What Is a Maximum or a Minimum?

by Sophia

WHAT'S COVERED

In this lesson, you will learn about the different kinds of maximum and minimum points on a graph of a function. Specifically, this lesson will cover:

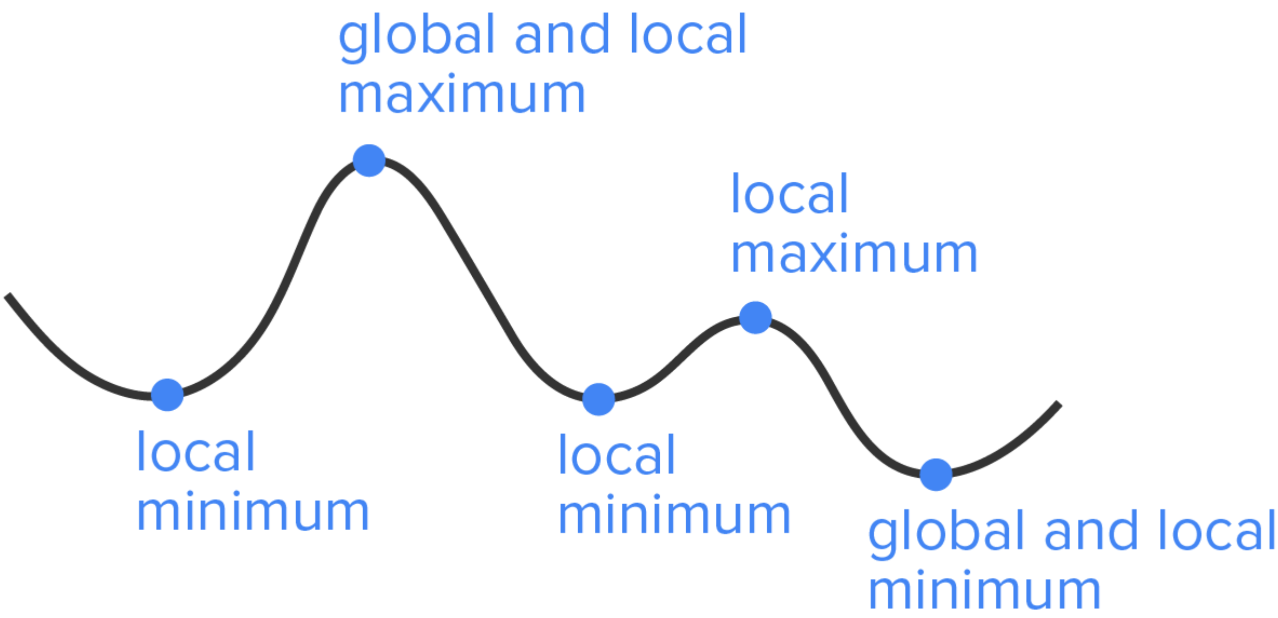
1. Definitions of Global and Local Maximum and Minimum Points

One of the main uses of derivatives is to find minimum and maximum values of a function, or more simply put, **extreme values** (or **extrema**) of a function.

A function can have any one of the following:

- **Global (or Absolute) Maximum**: A function $f(x)$ has a global (or absolute) maximum at $x=a$ if $f(a) \ge f(x)$ for all *x*. In other words, $f(a)$ is the largest value of a function $f(x)$, and occurs when $x=a$.
- **Global (or Absolute) Minimum**: A function $f(x)$ has a global (or absolute) minimum at $x=a$ if $f(a) \le f(x)$ for all *x*. In other words, $f(a)$ is the smallest value of a function $f(x)$, and occurs when $x=a$.
- **Local (or Relative) Maximum**: A function $f(x)$ has a local (or relative) maximum at $x=a$ if $f(a) \ge f(x)$ for all *x* close to $x=a$. In other words, $f(a)$ is the largest value of a function $f(x)$ for values near $x=a$.
- **Local (or Relative) Minimum**: A function $f(x)$ has a local (or relative) minimum at $x=a$ if $f(a) \le f(x)$ for all *x* close to $x=a$. In other words, $f(a)$ is the smallest value of a function $f(x)$ for values near $x=a$.

The graph shown here summarizes the differences between local and global extrema. Note that the second labeled point (from left to right) is both a local maximum and a global maximum because it meets both conditions: it is both the highest point on the graph and it is the highest point when compared to points immediately to the left and right.



TERMS TO KNOW

Extreme Values

The minimum or maximum values of a function.

Extrema

Another word for extreme values.

Global (or Absolute) Maximum

A function $f(x)$ has a global (or absolute) maximum at $x=a$ if $f(a) \geq f(x)$ for all *x*. In other words, $f(a)$ is the largest value of a function $f(x)$, and occurs when $x=a$.

Global (or Absolute) Minimum

A function $f(x)$ has a global (or absolute) minimum at $x=a$ if $f(a) \leq f(x)$ for all *x*. In other words, $f(a)$ is the smallest value of a function $f(x)$, and occurs when $x=a$.

Local (or Relative) Maximum

A function $f(x)$ has a local (or relative) maximum at $x=a$ if $f(a) \geq f(x)$ for all *x* close to $x=a$. In other words, $f(a)$ is the largest value of a function $f(x)$ for values near $x=a$.

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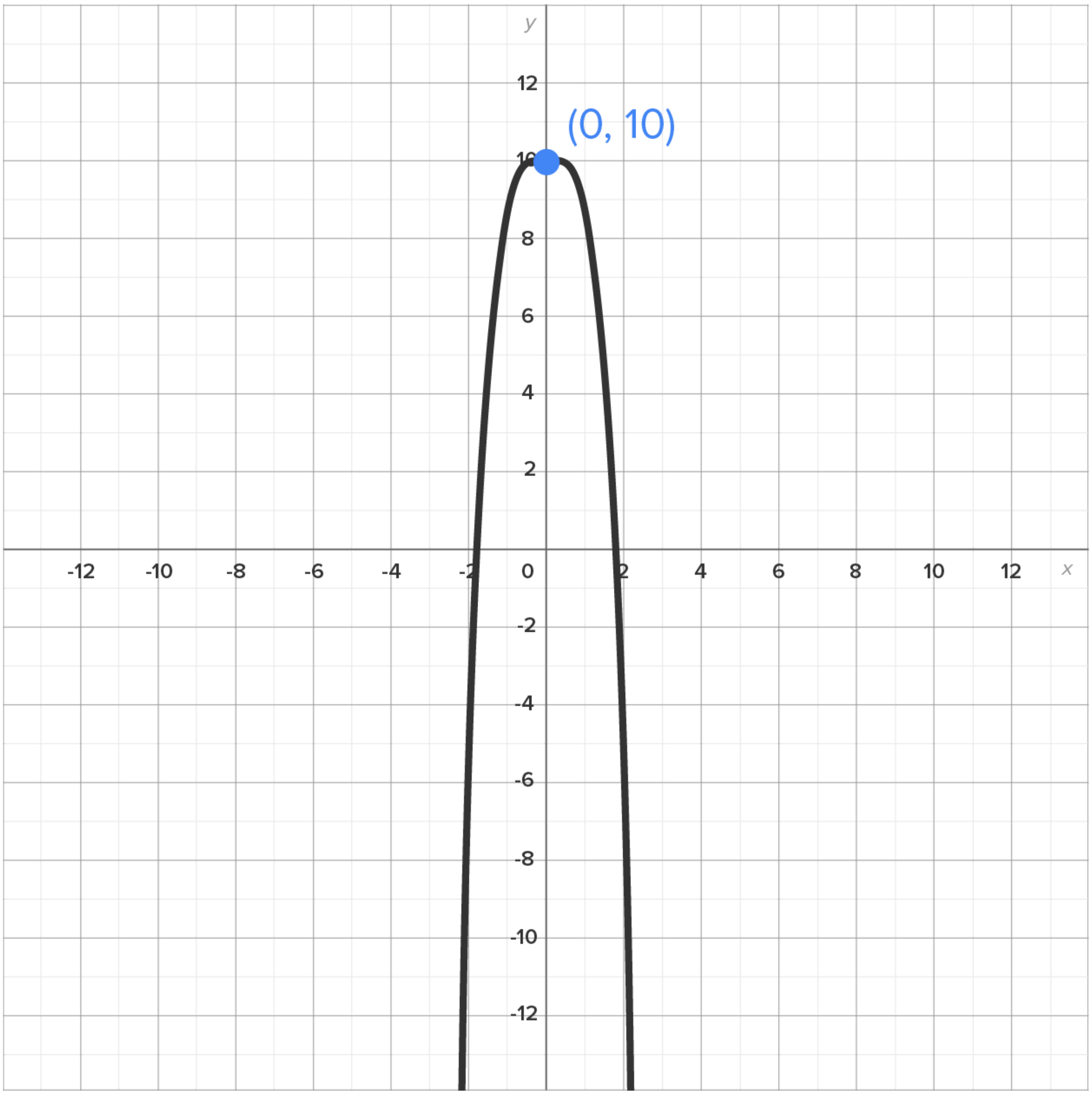
2. Finding Global and Local Maximum and Minimum Points

EXAMPLE Consider the function $f(x) = -x^4 + 10$ as shown in the graph below.

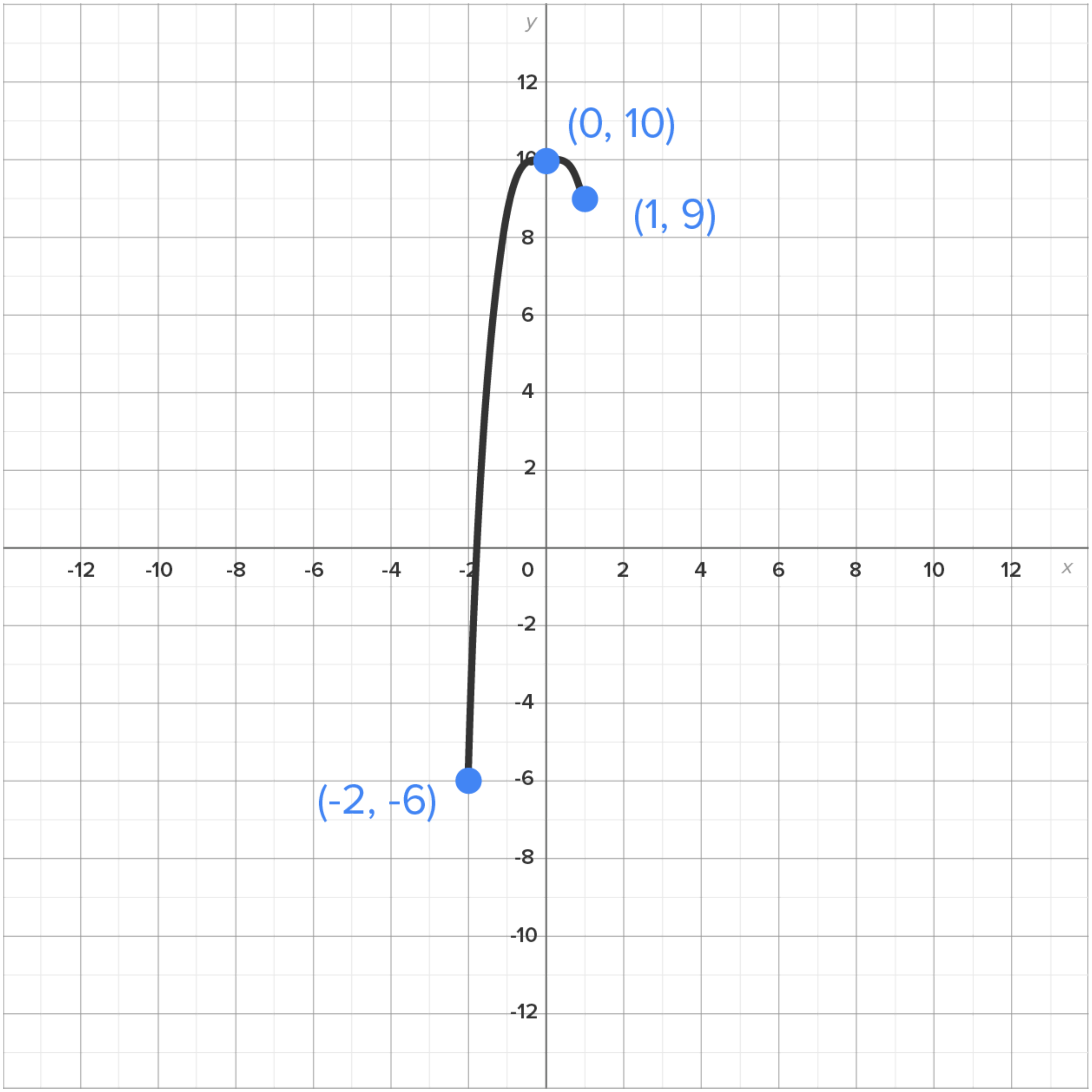


The highest point on the graph is (0, 10), while there is no lowest point. It is also the highest point when compared to other points around it.

Therefore, we say that $f(x)$ has a global maximum and local maximum at $x = 0$, and its value is 10.

There is no local or global minimum point.

EXAMPLE Now consider the function $f(x) = -x^4 + 10$, but contained on the interval $[-2, 1]$.



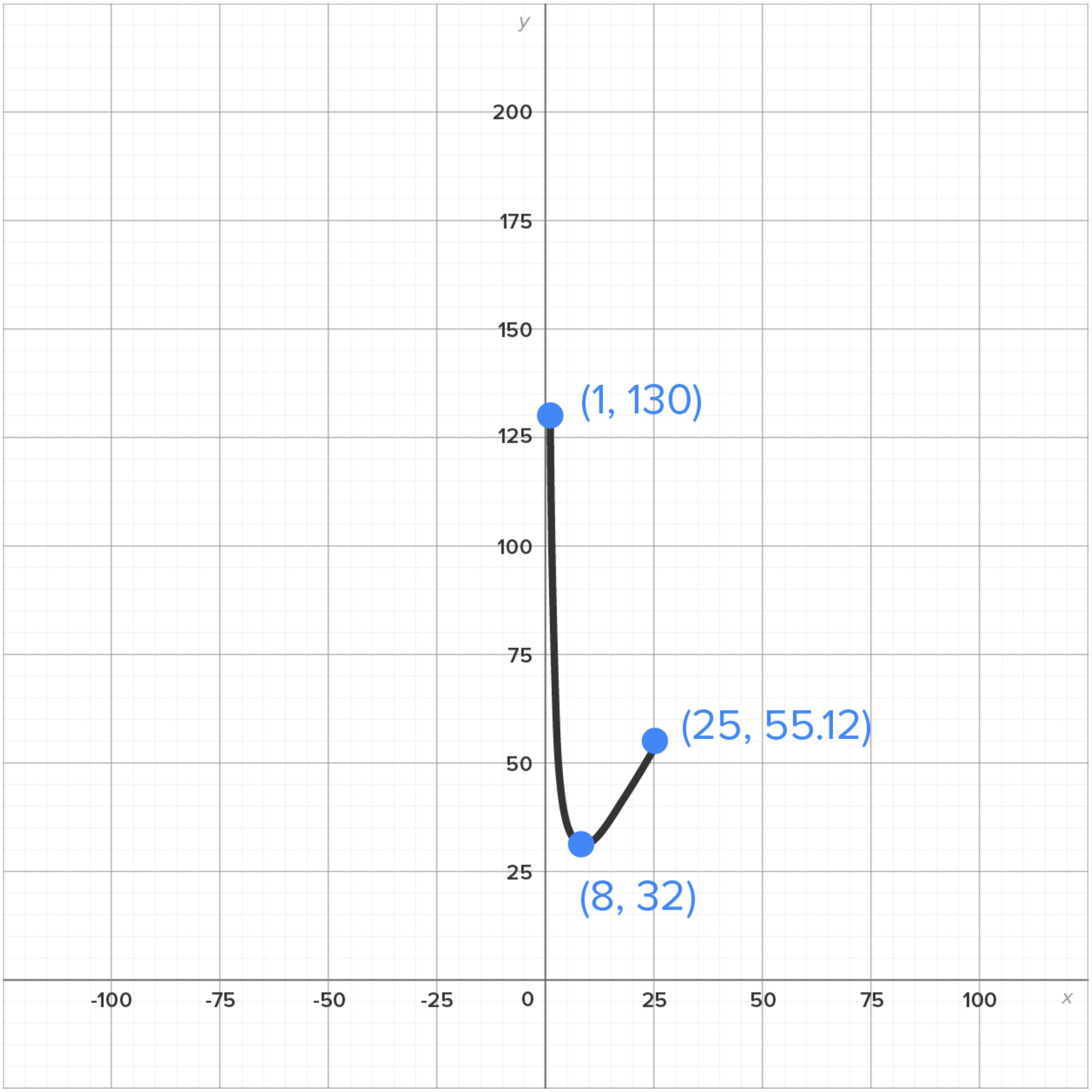
The highest point on the graph is (0, 10), which is also the highest point around (0, 10). Therefore, at $x = 0$, both a local and global maximum occurs and is equal to 10.

The lowest point on the graph is (-2, -6), which means that $f(x)$ has a global minimum at $x = -2$, which is equal to -6.

Neither (-2, -6) nor (1, 9) are considered local minimum values. This is because there is no graph on the other side of the points to compare.

In other words, having a local extreme point at $x = a$ means that $f(a)$ is the most extreme value on an open interval containing a (both sides of $x = a$).

TRY IT



Use the graph to determine all global and local maximum and minimum values of the function. +

The global maximum is 130 and it occurs at $x = 1$, and both a local and global minimum is 32 at $x = 8$.

SUMMARY

In this lesson, you learned that one of the main uses of derivatives is to find minimum and maximum values of a function. A function can have several types of extreme values, which can be identified from a graph. These **points** include: **global (or absolute) maximum**, **global (or absolute) minimum**, **local (or relative) maximum**, and **local (or relative) minimum**. Next, you explored using graphs to **find all global and local maximum and minimum values** of each respective function.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 3 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.

TERMS TO KNOW

Extrema

Another word for extreme values.

Extreme Values

The minimum or maximum values of a function.

Global (or Absolute) Maximum

A function $f(x)$ has a global (or absolute) maximum at $x = a$ if $f(a) \geq f(x)$ for all *x*. In other words, $f(a)$ is the largest value of a function $f(x)$, and occurs when $x = a$.

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