Southern New Hampshire University

PHY-150 Lab 7-1: Momentum

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1 Activity 1: Elastic Collision with Equal Masses

1.1 Data tables

Cart A mass (in kg)	Distance in metres m	Time in s	Average Time $< t >$	Velocity $(m/s)v_A$
b_a	0.2 m	Trial 1: .533 Trial 2: 1.07 Trial 3: .867	0.82	38.96 18.23 22.74

Table 1: **Cart A (before collision)**

Cart A mass (in kg)	Distance in metres m	Time in s	Average Time $< t >$	Velocity $(m/s)v_{A'}$
b_a	0.2 m	Trial 1: 1.13 Trial 2: 1.7 Trial 3: 3.97	2.27	6.22 9.29 .21

Table 2: **Cart A (after collision)**

Cart A mass (in kg)	Distance in metres m	Time in s	Average Time $< t >$	Velocity $(m/s)v_{B'}$
b_b	0.3 m	Trial 1: 1.47 Trial 2: 1.1 Trial 3: 1.47	1.35	13.54 14.93 14.98

Table 3: **Cart B after collision**

1.2 Calculations

Note that I did not measure the carts (b) and assume that $b_a = b_b = .001[kg]$.

Apply the law of conservation of momentum to the two-cart system by calculating the momentum before and after the collision.[1]

1. Calculate the momentum of the system before the collision (the left side of the equation) and after the collision (the right side of the equation).

[1]See Appendix A for useful formulæand Appendix B for approach to measurements.

My solution: First, I need to work out the "theoretical" velocity for cart A with the following equation:

$$v_{\text{cart-A}}(t) = .934gt$$

Knowing that $v_0 = 0[m/s]$ and that $t = .82[s]^2$, I can work out the velocity of the first cart

$$v_{cart-A} = .934 \cdot 9.8[m/s^2] \cdot .82[s] = 7.51[m/s]$$

and finally the momentum both before the collision and after the collision:

(a) Momentum of Cart A before collision: $p_A = m_A v_A = .001[kg] \cdot 7.51[m/s] 7.51 \times 10^{-3}[kg \cdot m/s]$

(b) Momentum of Cart B before collision: $p_B = 0[m/s]$

(c) The resulting momentum is $p_A + p_B = 7.51 \times 10^{-3}[kg \cdot m/s]$

The law of conservation of energy implies that

$$p_A + p_B = p_{A'} + p_{B'}$$

Ergo, predicted momentum after collision is $p_{A'} + p_{B'} = 7.51 \times 10^{-3}[kg \cdot m/s]$.

2. Calculate the percent difference between the two values.

My solution: The average of the trials of each velocity is:

- $< v_A >= 26.64[m/s]$
- $< v_{A'} >= 5.24[m/s]$
- $< v_{B'} >= 14.48[m/s]$

The predicted momentums versus the actual momentums are:

	$p_A + p_B$ (Pre-collision)	$p_{A'} + p_{B'}$ (Post-collision)
Predicted	7.51×10^{-3}	7.51×10^{-3}
Actual	$.001[kg] < 31.88 > [m/s]$	$.001[kg] < 14.48 > [m/s]$
% error	123%	63%

3. Explain any difference in the values before and after the collision.

My solution: The error margins seem to be a lot higher than acceptable—implying that the experiment did not go as expected. I am not quite sure why this is the case.

[2] From the Cart A (before collision) table.

2 Activity 2: Elastic Collision: Mass Added to Cart A

2.1 Data tables

Cart A mass (in kg)	**Distance in metres m**	**Time in s**	**Average Time $< t >$**	**Velocity $(m/s)v_A$**
$.1342 + b_a$	0.2 m	Trial 1: 1.18 Trial 2: .73 Trial 3: .85	.92	17.68 26.45 22.78

Table 4: **Cart A (before collision)**

Cart A mass (in kg)	**Distance in metres m**	**Time in s**	**Average Time $< t >$**	**Velocity $(m/s)v_{A'}$**
$.1342 + b_a$	0.2 m	Trial 1: 1.44 Trial 2: 1.7 Trial 3: .85	1.33	25.99 16.5 22.78

Table 5: **Cart A (after collision)**

Cart A mass (in kg)	**Distance in metres m**	**Time in s**	**Average Time $< t >$**	**Velocity $(m/s)v_{B'}$**
b_b	0.3 m	Trial 1: 1.14 Trial 2: 1.03 Trial 3: .98	1.05	26.38 30.51 29.87

Table 6: **Cart B after collision**

2.2 Calculations

Note that I did not measure the carts (b) and assume that $b_a = b_b = .001[kg]$.

Apply the law of conservation of momentum to the two-cart system by calculating the momentum before and after the collision.[3]

[3]See Appendix A for useful formulæand Appendix B for approach to measurements.

1. Calculate the momentum of the system before the collision (the left side of the equation) and after the collision (the right side of the equation).

   My solution: As with the previous problem, I first need to work out the theoretical velocity with my derived formula:

   $$v_{\text{cart-A}}(t) = .934gt + v_0 \ [m/s]$$

   The initial velocity is $v_0 = 0[m/s]$ and the time elapsed before collision is $t = .92[s]$[4] The velocity then works out to:

   $$v_{\text{cart-A}}(t) = .934 \cdot 9.8[m/s^2] \cdot .92[s] = 8.42 \ [m/s]$$

   I can now work out the momentum before and after the collision for both carts:

   (a) Momentum of Cart A before collision: $p_A = m_A v_A = (.1342 + .001)[kg] \cdot 8.42[m/s] = 1.13[kg \cdot m/s]$

   (b) Momentum of Cart B before collision: $p_B = 0[kg \cdot m/s]$

   (c) The resulting momentum is $p_A + p_B = 1.13[kg \cdot m/s] + 0[kg \cdot m/s] = 1.13[kg \cdot m/s]$

   The law of conservation of energy implies that:

   $$p_A + p_B = p_{A'} + p_{B'}$$

   Ergo, the resulting momentum of the system is predicted to be: $1.13[kg \ m/s]$

2. Calculate the percent difference between the two values.

   My solution: The average of each trial velocity is:

   - $v_A = 22.30[m/s]$
   - $v_{A'} = 21.76[m/s]$
   - $v_{B'} = 28.92[m/s]$

   Likewise, the cart sizes are unequal. So, the actual momentums are:

   - $p_A = m_A v_A = 3[kg \cdot m/s]$

[4] From the Cart A (before collision) table.

- $p_B = 0[kg \cdot m/s]$
- $p_{A'} = m_{A'}v_{A'} = 2.92[kg \cdot m/s]$
- $p_{B'} = m_{B'}v_{B'} = 2.89 \times 10^{-2}[kg \cdot m/s]$

The predicted momentums versus actual momentums are:

	$p_A + p_B$ (Pre-collision)	$p_{A'} + p_{B'}$ (Post-collision)
Predicted	$1.13[kg \cdot m/s]$	$1.13[kg \cdot m/s]$
Actual	$3.00[kg \cdot m/s]$	$2.95[kg \cdot m/s]$
% error	91%	89%

3. Explain any difference in the values before and after the collision.

   My solution: The error per centages seem to be better compared to the last activity. But they are still unacceptably high. I am not quite sure why.

3 Activity 3. Elastic Collision: Mass Added to Cart B

3.1 Data tables

Note that I did not measure the carts (b) and assume that $b_a = b_b = .001[kg]$.

Cart A mass (in kg)	**Distance in metres m**	**Time in s**	**Average Time $< t >$**	**Velocity $(m/s)v_A$**
$.1342 + b_a$	0.2 m	Trial 1: .833 Trial 2: .834 Trial 3: .9	.86	23.64 23.01 21.53

Table 7: **Cart A (before collision)**

Cart A mass (in kg)	**Distance in metres m**	**Time in s**	**Average Time $< t >$**	**Velocity $(m/s)v_{A'}$**
$.1342 + b_a$	0.2 m	Trial 1: 1 Trial 2: .834 Trial 3: 0.6	.81	.19 23.01 .09

Table 8: **Cart A (after collision)**

Cart A mass (in kg)	Distance in metres m	Time in s	Average Time $< t >$	Velocity $(m/s)v_{B'}$
$.2982 + b_b$	0.3 m	Trial 1: 1.07 Trial 2: .833 Trial 3: .633	.85	8.89 9.12 9.21

Table 9: **Cart B after collision**

3.2 Calculations

Apply the law of conservation of momentum to the two-cart system by calculating the momentum before and after the collision.[5]

1. Calculate the momentum of the system before the collision (the left side of the equation) and after the collision (the right side of the equation).

   My solution: As with the previous two activities, I need to first work out the theoretical velocity with the following formula that I derived:

   $$v_{\text{cart-A}}(t) = .934gt + v_0$$

   The initial velocity is $v_0 = 0[m/s]$ and the time elapsed is $t = .86$,[6] which works out to

   $$v_{\text{cart-A}}(t) = .934 \cdot 9.8[m/s^2] \cdot .86[s] = 7.87[m/s]$$

   With this velocity at hand, I can now work out the momentum of both Cart A and Cart B before the collision:

   (a) Momentum of Cart A before collision: $p_A = m_A v_A = (.1342 + .001)[kg] \cdot 7.87[m/s] = 1.06[kg \cdot m/s]$

   (b) Momentum of Cart B before collision: $p_B = 0[kg \cdot m/s]$

   (c) The resulting momentum is: $p_A + p_B = 1.06[kg \cdot m/s] + 0[kg \cdot m/s]$

   The law of conservation of energy applied to situation implies that:

   $$p_A + p_B = p_{A'} + p_{B'}$$

   Ergo, the momentum after collision would be: $p_{A'} + p_{B'} = 1.06[kg \cdot m/s]$

[5] See Appendix A for useful formulæand Appendix B for approach to measurements.
[6] From the Cart A (before collision) table.

2. Calculate the percent difference between the two values.

   My solution: The average of each trial velocity is:

   - $v_A = 22.73[m/s]$
   - $v_{A'} = 7.76[m/s]$
   - $v_{B'} = 9.07[m/s]$

   Likewise, the cart sizes are unequal. So, the actual momentums are:

   - $p_A = m_A v_A = 3.05[kg \cdot m/s]$
   - $p_B = 0[kg \cdot m/s]$
   - $p_{A'} = m_{A'} v_{A'} = 1.04[kg \cdot m/s]$
   - $p_{B'} = m_{B'} v_{B'} = 2.71[kg \cdot m/s]$

   The predicted momentums versus actual momentums are:

	$p_A + p_B$ (Pre-collision)	$p_{A'} + p_{B'}$ (Post-collision)
Predicted	$1.06[kg \cdot m/s]$	$1.06[kg \cdot m/s]$
Actual	$3.05[kg \cdot m/s]$	$3.75[kg \cdot m/s]$
% error	97%	112%

   My solution:

3. Explain any difference in the values before and after the collision.

   My solution: As with the previous activitites, the per cent errors are unacceptably high. I hope to elaborate on this later.

4 Questions with regard to Momentum

1. **Question:** The law of conservation of momentum states that the total momentum before a collision equals the total momentum after a collision provided there are no outside forces acting on the objects in the system. What outside forces are acting on the present system that could affect the results of the experiments?

   My answer: Unfortunately, there was a lot of measurement error in my experiment. While I cannot quite think of exactly what may have influenced the results of my experiment, I can probably think of some things that contributed to measurement error:

- When taking video footage and processing it with *Tracker*,[7] I could have coded the (t, x) information due to slight vision impairments.
- My theoretical velocity formula, $v_{\text{cart-A}} = .934gt + v_0[m/s]$, did not take friction into account. This may have biased[8] the results.
- I did not measure the cart and assumed that it weighed .001 kilograms. This may have further biased the results.

2. **Question:** What did you observe when Cart A containing added mass collided with Cart B containing no mass? How does the law of conservation of momentum explain this collision?

   My answer: I noticed that Cart B was able to move further. This is probably because the mass of Cart A positively influenced the velocity of Cart B due to its bigger mass, and as per the law of conservation of energy, the bigger mass influenced Cart B's velocity.

3. **Question:** In one of the experiments, Cart A may reverse direction after the collision. How is this accounted for in your calculations?

   My answer: I did not account for this, and just observed the magnitude of the direction. My theoretical velocity formula did not take that into account.

A Useful formulæ

- Momentum before the collision: $m_A v_A + m_B v_B$
- Momentum after the collision: $m_A v_{A'} + m_B v_{B'}$
- Law of conservation of energy applied to momentum:

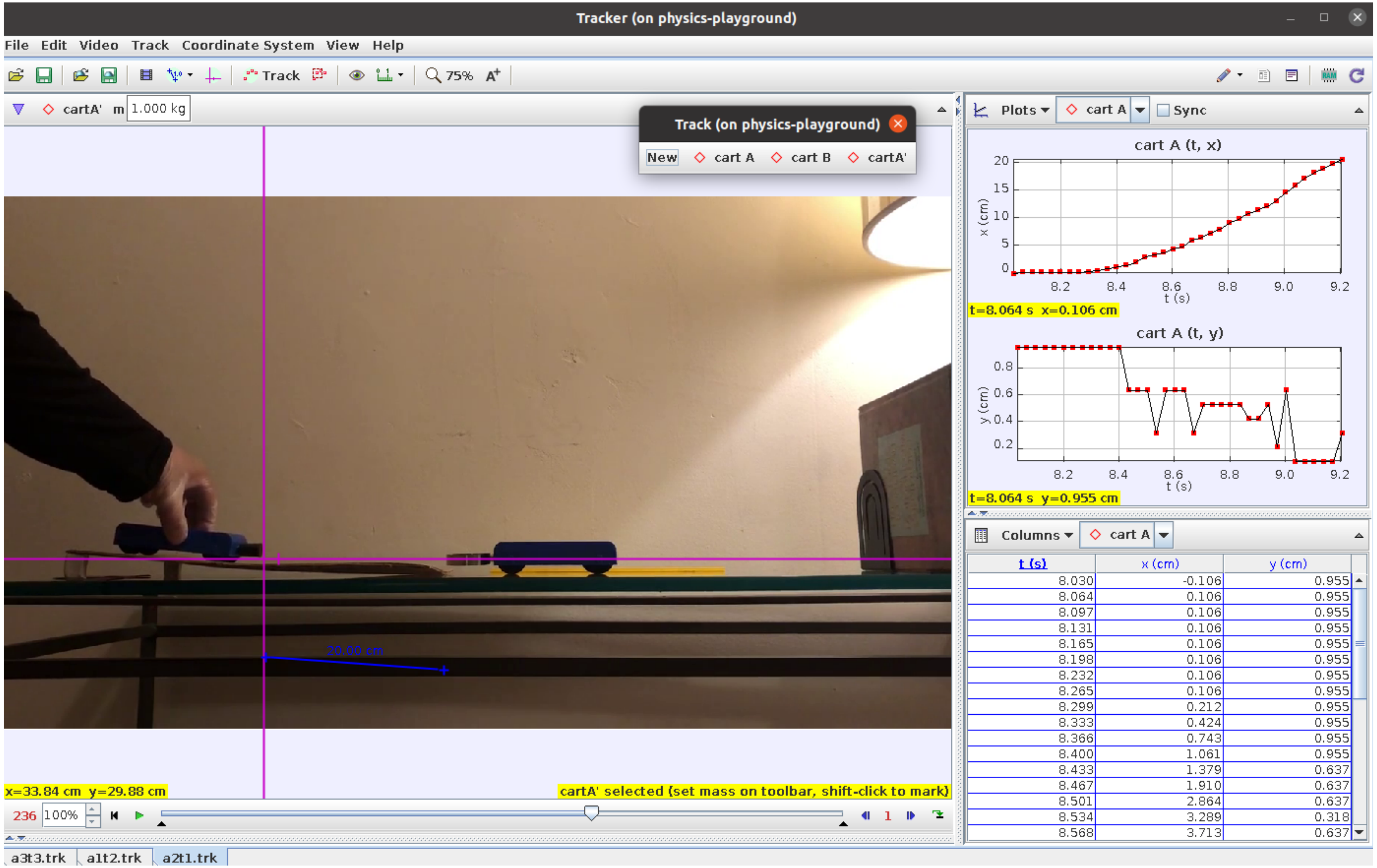
$$m_A v_A + m_B v_B = m_A v_{A'} + m_B v_{B'}$$

- Percent difference:

$$\%_{\text{diff}} = \frac{\text{first value} - \text{second value}}{\frac{\text{first value}+\text{second value}}{2}} \times 100\%$$

[7]See Appendix B for more information on measurement.

[8]I use "bias" like how a statistican would use it, i.e. as the difference between the true parameter and the measured parameter, $\theta_{\text{bias}} = \theta_{\text{true}} - \theta_{\text{error}}$



t (s)	x (cm)	y (cm)
8.030	-0.106	0.955
8.064	0.106	0.955
8.097	0.106	0.955
8.131	0.106	0.955
8.165	0.106	0.955
8.198	0.106	0.955
8.232	0.106	0.955
8.265	0.106	0.955
8.299	0.212	0.955
8.333	0.424	0.955
8.366	0.743	0.955
8.400	1.061	0.955
8.433	1.379	0.637
8.467	1.910	0.637
8.501	2.864	0.637
8.534	3.289	0.318
8.568	3.713	0.637

Figure 1: **An example of “tracking” the movement of the cart.**

B Measurement

With regard to the cart, their masses are unknown, and b_a and b_b are used to represent the mass of cart A and cart B respectively. When doing calculations, I will assume that both carts are equal and that $b_a = b_b = .001$ $[kg]$.

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With regards to the washers used to increase the mass of the carts, I know from education and experience that when subjecting the washers to measurement, they will report slightly different results—- that infamous "measurement error."

To deal with measurement error, I have measured the masses of the washers three times, with different washer combinations, for all sixteen (16) combinations for each number of washers possible. I then averaged out each number of washers by number of washers and have compiled them into a table.

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When extracting distance travelled versus time taken, I recorded video footage with a smartphone camera and then imported the videos onto a laptop for analysis. I used the software tool Tracker to get distance-versus-time data (see fig. 1).

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In terms of measuring the theoretical velocity of cart A coming down from the ramp, I used the following formula as a basis for deriving the velocity function:

$$v_{\text{cart-A}}(t) = \int m^{-1}\Big[mg\cos(20°)\Big]\ dt\ [m/s]$$

Integrating over the basic model and simplifying, the velocity function works out to

$$v_{\text{cart-A}}(t) = .934gt + v_0\ [m/s]$$

Note that my theoretical yardstick **does not** take the effects of friction into account.