Sophia

Functions Defined by Graphs and Tables of Values

by Sophia

WHAT'S COVERED

In this lesson, you will see how functions can be represented with graphs and tables (not only by equations). Specifically, this lesson will cover:

1. Functions Represented by Tables

Consider the table shown below, which shows the revenue (in thousands of dollars) earned by a particular company after each year for their first 8 years of business.

Year	1	2	3	4	5	6	7	8
Revenue	61	70.5	82	91	112.5	107.5	118.5	134.5

Thinking of the input-output relationship, it makes the most sense to label the year as input and the revenue as output. Since each input (1, 2, ..., 8) corresponds to one output, this table of values represents a function where the input is “Year” and the output is “Revenue.” We say that this defines revenue as a function of the year.

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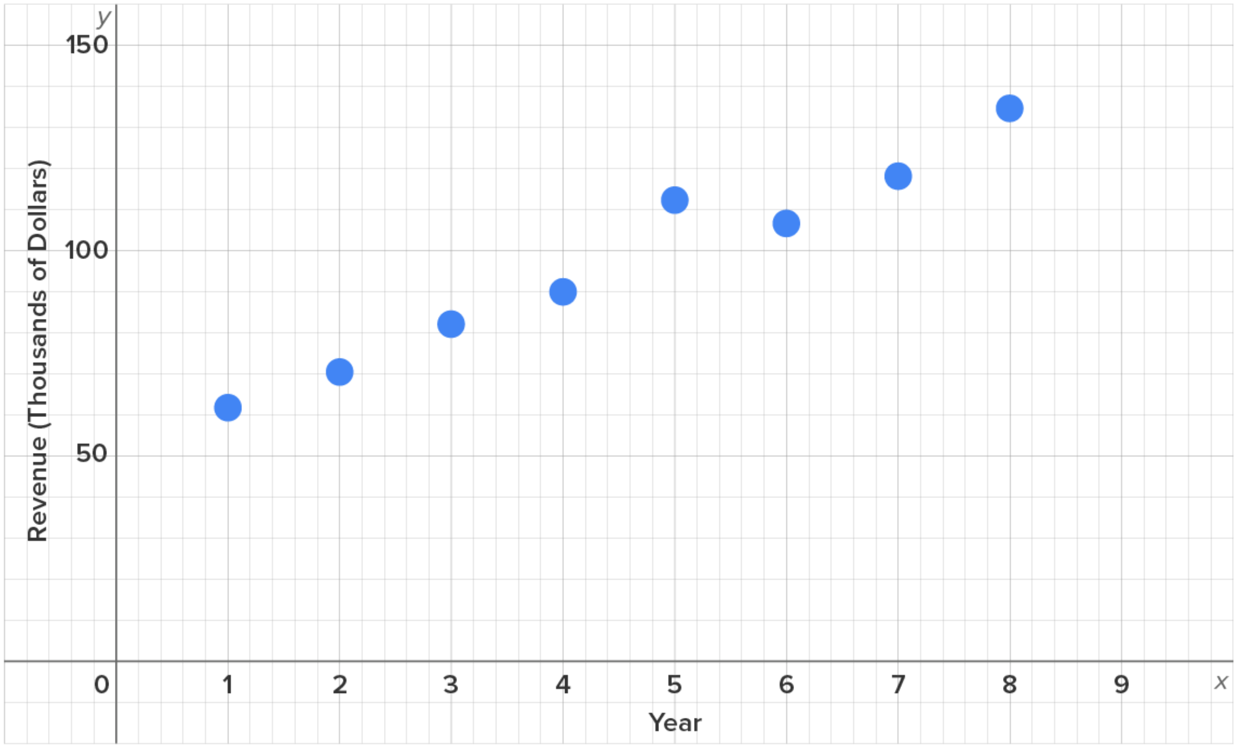
2. Functions Represented by Graphs

2a. From Tables of Values

Let’s take the table of values from the first section and graph the points:



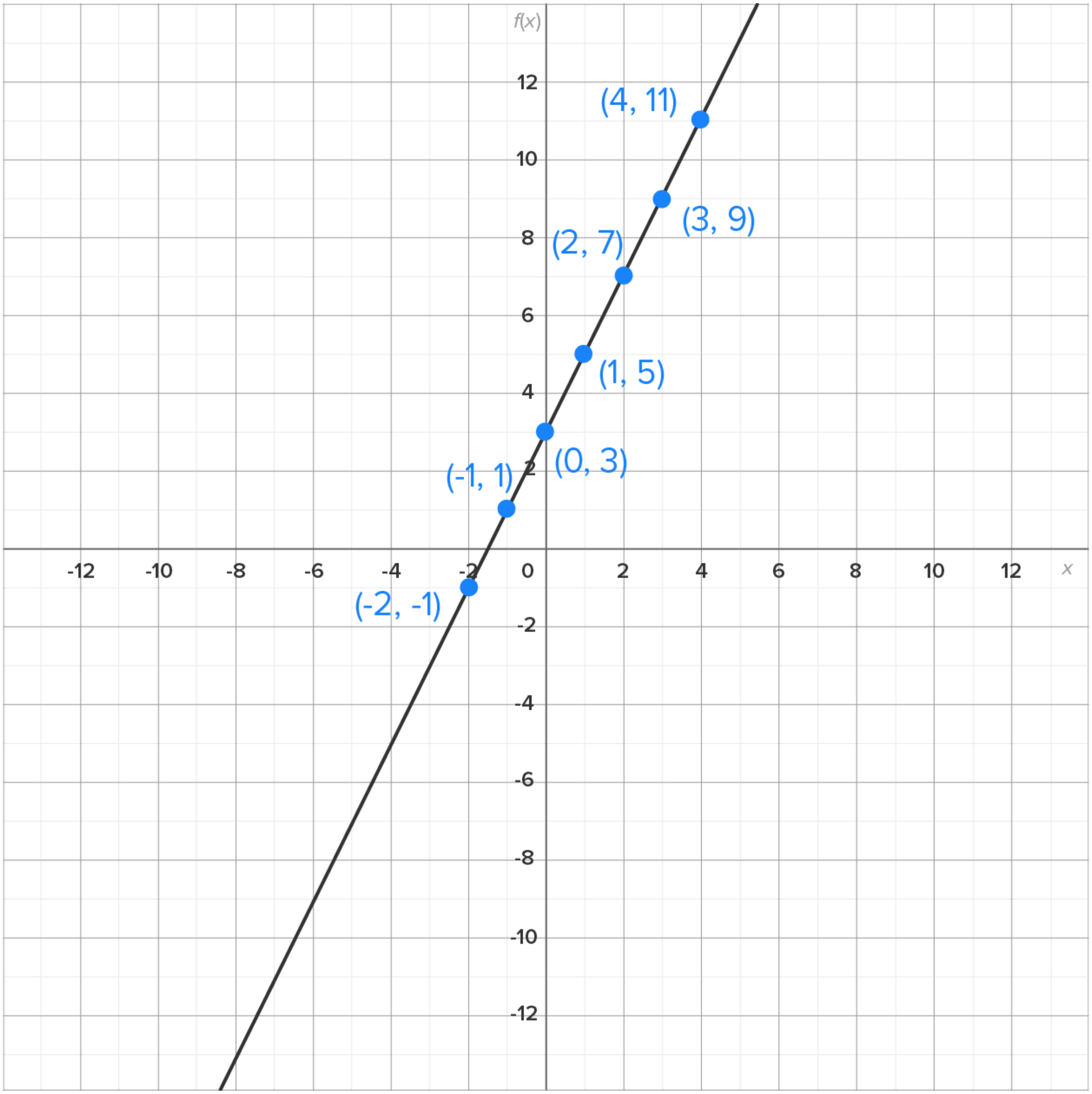
One could say that the graph looks *almost* linear or *roughly* linear, but not perfectly linear.

2b. From Equations

Consider the function $f(x) = 2x + 3$. The following table of values shows some input-output pairs for this function:

x	-2	-1	0	1	2	3	4
$f(x)$	-1	1	3	5	7	9	11

Now we plot the ordered pairs to form the graph (remember, we connect the dots since there are many other points on the graph; we just chose easy ones).



HINT

Note that this looks exactly like the graph of $y = 2x + 3$. In a previous lesson, you learned that $f(x)$ is simply a replacement for *y*.

Thus, graphing a function in the form "$f(x) = \ldots$" is identical to graphing an equation of the form "$y = \ldots$".

TRY IT

Consider the function $g(x) = x^2$.

Define input-output pairs for this function. +

x	-3	-2	-1	0	1	2	3
$g(x)$	9	4	1	0	1	4	9

Plot the input-output pairs to create a graph.

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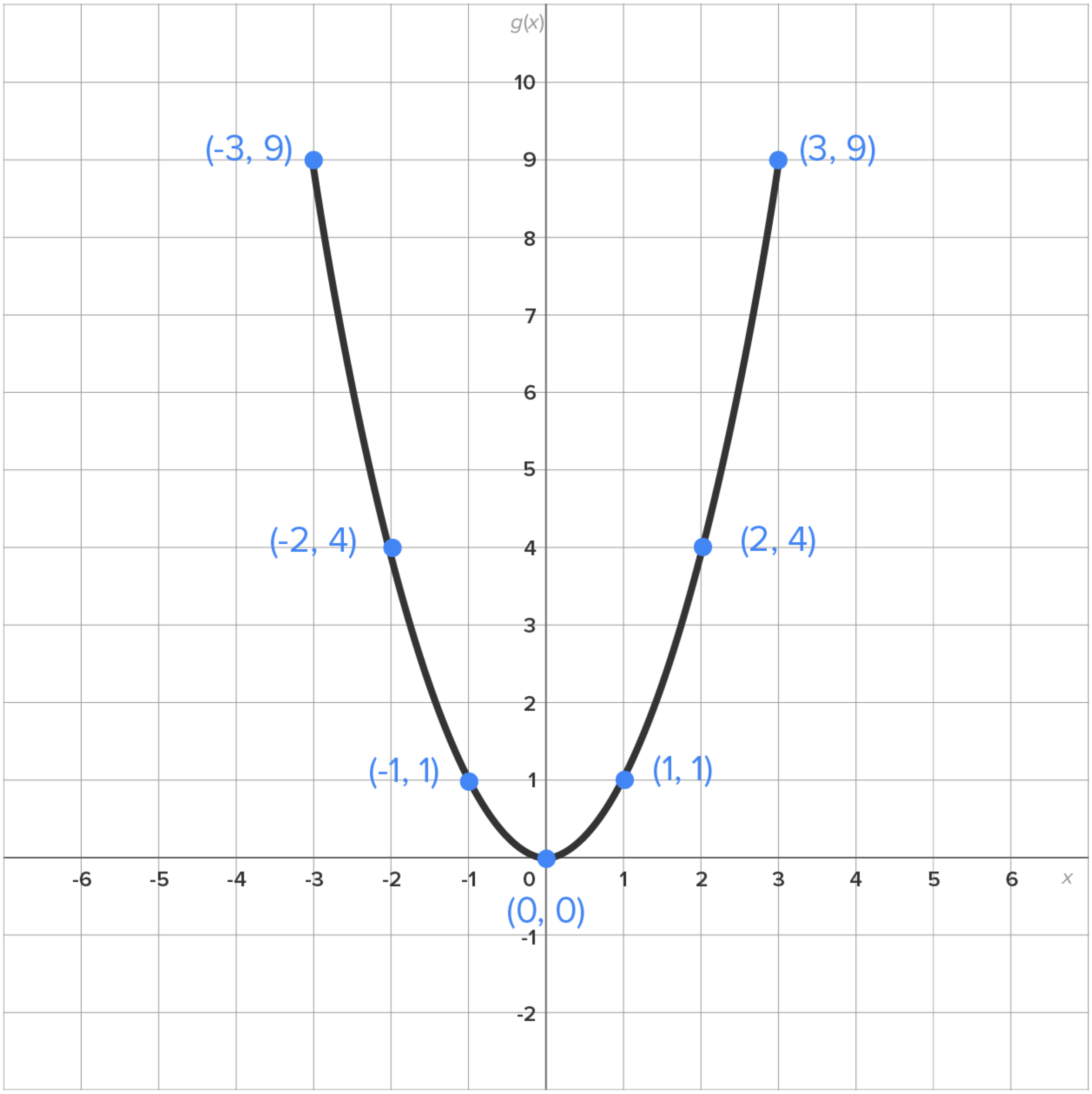


3. Using a Graph to Determine if It Represents a Function

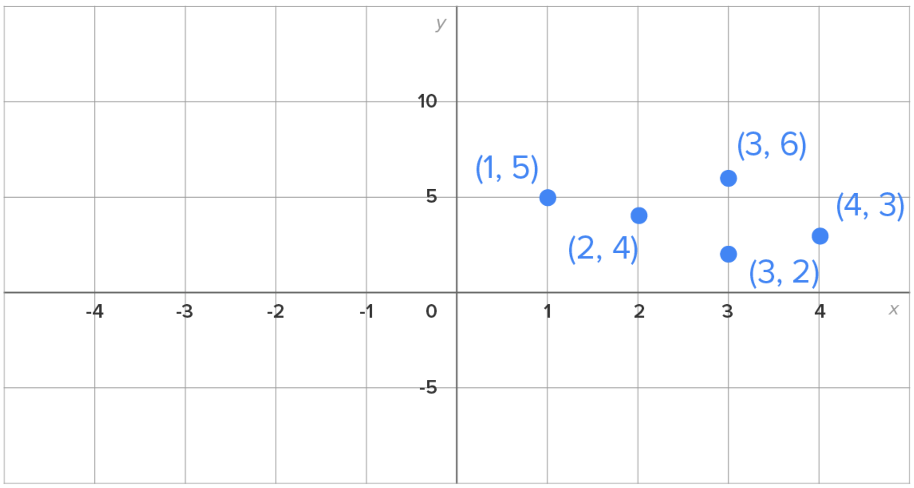
Something to think about: what would a graph look like if it *wasn't* a function?

Remember that if there is an input that corresponds to two or more outputs, then the relationship is not a function.

The equations $f(x)=2x+3$ and $g(x)=x^2$ are both functions, as shown in the last two examples.

Now consider this table of values, and the graph of the ordered pairs to its right:

x	*y*
1	5
2	4
3	6
3	2
4	3



From the table of values, we can see that this is not a function, since $x=3$ corresponds to two different outputs, $y=6$ and $y=2$.

On the graph, let's pay attention to the points (3, 2) and (3, 6). Notice that these points could be connected by a vertical line. This only happens when there are two points with the same x-coordinate. Thus, we have a simple test to determine whether or not a graph defines *y* as a function of *x*.

KEY CONCEPT

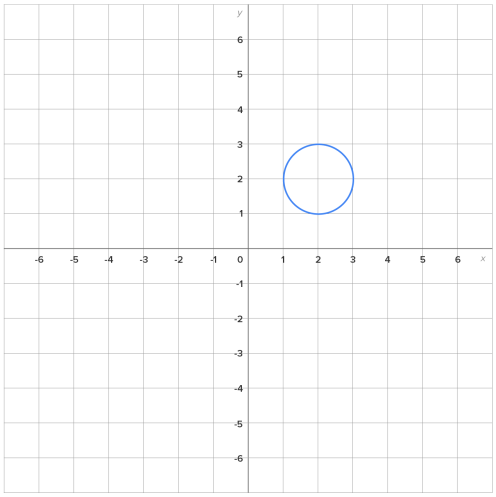
The Vertical Line Test

Given a graph, if a vertical line can be drawn and intersects more than once with the graph, then the graph does not define *y* as a function of *x*.

TRY IT

Consider the following graphs and determine which of these graphs defines *y* as a function of *x*.

Graph 1



Graph 1: Function or Not a Function?

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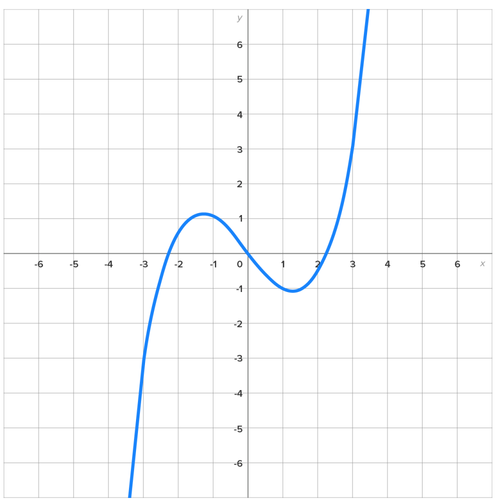
This is not a function. There exists a vertical line that passes through two points when drawn through the graph.

Graph 2



Graph 2: Function or Not a Function?

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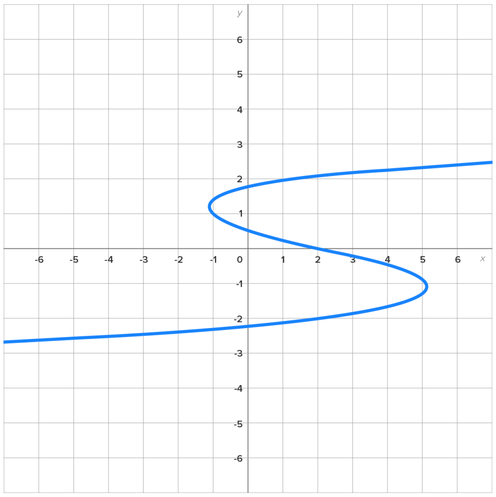
This is a function. Any vertical line will pass through one point on this graph.

Graph 3



Graph 3: Function or Not a Function?

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This is not a function. There are some places where a vertical line will pass through three points on the graph.

SUMMARY

In this lesson, you learned about the various ways that functions can be represented, including **functions represented by tables** and **functions represented by graphs**, including **from tables of values** and **from equations**. Understanding that if there is an input that corresponds to two or more outputs, then the relationship is not a function, you learned how to **use a graph to determine if it represents a function**, by utilizing the vertical line test: given a graph, if a vertical line can be drawn and intersects more than once with the graph, then the graph does not define y as a function of x.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 0 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.