

GENERAL TOPOLOGY
EXERCISES FOR SESSION 7 (TUE 19.3)

Exercise 1. *Prove that if $A \subset X$ is connected, then \bar{A} is connected.*

Exercise 2. *Let A be connected. Is $\text{int}(A)$ necessarily connected?*

Exercise 3. *Prove the **Border Crossing Theorem**: Let $E \subset X$ be connected and $A \subset X$. Prove that if E intersects A and A^c , then it intersects ∂A .*

Exercise 4. *Suppose that in the space X for every pair of points $x, y \in X$ there exists a connected set $E(x, y)$ which contains both x and y . Prove that X is connected.*

Exercise 5. *Prove that a space X is locally connected if and only if the components of every open subset $U \subset X$ are also open sets.*

Exercise 6. *Prove that none of the sets $[0, 1]$, $[0, 1)$ and $(0, 1)$ are homeomorphic. (Hint: How can you use connectedness?)*

Exercise 7. *Show that if a metric space X is connected and has more than one point, it is uncountable. (Hint: Find a surjection from X to an interval in \mathbb{R} .)*

Exercise 8. *Let $A \subset X$ and suppose that both A and X are connected. Show that if U and V form a separation of $X \setminus A$, then both $A \cup U$ and $A \cup V$ are connected.*

Exercise 9. *Show that if $A \subset \mathbb{R}^2$ is countable, then $\mathbb{R}^2 \setminus A$ is path connected.*