Rolle's Theorem

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WHAT'S COVERED

In this lesson, you will learn about Rolle’s theorem, a seemingly simple yet powerful theorem whose consequences are used in Unit 5 (Antiderivatives). Specifically, this lesson will cover:

1. Introduction to Rolle’s Theorem

Let’s say we have a function that passes through the points (1, 6) and (5, 6).

TRY IT

Take a piece of paper and draw the points (1, 6) and (5, 6). Connect the two points with a curve that is continuous and differentiable (something other than a horizontal line between them). This means that the graph has no break and no sharp turn.

What do you notice about your curve? Does your curve contain at least one horizontal tangent? +

Hopefully you have at least one horizontal tangent. As it turns out, under certain circumstances, this will always happen.

BIG IDEA

Rolle’s theorem:

Let $f(x)$ be continuous on the closed interval $[a, b]$ with $f(a) = f(b)$, and differentiable on the open interval (a, b).

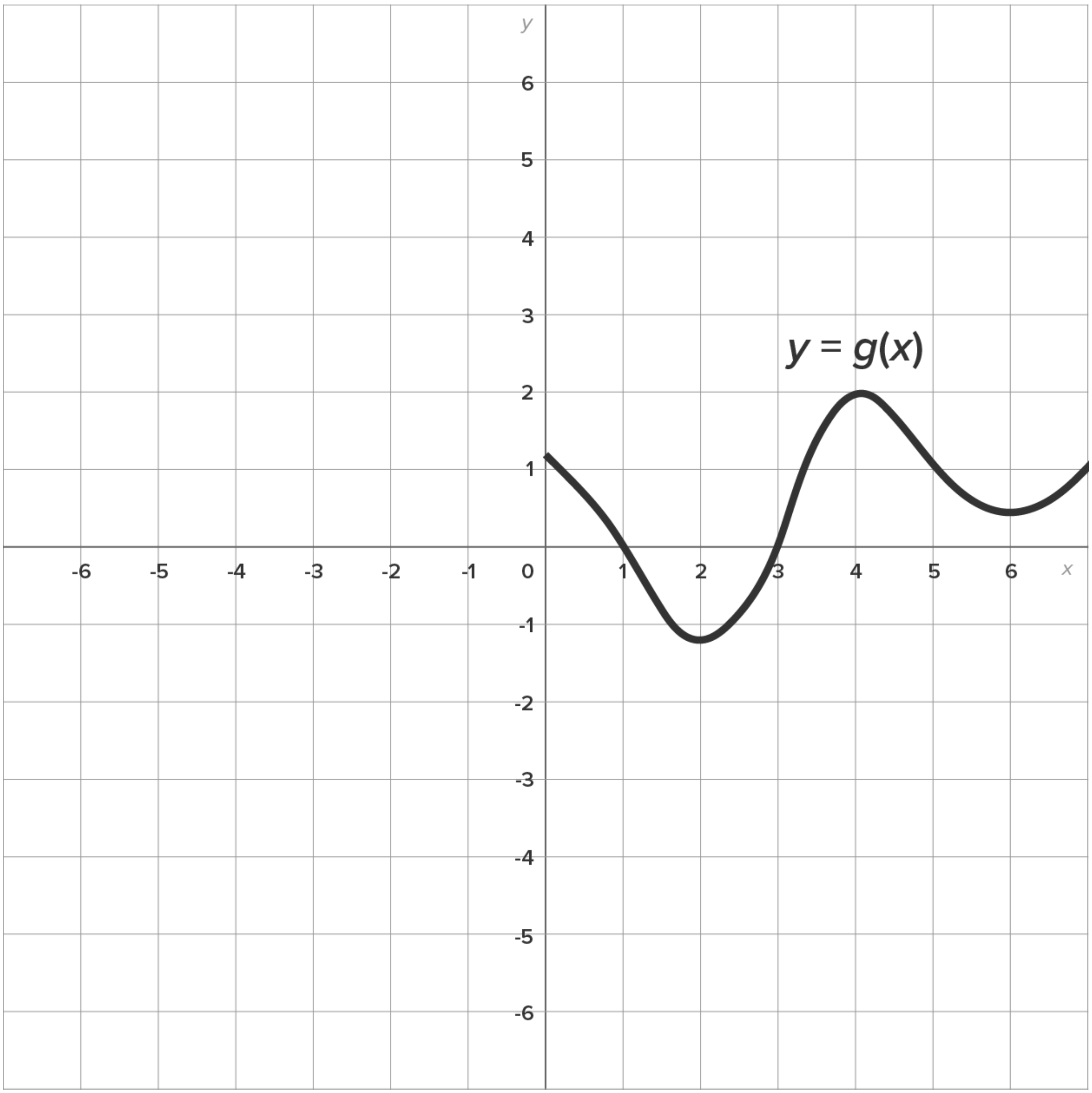
Then, there is at least one value of *c* between *a* and *b* for which $f'(c) = 0$.

2. Applying Rolle's Theorem

Now, let's look at a few examples of how Rolle's theorem can be applied.

EXAMPLE Here is the graph of some function $y=g(x)$, where $g(0)=g(7)$.

Since $g(x)$ is continuous and differentiable, it follows by Rolle's theorem that there is at least one value of *c* between 0 and 7 where $f'(c)=0$.



In the graph, we can see there are three x-values where a horizontal tangent line occurs: $x=2$, $x=4$, and $x=6$. Therefore, the guaranteed values of *c* are 2, 4, and 6.

EXAMPLE Consider the function $f(x)=3x+\frac{3}{x}$ on the interval $\left[\frac{1}{2}, 2\right]$.

First, check requirements for Rolle's theorem.

$f(x)$ is continuous on any interval not including 0, and therefore is continuous on $\left[\frac{1}{2}, 2\right]$.

$f(x)$ is differentiable everywhere except where $x=0$, so $f(x)$ is certainly differentiable on $\left(\frac{1}{2}, 2\right)$.

$f\left(\frac{1}{2}\right)=3\left(\frac{1}{2}\right)+\frac{3}{\left(\frac{1}{2}\right)}=\frac{15}{2}$ and $f(2)=3(2)+\frac{3}{2}=\frac{15}{2}$. Therefore, $f(a)=f(b)$.

Thus, the conditions of Rolle's theorem have been met and there is at least one value of c between $\frac{1}{2}$ and 2 such that $f'(c)=0$.

To find all values of c, take the derivative, then set equal to 0, then solve.

$f(x)=3x+\frac{3}{x}$	Start with the original function.
$f(x)=3x+3x^{-1}$	Rewrite to use the power rule.
$f'(x)=3-3x^{-2}$	Take the derivative.
$f'(x)=3-\frac{3}{x^2}$	Rewrite with positive exponents.
$3-\frac{3}{x^2}=0$	Set equal to 0.
$3=\frac{3}{x^2}$	Add $\frac{3}{x^2}$ to both sides.
$3x^2=3$	Multiply both sides by x^2.
$x^2=1$	Divide both sides by 3.
$x=\pm 1$	Take the square root of both sides.

Since we want all values on the interval $\left(\frac{1}{2}, 2\right)$, the value guaranteed by Rolle's theorem is $c=1$. (In other words, since $c=-1$ is not on the interval $\left(\frac{1}{2}, 2\right)$, it is not considered.)

WATCH

In this video, we will find all values of c guaranteed by Rolle's theorem for $f(x)=20\sqrt{x}-2x$ on the interval $[16, 36]$.

SUMMARY

In this lesson, you learned that when a function is continuous on a closed interval $[a, b]$, differentiable on the open interval (a, b), and $f(a) = f(b)$, then **Rolle's theorem** guarantees that there is a value of *c* between *a* and *b* such that $f'(c) = 0$, which means that there is a guaranteed horizontal tangent line at *c*. Then, you examined a few examples involving the **application of Rolle's theorem**.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 3 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.