GENERAL TOPOLOGY
EXERCISES FOR SESSION 3 (TUE 4.3)

Exercise 1. *Let $A \subset X$ and suppose that there is a sequence of points $(x_n) \subset A$ converging to some $a \in X$. Show that $a \in \bar{A}$.*

Exercise 2. *Suppose that $\mathbb{R}$ is equipped with the cofinite topology (A is open if $A = \emptyset$ or A^c is finite). Find all limit points of the sequence $1, 2, 3, \ldots$.*

Exercise 3. *Let X be Hausdorff and $A \subset X$. Prove that A is Hausdorff in the relative topology.*

Exercise 4. *Let $\mathbb{R}$ be equipped with the topology given by the basis $\{(-R, R) : R > 0\}$. Show that a sequence has a limit if and only if it is bounded (i.e. $|x_n| \leq M$ for some $M > 0$ and all n).*

Exercise 5. *Show that in a T_1-space X, a point x is an accumulation point of a set A if and only if every neighbourhood of x contains infinitely many points in A.*

Exercise 6. *Let $A \subset X$ and let $(x_n) \subset A$ be a converging sequence in the relative topology on A. Show that (x_n) also converges in the topology of X.*

Exercise 7. *Prove the second part of Theorem 2.4 in the lecture notes: If X is first-countable, $A \subset X$ and $a \in \bar{A}$, then there is a sequence in A converging to a.*

Exercise 8. *Show that if $A \subset X$ is closed, then the reverse of the statement in Exercise 6 is true: If $(x_n) \subset A$ converges in X, then it converges in the subspace A with the relative topology.*

Is there an example of a space X and a subspace A which is not closed but this still holds?