$$f(x) \geq 0, \quad \int_{-\infty}^{+\infty} f(x)\,dx = 1$$

$$f(x) = Cx^3 \cdot I_{\{-1 \leq x \leq 2\}} = \begin{cases} Cx^3, & -1 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

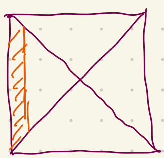
$$f(x) = \frac{c}{x^2} I_{0<x<1}$$

$\int_0^1 \frac{c}{x^2}\,dx$ diverges

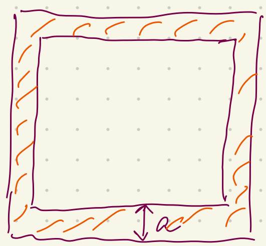
$$f(x) = \frac{cx^2}{x^4+6}$$

$$\int_{-\infty}^{+\infty} \frac{cx^2}{x^4+6}\,dx$$

$\frac{1}{x^\alpha}$ $\alpha > 1$

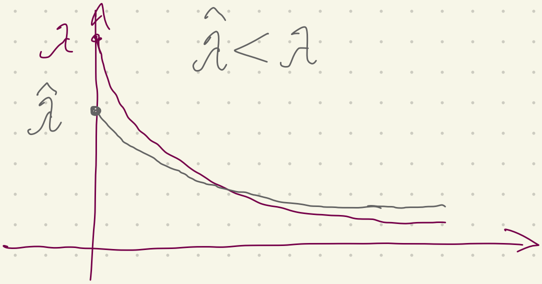


$P(\xi < x)$



$\xi \sim Exp(\lambda)$ $\quad \underline{\lambda > 0}$

$$f_\xi(x) = \lambda e^{-\lambda x} \cdot I_{x>0} = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$



$$f_\xi''(x) = \lambda^3 e^{-\lambda x} \cdot I_{x>0}$$

$$\int_0^{+\infty} \lambda e^{-\lambda x} dx = -e^{-\lambda x} \Big|_{x=0}^{+\infty} = 1$$

$$P(\xi > T + t \mid \xi > T) = P(\xi > t) \qquad t > 0, \quad T > 0$$

$$P(\xi > t) = \int_t^{+\infty} f_\xi(x) dx = \int_t^{+\infty} \lambda e^{-\lambda x} dx = -e^{-\lambda x} \Big|_{x=t}^{+\infty} = e^{-\lambda t}$$

$$\frac{P(\xi > T + t \;\&\; \xi > T)}{P(\xi > T)} = \frac{P(\xi > T + t)}{P(\xi > T)} = \frac{e^{-\lambda(T+t)}}{e^{-\lambda T}} = e^{-\lambda t}$$

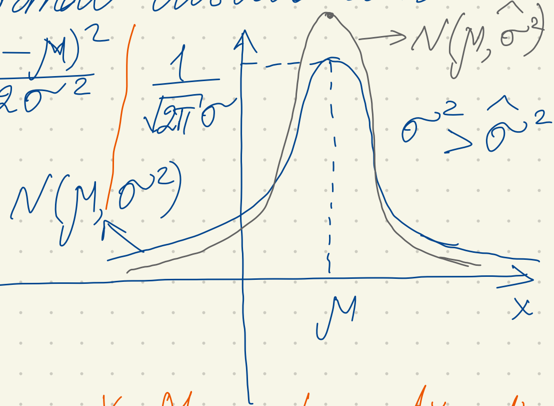
Normal distribution

$\sigma > 0$

$\xi \sim N(\mu; \sigma^2)$

$N(0;1)$ – standard normal distribution

$$f_\xi(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



$$\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx =$$

$$\frac{x-\mu}{\sigma} = t \Rightarrow \frac{dx}{\sigma} = dt$$

$$= \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = 1$$

$$E\xi = \int_{-\infty}^{+\infty} x \cdot \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx =$$

$$-e^{-t^2/2}\Big|_{-\infty}^{+\infty}$$

$$= \int_{-\infty}^{+\infty} (\sigma t + \mu) \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = \sigma \underbrace{\cancel{\int_{-\infty}^{+\infty} \frac{t e^{-t^2/2}}{\sqrt{2\pi}} dt}}_{\to 0} +$$

$$+ \mu \int_{-\infty}^{+\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}} dt = \mu$$

$$E\xi^2 = \int_{-\infty}^{+\infty} x^2 \cdot \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \sigma^2 \int_{-\infty}^{+\infty} \frac{t^2 e^{-t^2/2}}{\sqrt{2\pi}} dt +$$

$$+ \underbrace{\cancel{2\sigma\mu \int_{-\infty}^{+\infty} \frac{t e^{-t^2/2}}{\sqrt{2\pi}} dt}}_{\to 0} + \underbrace{\cancel{\mu^2 \int_{-\infty}^{+\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}} dt}}_{\to \mu^2} = \sigma^2 + \mu^2$$

$$\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}}\, t^2 e^{-t^2/2}\, dt = -\frac{1}{\sqrt{2\pi}}\, t e^{-t^2/2}\Big|_{t=-\infty}^{+\infty} +$$

$$t^2 e^{-t^2/2}\, dt = t e^{-t^2/2}\, d\frac{t^2}{2} = -t\, de^{-t^2/2}$$

$$+\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}}\, e^{-t^2/2}\, dt = 0 + 1 = 1$$

$$\mathrm{Var}\,\xi = E\xi^2 - (E\xi)^2 = (\sigma^2 + \mu^2) - \mu^2 = \sigma^2$$

$$N(\mu; \sigma^2)$$

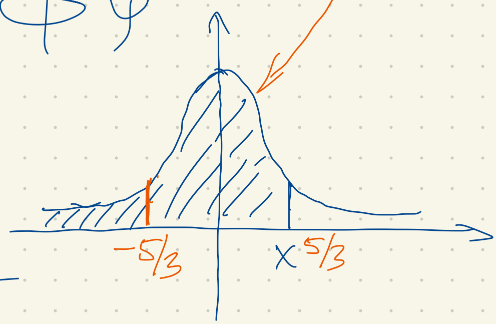
$\xi \sim N(-2; 9)$; $P(-7 < \xi < 3)$ $\qquad \frac{x+2}{3} = t$

$$P = \int_{-7}^{3} \frac{1}{\sqrt{2\pi}\cdot 3}\, e^{-\frac{(x+2)^2}{18}}\, dx =$$

$$= \int_{-5/3}^{5/3} \frac{1}{\sqrt{2\pi}}\, e^{-t^2/2}\, dt \; =$$

$N(0;1)$ $\quad \frac{1}{\sqrt{2\pi}}\, e^{-t^2/2}$

$\Phi \quad \varphi$



$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}}\, e^{-t^2/2}\, dt$$

$$\Phi_0(x) = \int_{0}^{x} \frac{1}{\sqrt{2\pi}}\, e^{-t^2/2}\, dt$$

$$\Phi_0(x) = \Phi(x) - \frac{1}{2}$$

$$= 2\int_{0}^{5/3} \frac{1}{\sqrt{2\pi}}\, e^{-t^2/2}\, dt = 2\Phi_0\left(\frac{5}{3}\right)$$

$$P\left(|\xi - E\xi| < \underset{4}{3}\sqrt{Var\,\xi}\right) = ?, \quad \xi \sim N(\mu; \sigma^2)$$

$$= P(|\xi - \mu| < 3\sigma) = P(-3\sigma < \xi - \mu < 3\sigma) =$$

$$= P(\mu - 3\sigma < \xi < \mu + 3\sigma) = \int_{\mu-3\sigma}^{\mu+3\sigma} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx =$$

$$\frac{x-\mu}{\sigma} = t$$

$$= \int_{-3}^{3} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = 2\int_{0}^{3} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = 2\Phi_0(\underset{4}{3}) \approx$$

$$\approx 2 \cdot 0,4986 = 0,9972$$

$$\approx 2 \cdot 0,499968$$

$\eta = a\xi + b \quad , \quad a \neq 0$ $\quad E\eta = a\,E\xi + b = a\mu + b$

$Var\,\eta = a^2 Var\,\xi = a^2\sigma^2$

1) $a > 0$

$F_\eta(x) = P(\eta < x) = P(a\xi + b < x) = P\left(\xi < \frac{x-b}{a}\right) =$

$= F_\xi\left(\frac{x-b}{a}\right)$

$f_\eta(x) = \frac{1}{a} f_\xi\left(\frac{x-b}{a}\right)$

2) $a < 0$

$F_\eta(x) = P\left(\xi > \frac{x-b}{a}\right) = P\left(\xi \geq \frac{x-b}{a}\right) =$

$1 - P\left(\xi < \frac{x-b}{a}\right) = 1 - F_\xi\left(\frac{x-b}{a}\right)$

$f_\eta(x) = -\frac{1}{a} f_\xi\left(\frac{x-b}{a}\right)$

$\xi \sim N(\mu; \sigma^2)$

$f_\xi(t) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(t-\mu)^2}{2\sigma^2}}$

$f_\eta(x) = \frac{1}{|a|} f_\xi\left(\frac{x-b}{a}\right) =$

$= \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{\left(\frac{x-b}{a} - \mu\right)^2}{2\sigma^2}} \cdot \frac{1}{|a|} = \frac{1}{\sqrt{2\pi}\,\sigma|a|} \cdot e^{-\frac{(x-b-a\mu)^2}{2\sigma^2 a^2}}$

$\sim N(b + a\mu; \sigma^2 a^2)$

$\eta \backslash \xi$	1	2	3	
-1	$1/12$	$3/12$	$5/12$	$9/12$
1	$1/12$	$1/12$	$1/12$	$3/12$
	$2/12$	$4/12$	$6/12$	

$$\begin{pmatrix} \xi \\ \eta \end{pmatrix}$$

$$\xi \sim \begin{pmatrix} 1 & 2 & 3 \\ 1/6 & 2/6 & 3/6 \end{pmatrix}$$

$$\eta \sim \begin{pmatrix} -1 & 1 \\ 3/4 & 1/4 \end{pmatrix}$$

$$E\xi = \frac{7}{3}$$

$$E\eta = -\frac{1}{2}$$

$$P(\xi_1 = a_1, \xi_2 = a_2, \dots, \xi_n = a_n)$$

$$Cov(\xi, \eta) = E(\xi\eta) - E\xi \cdot E\eta$$

$$\xi\eta \sim \begin{pmatrix} 1 & 2 & 3 & -1 & -2 & -3 \\ 1/12 & 1/12 & 1/12 & 1/12 & 3/12 & 5/12 \end{pmatrix}$$

$$E(\xi\eta) = 1 \cdot \frac{1}{12} + 2 \cdot \frac{1}{12} + 3 \cdot \frac{1}{12} - 1 \cdot \frac{1}{12} - 2 \cdot \frac{3}{12} - 3 \cdot \frac{5}{12} =$$

$$= -\frac{16}{12} = -\frac{4}{3}$$

$$Cov(\xi, \eta) = -\frac{4}{3} + \frac{7}{3} \cdot \frac{1}{2} = -\frac{1}{6}$$

correlation coefficient

$$\boxed{\rho_{\xi,\eta} = \frac{Cov(\xi, \eta)}{\sqrt{Var\,\xi \cdot Var\,\eta}}}$$

$$E(\xi + c\eta)^2 \geq 0$$

$$E\xi^2 + 2c \cdot E(\xi\eta) + c^2 \cdot E\eta^2 \geq 0 \quad \forall c$$

a quadratic inequality with respect to c

$$\frac{\mathcal{D}}{4} = (E(\xi\eta))^2 - E\xi^2 \cdot E\eta^2 \leq 0$$

$$|E(\xi\eta)| \leq \sqrt{E\xi^2 \cdot E\eta^2}$$

The equality can be reached if $\exists c: E(\xi + c\eta)^2 = 0$

$$\xi + c\eta = 0$$

$$P(\xi + c\eta = 0) = 1$$

$$\xi \to \xi - E\xi, \quad \eta \to \eta - E\eta$$

$$|Cov(\xi, \eta)| \leq \sqrt{Var\,\xi \cdot Var\,\eta}$$

$$\left| \frac{Cov(\xi, \eta)}{\sqrt{Var\,\xi \cdot Var\,\eta}} \right| \leq 1$$

$$|\rho_{\xi,\eta}| \leq 1$$

$$\xi - E\xi = c(\eta - E\eta)$$

$$\xi = c \cdot \eta + \mathcal{D}$$

$c = const$, $\mathcal{D} = const$

$$c > 0 \Rightarrow \rho_{\xi,\eta} = 1$$

$$c < 0 \Rightarrow \rho_{\xi,\eta} = -1$$