The Linear Approximation Error | f(x) – L(x) |

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WHAT'S COVERED

In this lesson, you will investigate the error in using a linear approximation to approximate the value of a function. Specifically, this lesson will cover:

1. Calculating the Error in Using *L*(*x*) to Approximate *f*(*x*)

It is no surprise that using the linear approximation will produce some error.

The **linear approximation error** is the difference between the actual function value and the value obtained through the linear approximation.

FORMULA TO KNOW

Linear Approximation Error

$\text{Error} = |f(x) - L(x)|$

HINT

The error formula has an absolute value since we are only concerned with the size of the error, not necessarily which of $L(x)$ or $f(x)$ is larger. With absolute value, the difference is nonnegative regardless.

EXAMPLE Consider the function $f(x) = \ln x$ near $x = 1$. We'll use the linear approximation to estimate $\ln 1.2$, then find the linear approximation error.

To find the linear approximation, first find the derivative: $f(x) = \frac{1}{x}$

Now, form the linear approximation:

$L(x) = f(a) + f'(a)(x - a)$	Use the equation for the tangent line.
$L(x) = f(1) + f'(1)(x - 1)$	$a = 1$
$L(x) = 0 + 1(x - 1)$	$f(1) = 0$ and $f'(1) = 1$
$L(x) = x - 1$	Simplify.

The linear approximation tells us that $\ln 1.2 \approx L(1.2) = 1.2 - 1 = 0.2$.

The actual value of $\ln 1.2$ is 0.1823 (to 4 decimal places).

Then, the linear approximation error is $|f(1.2) - L(1.2)| \approx |0.1823 - 0.2| = 0.0177$.

TRY IT

Consider the function $f(x) = x^4$. Use the linear approximation at $x = 1$ to estimate $f(1.06)$. Then, find the linear approximation error to 4 decimal places.

What is the linear approximation? +

$L(1.06) = 1.24$

What is the linear approximation error? +

0.0225

2. Analyzing Error as *x* Gets Further Away From *a*

In 3.4.1, we used $L(x) = 4 + \frac{1}{8}(x - 16)$ to approximate $f(x) = \sqrt{x}$ for values of *x* near 16.

Here is a table of values (rounded to four decimal places) to illustrate what happens to the error as *x* moves away from 16:

x	$L(x) = 4 + \frac{1}{8}(x - 16)$	$f(x) = \sqrt{x}$	Error $= \lvert f(x) - L(x) \rvert$

16.25	4.0313	4.0311	0.0002
16.5	4.0625	4.0620	0.0005
16.75	4.0938	4.0927	0.0011
17	4.1250	4.1231	0.0019
17.25	4.1563	4.1533	0.0030
17.5	4.1875	4.1833	0.0042
17.75	4.2188	4.2131	0.0057
18	4.2500	4.2426	0.0074

As you can see, even though the errors are relatively small, they are increasing as *x* increases from 16. We would see a similar pattern if *x* were to decrease from 16 (15.75, 15.5, etc.).

SUMMARY

In this lesson, you learned how to **calculate the linear approximation error, or the error in using *L*(*x*) to approximate *f*(*x*).** The linear approximation error is found by calculating the positive difference between the linear approximation and the actual value. You also learned that when considering the linear approximation $L(x) = f(a) + f'(a)(x - a)$, the **error gets larger as *x* gets further from *a*.**

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 2 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.

FORMULAS TO KNOW

Linear Approximation Error

$\text{Error} = |f(x) - L(x)|$