

Using Properties of Limits

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WHAT'S COVERED

In this lesson, you will utilize limit properties to evaluate more complex limits. Specifically, this lesson will cover:

1. Limit Properties
2. Evaluating Limits Using Limit Properties

1. Limit Properties

Suppose we know that $\lim_{x \rightarrow a} f(x) = L$ and $\lim_{x \rightarrow a} g(x) = M$. Then, we can establish the following properties of limits:

Property of Limits	Formula
Limit of a Constant	$\lim_{x \rightarrow a} k = k$
Limit of a Sum or Difference	$\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x) = L \pm M$
Limit of a Product	$\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x) = LM$
Limit of a Quotient	$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} = \frac{L}{M}$ (as long as $M \neq 0$)
Constant Multiple	$\lim_{x \rightarrow a} k \cdot f(x) = k \cdot \lim_{x \rightarrow a} f(x) = kL$
Limit of a Power	$\lim_{x \rightarrow a} [f(x)]^n = \left[\lim_{x \rightarrow a} f(x) \right]^n = L^n$
Limit of an nth Root	$\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)} = \sqrt[n]{L}$ (If n is even, then $L > 0$.)

2. Evaluating Limits Using Limit Properties

Here are a few examples of using the limit properties to evaluate limits.

⇒ **EXAMPLE** Given $\lim_{x \rightarrow 3} f(x) = 20$ and $\lim_{x \rightarrow 3} g(x) = 4$, evaluate $\lim_{x \rightarrow 3} [2f(x) + 3]$, $\lim_{x \rightarrow 3} \frac{f(x)}{1 + g(x)}$, and $\lim_{x \rightarrow 3} \sqrt{f(x) - g(x)}$.

$$\begin{aligned} & \lim_{x \rightarrow 3} [2f(x) + 3] && \text{Start with the original limit.} \\ &= \lim_{x \rightarrow 3} 2f(x) + \lim_{x \rightarrow 3} 3 && \text{Apply the sum/difference property.} \\ &= 2 \cdot \lim_{x \rightarrow 3} f(x) + \lim_{x \rightarrow 3} 3 && \text{Apply the constant multiple property.} \\ &= 2(20) + 3 && \text{Apply the limit of a constant property: } \lim_{x \rightarrow 3} f(x) = 20 \\ &= 43 && \text{Simplify the expression.} \end{aligned}$$

$$\begin{aligned} & \lim_{x \rightarrow 3} \frac{f(x)}{1 + g(x)} && \text{Start with the original limit.} \\ &= \frac{\lim_{x \rightarrow 3} f(x)}{\lim_{x \rightarrow 3} [1 + g(x)]} && \text{Apply the quotient property.} \\ &= \frac{\lim_{x \rightarrow 3} f(x)}{\lim_{x \rightarrow 3} 1 + \lim_{x \rightarrow 3} g(x)} && \text{Apply the sum/difference property.} \\ &= \frac{20}{1 + 4} && \text{Apply the limit of a constant property: } \lim_{x \rightarrow 3} f(x) = 20, \lim_{x \rightarrow 3} g(x) = 4 \\ &= 4 && \text{Simplify the expression.} \end{aligned}$$

$$\begin{aligned} & \lim_{x \rightarrow 3} \sqrt{f(x) - g(x)} && \text{Start with the original limit.} \\ &= \sqrt{\lim_{x \rightarrow 3} [f(x) - g(x)]} && \text{Apply the nth root property.} \\ &= \sqrt{\lim_{x \rightarrow 3} f(x) - \lim_{x \rightarrow 3} g(x)} && \text{Apply the sum/difference property.} \\ &= \sqrt{20 - 4} && \lim_{x \rightarrow 3} f(x) = 20, \lim_{x \rightarrow 3} g(x) = 4 \\ &= \sqrt{16} && \text{Simplify the expression.} \\ &= 4 && \text{Simplify the expression.} \end{aligned}$$



Suppose $\lim_{x \rightarrow a} f(x) = -4$ and $\lim_{x \rightarrow a} g(x) = 12$ and you want to find $\lim_{x \rightarrow a} \frac{[f(x)]^2}{g(x)}$.

Evaluate this limit.

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$$\lim_{x \rightarrow a} \frac{[f(x)]^2}{g(x)} = \frac{4}{3}$$



SUMMARY

In this lesson, you learned that **limit properties** can be helpful in evaluating limits where other related limits are known. You also explored several examples of **evaluating limits using limit properties**. Future topics will illustrate the need to use these properties.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 1 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.