

Extreme Value Theorem - Endpoint Extremes

by Sophia



WHAT'S COVERED

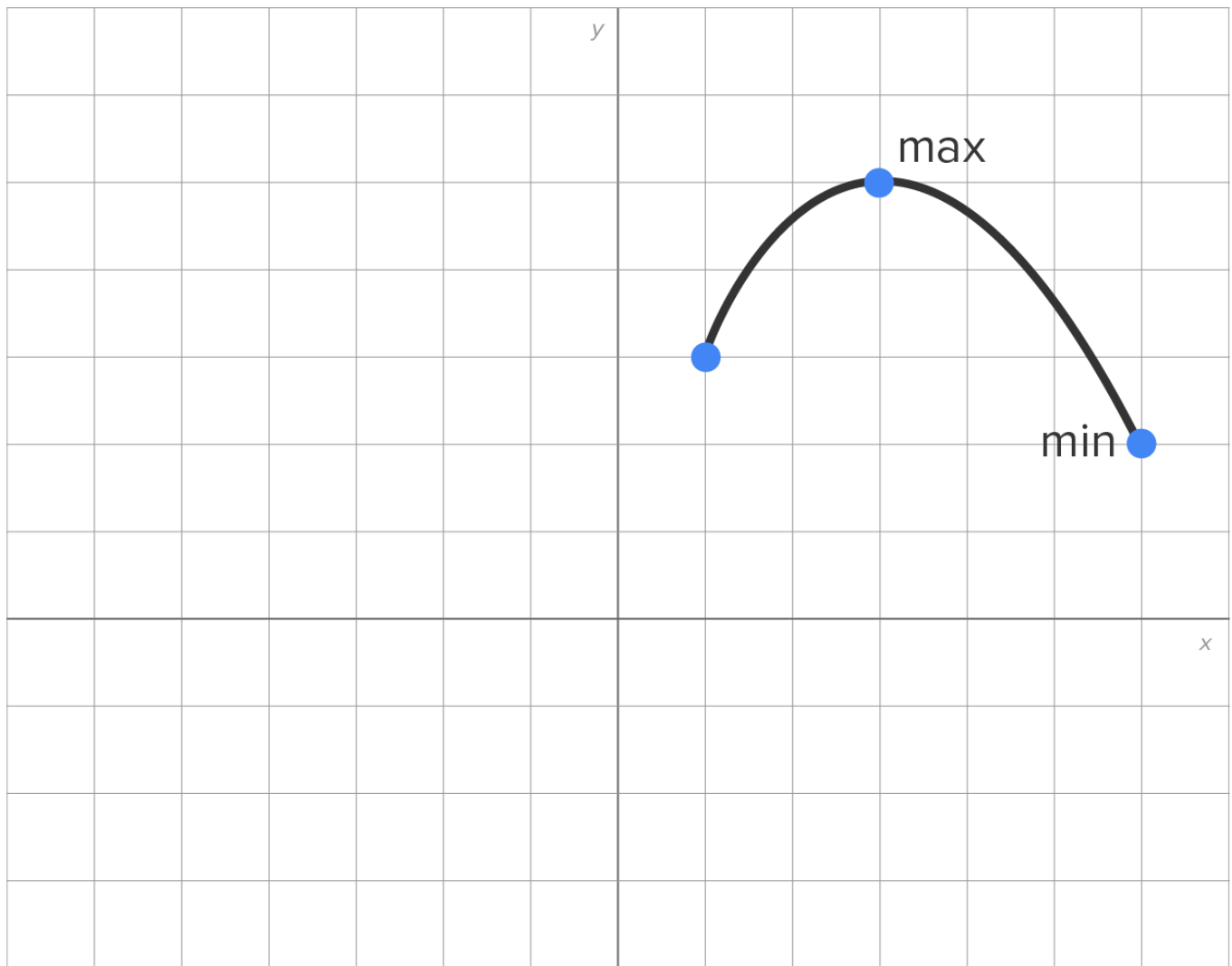
In this lesson, you will use critical numbers and endpoint analysis to determine the maximum and minimum values of a continuous function on some closed interval. Specifically, this lesson will cover:

1. [The Extreme Value Theorem](#)
2. [Finding Extreme Values of a Continuous Function on a Closed Interval](#)

1. The Extreme Value Theorem

If a function $f(x)$ is continuous on a closed interval $[a, b]$, then $f(x)$ is guaranteed to have global maximum and global minimum values on the interval $[a, b]$. This is known as the **extreme value theorem**.

Here is an illustration of the extreme value theorem:



f continuous closed interval

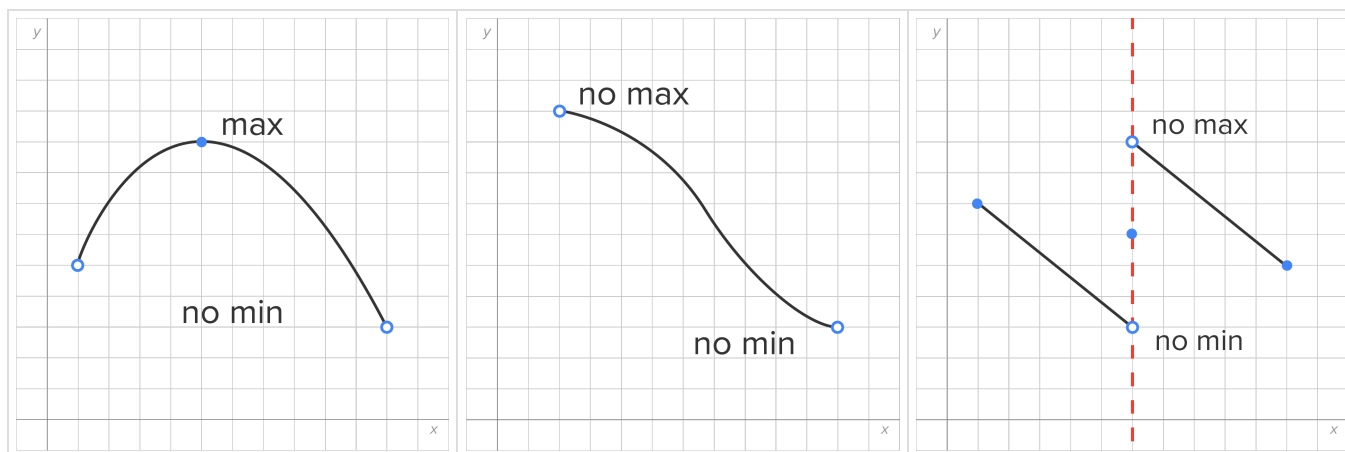
- The function is continuous on the interval $[a, b]$.
- The maximum point occurs inside the interval.
- The minimum occurs at an endpoint.

The following are examples of situations in which one of the criteria is violated.

f Continuous Open Interval

f Continuous Open Interval

f Not Continuous Closed Interval



TERM TO KNOW

Extreme Value Theorem

If $f(x)$ is a continuous function on some closed interval $[a, b]$, then $f(x)$ has global maximum and global minimum values on the interval $[a, b]$.

2. Finding Extreme Values of a Continuous Function on a Closed Interval

As a result of the theorem, here is what we need to do in order to find the global minimum and maximum values of $f(x)$ on a closed interval $[a, b]$.

1. Find all critical numbers of $f(x)$ that are in the interval $[a, b]$.
2. Evaluate $f(x)$ at each endpoint and each critical number. The largest value of f is the global maximum and the smallest value of f is the global minimum.

⇒ **EXAMPLE** Find the global maximum and minimum points of the function $f(x) = x^3 - 6x^2 + 5$ on the interval $[-1, 3]$.

First, find the critical numbers.

$$f(x) = x^3 - 6x^2 + 5 \quad \text{Start with the original function.}$$

$$f'(x) = 3x^2 - 12x \quad \text{Take the derivative.}$$

$$3x^2 - 12x = 0 \quad \text{Since } f'(x) \text{ is a polynomial, it is never undefined. Set } f'(x) = 0 \text{ and solve for } x.$$

$$3x(x - 4) = 0$$

$$x = 0, x = 4$$

Therefore, the critical numbers are $x = 0$ and $x = 4$.

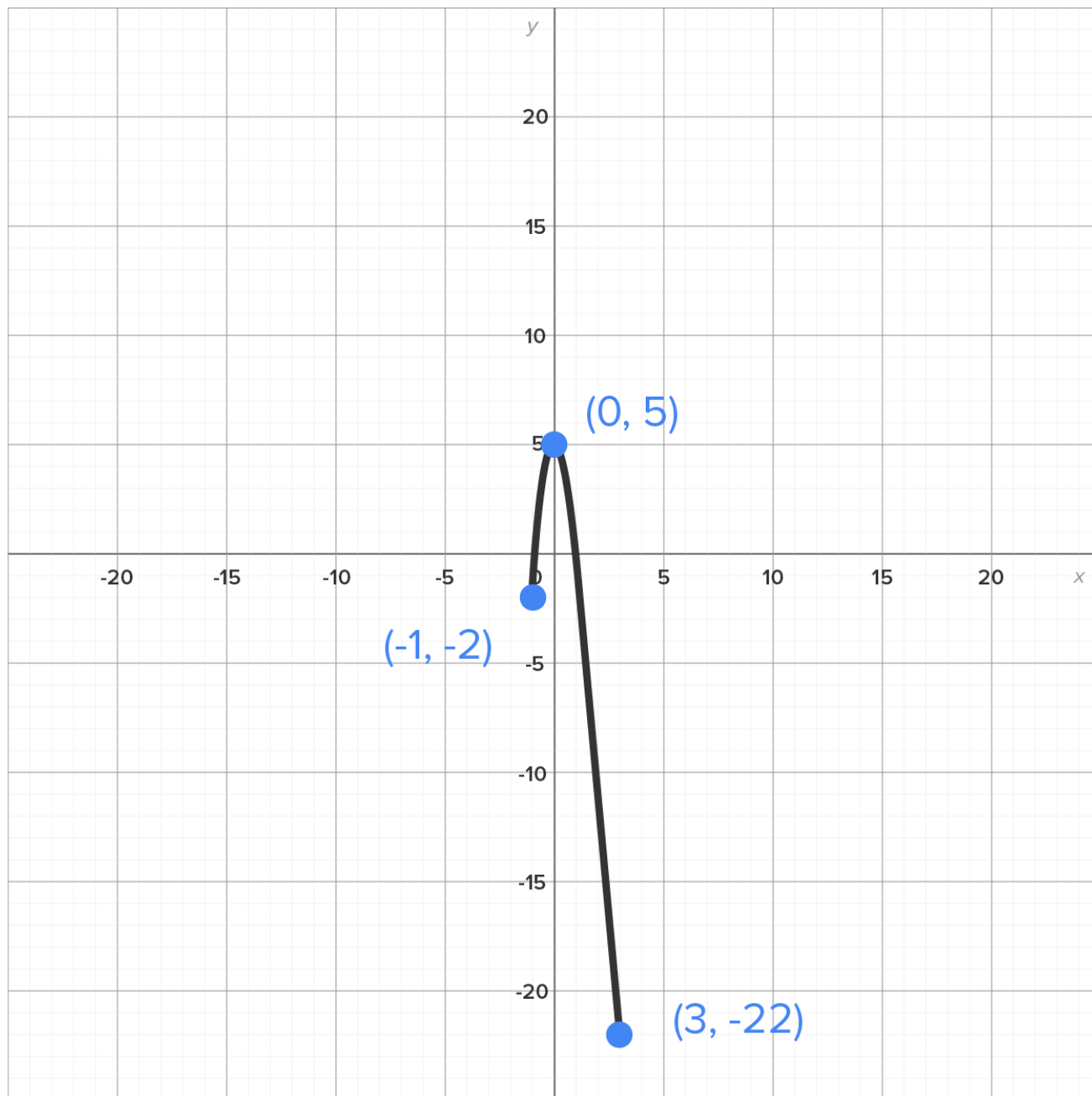
However, since only the closed interval $[-1, 3]$ is considered, the critical value $x = 4$ is not used.

Now, evaluate $f(x)$ at the endpoints, $x = -1$ and $x = 3$, and the remaining critical number, $x = 0$.

x	$f(x)$	Result
-1	$(-1)^3 - 6(-1)^2 + 5 = -2$	Neither a Global Maximum or Global Minimum
0	$(0)^3 - 6(0)^2 + 5 = 5$	Global Maximum
3	$3^3 - 6(3)^2 + 5 = -22$	Global Minimum

In conclusion, the global maximum occurs at the point $(0, 5)$ and the global minimum occurs at the point $(3, -22)$. In other words, the global maximum value is 5 and occurs when $x = 0$; and the global minimum value is -22 and occurs when $x = 3$.

The graph of the function on $[-1, 3]$ is shown below, which confirms the results.



WATCH

In this video, we'll find the global minimum and maximum values of $f(x) = 10\sqrt{x} - x$ on the interval $[16, 64]$.



SUMMARY

In this lesson, you learned that when $f(x)$ is continuous on a closed interval, **the extreme value theorem** guarantees a global minimum value and a global maximum value at some location within the closed interval. Then, you applied this theorem to **find extreme values of a continuous function on a closed interval**, by first finding all critical numbers of $f(x)$ that are in the interval $[a, b]$, then evaluating $f(x)$ at

each endpoint and each critical number. This concept is going to be very useful once we use derivatives to solve optimization problems.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 3 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.



TERMS TO KNOW

Extreme Value Theorem

If $f(x)$ is a continuous function on some closed interval $[a, b]$, then $f(x)$ has global maximum and global minimum values on the interval $[a, b]$.