Set Up and Simplify a Difference Quotient

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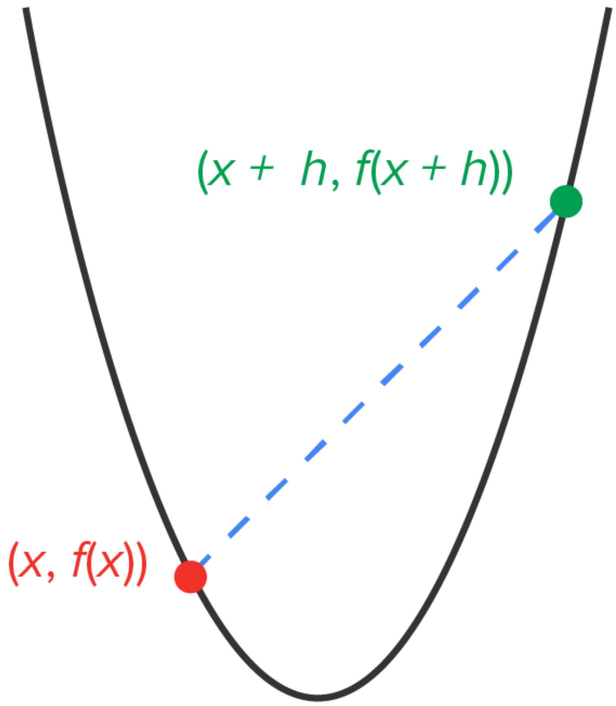
WHAT'S COVERED

In this lesson, you will learn what a difference quotient is, then set up and simplify a difference quotient for specified functions. Specifically, this lesson will cover:

1. What a Difference Quotient Represents

Consider the graph of a function $y = f(x)$.



In this picture, $(x, f(x))$ is any point on the graph. $(x+h, f(x+h))$ is the resulting point when x is increased by h.

The slope of the line between these points is as follows:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}$$

Therefore, the quantity $\frac{f(x+h) - f(x)}{h}$ gives the slope of the line between $(x, f(x))$ and $(x+h, f(x+h))$. This quantity is called the **difference quotient** for $f(x)$ and can be found with the following formula.

FORMULA TO KNOW

Difference Quotient

$$\frac{f(x+h) - f(x)}{h}$$

HINT

Since the slope between two points is also known as the average rate of change between the points, we also say that the difference quotient is the average rate of change between $(x, f(x))$ and $(x+h, f(x+h))$.

TERM TO KNOW

Difference Quotient

An expression that represents the average rate of change between two points on a curve between input values x and $x+h$.

2. Evaluating a Difference Quotient

Evaluating a difference quotient is an algebraic process. Let's take a look at how to evaluate the difference quotient for a few different types of functions.

2a. Linear Functions

Evaluate the difference quotient for $f(x)=3x-7$.

$\frac{f(x+h)-f(x)}{h}$	Difference Quotient Formula
$\frac{[3(x+h)-7]-[3x-7]}{h}$	Substitute the function.
$\frac{[3x+3h-7]-[3x-7]}{h}$	Simplify the first quantity in brackets.
$\frac{3x+3h-7-3x+7}{h}$	Distribute the subtraction sign.
$\frac{3h}{h}$	Simplify the numerator.
3	Divide out the common factor.

The difference quotient for the function $f(x)=3x-7$ is 3.

THINK ABOUT IT

This answer makes sense since $f(x)=3x-7$ is a linear function with slope 3. This means that the slope of this line through any two points is 3.

2b. Quadratic Functions

Evaluate the difference quotient for $f(x)=x^2+3x-2$.

$\frac{f(x+h)-f(x)}{h}$	Difference Quotient Formula
$\frac{[(x+h)^2+3(x+h)-2]-[x^2+3x-2]}{h}$	Substitute the function.
$\frac{[x^2+2xh+h^2+3x+3h-2]-[x^2+3x-2]}{h}$	Simplify the first quantity in brackets.

$$\frac{x^2+2xh+h^2+3x+3h-2-x^2-3x+2}{h}$$ Distribute the subtraction sign.

$$\frac{2xh+h^2+3h}{h}$$ Simplify the numerator.

$$\frac{h(2x+h+3)}{h}$$ Factor the numerator.

$$2x+h+3$$ Divide out the common factor.

The difference quotient for the function $f(x)=x^2+3x-2$ is $2x+h+3$.

HINT

You may also remember from your background that $\frac{2xh+h^2+3h}{h}$ can be written as $\frac{2xh}{h}+\frac{h^2}{h}+\frac{3h}{h}$, which also yields $2x+h+3$ after performing the division.

2c. Higher-Power Polynomial Functions (Degree 3 or Higher)

The algebra can get quite complicated when evaluating difference quotients, especially when the highest power is more than 2.

WATCH

Here is a video that walks you through a difference quotient for a cubic function. The function used here is $f(x)=x^3-4x+2$.

2d. Rational Functions

When $f(x)$ is either a rational function or a radical function, more complex algebraic techniques are needed. These are illustrated in the next two videos.

WATCH

Here is a video that shows the required simplification techniques when evaluating a difference quotient for a rational function. The function used here is $f(x)=\frac{2}{x+4}$.

2e. Radical Functions

WATCH

Here is a video that shows the required simplification techniques when evaluating a difference quotient for a radical function. The function used here is $f(x)=\sqrt{x+3}$.

SUMMARY

In this lesson, you learned **what a difference quotient represents**, understanding that it is used to find an expression for the average rate of change between two points. You learned that **evaluating a**

difference quotient is an algebraic process, and depending on the function, basic simplification or more advanced methods such as simplifying a complex fraction or rationalizing the numerator may be required. You explored how to evaluate the difference quotient for several different types of functions, including **linear functions**, **quadratic functions**, **higher-power polynomial functions (degree 3 or higher)**, **rational functions**, and **radical functions**.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 0 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.

TERMS TO KNOW

Difference Quotient

An expression that represents the average rate of change between two points on a curve between input values x and $x + h$.

FORMULAS TO KNOW

Difference Quotient

$$\frac{f(x+h)-f(x)}{h}$$