Exercise 1,2,3

(a)
$$C \to B$$

 $AC \to B$
 $A \to B$

(b)
$$\alpha \to \beta, \beta \subseteq \alpha \Rightarrow \alpha \to \beta \cup \gamma$$

 $\alpha \to \gamma$
 $\alpha \to \beta \Rightarrow \alpha \cup \gamma \to \beta \cup \gamma, because $\gamma \subseteq \alpha, then \ \alpha \cup \gamma \to \alpha, then \ \alpha \to \beta \cup \gamma$$

	x_i	$\alpha_i \to \beta_i$
(a)	lectureId, studentId	$lectureId \rightarrow topicprofessorId$
	lectureId, studentId, topic, professorId,	$professorId \rightarrow office$
	lectureId, studentId, topic, office, professorId	$studentId \rightarrow studentName$
	lectureId, studentId, topic, office, professorId, studentName	$lectureId, studentId \rightarrow grade$
	lectureId, topic, professorId, office, studentName, grade, studentId	

Minimal keys are **lectureId** and **studentId**

(a.i) R = A,B,C,D,E,F,G,H,I,J
$$F = C \to BH, I \to DG, J \to C, AJ \to E, A \to FI$$

$$\begin{array}{c|cc} x_i & \alpha_i \rightarrow \beta \\ \hline A,J & A \rightarrow FI \\ A,F,I,J & J \rightarrow C \\ A,C,F,I,J & C \rightarrow BH \\ A,B,C,F,H,I,J & I \rightarrow DG \\ A,B,C,D,G,H,I,J & AJ \rightarrow E \\ A,B,C,D,E,F,G,H,I,J & \end{array}$$

 \mathbf{A} , \mathbf{J} are minimal keys

- (a.ii) $R_1\{\underline{A}, F, \underline{I}, D, G\}$ $R_2\{\underline{J}, \underline{C}, B, H\}$ $R_3\{\underline{A}, \underline{J}, E\}$
- (a.iii) $R_{11}\{\underline{A}, F, I\}$ $R_{12}\{\underline{I}, D, G\}$ $R_{21}\{\underline{J}, C\}$ $R_{22}\{\underline{C}, B, H\}$ $R_{3}\{\underline{A}, \underline{J}, E\}$ This R is in BCNF, because all FDs are implied by its key constrains
 - (b) BCNF is violated because of $MVP \to Team$ dependency. Relations $R_1\{Matchday \to Team\}$, $R_2\{Team \to MVP\}$ is in BCNF