

$$9. \bullet \sum_{i=1}^n (2 \cdot i - 1) = n^2$$

$$\sum_{i=1}^n (2 \cdot i - 1) = 2n - 1 + \sum_{i=1}^{n-1} (2 \cdot i - 1)$$

$$= 2n - 1 + (n - 1)^2 =$$

$$= n^2 - 2n + 1 + 2n - 1 = n^2$$

$$\sum_{i=1}^n (2 \cdot i - 1) = n^2$$

$$\sum_{k=1}^n k^3 = \left(\sum_{k=1}^n k \right)^2$$

$$n = m$$

$$\sum_{k=1}^n k^3 = \sum_{k=1}^{n-1} k^3 + n^3$$

$$= n^3 + \left(\sum_{k=1}^{n-1} k^3 \right)$$

$$= n^3 + \left(\sum_{k=1}^{n-1} k \right)^2$$

$$= n^3 + \left(\frac{(n-1)n}{2} \right)^2 =$$

$$= n^3 + \frac{1}{4} n^2 (n-1)^2 =$$

$$= n^3 + \frac{1}{4} n^2 (n^2 - 2n + 1) =$$

$$= n^3 + \frac{1}{4} (n^4 - 2n^3 + n^2) =$$

$$= n^3 + \frac{1}{4} n^4 - \frac{1}{2} n^3 + \frac{1}{4} n^2 =$$

$$= \frac{1}{4} n^4 + \frac{1}{2} n^3 + \frac{1}{4} n^2 =$$

$$= \frac{1}{4} n^2 (n^2 + 2n + 1) =$$

$$= \frac{1}{4} n^2 (n+1)^2 = \left(\frac{n(n+1)}{2} \right)^2 = \left(\sum_{k=0}^n k \right)^2$$