

GENERAL TOPOLOGY

HOMEWORK FOR WEEK 1

DEADLINE: MON 4.3, 23:59

Exercise 1. Let A be an open set in a topological space X . Prove that $\text{int } \partial A = \emptyset$.

Exercise 2. Find the mistake in the following reasoning by ChatGPT. Note that the statement it is trying to prove is not true in the first place.



You

Let A_i be some sets in a topological space X . Prove that the closure of their union is contained in the union of their closures.



ChatGPT

To prove that the closure of the union of sets A_i is contained in the union of their closures, we can proceed by showing that any accumulation point of the union of sets A_i is also an accumulation point of the union of their closures.

Let x be an accumulation point of $\bigcup A_i$. This means that every neighborhood of x contains a point of $\bigcup A_i$ different from x .

Since x is an accumulation point of $\bigcup A_i$, there exists an index j such that x is an accumulation point of A_j . This means that every neighborhood of x contains a point of A_j different from x .

Since x is an accumulation point of A_j , it follows that x belongs to the closure of A_j , denoted $\overline{A_j}$. Therefore, x is also a point in the union of closures of sets A_i , i.e., $x \in \bigcup \overline{A_i}$.

Thus, we have shown that any accumulation point of $\bigcup A_i$ is also a point of $\bigcup \overline{A_i}$, which implies that the closure of $\bigcup A_i$ is contained in the union of their closures.

Hence, we have proved that:

$$\overline{\bigcup_i A_i} \subseteq \bigcup_i \overline{A_i}$$