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What Is a Function?

by Sophia

WHAT'S COVERED

In this lesson, you will determine which relations are functions and which are not. Specifically, this lesson will cover:

1. What Is a Function?

In mathematics, a **function** is a relationship that is designed so each input is assigned to only one output. In mathematics, we say that the output is a function of its input(s).

Here are some examples of functions you have seen:

Situation	Formula	Inputs/Outputs
Area of a Rectangle	$A = LW$	Inputs: L = Length, W = Width Output: A = Area A is a function of L and W.
Perimeter of a Square	$P = 4S$	Input: S = Side Length Output: P = Perimeter P is a function of S.
Volume of a Right Circular Cylinder	$V = \pi r^2 h$	Inputs: r = radius, h = height Output: V = Volume V is a function of r and h.

To get another perspective of what a function is, think about your calculator. When you press the square root key, the calculator returns the square root of the number you input (if your input is 16, the output is 4). The calculator never returns more than one value; this is the essence of a function.

TERM TO KNOW

Function

A correspondence between a set of inputs (x) and a set of outputs (y) such that each input corresponds to at most one output.

2. Functions Defined by Sets of Ordered Pairs

A relationship can be thought of as a collection of ordered pairs (x, y), where x is the input and y is the output.

EXAMPLE Let x = the temperature reading in Celsius, and let y = the corresponding temperature reading in Fahrenheit. A few ordered pairs represented by the situation would be (0, 32), (100, 212), (37, 98.6), and (-40, -40).

This relationship is a function since there is no Celsius temperature (x) that corresponds to more than one Fahrenheit temperature (y). In other words, if a Celsius temperature is given, there is one definitive Fahrenheit temperature.

EXAMPLE A teacher gave 5 homework assignments in preparation for a 20-point quiz.

- Let x = the number of assignments completed.
- Let y = the number of points earned on the quiz.

Ordered pairs in the form (x, y) are recorded for several students. Here is a list of several results: {(4, 15), (3, 12), (5, 18), (4, 10), (3, 16), (2, 10), (5, 14), (4, 16)}

This relationship is not a function since there are several inputs that correspond to multiple outputs. For example, the input value $x = 4$ corresponds to outputs 15, 10, and 16.

In a real-life sense, this means that there is no way to definitely predict a student's score by knowing the number of assignments completed, since there are several results for a given input.

3. Functions Defined by Equations

3a. Identifying Which Equations Are Functions

In this course, we will mostly be sticking with equations with two variables. In general, we will use the variables x and y, where x is the input variable and y is the output variable. When the equation is determined to be a function, we say that y is a function of x.

Our first goal is to determine which equations are functions (in other words, which equations produce one y-value for each choice of x-value). Here are a few examples to help you contrast which relations are functions vs. which are *not* functions.

Equation	Function or Not a Function?
$y = 2x + 3$	This is a function since each input (x) corresponds to only one output (y). In other words, whatever is chosen for x will result in just one value of y. Some examples of ordered pairs are (0, 3), (1, 5), (2, 7), and (3, 9).
$x^2 + y^2 = 25$	This is not a function because there are points that have the same x-coordinate but different y-coordinates. For example, (0, 5) and (0, -5) both satisfy the equation.
$y = x^2 - 4x - 2$	This is a function since each input (x) corresponds to only one output (y). In other words, whatever is chosen for x will result in just one value of y. Some examples of ordered pairs are (1, -5), (2, -6), (3, -5), and (4, -2).

BIG IDEA

Generally speaking, any equation that can be written in "$y=$" form without using "$\pm$" is a function.

EXAMPLE The equation $2x - 3y = 6$ is a linear function since the slope-intercept form is $y = \frac{2}{3}x - 2$.

3b. Common Functions From Algebra

In your mathematical career so far, you have been exposed to many types of equations that were actually functions. Here is a list of some of the most notable (and there are more to come in this unit as well).

Type of Function	Equation
Constant Function	$y = b$ (b is a constant)
Linear Function	$y = mx + b$
Quadratic Function	$y = ax^2 + bx + c$
Cubic Function	$y = ax^3 + bx^2 + cx + d$
Polynomial Function (degree n)	$y = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$
Radical Function (nth Root)	$y = \sqrt[n]{x}$

SUMMARY

In this lesson, you learned **what a function is**, which is a relationship (an equation or formula) that is designed so that each input has at most one output. You learned that there are **functions defined by sets of ordered pairs**—a relationship which is a non-empty collection of ordered pairs (x, y), where x is

the input and *y* is the output—and there are **functions defined by equations**, where *x* is the input variable and *y* is the output variable, and we say that *y* is a function of *x*. You also learned how to **identify which equations are functions**, or in other words, which equations produce one y-value for each choice of x-value. Finally, you explored a list of **common functions from algebra**, noting that many equations used in the past are actually functions—which is where we will begin our exploration using a calculus perspective.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 0 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.

TERMS TO KNOW

Function

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