GENERAL TOPOLOGY
EXERCISES FOR SESSION 8 (WED 19.3)

Exercise 1. *Show that a closed subspace of a normal space is normal.*

Exercise 2. *Show that every compact Hausdorff space is normal. (Hint: Theorem 3.4)*

Exercise 3. *Show that if X is normal and $F, G \subset X$ are closed and disjoint, then there are open neighbourhoods U, V of F, G respectively such that $\bar{U} \cap \bar{V} = \emptyset$.*

Exercise 4. *Show that every locally compact Hausdorff space is regular. You can follow this scheme:*

- *Pick a closed set $F \subset X$ and $x \notin F$.*
- *Consider a neighbourhood V of x such that $\bar{V}$ is compact.*
- *Handle the situation inside $\bar{V}$ first.*
- *Then you can solve the general situation.*

Exercise 5. *Verify that the one-point compactification X^* defined in Definition 3.3 is actually compact.*

Hint: First of all, you should prove that for every set $U \in \mathcal{T}_\infty$, the set $U \setminus \{\infty\} \subset X$ is actually an open set in the topology $\mathcal{T}$ of X (here you will need that X is Hausdorff). This will make working with a cover of sets in $\mathcal{T} \cup \mathcal{T}_\infty$ easier.

Exercise 6. *Show that the one-point compactification of $\mathbb{Z}_+$ is homeomorphic with the subspace $\{0\} \cup \left\{\frac{1}{n} : n \geq 1, n \in \mathbb{Z}\right\}$ of $\mathbb{R}$.*

Exercise 7. *A space X is said to be **completely normal** if every subspace of X is normal. Prove that X is completely normal if and only if for every pair of sets $A, B \subset X$ with $\bar{A} \cap B = \emptyset = A \cap \bar{B}$, there exist disjoint neighbourhoods of A and B.*

Exercise 8. *Are metrizable spaces completely normal?*