# GENERAL TOPOLOGY
# HOMEWORK FOR WEEK 5 AND 6

DEADLINE: FRI 4.4, 23:59.

Some hints are on the next page.

**Exercise 1.** *Let $X$ be separable and $f : X \to Y$ be a continuous surjection. Prove that $Y$ is separable.*

**Exercise 2.** *Prove that a compact metrizable space is second-countable.*

**Exercise 3.** *Let $X$ have two topologies $\mathcal{T}_1$ and $\mathcal{T}_2$ such that $\mathcal{T}_1 \subset \mathcal{T}_2$. Suppose that $\mathcal{T}_1$ is Hausdorff and $\mathcal{T}_2$ is compact. Prove that $\mathcal{T}_1 = \mathcal{T}_2$.*

**Exercise 4.** *Let $X$ be locally compact and second-countable. Prove that there exists a sequence of open sets $U_1, U_2, \ldots$ such that:*

- $\overline{U_j}$ *is compact for all* $j$.
- $\overline{U_j} \subset U_{j+1}$ *for all* $j$.
- $X = \bigcup_j U_j$.

**Exercise 5.** *Answer the course feedback form on MyCourses which opens on 28.3. The feedback is anonymous, but we will see the total list of people who have answered and give 1 point each for doing so.*

**Hint for Exercise 1:** This one is straightforward. Good luck!

**Hint for Exercise 2:** You may need to look at smaller and smaller balls.

**Hint for Exercise 3:** You can think about this via closed sets. Alternatively, there is a fancy way using an exercise from Session 6 (compactness).

**Hint for Exercise 4:** Prove that if $\mathcal{B}$ is your countable basis, then the subcollection $\mathcal{B}' = \{B \in \mathcal{B} : \bar{B} \text{ is compact}\}$ is still a basis. Find the sets $U_j$ using the elements in $\mathcal{B}'$. Be careful with details, this one is not easy.

**Hint for Exercise 5:** Click the black "Course feedback" button on the top right in MyCourses.