



**PHY-150 Lab 3-2: A hypothetical supply drop mission**

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**Abstract**

This report is a description of a supply drop mission as a response to a natural disaster. Here, I shall demonstrate the usefulness of formalised Newtonian physics in “the real world” by using it to describe the specifics on how the supply drop *ought to* be carried out with through the most efficient manner possible.

The report begins on the next page...

# A&L ENGINEERING SUPPLY DROP PLAN

## DIAGRAM

**Prompt 1:** Create a diagram describing the horizontal and vertical motion of the payload. Remember that your diagram should visually represent the motion of the payload.

**Answer:** See Fig. 1.

## INITIAL CALCULATIONS

**Prompt 2:** Provide your calculations, including all relevant steps are included and your units are labeled.

**Answer:** In regard to the horizontal motion (along the x-axis), the equation of motion is:

$$x(t) = v_{ix}t + x_i$$

The  $v_{ix} = 250$  mi/hr,  $x_i = -100$  mi and  $x(t) = 0$  mi. I need to solve for  $t$  in hours:

$$x(t) = v_{ix}t + x_i$$

$$\Rightarrow t = [x(t) - x_i] \div v_{ix}$$

$$\Rightarrow t = [0 \text{ mi} - (-100 \text{ mi})] \div 250 \text{ mi/hr} = 0.4 \text{ hr}$$

Or 24 minutes.

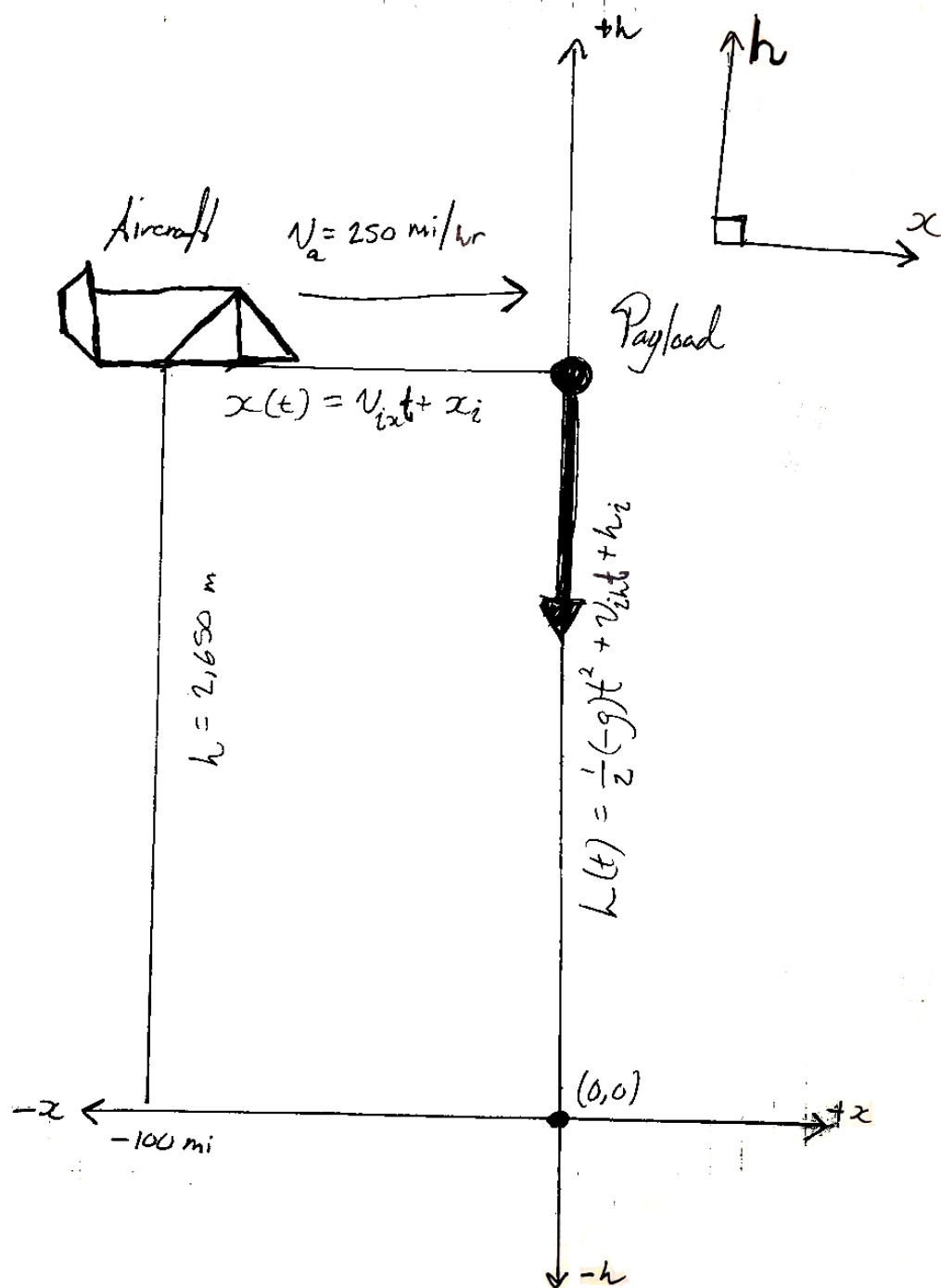


Figure 1: Free-body diagram for Scenario 1.

**Answer (Prompt 2 cont.):** In regards to the vertical motion of the dropping payload, the equation of motion is:

$$h(t) = \frac{-gt^2}{2} + v_{ih}t + h_i$$

Knowing that  $g = 9.8 \text{ m/s}^2$ ,  $v_{ih} = 0 \text{ m/s}$ ,  $h_i = 2650 \text{ m}$  and that  $h(t) = 0 \text{ m}$ , I can work out the solutions by finding a  $t$  s.t.  $h(t) = 0 \text{ m}$ . This can be easily done with the following solutions to the parabola:

$$t_1 = \frac{-v_{ih} - \sqrt{v_{ih}^2 - 4(-g)h_i}}{2(-g)}$$

$$t_2 = \frac{-v_{ih} + \sqrt{v_{ih}^2 - 4(-g)h_i}}{2(-g)}$$

Both the  $t_1$  and  $t_2$  have a magnitude of 23.2555 s, so it can be concluded that the  $t$  that satisfies the  $h(t)$  s.t. it is equal to zero is about 23.26 seconds.<sup>a</sup>

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<sup>a</sup>**Note that** Dr. Brazell used the formula

$$t = \sqrt{\frac{2\Delta h}{g}} = \sqrt{\frac{2 \times 2650 \text{ m}}{9.8 \text{ m/s}^2}} \approx 23.255 \text{ s}$$

to arrive at approx. 23.26 seconds.

## Description

**Prompt 3:** Describe the components of the kinematics equations used in your initial calculations below.

**Answer:** It should be noted that, before I continue, the location of the site to receive the supply drop is located at the origin of the Cartesian plot, or at  $(0\text{m}, 0\text{m})$ .

The horizontal x-axis, the  $x(t)$ , it describes the motion of the aircraft as it arrives to the site. The  $v_a$  is the velocity of the aircraft in miles per hour, the  $x_i$  is how far away the aircraft is from the site and the  $x(t)$  is the point on the graph where the site is located. The calculations regarding the solution to  $t$  for the function  $x(t)$  are how long it will take for the aircraft to arrive at the site.

**Answer (Prompt 3 cont.):** In regards to the vertical h-axis, the  $h(t)$ , it describes the motion of the falling payload to the site. The  $g$  is a gravitational constant, and specifically for planet Earth, it is set to  $g = 9.8 \text{ m/s}^2$ . The  $h_i$  is the height between the supply drop and the site, which is  $h_i = h = 2650 \text{ m}$ .

Finally, the velocity of the payload is represented as  $v_{ih}$  and is initially at zero metres per second, or  $v_{ih} = 0 \text{ m/s}$ . The calculations regarding the solution to  $t$  for the function  $h(t)$  are how long it will take for the payload to be dropped from the aircraft onto the site in seconds.

## MODIFIED SCENARIO ONE

**Prompt 4:** Create a diagram showing the first modified scenario. Then adjust your initial calculations to incorporate the changing variables from the scenario and describe how these changed variables affect your calculations.

### Diagram

**Answer:** See Fig. 2.

### Description

**Answer:** This is just a slight adjustment to the first scenario. The aircraft's velocity is affected by headwind ( $w$ ) that has a velocity of  $v_w = 15 \text{ mi/hr}$ . The resulting velocity of the aircraft is:

$$v_s = v_a - v_w = 235 \text{ mi/hr}$$

The  $v_s$  can be substituted in the solution to  $t$  for the function  $x(t)$  to arrive at the time of arrival— which is approx. 26 minutes. The headwind slowed down slightly slowed down the aircraft.

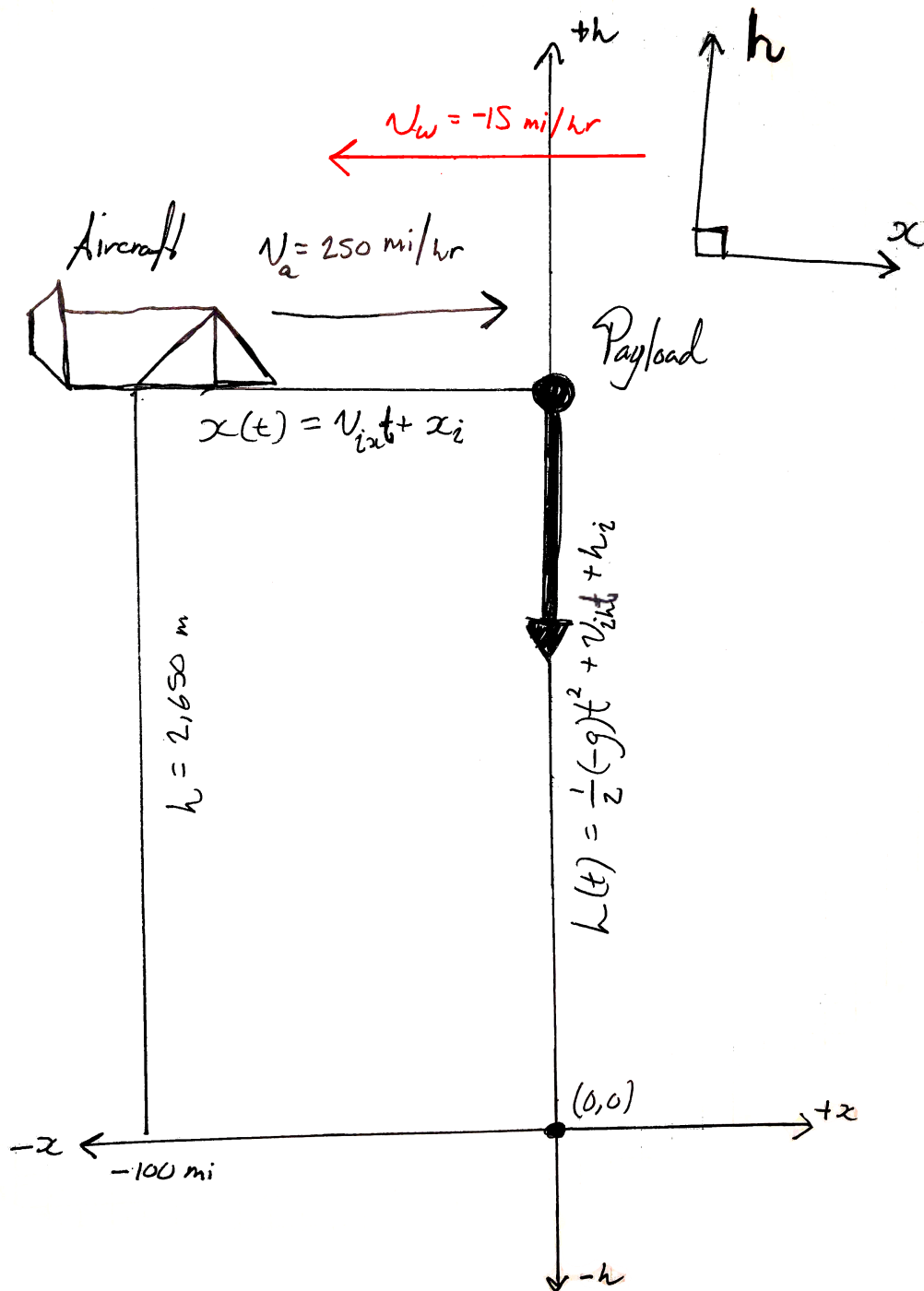


Figure 2: Free-body diagram for Scenario 2.

## MODIFIED SCENARIO TWO

**Prompt 5:** Create a diagram showing the second modified scenario. Then adjust your initial calculations to incorporate the changing variables from the scenario and describe how these changed variables affect your calculations.

(See next page for answer.)

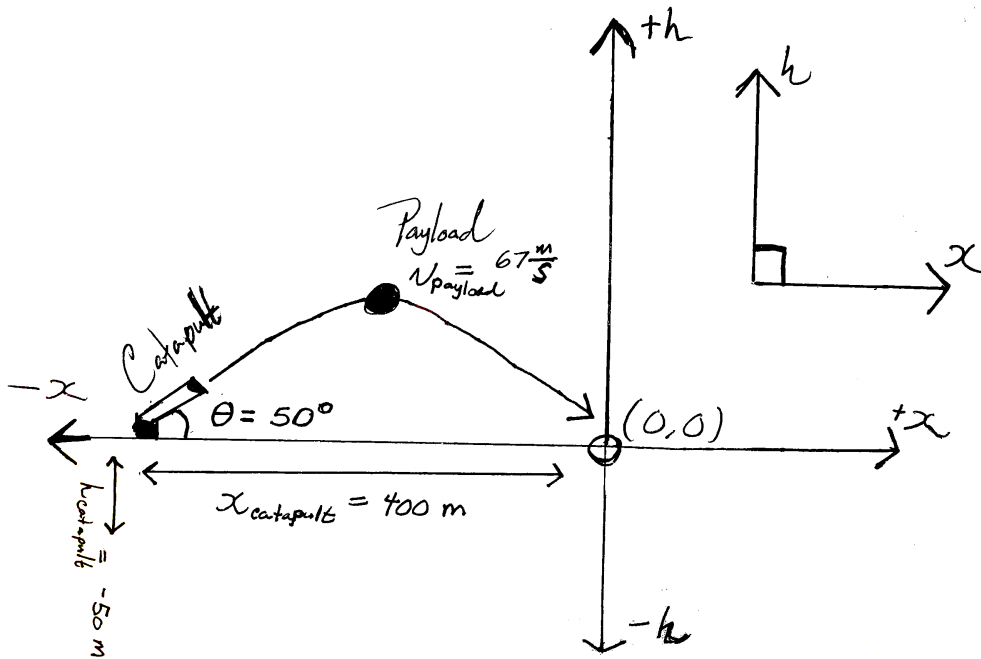


Figure 3: Free-body diagram for Scenario 3.

## Diagram

**Answer:** See Fig. 3.

## Description

**Answer:** This version of the scenario replaces the aircraft with a catapult where its angle of elevation needs to be accounted for. Ergo, the velocity of both the  $x$  and  $h$  axis are  $v_x = v_i \sin \theta$  and  $v_h = v_i \sin \theta$ . We need to work out the total flight time, maximum height of the payload, and range that it will cover.

The angle of elevation is  $\theta = 50^\circ$ , the velocity of the payload is  $v_i = v_{\text{payload}} = 67 \text{ m/s}$  and the catapult 400 metres away from the site and 50 metres below the site, so it will have a  $h_i$  of  $h_i = h_{\text{catapult}} = -50 \text{ m}$  and an  $x_i$  of  $x_i = x_{\text{catapult}} = -400 \text{ m}$ .

The solutions of total flight time, range of payload<sup>a</sup> and payload's maximum height are respectively:<sup>b</sup>

$$t = \frac{2v_i \sin \theta}{g} = \frac{2 \times 67 \text{ m/s} \times \sin 50^\circ}{9.8 \text{ m/s}^2} \approx 10.48 \text{ s}$$

$$\Delta x = (v_i \cos \theta) = (67 \text{ m/s} \times \cos 50^\circ) \times 10.48 \approx 45.13 \text{ m}$$

$$\Delta h_{\text{max}} = \frac{(v_i \sin \theta)^2}{2g} = \frac{(67 \text{ m/s} \times \sin 50^\circ)^2}{2 \times 9.8 \text{ m/s}^2} \approx 134.40 \text{ m}$$

It will take about 10.48 seconds for the payload to be catapulted, it will have a maximum height of 134.40 metres and travel a distance of about 45.13 metres. It will not reach the site.<sup>c</sup>

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<sup>a</sup>**Note that** the  $t$  for the range of the payload comes from the total flight time.

<sup>b</sup>Formulæ supplied by Dr. Bruce Brazell.

<sup>c</sup>  $\because$  the catapult is 400 metres away from the site ( $x_i = -400 \text{ m}$ ) and the distance that it has not met is  $x_i + \Delta x = -400 \text{ m} + 45.13 \text{ m} = -354.87 \text{ m}$  which is **not** located on the origin.