

Equations of Circles

by Sophia



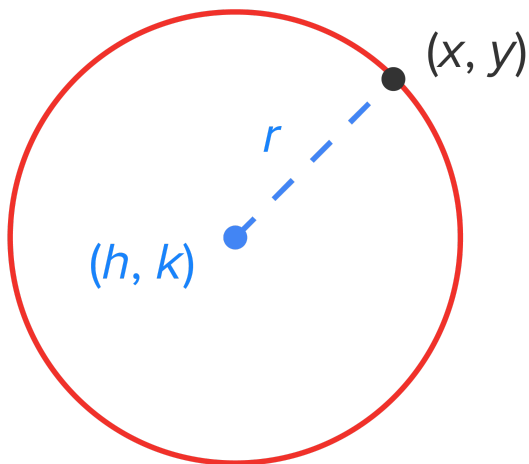
WHAT'S COVERED

In this lesson, you will learn how to write the equation of a circle. Specifically, this lesson will cover:

1. Parts of a Circle
2. Standard Form Equation of a Circle

1. Parts of a Circle

Consider the circle shown below.



In the image above:

- The center of the circle is labeled (h, k) .
- (x, y) represents any point on the circle.

- The radius of the circle is r , which is the distance from (h, k) to (x, y) .

So, how can we calculate this distance?

Using the distance formula from the previous lesson, we can set up a relationship. The distance between (h, k) and (x, y) is the radius, r , and can be found with the following formula:



FORMULA TO KNOW

Radius of a Circle

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

Where:

(h, k) is the center and (x, y) is a point on the circle.

2. Standard Form Equation of a Circle

If we take the radius formula from the above section and square it, we get the following equation:

$$(x-h)^2 + (y-k)^2 = r^2$$

This actually is the standard form of the equation of a circle. So, if a circle has center (h, k) and radius r , the equation for all points on the circle is $(x-h)^2 + (y-k)^2 = r^2$.



FORMULA TO KNOW

Standard Form Equation of a Circle

$$(x-h)^2 + (y-k)^2 = r^2$$

Where:

(h, k) is the center and r is the radius.

⇒ **EXAMPLE** Write the equation of a circle whose center is $(0, -2)$ and which has a radius of 3.

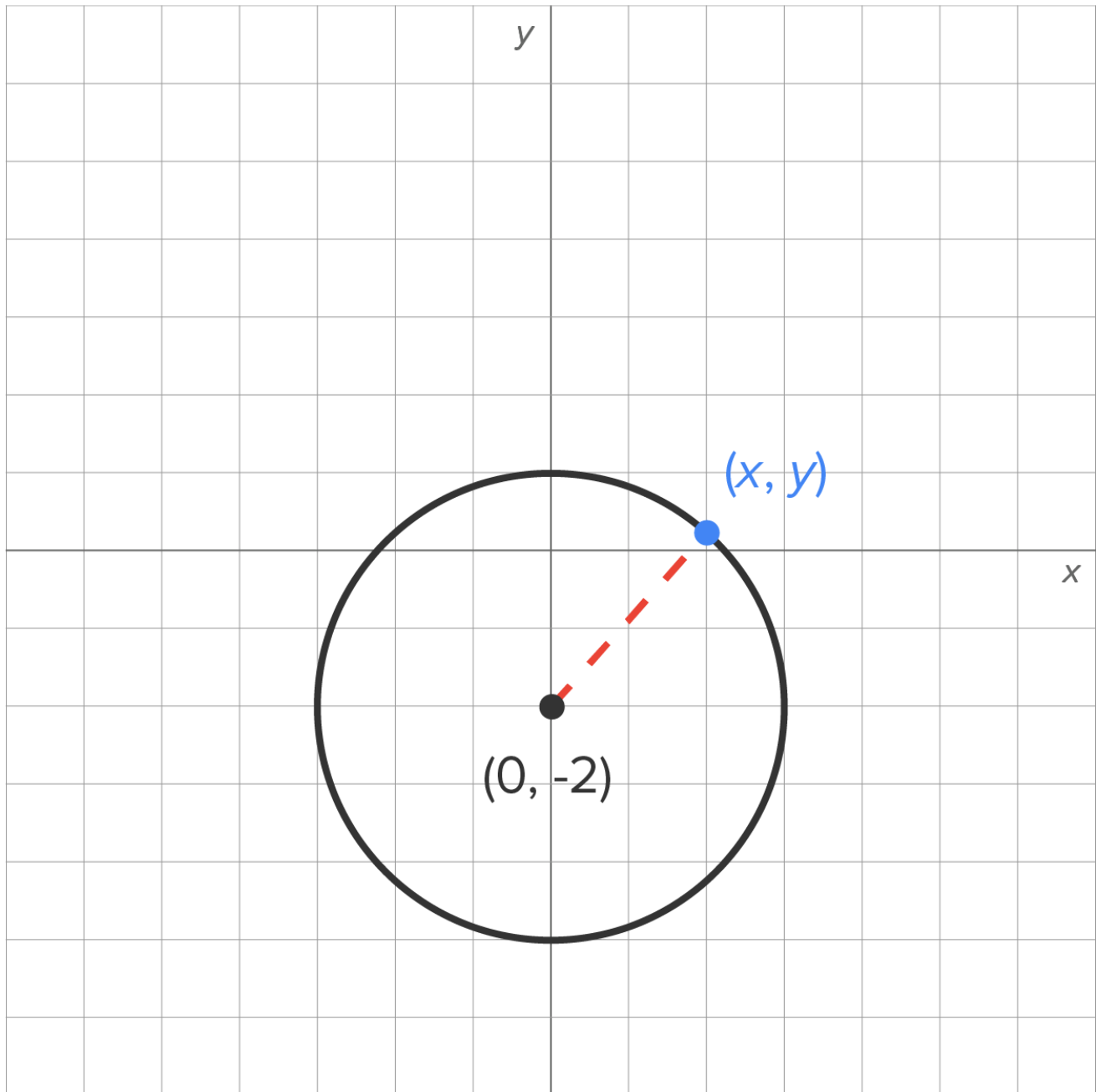
$$(x-h)^2 + (y-k)^2 = r^2 \quad \text{Standard Form Equation of a Circle}$$

$$(x-0)^2 + (y-(-2))^2 = 3^2 \quad \text{Substitute known values: } h=0, k=-2, \text{ and } r=3.$$

$$x^2 + (y+2)^2 = 9 \quad \text{Simplify the equation.}$$

The equation of the circle whose center is $(0, -2)$ and which has a radius of 3 is $x^2 + (y+2)^2 = 9$ and has the

following graph:



TRY IT

A circle has the equation $(x - 1)^2 + (y + 2)^2 = 16$.

What is the center?

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Matching the equation to the standard form, we see that $h = 1$ and $k = -2$, so the center of the circle is $(1, -2)$.

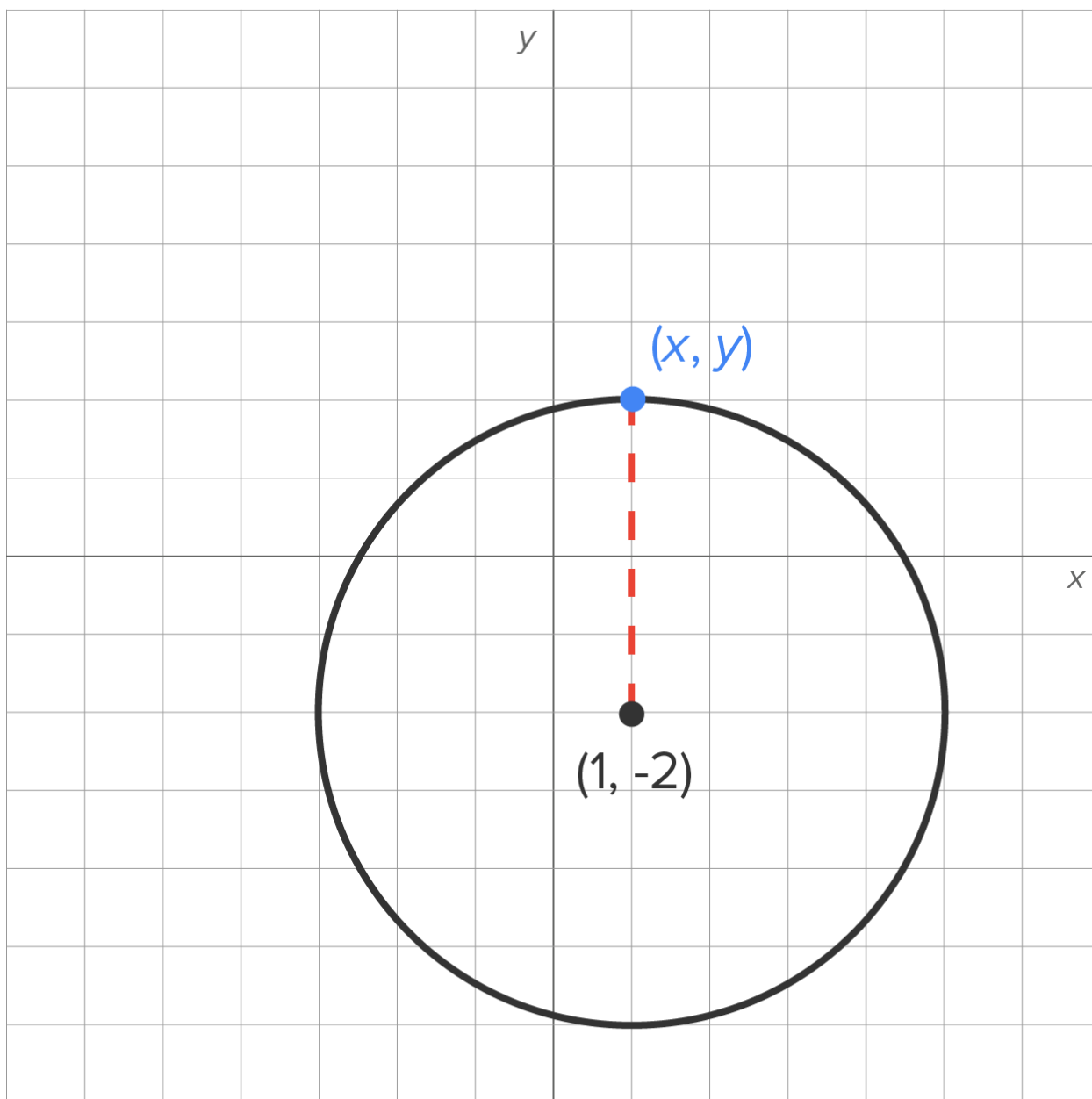
What is the radius?

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Also from the equation, we see that the right side of the equation, 16, corresponds with r^2 , which means the radius, r , is 4.

What does this graph look like?

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SUMMARY

In this lesson, you learned about the **parts of a circle**, including the center of the circle, the representation of any point on the circle, and the radius, which is the distance between the center and a given point on the circle. You learned how to calculate this distance using the formula for the radius of a

circle. You also learned how to write the equation of a circle using the **standard form equation of a circle**.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 0 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.



FORMULAS TO KNOW

Radius of a Circle

$$r = \sqrt{(x - h)^2 + (y - k)^2}$$

Where: (h, k) is the center and (x, y) is a point on the circle.

Standard Form Equation of a Circle

$$(x - h)^2 + (y - k)^2 = r^2$$

Where: (h, k) is the center and r is the radius.