Distance Between Two Points

by Sophia

WHAT'S COVERED

In this lesson, you will learn how to find the distance between two points on a number line and also on the xy-plane. Specifically, this lesson will cover:

1. The Distance Between Two Numbers on a Number Line

Suppose you want to calculate the **distance** between two locations on a number line, as shown below.

The distance between these two points is $b - a$, but that is assuming that b is larger than a.

In general, just so we don't have to worry about which number is larger, the distance between two numbers a and b is $dist(a, b) = |b - a|$. The absolute value is used to ensure that the result is not negative.

FORMULA TO KNOW

Distance on a Number Line

$dist(a, b) = |b - a|$

TRY IT

Find the distance between *a* and *b* in each example below.

What is the distance when $a = 13$ and $b = 5$? +

The distance between 13 and 5 is 8.

$dist(13,5) = |5 - 13| = |-8| = 8$

What is the distance when $a = -21$ and $b = 9$? +

The distance between -21 and 9 is 30.

$dist(-21,9) = |9 - (-21)| = |9 + 21| = |30| = 30$

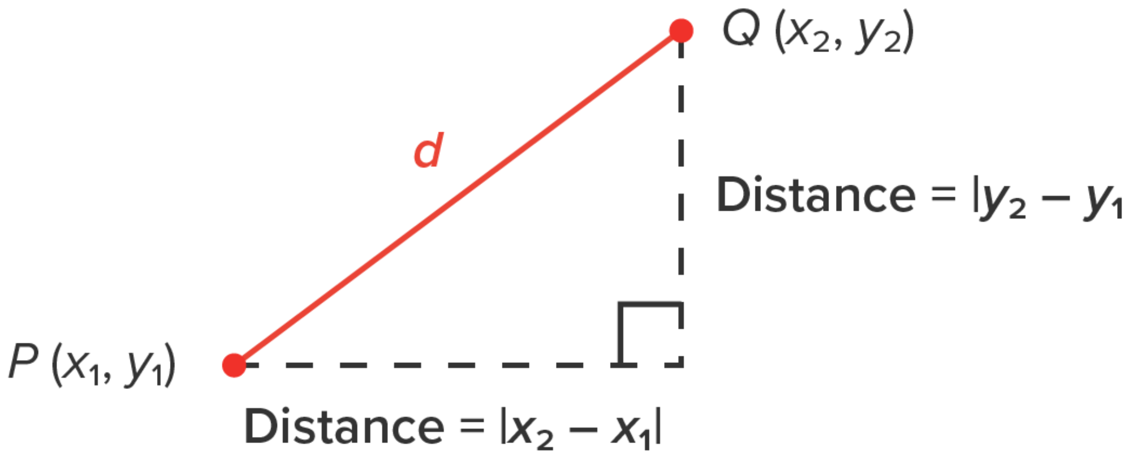
TERM TO KNOW

Distance

The length of a line segment between two points.

2. The Distance Between Two Points in the xy-Plane

The following image shows two points, *P* and *Q*, and the distance between them in the xy-plane, *d*. Let's find a formula for the distance between these two points.



In the image above:

- The vertical side is the distance between the y-coordinates, which is $|y_2 - y_1|$.
- The horizontal side is the distance between the x-coordinates, which is $|x_2 - x_1|$.
- The distance between the points is labeled as *d*.

Notice that we have three sides of a right triangle. This means that the Pythagorean theorem can be used to relate the sides to each other. Recall that the Pythagorean theorem states that $(leg_1)^2 + (leg_2)^2 = (hypotenuse)^2$, where a leg is defined as a side that makes up the right angle and the hypotenuse is the side opposite the right angle (the longest side).

Applying the Pythagorean theorem to our image, we have $|x_2 - x_1|^2 + |y_2 - y_1|^2 = d^2$.

HINT

Notice that the first two terms are squares of absolute values. Since squaring also guarantees a nonnegative result, there is no need to include the absolute value. Thus, the relationship actually can be rewritten as $(x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2$.

To write an expression for the distance, *d*, take the square root of both sides to get the following formula:

FORMULA TO KNOW

Distance in the xy-Plane

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

HINT

You might remember from algebra that taking the square root of both sides results in a positive solution and a negative solution. Since distance is always nonnegative, only the positive square root is considered.

EXAMPLE Calculate the exact distance between the points (4, 5) and (8, 1).

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	Distance Formula
$d=\sqrt{(8-4)^2+(1-5)^2}$	Substitute known quantities: $x_1=4, y_1=5, x_2=8, y_2=1$.
$d=\sqrt{4^2+(-4)^2}$	Evaluate subtraction inside parentheses.
$d=\sqrt{16+16}$	Square values.
$d=\sqrt{32}$	Add values under the square root.
$d=\sqrt{16\cdot 2}$	Rewrite the square root with any perfect square factors.
$d=\sqrt{16}\sqrt{2}$	Apply the product property of square roots.
$d=4\sqrt{2}$	Simplify the radical.

The distance between the points (4, 5) and (8, 1) is $4\sqrt{2}$, or about 5.66 units.

WATCH

The following video further illustrates the use of the distance formula.

SUMMARY

In this lesson, you learned how to calculate **the distance between two numbers on a number line** by calculating the absolute value of their difference. Next, you applied this idea, along with the Pythagorean theorem, to arrive at the distance formula to calculate **the distance between two points in the xy-plane**.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 0 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.

TERMS TO KNOW

Distance
The length of a line segment between two points.

FORMULAS TO KNOW

Distance in the xy-Plane

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance on a Number Line

$$dist(a, b) = |b - a|$$