

# GENERAL TOPOLOGY

## HOMEWORK FOR WEEK 5 AND 6

DEADLINE: SAT 5.4, 23:59.

Some hints are on the next page.

**Exercise 1.** *Let  $X$  be separable and  $f : X \rightarrow Y$  be a continuous surjection. Prove that  $Y$  is separable.*

**Exercise 2.** *Prove that a compact metrizable space is second-countable.*

**Exercise 3.** *Let  $X$  have two topologies  $\mathcal{T}_1$  and  $\mathcal{T}_2$  such that  $\mathcal{T}_1 \subset \mathcal{T}_2$ . Suppose that  $\mathcal{T}_1$  is Hausdorff and  $\mathcal{T}_2$  is compact. Prove that  $\mathcal{T}_1 = \mathcal{T}_2$ .*

**Exercise 4.** *Let  $X$  be locally compact and second-countable. Prove that there exists a sequence of open sets  $U_1, U_2, \dots$  such that:*

- $\overline{U_j}$  is compact for all  $j$ .
- $\overline{U_j} \subset U_{j+1}$  for all  $j$ .
- $X = \bigcup_j U_j$ .

**Exercise 5.** *Answer the course feedback form on MyCourses which opens on 28.3. The feedback is anonymous, but we will see the total list of people who have answered and give 1 point each for doing so.*

**Hint for Exercise 1:** This one is straightforward. Good luck!

**Hint for Exercise 2:** You may need to look at smaller and smaller balls.

**Hint for Exercise 3:** You can think about this via closed sets. Alternatively, there is a fancy way using an exercise from Session 6 (compactness).

**Hint for Exercise 4:** Prove that if  $\mathcal{B}$  is your countable basis, then the subcollection  $\mathcal{B}' = \{B \in \mathcal{B} : \bar{B} \text{ is compact}\}$  is a countable cover for the whole space. Find the sets  $U_j$  using the elements in  $\mathcal{B}'$ . Be careful with details, this one is not easy.

**Hint for Exercise 5:** Click the black "Course feedback" button on the top right in MyCourses.