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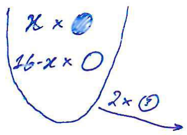
Task 1. Set 2

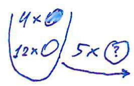


$$P_1 = \frac{x}{16}\cdot\frac{16-x}{15} + \frac{16-x}{16}\cdot\frac{x}{15} = \frac{x(16-x)}{8\cdot 15} = \frac{2}{5} \Rightarrow 16x - x^2 - 48 = 0$$

$$x^2 - 16x + 48 = 0$$

$$x = \left[\begin{matrix} 4 \\ 12 \end{matrix}\right.$$

if $x = 4$:



$$\frac{12}{16}\cdot\frac{11}{15}\cdot\frac{10}{14}\cdot\frac{9}{13}\cdot\frac{8}{12} = 0,18131$$

if $x = 12$:

$-//- = 0,18131$

Answer: 0,18131

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Task 2. Set2

$X \sim Exp(4)$
$Y \sim Exp(3)$

So, it is impossible to solve the task because there is no information about dependasies between X and Y

assume X, Y - independent.

$$\begin{cases} 3X - 5Y = A \\ B = X \end{cases} \Rightarrow \begin{cases} X = B \\ Y = \frac{3B-A}{5} \end{cases} \quad f_{A,B}(a,b) = f_{X,Y}\left(b, \frac{3b-a}{5}\right)\left|\frac{\partial(x,y)}{\partial(a,b)}\right|$$

$$|J| = \left\| \begin{matrix} 0 & 1 \\ -\frac{1}{5} & ? \end{matrix} \right\| = \frac{1}{5} \qquad f_{A,B}(a,b) = f_X(b) \cdot f_Y\left(\frac{3b-a}{5}\right) \cdot \frac{1}{5} =$$

$$= 4e^{-4b} \cdot 3e^{-3\left(\frac{3b-a}{5}\right)} \cdot \frac{1}{5} \cdot I_{b>0} \cdot I_{3b>a}$$

$$f_A(a) = \frac{12}{5}\int_{\frac{a}{3}}^{+\infty} e^{-4b - \frac{9b - 3a}{5}}\, db = \frac{12}{5}\int_{\frac{a}{3}}^{+\infty} e^{b(-4-\frac{9}{5})} \cdot e^{\frac{3}{5}a}\, db = \frac{12}{29} e^{\frac{3}{5}a} \int_{\frac{a}{3}}^{+\infty} e^{-\frac{29}{5}b}\, d\left(-\frac{29}{5}b\right) =$$

$$= -\frac{12}{29} e^{\frac{3}{5}a} \int_{\frac{a}{3}}^{+\infty} d e^{-\frac{29}{5}b} = \frac{12}{29} e^{\frac{3}{5}a} \cdot e^{-\frac{29}{5}\cdot\frac{a}{3}}$$

~~Answer~~

$$f_{3X-5Y}(t) = \frac{12}{29} e^{\frac{3}{5}t} \cdot e^{-\frac{29}{5}\cdot\frac{a}{3}}$$

Answer

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Task 3. set 2

$\xi_1, \dots \xi_n \sim Bin(3, p)$ $N = 750$

i	0	1	2	3
ϑ_i	7	85	301	357
e_i	6	72	288	384

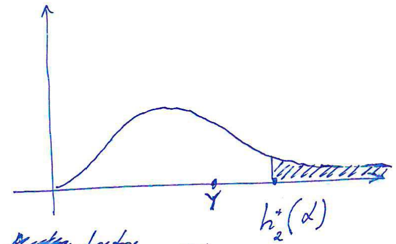
$H_0: p = 0{,}8$

$\alpha = 0{,}02$

$$Y = \sum_{i=0}^{3} \frac{(\vartheta_i - \overbrace{NC_3^i p^i q^{3-i}}^{e_i})^2}{NC_3^i p^i q^{3-i}}$$

$Y_* \approx 5$

$N \to \infty: \ Y \sim \chi_2^2$



$h_2^+(\alpha) \approx 7{,}82 > 5 \Rightarrow H_0$ do not contradict given data

Answer

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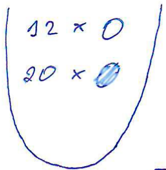
Problem 4 . set 2.



$\cancel{\xi}$ $\xi = \zeta = \#$ balls of the same color by the moment ....

ξ - non negative integer

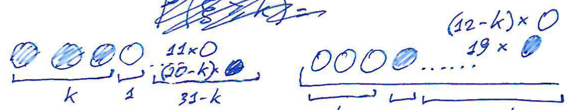
C^{12}_{32} - all possible outputs

$$E\xi = \sum_{k=1}^{\infty} P(\xi \geq k)$$

$\cancel{P(\xi \geq 21) = 0}$

$\cancel{P(\xi \geq k) = \frac{C^{12}_{32-k}}{C^{12}_{32}}, k \in [13;20]}$

$\cancel{P(\xi \geq k) =}$



$$P(\xi = 21) = 0$$

$$P(\xi = k) = \frac{C^{11}_{31-k}}{C^{12}_{32}}, k \in [13;20]$$

$$P(\xi = k) = \frac{C^{11}_{31-k} + C^{19}_{31-k}}{C^{12}_{32}}, k \in [12;1]$$

$$P(\xi = 0) = 0$$

$\Rightarrow$

$$E\xi = \sum_{i=0}^{n} i\, P(\xi = i) \approx 2,1099$$

Answer

$$E\xi^2 = \sum_{i=0}^{n} i^2 \cdot P(\xi = i) \approx 6,857$$

$$Var\,\xi = E\xi^2 - (E\xi)^2 \approx 2,4055$$

Answer

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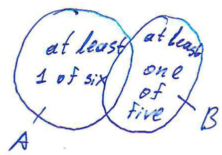
Task 7. set 2

$N = 11$ ~~number was drawn~~ ~~P(get at least 1 of six and 1 of five) = 1 - P(get only)~~



$$P(A) = 1 - \left(\frac{5}{6}\right)^{11}$$

$$P(B) = 1 - \left(\frac{5}{6}\right)^{11}$$

$$P(A+B) = 1 - \left(\frac{4}{6}\right)^{11}$$

$$P(AB) = P(A) + P(B) - P(A+B) = 1 - 2\left(\frac{5}{6}\right)^{11} + \left(\frac{4}{6}\right)^{11} \approx 0{,}7423$$

Answer

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Task 8. set 2

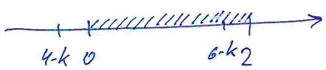
$\xi \sim U(0;2)$ $\qquad f_\xi(x) = \frac{1}{2} \cdot I_{0<x<2}$

$\eta \sim Poisson(0,5)$ $\qquad P(\eta = k) = \frac{0,5^k}{k!} e^{-0,5}$

ξ, η - indep.

if $\eta = k \Rightarrow P(\eta + \xi \in (4;6)) = P(4 < k + \xi < 6) = P(4-k < \xi < 6-k) =$

$$= \int_{4-k}^{6-k} \frac{1}{2} \cdot I_{0<x<2}\, dx = (6-k)\, I_{0 \le 6-k \le 2} + (2-(4-k))\, I_{2>4-k>0} =$$



$$= \begin{cases} \frac{6-k}{2}, & 4 \le k < 6 \\ \frac{k-2}{2}, & 2 < k < 4 \\ 0, & \text{otherwise} \end{cases}$$

$$P(4 < \eta + \xi < 6) = P(\eta = 3) \cdot \frac{(3-2)}{2} + P(\eta = 4) \cdot \frac{(6-4)}{2} + P(\eta = 5) \cdot \frac{(6-5)}{2} =$$

$$= \frac{P(\eta = 3) + 2P(\eta = 4) + P(\eta = 5)}{2} \approx 0,0079765$$

Answer