

Equations of Lines

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WHAT'S COVERED

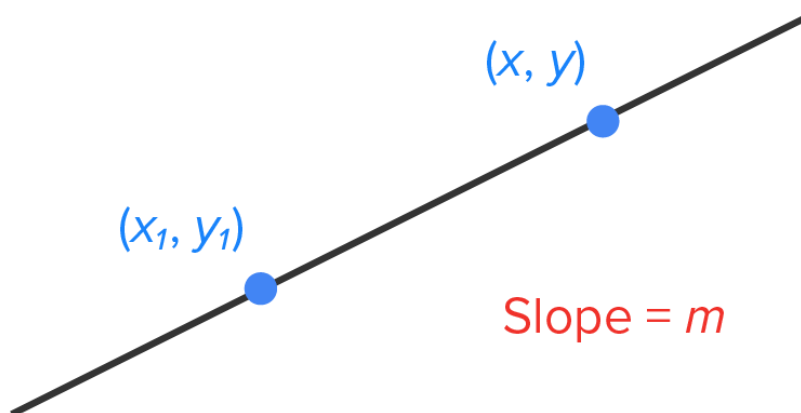
In this lesson, you will be able to write the equation of a line when given the appropriate information. Specifically, this lesson will cover:

1. Point-Slope Form
2. Slope-Intercept Form

1. Point-Slope Form

A line has the property that the slope between any two points on that line is always the same (we call it m).

Let (x, y) represent any point on a line and (x_1, y_1) a specific point on the line.



Using the slope formula, we know $m = \frac{y - y_1}{x - x_1}$. If we multiply both sides by $x - x_1$ this gives us the point-slope form of a linear equation:



FORMULA TO KNOW

Point-Slope Form

$$y - y_1 = m(x - x_1)$$



HINT

Typically, the point (x_1, y_1) and the slope m are substituted into this equation, then the final answer is solved for y .

⇒ **EXAMPLE** Use point-slope form to write the equation of the line that contains the point $(-1, 4)$ and has slope 3.

$$y - y_1 = m(x - x_1) \quad \text{Point-Slope Form}$$

$$y - 4 = 3(x - (-1)) \quad \text{Substitute the value for } m \text{ and the known point for } x_1 \text{ and } y_1.$$

$$y - 4 = 3(x + 1) \quad \text{Simplify the subtraction inside the parentheses.}$$

$$y - 4 = 3x + 3 \quad \text{Use the distributive property to simplify the right-hand side.}$$

$$y = 3x + 7 \quad \text{Add 4 to both sides.}$$

The equation of the line is $y = 3x + 7$.

2. Slope-Intercept Form

Another form of a line you may be familiar with is $y = mx + b$, which is the slope-intercept form of a line. The variable m is the slope, where the variable b is the y -coordinate of the y -intercept. Thus, another way to think about the line in the previous section is that it has slope 3 and y -intercept $(0, 7)$.



FORMULA TO KNOW

Slope-Intercept Form

$$y = mx + b$$

⇒ **EXAMPLE** Write the equation of the line that contains the points $(1, 5)$ and $(4, 7)$ in slope-intercept form.

First, label the variables: $x_1 = 1, y_1 = 5, x_2 = 4, y_2 = 7$.

Then, the slope of the line is $m = \frac{7 - 5}{4 - 1} = \frac{2}{3}$.

You can then use point-slope form, along with either given point and the slope you just found. In this example, the point (1, 5) is used.

$$y - y_1 = m(x - x_1) \quad \text{Point-Slope Form}$$

$$y - 5 = \frac{2}{3}(x - 1) \quad \text{Substitute the value for } m \text{ and the known point for } x_1 \text{ and } y_1.$$

$$y - 5 = \frac{2}{3}x - \frac{2}{3} \quad \text{Distribute the right-hand side.}$$

$$y = \frac{2}{3}x + \frac{13}{3} \quad \text{Add 5 to both sides.}$$

Thus, the equation of the line is $y = \frac{2}{3}x + \frac{13}{3}$. This tells us that the line has a slope of $\frac{2}{3}$ and a y-intercept $\left(0, \frac{13}{3}\right)$.



WATCH

The following video illustrates how to write the equation of a line.



SUMMARY

In this lesson, you learned that a line has the property that the slope between any two points on that line is always the same (m). You learned that given the slope and a point on the line (or two points contained on the line), you can use the **point-slope form** to write its equation. You also learned how to write the equation of a line using the **slope-intercept form**.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 0 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.



FORMULAS TO KNOW

Point-Slope Form

$$y - y_1 = m(x - x_1)$$

Slope-Intercept Form

$$y = mx + b$$