Finding Maximums and Minimums of a Function

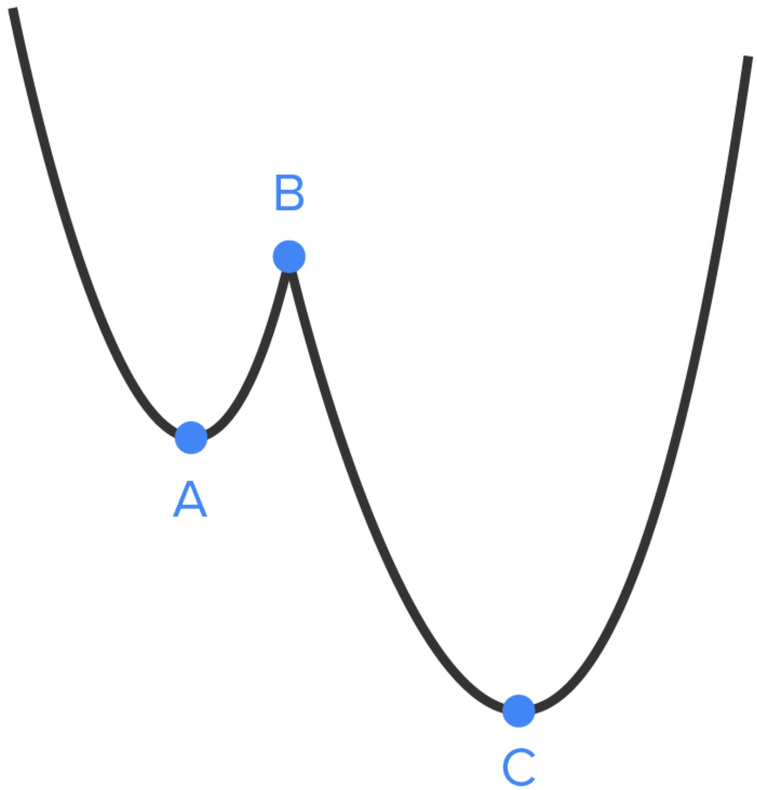
by Sophia

WHAT'S COVERED

In this lesson, you will use derivatives and critical numbers to find local maximum and local minimum values. Specifically, this lesson will cover:

1. The Relationship Between Critical Numbers and Local Extrema

Consider the graph of a function $y = f(x)$, shown here:



- At point *A*, $f'(x)=0$.
- At point *B*, $f'(x)$ is undefined.
- At point *C*, $f'(x)=0$.

As discussed in the previous tutorial, values of *x* in the domain of $f(x)$ where $f'(x)=0$ or $f'(x)$ is undefined are called critical numbers.
Therefore, critical numbers can tell us where local maximum or minimum values could occur.

However, the only way to find out is through further analysis, which will be covered in challenge 4.3.

STEP BY STEP

To identify relative extrema:

1. Find all critical values.
2. Use a graph of the function to determine which critical numbers correspond to which relative extreme points.

Now that we know the connection between critical numbers and extrema, let's look at a few examples.

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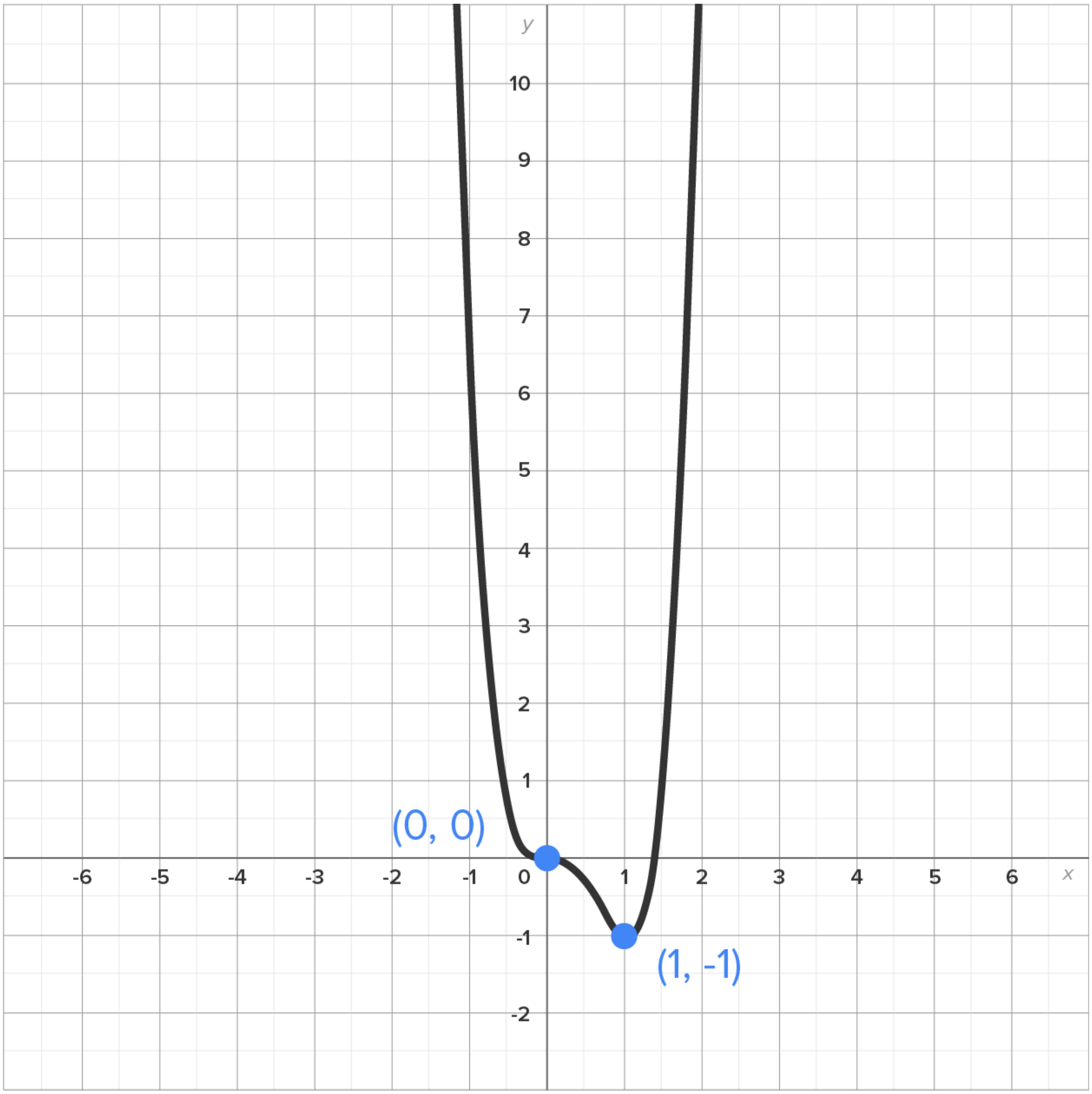
2. Finding Local Extrema

EXAMPLE Consider the function $f(x)=3x^4-4x^3$. First, find all critical numbers:

$f(x)=3x^4-4x^3$	Start with the original function; the domain is all real numbers.
$f'(x)=12x^3-12x^2$	Take the derivative.
$12x^3-12x^2=0$	Since $f'(x)$ is a polynomial, it is never undefined. Set $f'(x)=0$ and solve.
$12x^2(x-1)=0$	Factor.
$12x^2=0,\ x-1=0$	Set each factor equal to 0.
$x=0,\ x=1$	Solve.

Thus, the critical numbers are $x=0$ and $x=1$.

Now, the graph of $f(x)$ is shown.



The point (0, 0) is neither a local maximum nor a local minimum, while a local minimum (also a global minimum) occurs at (1, -1).

TRY IT

Consider the function $f(x) = -\frac{1}{2}x^4 + 9x^2 + 10$.

Find all critical numbers of *f*, then determine the local minimum and maximum points by using a graph. +

Critical numbers are $x = 0$, -3, and 3.

Local minimum is at (0, 10) and local maximums are at (-3, 50.5) and (3, 50.5).

SUMMARY

In this lesson, you learned about **the relationship between critical numbers and local extrema**, which is that critical numbers help locate the coordinates of local maximum and minimum points. Understanding this connection between critical numbers and extrema, you practiced **finding local extrema** of a function by first determining all critical numbers. As you saw with one example, a critical number at $x = c$ doesn't automatically imply that there is a local maximum or minimum at $x = c$.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 3 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.