Kinematics

Carolina Distance Learning
Investigation Manual

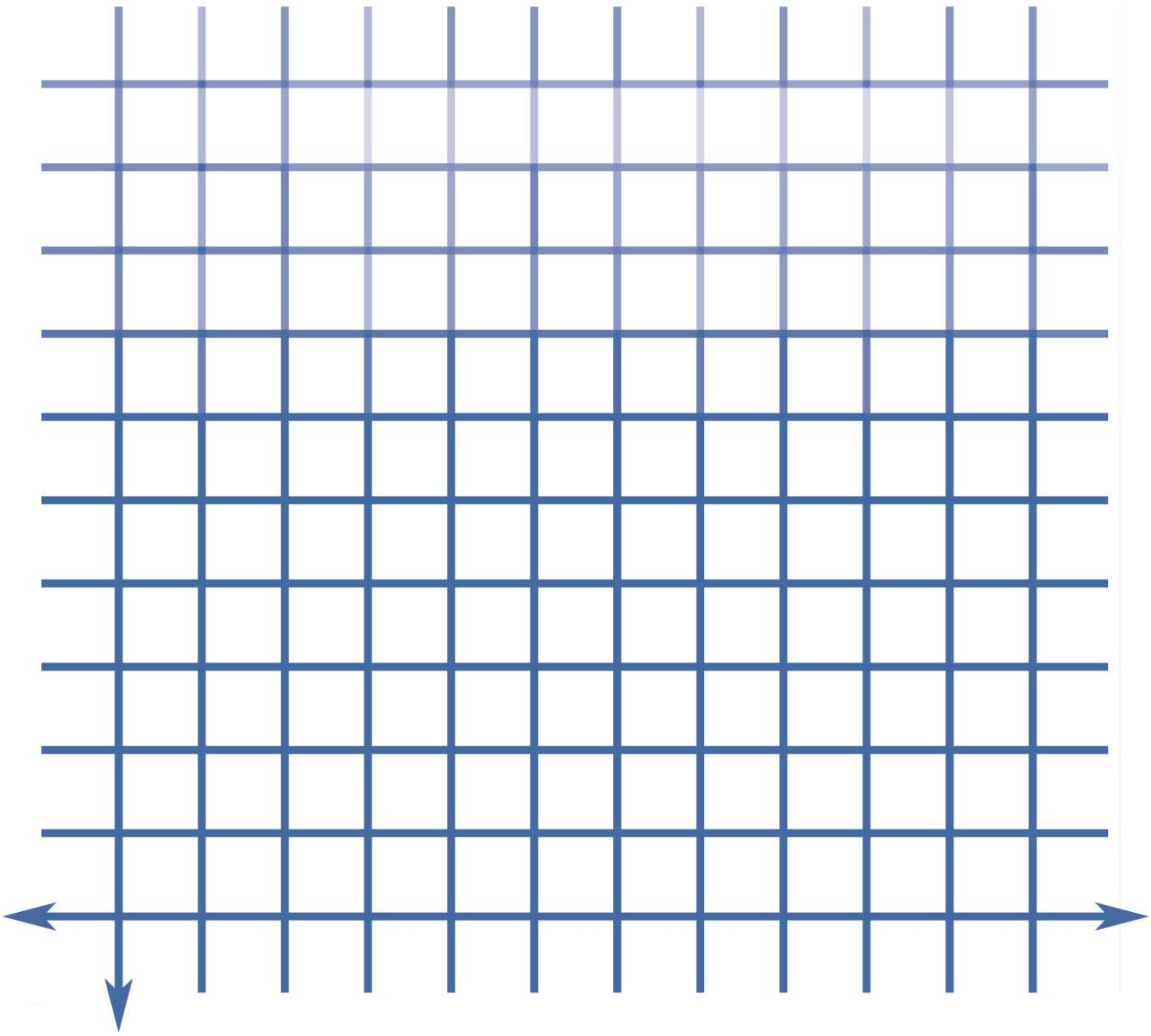


Table of Contents

Overview

Kinematics is the branch of physics that deals with the analysis of the motion of objects without concern for the forces causing the motion. Scientists have developed equations that describe the movement of objects within certain parameters, such as objects moving with a constant velocity or a constant acceleration. Using these equations, the future position and velocity of an object can be predicted. This investigation will focus on objects moving with a constant velocity or a constant acceleration. Data will be collected on these objects, and the motion of the objects will be analyzed through graphing these data.

Objectives

- Explain linear motion for objects traveling with a constant velocity or constant acceleration
- Utilize vector quantities such as displacement and acceleration, and scalar quantities such as distance and speed.
- Analyze graphs that depict the motion of objects moving at a constant velocity or constant acceleration.
- Use equations of motion to analyze and predict the motion of objects moving at a constant velocity or constant acceleration.

Time Requirements

Preparation..........5 minutes
Activity 1 ....15 minutes
Activity 2 ....20 minutes
Activity 3 ....20 minutes
Activity 4 ....10 minutes
Activity 5 ....20 minutes

Background

Mechanics is the branch of physics that that studies the motion of objects and the forces and energies that affect those motions. Classical Mechanics refers to the motion of objects that are large compared to subatomic particles and slow compared to the speed of light. The effects of quantum mechanics and relativity are negligible in classical mechanics. Most objects and forces encountered in daily life can be described by classical mechanics, such as the motion of a baseball, a train, or even a bullet or the planets. Engineers and other scientists apply the principles of physics in many scenarios. Physicists and engineers often collect data about an object and use graphs of the data to describe the motion of objects.

Kinematics is a specific branch of mechanics that describes the motion of objects without reference to the forces causing the motion. Examples of kinematics include describing the motion of a race car moving on a track or an apple falling from a tree, but only in terms of the object's position, velocity, acceleration, and time without describing the force from the engine of the car, the friction between the tires and the track, or the gravity pulling the apple. For example, it is possible to predict the time it would take for an object dropped from the roof of a building to fall to the ground using the following kinematics equation:

$$\boldsymbol{s} = \frac{1}{2}\,\boldsymbol{a}\,t^2$$

Where $\boldsymbol{s}$ is the displacement from the starting position at a given time, $\boldsymbol{a}$ is the acceleration of the object, and t is the time after the object is dropped. The equation does not include any variables for the forces acting on the object or the mass or energy of the object. As long as the some initial conditions are known, such an object's position, acceleration, and velocity at a given time, the motion or position of the object at any future or previous time can be calculated by applying kinematics. This method has many useful applications. One could calculate the path of a projectile such as a golf ball or artillery shell, the time or distance for a decelerating object to come to rest, or the speed an object would be traveling after falling a given distance.

Early scientists such as Galileo Galilee (1564-1642), Isaac Newton (1642-1746) and Johannes Kepler (1571-1630) studied the motion of objects and developed mathematical laws to describe and predict their motion. Until the late sixteenth century, the idea that heavier objects fell faster than lighter objects was widely accepted. This idea had been proposed by the Greek philosopher Aristotle, who lived around the third century B.C. Because the idea seemed to be supported by experience, it was generally accepted. A person watching a feather and a hammer dropped simultaneously from the same height would certainly observe the hammer falling faster than the feather. According to legend, Galileo Galilee, an Italian physicist and mathematician, disproved this idea in a dramatic demonstration by dropping objects of different mass from the tower of Pisa to demonstrate that they fell at the same rate. In later experiments, Galileo rolled spheres down inclined planes to slow down the motion and get more accurate data. By analyzing the ordinary motion of objects and graphing the results, it is possible to derive some simple equations that predict their motion.

To study the motion of objects, a few definitions should be established. A **vector** refers to a number with a direction and magnitude (or size). Numbers that have a magnitude but not a direction are referred to as a **scalar**. In kinematics, vectors are important, because the goal is to calculate the location and direction of movement of the object at any time in the future or past. For example if an object is described as being 100 miles from a given position traveling at a speed of 50 miles per hour, that could mean the object will reach the position in 2 hours. It could also mean the object could be located up to 100 miles farther away in 1 hour, or somewhere between 100 and 200 miles away depending on the direction. The quantity **speed,** which refers to the rate of change in position of an object, is a scalar quantity because no direction of travel is defined. The quantity **velocity,** which refers to both the speed and direction of an object, is a vector quantity.

Distance, or the amount of space between two objects, is a scalar quantity. **Displacement,** which is distance in a given direction, is a vector quantity. If a bus travels from Washington D.C. to New York City, the distance the bus traveled is approximately 230 miles. The displacement of the bus is (roughly) 230 miles North-East. If the bus travels from D.C to New York and back, the distance traveled is roughly 460 miles, but the displacement is zero because the bus begins and ends at the same point.

It is important to define the units of scalar and vector quantities when studying mechanics. A person giving directions from Washington D.C. to New York might describe the distance as being approximately 4 hours. This may be close to the actual travel time, but this does not indicate actual distance.

To illustrate the difference between distance and displacement, consider the following diagrams in Figures 1-3.

Consider the number line in **Figure 1**. The displacement from zero represented by the arrowhead on the number line is -3, indicating both direction and magnitude. The distance from zero indicated by the point on the number line equals three, which is the magnitude of the displacement. For motion in one dimension, the + or– sign is sufficient to represent the direction of the vector.

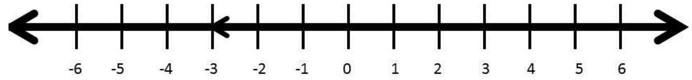


Figure 1.

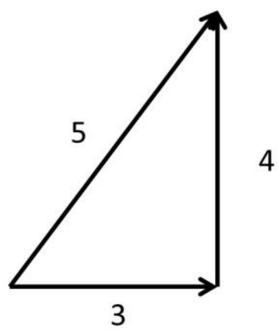


Figure 2

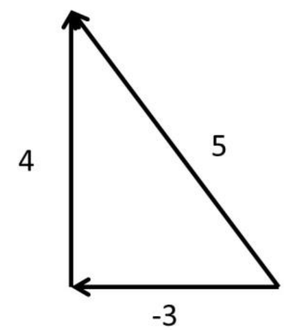


Figure 3

The arrows in Figures 2 and 3 represent displacement vectors for an object. The long lines represent a displacement with a magnitude of five. This displacement vector can be resolved into two component vectors along the x and y axes. In all four diagrams the object is moved some distance in either the positive or negative x direction, and then some distance in the positive y direction; however, the final position of the object is different in each diagram. The total distance between the object's initial and final position in each instance is 5 meters, however to describe the displacement, **s**, from the initial position more information is needed.

In **Figure 2**, the displacement vector can be given by 5 meters (m) at 53.1°. This vector is found by vector addition of the two component vectors, 3 m at 0° and 4 m at 90°, using conventional polar coordinates that assign 0° to the positive x direction and progress counterclockwise towards 360°. The displacement in **Figure 3** is 5 m at 143.1°. In each case the magnitude of the vector is length of the arrow, that is, the distance that the object travels. Most texts will indicate that a variable represents a vector quantity by placing an arrow over the variable or placing the variable in bold.

To indicate the magnitude of a vector, absolute value bars are used. For example the magnitude of the displacement vector in each diagram is 5 m. In Figure 2 the displacement is given by:

$$\boldsymbol{s} = 5 \text{ m at } 51.3°$$

The magnitude of this vector may be written as:

$$|\boldsymbol{s}| = d = 5 \text{ m}$$

The displacement vector in **Figure 2**, ***s*** = 5 m at 53.1°, can be resolved into the component vectors 3 m at 0° and 4 m at 90°.

Two more terms that are critical for the study of kinematics are velocity and acceleration. Both terms are vector quantities.

Velocity ($\boldsymbol{v}$) is defined as the rate of change of the position of an object. For an object moving in the x direction, the magnitude of the velocity (speed) may be described as:

$$\boldsymbol{v} = \frac{x_2 - x_1}{\Delta t}$$

Where x_2 is the position at time t_2 and x_1 is the position of the object at time t_1. The variable Δt represents the time interval t_2 - t_1. The symbol, Δ, is the Greek symbol delta, and refers to a change or difference. Δt is read, "delta t". Time in the following examples is provided in seconds (s). Please be sure that you do not confuse the "s" unit for seconds, and the "$\boldsymbol{s}$" unit for displacement in these formulas.

For example if an object is located at a position designated x_1 = 2 m and moves to position x_2 = 8 m over a time interval Δt = 2 s, then the average speed could be calculated:

$$\frac{8m - 2m}{2s} = 3m/s$$

The velocity could for this object could be indicated as:

$$\boldsymbol{v} = 3\ m/s$$

Because velocity is a vector quantity, the positive sign indicates that the object was traveling in the positive x direction, at a speed of 3 m/s.

Acceleration is defined as the rate of change of velocity. The magnitude of acceleration may be described as:

$$\boldsymbol{a} = \frac{\boldsymbol{v_2} - \boldsymbol{v_1}}{\Delta t}$$

For example, an object with an initial velocity v_1 **=** 10 m/s slows to a final velocity of v_2 = 1 m/s over an interval of 3 s.

$$\frac{1\,{}^{m}/_{\mathrm{s}} - 10\,{}^{m}/_{s}}{3\mathrm{s}} = -3\,{}^{m}/_{\mathrm{s}}/_{\mathrm{s}}$$

The object has an average acceleration of –3 meters per second per second, which can also be written as –3 meters per second squared, or –3 ${}^{m}/_{\mathrm{s}^2}$.

Because only the initial and final positions or velocities over a given time interval are used in these equations, the calculated values indicate the *average* velocity or acceleration. Calculating the *instantaneous* velocity or acceleration of an object requires the application of calculus. Only average velocity and acceleration are considered in this investigation.

Materials

Included in the Central Materials kit:
Tape Measure
Rubber Bands
Protractor

Included in the Mechanics Module materials kit
Constant Velocity Vehicle
Steel Sphere
Acrylic Sphere
Angle Bar
Foam Board
Block of Clay

Needed, but not supplied:
Scientific or Graphing Calculator
or Computer with Spreadsheet Software
Permanent Marker
Masking Tape
Stopwatch, or smartphone able to record video

Reorder Information: Replacement supplies for the Kinematics investigation can be ordered from Carolina Biological Supply Company, kit 580404 Mechanics Module. Call 1-800-334-5551 to order.

Safety

Safety goggles should be worn while conducting this investigation.

Read all the instructions for this laboratory activity before beginning. Follow the instructions closely and observe established laboratory safety practices.

Do not eat, drink, or chew gum while performing this activity. Wash your hands with soap and water before and after performing the activity. Clean up the work area with soap and water after completing the investigation. Keep pets and children away from lab materials and equipment.

Alternate Methods for Collecting Data using Digital Devices.

Much of the uncertainty in these experiments arises from human error in measuring the times of events. Some of the time intervals are very short, which increases the effect of human error due to reaction time.

Observing the experiment from a good vantage point that removes parallax errors and recording measurements for multiple trials helps to minimize error, but using a digital device as an alternate method of data collection may further minimize error.

Many digital devices, smart phones, tablets, etc. have cameras and software that allow the user to pause or slow down the video.

If you film the experiment against a scale, such as a tape measure, you can use your video playback program to record position and time data for the carts. This can provide more accurate data and may eliminate the need for multiple trials.

If the time on your device's playback program is not sufficiently accurate, some additional apps may be available for download.

Another option is to upload the video to your computer. Different video playback programs may come with your operating system or software suite or may be available for download.

Some apps for mobile devices and computer programs available for download are listed below, with notes about their features.

Hudl Technique: [*http://get.hudl.com/products/technique/*](http://get.hudl.com/products/technique/)

- *iPhone/iPad and Android*
- *FREE*
- *Measures times to the hundredth-second with slow motion features*

QuickTime http://www.apple.com/quicktime/download/

- Free
- Install on computer
- 30 frames per second
- Has auto scrubbing capability

Preparation

1. Collect materials needed for this investigation.
2. Locate and clear an area of level floor space in order to conduct the constant velocity experiment. The space should be free of obstruction and three to four meters long with a surface which will allow the vehicle to maintain traction but not impede the vehicle.

Activity 1: Graph and interpret motion data of a moving object

One way to analyze the motion of an object is to graph the position and time data. The graph of an object's motion can be interpreted and used to predict the object's position at a future time or calculate an object's position at a previous time.
Table 1 represents the position of a train on a track. The train can only move in one dimension, either forward (the positive x direction) or in reverse (the negative x direction).

Table 1

Time (x-axis), seconds	Position (y-axis), meters
0	0
5	20
10	40
15	50
20	55
30	60
35	70
40	70
45	70
50	55

1. Plot the data from Table 1 on a graph using the y-axis to represent the displacement from the starting position (y = 0) and the time coordinate on the x-axis.

2. Connect all the coordinates on the graph with straight lines.

Activity 2: Calculate the velocity of a moving object

In this activity you will graph the motion of an object moving with a constant velocity. The speed of the object can be calculated by allowing the Constant Velocity Vehicle to travel a given distance and measuring the time that it took to move this distance. As seen in Activity 1, this measurement will only provide the average speed. In this activity, you will collect time data at several travel distances, plot these data, and analyze the graph

1. Find and clear a straight path approximately two meters long.
2. Install the batteries and test the vehicle.

   Note: The vehicle should be able to travel two meters in a generally straight path. If the vehicle veers significantly to one side, you may need to allow the vehicle to travel next to a wall. The friction will affect the vehicle's speed, but the effect will be uniform for each trial.

3. Use your tape measure or ruler to measure a track two meters long. The track should be level and smooth with no obstructions. Make sure the surface of the track provides enough traction for the wheels to turn without slipping.
   Place masking tape across the track at 25 cm intervals.
4. Set the car on the floor approximately 5 cm behind the start point of the track.

   Note: Starting the car a short distance before the start point allows the vehicle to reach its top speed before the time starts and prevents the short period of acceleration from affecting the data.

5. Set the stopwatch to the timing mode and reset the time to zero.
6. Start the car and allow the car to move along the track.
7. Start the stopwatch when the front edge of the car crosses the start point.
8. Stop the stopwatch when the front edge of the car crosses the first 25 cm point.
9. Recover the car, and switch the power off. Record the time and vehicle position on the data table.
10. Repeat steps #5–9 for each 25 cm interval marked. Each trial will have a distance that is 25 cm longer than the previous trial, and the stopwatch will record the time for the car to travel the individual trial distance.
11. Record the data in **Data Table 1**.

Data Table 1

Time (s)	Displacement (m)
	0.00
	0.25
	0.50
	0.75
	1.00
	1.25
	1.50
	1.75
	2.00

12. Graph the time and displacement data points on graph paper.

13. Draw a line of best fit through the data points.

> **Note:** The points should generally fall in a straight line. If you have access to a graphing calculator or a computer with spreadsheet software, the calculator or spreadsheet can be programmed to draw the line of best fit, or trend line.

14. Calculate the slope of the line.

> **Note:** Based on the equation of a line that cross the y-axis at y = 0, the slope of the line, m, will be the velocity of the object.
> $$y = mx$$
> $$d = v\Delta t$$

15. Make a second data table, indicating the velocity of the car at any time.

Data Table 2

Time (s)	Velocity (m/s)
1	
2	
3	
4	
5	
6	
7	
8	

Note: Because the object in this example, the battery-powered car, moves with a constant speed, all the values for the velocity of the car in the second table should be the same. The value of the velocity for the car should be the slope of the line in the previous graph.

16. Graph the data points from the **Data Table 2** on a second sheet of graph paper. Label the y-axis Velocity and the x-axis Time.

Note: When the data points from this table are plotted on the second graph, the motion of the car should generate a horizontal line. On a velocity vs. time graph, an object moving with a constant speed is represented by a horizontal line.

17. Draw a vertical line from the x-axis at the point time = 2 seconds so that it intersects the line representing the velocity of the car.

18. Draw a second vertical line from the x-axis at the point time = 4 seconds so that it intersects the line representing the velocity of the car.

19. Calculate the area represented by the rectangle enclosed by the two vertical lines you just drew, the line for the velocity of the car, and the x-axis. An example is shown as the blue shaded area in **Figure 4**.

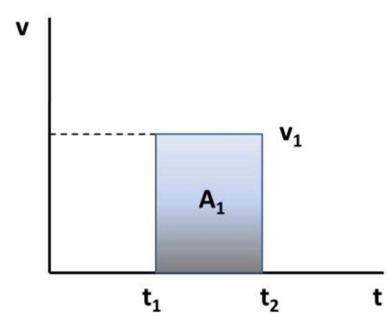


Figure 4

Note: In order to calculate the area of this rectangle, you must multiply the value for the time interval between time t=2 s and time t=4 s, by the velocity of the car. This area represents the distance traveled by the object during this time interval. This technique is often referred to as calculating the "area under the curve". The graph of velocity vs. time for an object that is traveling with a constant acceleration will not be a horizontal line, but using the same method of graphing the velocity vs. time and finding the "area under the curve" in a given time interval can allow the distance traveled by the object to be calculated.

Distance = velocity × time

In this equation, the time units (s) cancel out when velocity and time are multiplied, leaving the distance unit in meters.

Activity 3: Graph the motion of an object traveling under constant acceleration

Collecting data on freefalling objects requires accurate timing instruments or access to a building with heights of several meters where objects can safely be dropped over heights large enough to allow accurate measurement with a stopwatch. To collect usable data, in this activity you will record the time objects to roll down an incline. This reduces acceleration to make it easier to record accurate data on the distance that an object moves.

1. Collect the following materials:
   Steel Sphere
   Acrylic Sphere
   Angle Bar
   Clay
   Tape Measure
   Timing Device
   Protractor
2. Use the permanent marker and the tape measure to mark the inside of the angle bar at 1-cm increments.
3. Use the piece of clay and the protractor to set up the angle bar at an incline between 5° to 10°. Use the clay to set the higher end of the anglebar and to stabilize the system. (**Figure 5**)

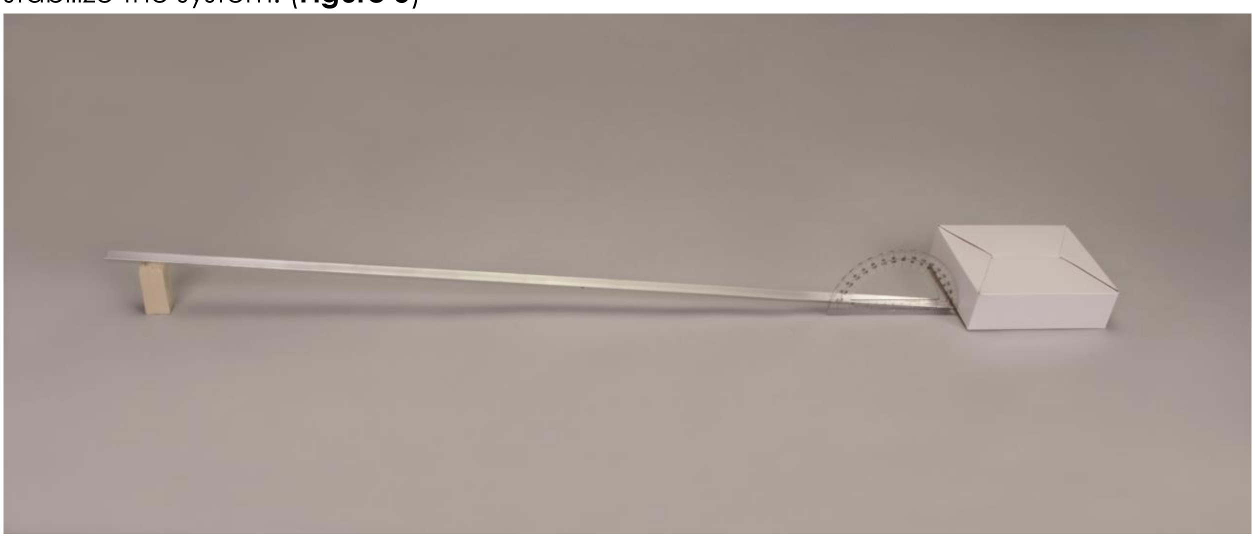


Figure 5

Set up the angle bar so that the lower end terminates against a book or a wall, to stop the motion of the sphere (**Figure 6.**)

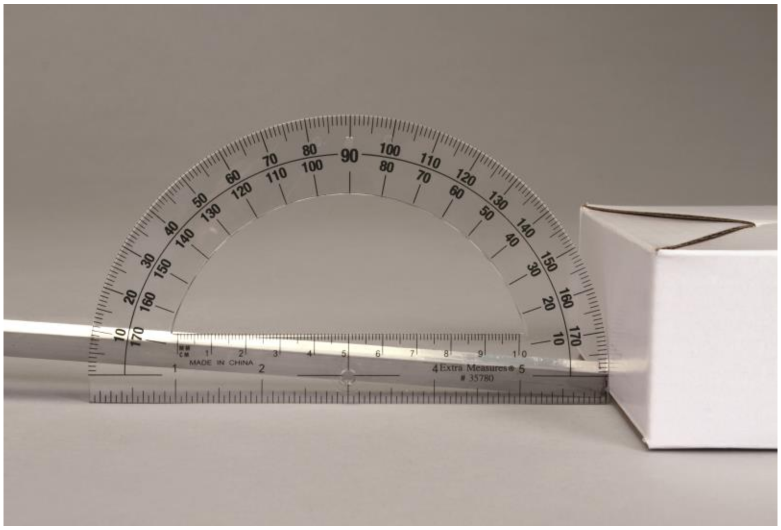


Figure 6

4. Place the steel sphere 10 cm from the lower end of the track.
5. Release the steel sphere and record the time it takes for the sphere to reach the end of the track.
6. Repeat steps #4–5 two more times for a total of three measurements at a starting point of 10 cm.
7. Repeat steps #4–6, increasing the distance between the starting point and the end of the track by 10 cm each time.
8. Record your data in Data Table 3.

> **Note:** You are recording the time it takes for the sphere to accelerate over an increasing distance. Take three measurements for each distance, and average the time for that distance. Record the time for each attempt and the average time in Table 4.

Data Table 3

Time (s)	Average time (s)	Average Time ^{2} (s^2)	Distance (m)
Trial 1 = Trial 2 = Trial 3 =			0.1
Trial 1 = Trial 2 = Trial 3 =			0.2
Trial 1 = Trial 2 = Trial 3 =			0.3
Trial 1 = Trial 2 = Trial 3 =			0.4
Trial 1 = Trial 2 = Trial 3 =			0.5
Trial 1 = Trial 2 = Trial 3 =			0.6
Trial 1 = Trial 2 = Trial 3 =			0.7
Trial 1 = Trial 2 = Trial 3 =			0.8

9. Calculate the average time for each distance and record this value in Table 4.

10. Create a graph of distance vs. time using the data from Table 4.

11. Complete Table 4 by calculating the square of the average time for each distance.

12. Create a graph of displacement vs. time squared from the data in Table 4.

Graphing the displacement vs time data from Table 4 will generate a parabola. When data points generate a parabola, it means the y value is proportional to the square of the x value, or:

$$\boldsymbol{y} \propto x^2$$

That means the equation for a line that fits all the data points looks like:

$$y = Ax^2 + Bx + C.$$

In our experiment, the **y-**axis is **displacement** and the **x-**axis is **time-**; therefore displacement is proportional to the time squared:

$$\boldsymbol{s} \propto t^2$$

So, we can exchange y in the equation with displacement ($\boldsymbol{s}$), to give a formula that looks like:

$$\boldsymbol{s} = At^2 + Bt + C.$$

We would know the displacement $\boldsymbol{s}$, at any time t. We just need to find the constants, A, B, and C.

The equation that describes the displacement of an object moving with a constant acceleration is one of the kinematics equations:

$$\boldsymbol{s} = \frac{1}{2}\boldsymbol{a}\Delta t^2 + \boldsymbol{v_1}\Delta t$$

The following section describes how to find this equation using the same method of finding the "area under the curve" covered in Activity 2.

Finding an Equation for the Motion of an Object with Constant Acceleration

The general form of a line is:

$$y = mx + b$$

Where m is the slope of the line, and b is the y-intercept, the point where the line crosses the y-axis. Because the first data point represents time zero and displacement zero, the y-intercept is zero and the equation for the line simplifies to:

$$y = mx$$

The data collected in Activity 3 showed that:

$$\boldsymbol{s} \propto t^2$$

This means that the displacement for the object that rolls down an inclined plane is can be represented mathematically as:

$$\boldsymbol{s} = kt^2 + c$$

Where k is an unknown constant representing the slope of the line, and c is an unknown constant representing the y-intercept.
The displacement of the sphere as it rolls down the incline can be calculated using this equation, if the constants k and c can be found.

Further experimentation indicates that the constant k for an object in freefall is one-half the acceleration. If the object is released from rest, the constant c will be zero.

So for an object that is released from rest, falling under the constant acceleration due to gravity, the displacement from the point of release is given by:

$$\boldsymbol{s} = \frac{1}{2}\boldsymbol{a}t^2$$

Where $\boldsymbol{s}$ is the displacement, t is the time of freefall, and $\boldsymbol{a}$ is the acceleration. For objects in freefall near Earth's Surface the acceleration due to gravity has a value of 9.8 $^{m}/_{s^2}$.

Another way to derive this equation, and find the values for k and c, is to consider the velocity vs. time graph for an object moving with a constant acceleration. Remember the velocity vs. time graph for the object moving with constant velocity from Activity 2. If velocity is constant, the equation of that graph would be:

$$\boldsymbol{v} = k$$

Where $\boldsymbol{v}$ represents the velocity, plotted on the y-axis, and k is the constant value of the velocity. Plotted against time on the x-axis, this graph is a horizontal line, as depicted in **Figure 7**.

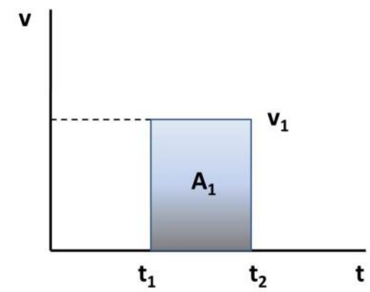


Figure 7

By definition, the shaded area is the distance traveled by the object during the time interval:

$$\Delta t = t_2 - t_1$$
$$\boldsymbol{v} = \frac{\boldsymbol{displacement}}{time} = \frac{\boldsymbol{s}}{\Delta t}$$
$$\therefore \boldsymbol{s} = \boldsymbol{v}\Delta t$$

If an object has a constant acceleration, then by definition:

$$\boldsymbol{a} = \frac{\boldsymbol{v_2} - \boldsymbol{v}_1}{\Delta t}$$

Or :

$$\boldsymbol{v}_2 = \boldsymbol{a}\Delta t + \boldsymbol{v_1}$$

This is equation is in the general form of a line y = mx + b, with velocity on the y-axis and time on the x-axis. The graph of this equation would look like the graph in **Figure 8**.

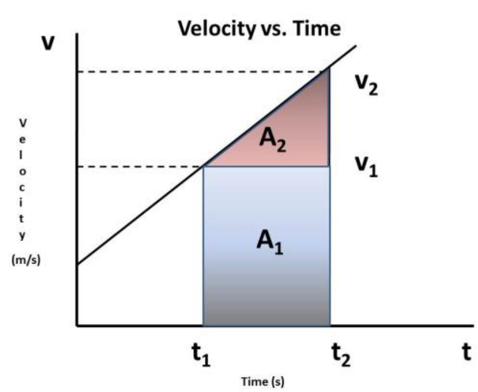


Figure 8

Similar to how the shaded area A_1 in Figure 7 represents the distance traveled by the object during the time interval $\Delta t = t_2 - t_1$, the shaded area A_2 combined with

A_1 equals the distance traveled by the object undergoing constant acceleration.

The area A_1 can be given by:

$$A_1 = \boldsymbol{v_1}\Delta t$$

The area A_2 can be given by:

$$A_2 = \frac{1}{2}(\boldsymbol{v}_2 - \boldsymbol{v_1})\Delta t$$

Because this is the area of the triangle, where the length of the base is Δt and the height of the triangle is $(\boldsymbol{v_2} - \boldsymbol{v_1})$,

Adding these two expressions and rearranging:

$$\boldsymbol{s} = \frac{1}{2}(\boldsymbol{v}_2 - \boldsymbol{v_1})\Delta t$$

And substituting:

$$\boldsymbol{v}_2 = \boldsymbol{a}\Delta t + \boldsymbol{v_1}$$

Gives this equation:

$$\boldsymbol{s} = \frac{1}{2}(\boldsymbol{a}\Delta t + v_1 + v_2\Delta t + v_1\Delta t)$$

Simplifying gives:

$$\boldsymbol{s} = \frac{1}{2}\boldsymbol{a}\Delta t^2 + \boldsymbol{v_1}\Delta t$$

This equation gives the theoretical displacement for an object undergoing a constant acceleration, $\boldsymbol{a}$, at any time t, where $\boldsymbol{s}$ is the displacement during the time interval, Δt, and $\boldsymbol{v}_1$ is the initial velocity.

If the object is released from rest, as in our experiment, $\boldsymbol{v}_1 = 0$ and the equation simplifies to:

$$\boldsymbol{s} = \frac{1}{2}\boldsymbol{a}\Delta t^2$$

Activity 4: Predict the time for a steel sphere to roll down an incline

Note: Read the following section carefully.

In this activity you will use the kinematics equation:

$$\boldsymbol{s} = \frac{1}{2}\boldsymbol{a}\Delta t^2$$

This will allow you to predict how long the sphere will take to roll down the inclined track.

First you must solve the previous equation for time:

$$t = \sqrt{\frac{2\boldsymbol{s}}{\boldsymbol{a}}}$$

If the object in our experiment was in freefall you would just need to substitute the distance it was falling for $\boldsymbol{s}$ and substitute the acceleration due to Earth's gravity for $\boldsymbol{a}$, which is

$$\mathbf{g} = 9.8 \text{ m/s}^2$$

In this experiment, however the object is **not** undergoing freefall, it is **rolling** down an incline.

The acceleration of an object **sliding**, without friction down an incline is given by:

$$\boldsymbol{a} = \mathbf{g}\text{SIN}\theta$$

Where θ is the angle between the horizontal plane (the surface of your table) and the inclined plane (the track), and **g** is the acceleration due to Earth's gravity.

When a **solid sphere** is **rolling down an incline** the acceleration is given by:

$$\boldsymbol{a} = 0.71\ \mathbf{g}\text{SIN}\theta$$

The SIN (trigonometric sine) of an angle can be found by measuring the angle with a protractor and using the SIN function on your calculator or simply by dividing the length of the side opposite the angle (the height from which the sphere starts) by the length of the hypotenuse of the right triangle (the length of the track). **Figure 9** shows the formula for deriving sines from triangles.

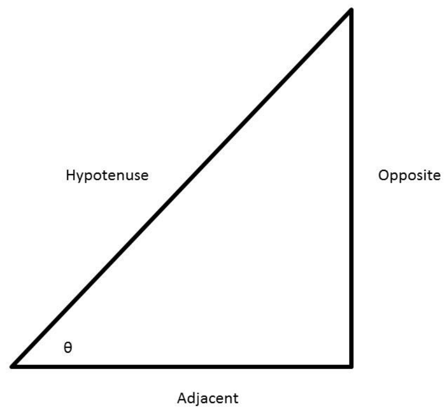


Figure 9

$$\sin\theta = \frac{opposite}{hypotenuse}$$

Activity 4: Procedure

1. Set up the angle bar as a track. Measure the length of the track and the angle of elevation between the track and the table.
2. Rearrange the kinematics equation to solve for time (second equation), and substitute the value 0.71 **g** SINθ for $\boldsymbol{a}$ (third equation). Use a distance of 80 cm for $\boldsymbol{s}$.

$$\boldsymbol{s} = \frac{1}{2}\boldsymbol{a}\Delta t^2$$

$$t = \sqrt{\frac{2\boldsymbol{s}}{\boldsymbol{a}}}$$

$$t = \sqrt{\frac{2\boldsymbol{s}}{(0.71\mathbf{g}\,\mathrm{SIN}\theta)}}$$

3. Release the steel sphere from the start point at the elevated end of the track and measure the time it takes for the sphere to roll from position $\boldsymbol{s}$ = 0 to a final position $\boldsymbol{s}$ = 80 cm.
4. Compare the measured value with the value predicted in Step 2. Calculate the percent difference between these two numbers.

5. *Repeat Activity 4 with the acrylic sphere. What effect does the mass of the sphere have on the acceleration of the object due to gravity?*

Activity 5: Demonstrate that a sphere rolling down the incline is moving under constant acceleration

1. Collect the piece of foam board. Use a ruler and a pencil to draw lines across the short dimension (width) of the board at 5 cm increments.
2. Collect rubber bands from the central materials kit. Wrap the rubber bands around the width of the foam board so that the rubber bands line up with the pencil marks you made at the 5 cm intervals. See Figure 10, left panel.
3. Use a book to prop up the foam board as an inclined plane at an angle from 5° to 10° from the horizontal.
4. Place the steel sphere at the top of the ramp and allow the sphere to roll down the foam board.

> **Note:** The sound as the steel sphere crosses the rubber bands will increase in frequency as the steel sphere rolls down the ramp, indicating that the sphere is accelerating. As the sphere continues to roll down the incline, it takes less time to travel the same distance.
>
> If the steel sphere is moving under a constant acceleration, then the displacement of the sphere from the initial position, if the sphere is released from rest, is given by:
>
> $$\boldsymbol{s} = \frac{1}{2}\boldsymbol{a}\Delta t^2$$
>
> The displacement at each time t should be proportional to t^2

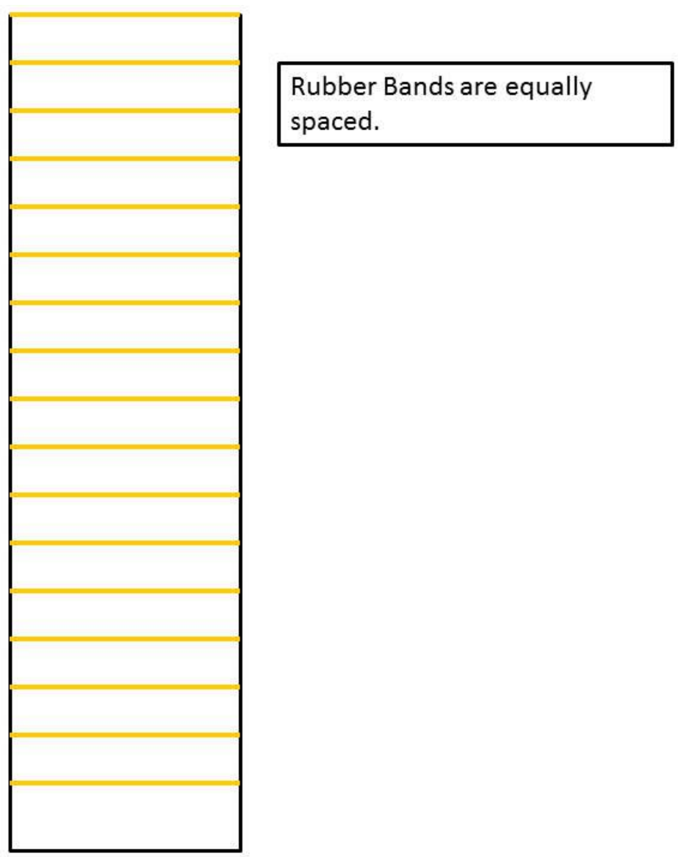
5. Remove the rubber bands from the foam board.
6. On the reverse side of the foam board, use a pencil to mark a line across the short dimension of the board 2 cm from the end. Label this line zero. Mark lines at the distances listed in Table 5. Each measurement should be made from the zero line. (see Figure 10).

Table 2

Displacement (cm)
1
4
9
16
25
36
49
64
81

7. Place rubber bands on the foam board, covering the pencil lines you just made.
8. Set the foam board up at the same angle as the previous trial.
9. Roll the steel sphere down the foam board.

Note: The sounds made as the sphere crosses the rubber bands on the foam board in the second trial should be at equal intervals. The sphere is traveling a greater distance each time it crosses a rubber band, but the time interval remains constant meaning the sphere is moving with a constant acceleration.



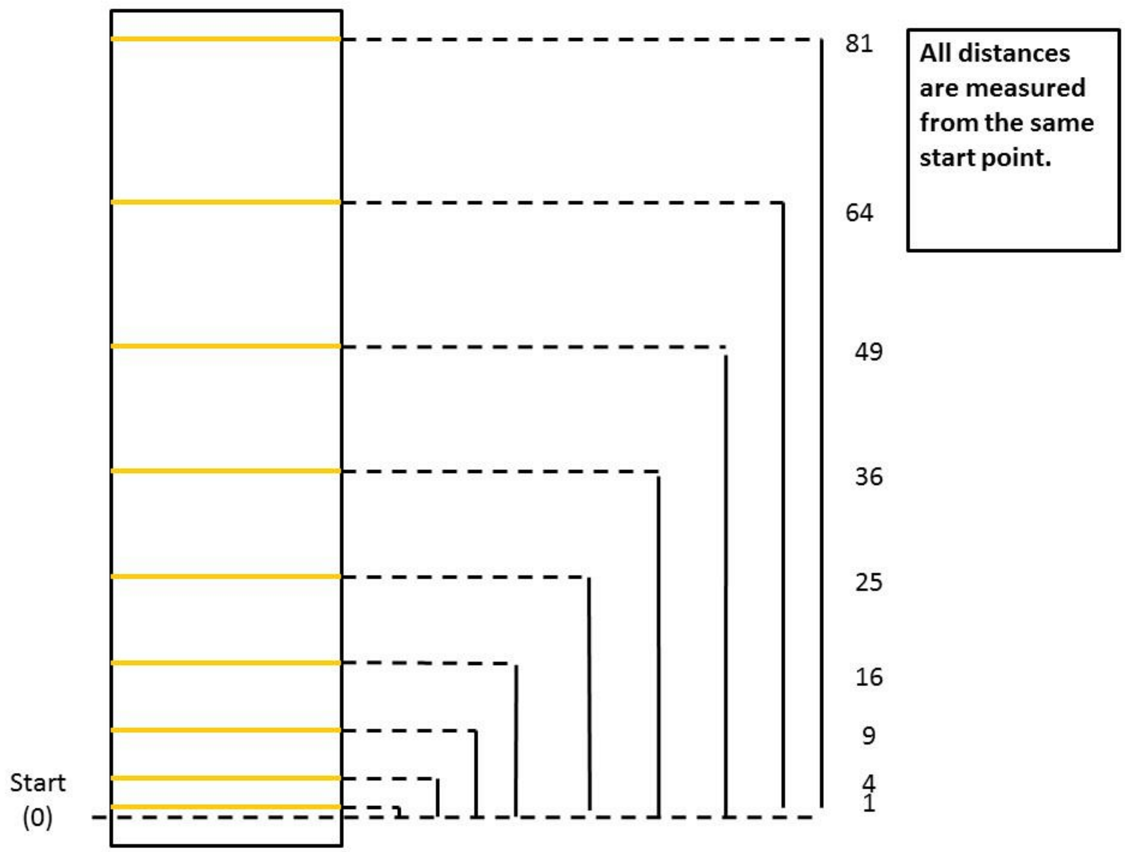


Figure 10

Note: For more information on the Trigonometry, Kinematics Equations, and Rotational Motion exercises, visit the Carolina Biological Supply website at the following links:

Basic Right Triangle Trigonometry

Derivation of the Kinematics Equations

The Ring and Disc Demonstration

Title: ______________________________

Label (*y*-axis): ______________________________

Label (*x*-axis): ______________________________