The Table Method

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WHAT'S COVERED

In this lesson, you will use tables to evaluate limits. Specifically, this lesson will cover:

1. Creating a Table of Values to Estimate a Limit

Let's consider again the function $f(x) = \frac{x^2-1}{x-1}$. This time though, we can use a table to estimate the value of $\lim_{x \to 1} \frac{x^2-1}{x-1}$.

First, we must create the table. To do this, we need to use a sequence of x-values that get closer to 1 from both the left and the right.

From the left, you could use $x = 0.9, 0.99, 0.999$.

From the right, you could use $x = 1.001, 1.01, 1.1$.

Now, place the information into one table, also leaving a place for $x = 1$ as shown below:
(Notice the "---" in the place for $x = 1$. This is because $f(x)$ is undefined there.)

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x) = \frac{x^2-1}{x-1}$				---			

Now, complete the table by substituting all x-values into the function.

x	0.9	0.99	0.999	1	1.001	1.01	1.1

$f(x)=\frac{x^2-1}{x-1}$	1.9	1.99	1.999	---	2.001	2.01	2.1

It appears that as *x* gets closer to 1 from either side, $f(x)$ gets closer to 2.

Thus, we can say $\lim_{x\to 1}\frac{x^2-1}{x-1}=2$, just as we said in the graphing example in the previous part of this challenge.

WATCH

The following video walks you through the process of evaluating the limit numerically as *x* approaches -2 of the rational function $f(x)=\frac{x^3+8}{x+2}$.

2. Using a Table of Values to Estimate a Limit

If a table is already created, we can use the information from the table to estimate the limit.

EXAMPLE Evaluate $\lim_{x\to 0}\sqrt{x}$. Here is a table of values that represent x-values around $x=0$.

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)=\sqrt{x}$	undef.	undef.	undef.	0	0.03162	0.1	0.31623

From the left side, there is no limit since $\sqrt{x}$ is undefined when $x<0$. From the right, it appears as if the limit is 0 since the values of $\sqrt{x}$ are trending toward 0.

Since the left-hand and right-hand sides do not match, we conclude that $\lim_{x\to 0}\sqrt{x}$ does not exist.

HINT

Be sure your calculator is set to *Radians* when creating a table for trigonometric functions like in the example below.

EXAMPLE Use a table of values to evaluate $\lim_{x\to 0}\frac{\sin 4x}{x}$. The table with the values of $f(x)$ is shown below:

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)=\frac{\sin 4x}{x}$	3.89418	3.99893	3.99999	---	3.99999	3.99893	3.89418

It appears as if $f(x)$ is getting closer to 4 from either side. Therefore, we conclude that $\lim_{x\to 0}\frac{\sin 4x}{x}=4$.

TRY IT

Consider the function $\frac{\sqrt{x}-2}{x-4}$. Answer the following questions to evaluate $\lim_{x \to 4} \frac{\sqrt{x}-2}{x-4}$.

Create a table of values for this function. +

x	3.9	3.99	3.999	0	4.001	4.01	4.1
$f(x)=\frac{\sqrt{x}-2}{x-4}$	0.25158	0.25016	0.25002	---	0.24998	0.24984	0.24846

What is the limit of the function as it approaches 4? +

$$\lim_{x \to 4} \frac{\sqrt{x}-2}{x-4} = 0.25$$

SUMMARY

In this lesson, you learned about another method to evaluate limits, by **creating a table of values to estimate a limit**. You also learned that it is very helpful to **use a table of values to estimate a limit**, since it shows patterns in how $f(x)$ changes as x approaches a number.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 1 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.