Derivatives of Non-Natural Logarithmic Functions

by Sophia

WHAT'S COVERED

In this lesson, you will find derivatives involving logarithmic functions that deal with bases other than *e*. Since there were reasons to use exponential functions with bases other than *e*, it makes sense to discuss the corresponding logarithmic functions. Specifically, this lesson will cover:

1. The Derivative of $y = \log_a x$ and $y = \log_a u$, Where u Is a Function of x

Consider the function $y = \log_a x$, where *a* is any positive number except 1.

If we apply the change of base formula, we have $\log_a x = \frac{\ln x}{\ln a} = \left(\frac{1}{\ln a}\right) \cdot \ln x$.

Then, $D[\log_a x] = D\left[\left(\frac{1}{\ln a}\right) \cdot \ln x\right] = \left(\frac{1}{\ln a}\right) \cdot \frac{1}{x} = \frac{1}{x \cdot \ln a}$.

So, we can say the derivative of a logarithm function with base *a* can be expressed with the following formula:

FORMULA TO KNOW

Derivative of a Logarithm Function, Base *a*

$$D[\log_a x] = \frac{1}{x \cdot \ln a}$$

When *x* is replaced with *u* (a function of *x*), the chain rule is used.

FORMULA TO KNOW

Derivative of a Composite Logarithm Function, Base a

$$D[\log_a u] = \frac{1}{u \cdot \ln a} \cdot u' = \frac{u'}{u \cdot \ln a}$$

HINT

Notice that this derivative formula is the same as the one for $\ln x$, but there is also a factor of $\ln a$ in the denominator.

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2. Derivatives of Functions Involving $\log_a x$ and $\log_a u$, Where u Is a Function of x

Since the derivatives of $\log_a x$ and $\ln x$ are similar, we'll look at various functions.

EXAMPLE Consider the function $f(x) = \log_2(x^3 + 5x)$. Find its derivative.

$f(x) = \log_2(x^3 + 5x)$	Start with the original function.
$f'(x) = \frac{1}{(x^3 + 5x)\ln 2} \cdot (3x^2 + 5)$	$D[\log_a u] = \frac{1}{u \cdot \ln a} \cdot u'$
$f'(x) = \frac{3x^2 + 5}{(x^3 + 5x)\ln 2}$	Write as a single fraction.

Thus, $f'(x) = \frac{3x^2 + 5}{(x^3 + 5x)\ln 2}$.

TRY IT

Consider the function $f(x) = x^2 \log_3(2x + 1)$.

Find its derivative. +

$$f'(x) = 2x\log_3(2x + 1) + \frac{2x^2}{(2x + 1)\ln 3}$$

WATCH

The following video illustrates the use of properties of logarithms to find the derivative of $f(x) = \log\left(\frac{x \cdot \sin x}{3x^2 + 1}\right)$.

SUMMARY

In this lesson, you learned to find **the derivatives of** $y = \log_a x$ **and** $y = \log_a u$**, where *u* is a function of *x*,** and **derivatives of functions involving** $\log_a x$ **and** $\log_a u$**, where *u* is a function of *x*.** Remember that the derivative of the "base *a*" logarithmic function is very similar to that of the natural logarithmic function with a factor of $\ln a$ in the denominator. With this function added to your toolbox, you now have the ability to find derivatives of any combination of polynomial, trigonometric, exponential, and logarithmic functions.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 2 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.

FORMULAS TO KNOW

Derivative of a Composite Logarithm Function, Base a

$$D\left[\log_a u\right] = \frac{1}{u \cdot \ln a} \cdot u' = \frac{u'}{u \cdot \ln a}$$

Derivative of a Logarithm Function, Base a

$$D\left[\log_a x\right] = \frac{1}{x \cdot \ln a}$$