Southern New Hampshire University

PHY-150 Project 7-2:
A design and analysis of a new roller coaster

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Contents

1 Free-body diagram analysis

Prompt: Create a diagram of a roller coaster track containing at least two peaks and two valleys. As you complete your report, you may wish to design a more complicated coaster.

However, it should still have two peaks and two valleys that meet the requirements below and that you are comfortable using in calculations and descriptions of energy and momentum. Your diagram should include the following information: *an initial height of 75 meters*, *at least two peaks and two valleys representing drops of over 20 meters*, and *a set mass for your roller coaster cart between 400 and 600 kilograms.*

My answer: The free-body diagram is depicted in Fig. 1. The x_i is the initial position of the car to be moved down the roller coaster, the v_1 and v_2 are **not** velocities, but rather stands for the first and second vallies and the p_1 and p_2 stand for the peaks. Finally, the k stands for the cart on the initial position, k_1 stands for the "first cart" to be hit by the initial cart and k_2 stands for the "second cart" to be hit by the k_1.

The initial height h is $h = 75$ m and the both the peaks p_1 and p_2, and vallies v_1 and v_2 are a length of 25 m and 60 m respectively.

Now with regards to the cart(s), I set it to $\sim$ 500 kg as a "happy medium" between 400 kg and 600 kg, but the k can be set to whatever precice value between this range if the analysis calls for it.

Finally, with regards to k_1 and k_2, they exist for certain problems—specifically the ones where I will assume the existence of carts of varying masses to work out inelastic calculations. The horizontal line that it on the carts are there to "point out" that these carts were added *deus ex machia* for inelastic collision problems. I will set them when solving those specific problems.

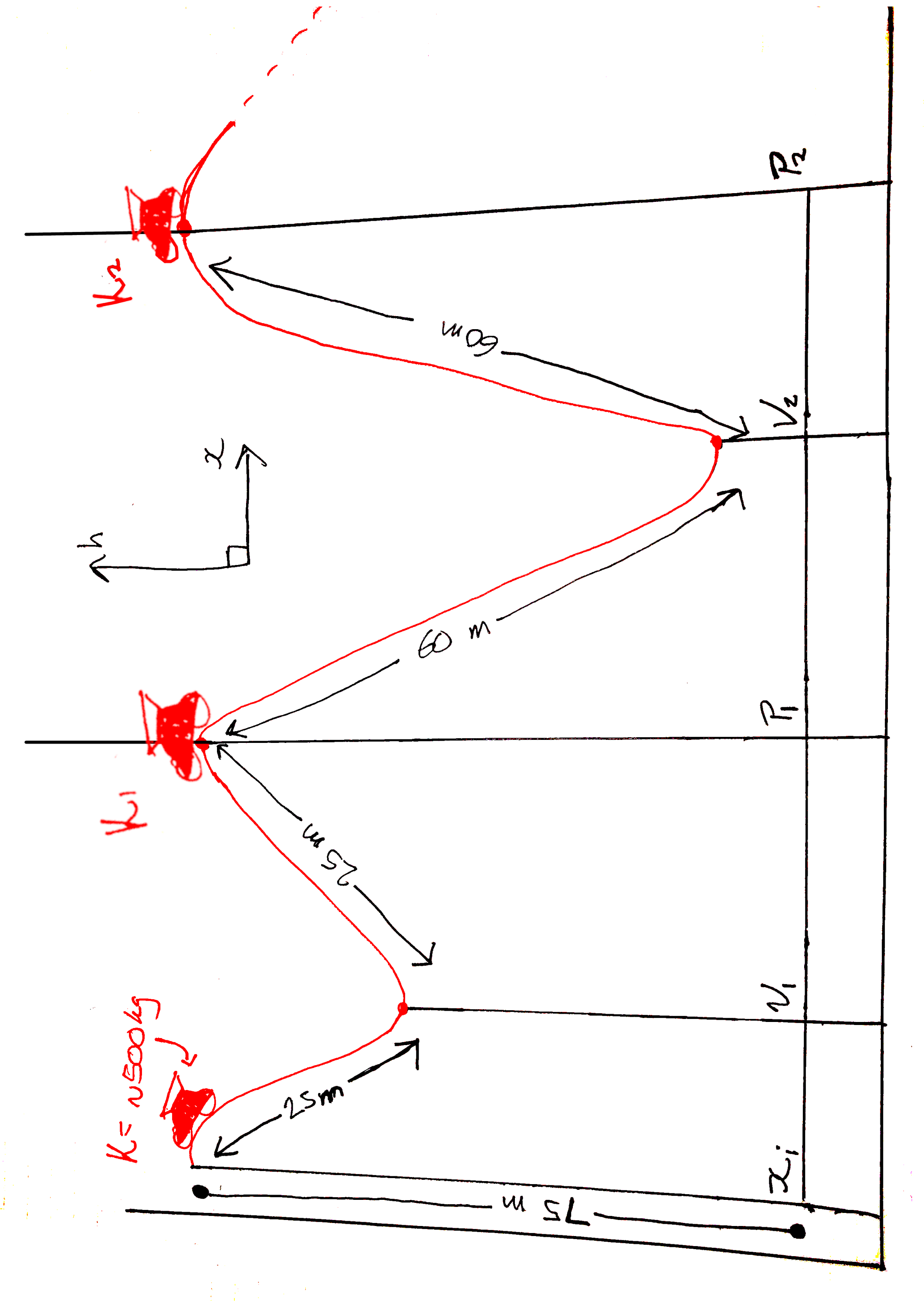

Figure 1: **Free-body diagram for a new roller coaster.**

2 Potental and kinetic energy, and momentum

Prompt: Calculate the kinetic energy, potential energy, and momentum of the cart at the initial drop for both peaks, and for both valleys. If your coaster has more than two peaks and two valleys, select which peaks and valleys you wish to use in your calculations and clearly mark them on your diagram.

In your calculations, be sure to explicitly state the equations you use and what values you will be substituting to calculate the final value.

My answer: First, I need to "lay out" some basic notions with regards to mechanical work and energy. Work is defined as the amount of mechanical force applied over a distance,[1] and energy is the quantifiable capacity to do work. Potential energy refers to the amount of work that can be done by a physical objective relative to its position and kinetic energy is the amount of work done by an objective because of its motion.[2] Force applied over a distance, potental energy and kinetic energy are equal to one another.[3]

Please note that I am **not** going to assume that there are other carts despite the force diagram depicting more carts at other peaks. I will also **not** assume friction is acting on the motion of the cart.

I will assume the cart has a mass of $m = 500$ kg. I will begin by outlining the method that I will use to solve for the potential and kinetic energies, as well as the momentum. Starting at x_i, its potental energy is

$$U_g = mgh\ [J] = 500\ \text{kg} \cdot 9.8\ \text{m/s}^2 \cdot 25\ \text{m} \approx 1.23 \times 10^6\ [J]$$

because the cart is assumed to be 500 kilograms and the h between the initial point (x_1) and the first valley that it must cross (v_1) is $75\ \text{m} - 25\ \text{m} = 50$ m.

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[1] More formally, let ΔX be the curve of the path of some physical body, $\vec{F}$ be a mechanical force and x be displacement. Then, work is defined as

$$W = \int_{\Delta X} \vec{F} \cdot d\vec{x}$$

[2] To "keep it simple," I will just introduce the formal definitions of potential and kinetic energy, which are $U_g = mgh\ [J]$ and $U_k = \frac{1}{2}mv^2\ [J]$ respectively.

[3] Or, more formally, $\vec{F}d\ [J] = mgh\ [J] = \frac{1}{2}mv^2\ [J]$.

Working out the kinetic energy of anything requires its velocity, which I do not have. But, knowing that $U_g = U_k$, I can, with confidence, conclude that the kinetic energy of the cart is equal to the potential energy of itself, or

$$U_k = U_g = mgh \approx 1.23 \times 10^6 \ [J]$$

Finally, the velocity of the cart starting from the x_i can easily be worked out with a bit of algebra[4] on the U_k, in particular the method of transposition of variables:

$$v_x = \sqrt{\frac{2U_k}{m}}$$

Knowing that $m = 500$ kg and that the $U_k \approx 1.23 \times 10^6$, I just need to "plug and chug"

$$v_{xi} = \sqrt{\frac{2 \cdot (1.23 \times 10^6 \ [J])}{500 \text{ kg}}} \approx 70.14 \text{ m/s}$$

The table below shows the potential and kinetic energy calculations,[5] as well as the momentum calculations for the cart for each "checkpoint" that it goes through. They were all worked out using "more-or-less" the same technique described above:

From	**To**	**P.E.**	**K.E.**	**Momentum**
Initial point (x_i)	Valley 1 (v_1)	$\sim 1.23 \times 10^6 \ [J]$	$\sim 1.23 \times 10^6 \ [J]$	$\sim 70.14 \ m/s$
Valley 1 (v_1)	Peak 1 (p_1)	$0 \ [J]$	$\sim 1.23 \times 10^6 \ [J]$	$\sim 70.14 \ m/s$
Peak 1 (p_1)	Valley 2 (v_2)	$2.94 \times 10^6 \ [J]$	$2.94 \times 10^6 \ [J]$	$108.44 \ m/s$
Valley 2 (v_2)	Peak 2 (p_2)	$0 \ [J]$	$2.94 \times 10^6 \ [J]$	$108.44 \ m/s$

2.1 Discussion of energy transfer

Prompt: Describe the energy transfers that occur as the cart moves along the track. This should be a narrative description of the energy transfers that occur at the initial launching point, peaks, and valleys.

[4] One can work out the v by starting with

$$U_k = \frac{mv^2}{2}$$

and solving for the v. I will skip the algebra for the sake of keeping this paper short and simple.

[5] "P.E." is potential energy and "K.E." is kinetic energy.

In your descriptions, address the following: *for each of the identified points, discuss how kinetic energy transferred to potential energy and vice versa*, *discuss what happens to the total energy of the cart as it moves along the track and why*, and *discuss how is the principle of conservation of energy applied in this situation.*

My answer: As I have previously mentioned, energy is the quantifiable capacity to do work. In this system where there are no other carts to interact with, nor is there any friction to alter the motion of the carts, the energy is conserved for the most part, and the cart will go from peak to valley, and ultimately end up on the final position, the second peak (p_2).

3 Inelastic collisions

Prompt: Calculate the momentum and kinetic energy of the cart before and after an inelastic collision.

In your calculations, be sure to explicitly state the equations you use and what values you will be substituting to calculate the final value.

My answer: *Inelastic energy* is a situation where the internal kinetic energy of a body in motion changes, or in practical terms regarding the problem that I am faced with, the cart in motion "sticks" to another cart in rest and its resulting velocity is effected. Specifically, when doing calculations on a cart that is rolling down the rails of a roller coaster, I will set $K_1 = 400\ kg$ and $K_2 = 600\ kg$ to analyse their effects.

Starting with $K_1 = 400\ kg$ and $K_2 = 600\ kg$, I will begin by "laying out" the equations of inelastic energy calculations to be employed when doing these calculations. First off, the formula for working out inelastic collisions is:

$$(m_A + m_B)v_{A+B} = m_A v_{Ai} + m_B v_{Bi}$$

I will apply this notion to the situation that I am faced with. I have a cart falling from the initial point (K) that is to "bump in" to the cart on the top of the first peak (K_1). Plugging these into the formula for inelastic collision, I get:

$$(m_{K'} + m_{K1'})v_{K+K1} = m_K v_K + m_{K1} v_{K1}$$

With a bit of algebra, the v_{K+K1} is worked out to

$$v_{K+K1} = \frac{m_K v_K + m_{K1} v_{K1}}{m_{K'} + m_{K1'}}$$

The K_1 is not experiencing any velocity until it comes into contact with the K, so it can be canceled out

$$v_{K+K1} = \frac{m_K v_K + \cancel{m_{K1} v_{K1}}}{m_{K'} + m_{K1'}} = \frac{m_K v_K}{m_{K'} + m_{K1'}}$$

What is needed to be discovered is the v_K, then the v_{K+K1} can be worked out. Fortunately this can be done by working out the potential and kinetic energies, and then the velocity from the kinetic energy of the initial cart. The potental energy is

$$U_g = mgh = 500\ kg \cdot 9.8\ m/s^2 \cdot 25\ m \approx 1.23 \times 10^6\ [J]$$

The kinetic energy is:

$$U_k = 1.23 \times 10^6 \because U_g = U_k$$

And the v_K is worked out to

$$v_K = \sqrt{\frac{2U_k}{m}} = \sqrt{\frac{2 \cdot (\sim 1.23 \times 10^6\ [J])}{500\ kg}} \approx 70.14\ m/s$$

Finally, the v_{K+K_1} can be worked out. Knowing that mass is constant, that the $m_K = 500\ kg$ and that the $m_{K1} = 400\ kg$, the v_{K+K1} is:

$$v_{K+K1} = \frac{m_K v_K}{m_{K'} + m_{K1'}} = \frac{500\ kg \cdot 70.14\ m/s}{500\ kg + 400\ kg} \approx 38.97\ m/s$$

So I can confidently state that, from the initial point (x_i) to the first valley (v_1), the:

- Potential energy is $U_g \approx 1.23 \times 10^6\ [J]$
- Kinetic energy is $U_k = U_g \approx 1.23 \times 10^6\ [J]$
- Resulting velocity is $v_{K+K1} = 38.97\ m/s$

With the same procedure as outlined above, I have worked out the solution to the second half of this problem, the $v_{K+K1+K2}$, or the resulting velocity of the carts in motion along with their potential and kinetic energies:

- The potential energy of the carts are: $U_g \approx 17536\ [J]$
- The kinetic energy of the carts are: $U_k \approx 17536\ [J]$
- The resulting velocity of the carts are: $v_{K+K1+K2} \approx 15.59\ m/s$

3.1 Discussion of inelastic collisions

Prompt: Describe the energy transfers that occur as a cart inelastically collides with an object of equal mass at rest. This should be a narrative description of the energy transfers that occur as the cart inelastically collides with a cart of equal mass.

In your descriptions, address the following: *the kinetic energy of each cart before and after the collision*, *the total energy of the system— now including both carts— as a result of the inelastic collision*, and *the principle of conservation of energy applied in this situation.*

My answer: The inelastic collisions assume that the carts will bump into each other and then "stick" to each other, increasing its mass and ultimately affecting its resulting velocity in the process. From my calculations, the total energy of the carts seems to *decrease* despite the carts getting more massive with each collision, and the resulting velocity seems to be decreasing as well.

4 Energy transfers with friction

Prompt: Calculate the work due to friction and frictional force. In your calculations, be sure to explicitly state the equations you use and what values you will be substituting to calculate the final value.

My answer: Friction is a force that decelerates a body when it slides against another body. In this case, friction is caused by the wheels of the cart sliding with the metalic rails of the roller coaster.

Note that I will **not** assume carts on the other peaks as my force diagram may suggest.

Specifically, I will use the *kinetic friction coefficient* $\mu_k = 0.02$ when doing my calculations on friction's effect on the cart's motion. I will assume maximum friction as the cart is rolling down the rails, and to work out the force of maximum force of kinetic friction, I used the formula:

$$f_k = \mu_k \cdot mg \cdot \cos\theta$$

I will set the *theta* to a constant of $\theta = 25°$ for the sake of simplicity. The force of kinetic friction with regards to the first cart K is then:

$$f_k = [0.02 \cdot (500\ kg \cdot 9.8\ m/s)] \cdot \cos 25° = 88.82\ [N]$$

Energy is measured with the Joule, which is the shorthand for the Newton-metre— or $\vec{F}d$ with $\vec{F}$ being mechanical force and d being displacement. Ergo energy can be alternatively be calculated by working out the mechanical force by the displacement of the physical body.

In this case, I need to work out the product of the resulting force by the distance traveled. The resulting force is

$$F_{\text{net}} = (mg) - f_s = (500\ kg \cdot 9.8\ m/s^2) - 88.82\ [N] = 4811.18\ [N]$$

Finally, both the potental and kinetic energy of the cart can be worked out by multiplying the F_{net} witht the distance traveled, with regards to the potential and kinetic energy from the initial point (x_i) to the first valley (v_1):

$$U_g = U_k = 4811.18\ [N] \cdot 25\ [m] = 120280\ [J]$$

With regards to the cart moving from the first valley to the first peak (p_1), its potential energy is equal to its kinetic energy and friction needs to be accounted for. To do this, I will consider gravity acting on the cart moving uphill and the frictional force. Gravitational pull towards a surface (W) is expressed as

$$W = -mg\ [N] \cdot \cos 25°$$

In this case, the mass of the cart is $m = 500\ kg$ and the frictional force has been previously worked out to $f_k = 88.82\ [N]$. The resulting force is then

$$F_{\text{net}} = -mg \cdot \cos 25° - f_k = -4529.73\ [N]?$$

I DID NOT FINISH THIS QUESTION

4.1 Discussion of energy transfers with a halting cart

Prompt: Describe the energy transfers that occur as the cart is brought to a stop. This should be a narrative description of the energy transfers—written to describe these concepts to a nontechnical audience—that occur as the cart is brought to a stop.

In your descriptions, address the following: *the kinetic energy of the cart system before and after it has been brought to a stop*, *the total energy of the system as a result of this change in motion*, and *the principle of conservation of energy is applied in this situation.*

My answer: Unfortuantely, due to the ambiguousness of the project's setup, I could not fully answer what the energy and momentum of the cart would be. I do predict that the cart of the roller coaster would not fully "make it" to the second peak, but I lack an analytical model to test this notion.

5 Acknowledgements

I used the following resources when doing this project:

- The *OpenStax College Physics* textbook.
- The following test item from *William & Mary University*: http://www.physics.wm.edu/Courses/Phys101.00/old_Tests /test2_3_99.html
- This *Teach Engineering* article: https://www.teachengineering.org/lessons/view/ duk_rollercoaster_music_less
- This *Coaster 101* article: https://www.coaster101.com/2011/10/24 /coasters-101-wheel-design