GENERAL TOPOLOGY
EXERCISES FOR SESSION 6 (WED 12.3)

Exercise 1. *Let $a \in X$ and suppose that $X \setminus U$ is compact whenever U is a neighbourhood of a. Prove that X is compact.*

Exercise 2. *Show that an open interval (a, b) on the real line is not compact.*

Exercise 3. *Let X be compact, Y be Hausdorff, and $f : X \to Y$ be continuous. Use the properties of compactness proved in the lectures/notes to:*

- *Prove that f is closed (that the image of a closed set is closed under f).*
- *Prove that if f is a bijection then it is a homeomorphism.*

Exercise 4. *Suppose that X is equipped with the cofinite topology ($U \subset X$ is open if $U = \emptyset$ or U^c is finite). Prove that every subspace $A \subset X$ is compact.*

Exercise 5. *Prove Theorem 3.3 in the lecture notes.*

Exercise 6. *Suppose that $\mathcal{B}$ is a basis for the topology on X. Suppose that whenever $\mathcal{D} \subset \mathcal{B}$ is a cover of X, it has a finite subcover. Prove that X is compact.*

Exercise 7. *Show that a closed subspace of a locally compact space is locally compact.*

Exercise 8. *Suppose that $f : \mathbb{R}^n \to \mathbb{R}^m$ is a continuous function such that $|f(x)| \to \infty$ as $|x| \to \infty$. Prove that f is a closed map.*

Exercise 9. *Let $X \neq \emptyset$ be compact and Hausdorff. Given a continuous function $f : X \to X$, show that there exists a nonempty closed subset $A \subset X$ such that $f(A) = A$.*