{\pi}{\tau}$$

$a = 0$

$$f_\xi(x) = \frac{\tau}{\pi(\tau^2 + x^2)}$$



$\eta = 1/\xi$

$$F_\eta(x) = P\Big(\underbrace{\frac{1}{\xi} < x}\Big) = \frac{1}{2} + P\Big(\xi > \frac{1}{x}\Big) = \frac{1}{2} + 1 - P\Big(\xi < \frac{1}{x}\Big) =$$
$$= \frac{3}{2} - F_\xi\Big(\frac{1}{x}\Big)$$

$x > 0$

$$\left[\begin{array}{l} \xi < 0 \\ \begin{cases} \xi > 0 \\ \xi > \frac{1}{x} \end{cases} \end{array}\right.$$

$x < 0$ $\quad \begin{cases} \xi < 0 \\ \xi > \frac{1}{x} \end{cases}$

$$F_\eta(x) = P\Big(\frac{1}{x} < \xi < 0\Big) = P\Big(\xi > \frac{1}{x}\Big) - \frac{1}{2} =$$
$$= 1 - P\Big(\xi < \frac{1}{x}\Big) - \frac{1}{2} = \frac{1}{2} - F_\xi\Big(\frac{1}{x}\Big)$$

$$f_\eta(x) = -f_\xi\Big(\frac{1}{x}\Big) \cdot \Big(-\frac{1}{x^2}\Big) = \frac{\tau}{\pi\big(\tau^2 + \frac{1}{x^2}\big)} \cdot \frac{1}{x^2} = \frac{\tau}{\pi(\tau^2 x^2 + 1)} =$$
$$= \frac{1/\tau}{\pi(x^2 + 1/\tau^2)}$$