

# Introduction to Cryptography

## Exercise Week 4

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### Exercise 1.

Prove Proposition 3.6 from the lecture:

Let  $\text{negl}_1$  and  $\text{negl}_2$  be negligible functions. Then,

1.  $\text{negl}_3(n) := \text{negl}_1(n) + \text{negl}_2(n)$  is negligible.
2. For any polynomial  $p$ , the function  $\text{negl}_4(n) := p(n) \cdot \text{negl}_1(n)$  is negligible.

### Exercise 2.

Prove that Definition 3.8 cannot be satisfied if  $\Pi$  can encrypt arbitrary-length messages and the adversary is *not* restricted to output equal-length messages in experiment  $\text{PrivK}_{\mathcal{A}, \Pi}^{\text{eav}}$ .

Hint: Let  $p$  be a polynomial. Why is it very likely that the resulting ciphertext from encrypting a string of length  $u + (u)d$  is of length greater than  $(u)d$ ?

### Exercise 3.

Prove or disprove whether each  $G_i$  is a pseudorandom generator for the following constructions:

- (a)  $G_1 : \{0, 1\}^n \rightarrow \{0, 1\}^{2n}, G_1(s) := s_1 || s_1 || \dots || s_n || s_n$ .
- (b)  $G_2 : \{0, 1\}^n \rightarrow \{0, 1\}^{2n}, G_2(s) := s || s$ .
- (c) Let  $G$  be a pseudorandom generator. Define  $G_3(s) := G(\bar{s})$  where  $\bar{s}$  is the complement of  $s$ .
- (d) Again let  $G$  be a PRG and define  $G_4(s) := \overline{G(s)}$ , i.e. the complement of  $G(s)$ .