Probability & Statistics. Midterm Examination. Problem Set 1. Part 1

Name _____ **Group Number** _____

Solutions for each part of this examination have to be written on separate sheets! Do not write solutions of two different parts of this examination on one sheet of paper. You can use a single sheet for several problems from one part of the examination (i.e. you can write problems 1, 2, 3, 4 on one sheet). Do not forget to sign each of the sheets that you hand in. You can use this sheet for solving problems as well.

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1. Random variable ξ has a normal distribution, and it is known that $E\xi = -4$, $E\big((\xi+3)(\xi-1)\big) = 14$. Determine the probability that $\xi \in (-10.33; 0.14)$.

2. Function $f(x) = \begin{cases} \frac{C}{1+x}, & 0 < x < 4, \\ 0, & \text{otherwise} \end{cases}$ is a probability density function of some random variable ζ.

   (a) Determine the value of constant C;

   (b) find the expected value and the variance of ζ.

3. A 6-digit number is chosen at random.

   (a) Find the probability that it contains at least one even digit.

   (b) Find the expected value for the quantity of even digits in this number.

4. Find the probability generating function of a geometrical distribution with parameter p. Derive formulae for expected value and variance of a geometrically distributed random variable using its probability generating function.

Probability & Statistics. Midterm Examination. Problem Set 1. Part 2

Name _____ **Group Number** _____

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5. Random variables X and Y are independent, and $EX = 1$, $EY = 11$, $\operatorname{var} X = 5$, $\operatorname{var} Y = 13$. Find the expected value and the variance of $Z = 4X - Y$.

6. Using Chebyshev's inequality estimate the probability $P\big(|\eta - E\eta| < 5\sqrt{\operatorname{var}\eta}\big)$.

Probability & Statistics. Midterm Examination. Problem Set 1. Part 3

Name _____ **Group Number** _____

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7. Three people enter the lift at the ground floor of an eleven-storey building[1]. Let ξ be a number of people who exit at the eighth floor, and ζ be a number of people who exit at the eighth floor or higher. Find the correlation coefficient between ξ and ζ.

8. Let us consider a 12×10 checkerboard. Initially, we are situated in the lower-left square, and our goal is to move to the upper-right square. We are only allowed to move one square to the right or one square upwards at a time. One of the possible itineraries is chosen at random. What is the probability that the last three steps are made along the longer side of the board?

9. Some probabilities of a joint distribution of random variables ξ and η are given in a table below.

$\eta\backslash\xi$	-1	0	1
-2	$\frac{3}{17}$	$\frac{4}{17}$	$\frac{1}{17}$
2	$\frac{1}{17}$	$\cdot$	$\cdot$

It is also known that $\operatorname{var}\eta - 4\operatorname{var}\xi = \frac{40}{17}$. Find the probability $P(\xi = 1, \eta = 2)$.

[1] There are ground floor, first floor, second floor, ..., eleventh floor.

Probability & Statistics. Midterm Examination. Problem Set 2. Part 1

Name _____ **Group Number** _____

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1. Random variable ξ has a normal distribution, and it is known that $E\xi = -11$, $E\big((2\xi+3)(\xi-2)\big) = 253$. Determine the probability that $\xi \in (-15; 0.16)$.

2. Function $f(x) = \begin{cases} \frac{C}{x-3}, & -5 < x < 2, \\ 0, & \text{otherwise} \end{cases}$ is a probability density function of some random variable ζ.

   (a) Determine the value of constant C;

   (b) find the expected value and the variance of ζ.

3. An 8-digit number is chosen at random.

   (a) Find the probability that it contains at least one odd digit.

   (b) Find the expected value for the quantity of odd digits in this number.

4. Find the probability generating function of a binomial distribution with parameters N and p. Derive formulae for expected value and variance of a geometrically distributed random variable using its probability generating function.

Probability & Statistics. Midterm Examination. Problem Set 2. Part 2

Name _____ **Group Number** _____

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5. Random variables X and Y are independent, and $EX = 6$, $EY = 19$, $\operatorname{var} X = 7$, $\operatorname{var} Y = 12$. Find the expected value and the variance of $Z = 3X - 2Y$.

6. Using Chebyshev's inequality estimate the probability $P\big(|\eta - E\eta| < 8\sqrt{\operatorname{var}\eta}\big)$.

Probability & Statistics. Midterm Examination. Problem Set 2. Part 3

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7. Four people enter the lift at the ground floor of a twelve-storey building[2]. Let ξ be a number of people who exit at the eighth floor, and ζ be a number of people who exit at the eighth floor or higher. Find the correlation coefficient between ξ and ζ.

8. Let us consider a 15×9 checkerboard. Initially, we are situated in the lower-left square, and our goal is to move to the upper-right square. We are only allowed to move one square to the right or one square upwards at a time. One of the possible itineraries is chosen at random. What is the probability that the last three steps are made along the shorter side of the board?

9. Some probabilities of a joint distribution of random variables ξ and η are given in a table below.

$\eta\backslash\xi$	-2	0	2
-1	$\cdot$	$\cdot$	$\frac{2}{13}$
1	$\frac{3}{13}$	$\frac{2}{13}$	$\frac{1}{13}$

It is also known that $7 \operatorname{var} \xi = 19 \operatorname{var} \eta$. Find the probability $P(\xi = -2, \eta = -1)$.

[2]There are ground floor, first floor, second floor, ..., twelfth floor.