Theorem 6.2 : *If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.*

This theorem can be proved by taking a line DE such that

$$\frac{AD}{DB} = \frac{AE}{EC}$$

and assuming that DE is not parallel to BC (see Fig. 6.12).

If DE is not parallel to BC, draw a line DE′ parallel to BC.

So,

$$\frac{AD}{DB} = \frac{AE'}{E'C} \quad \text{(Why ?)}$$

Therefore,

$$\frac{AE}{EC} = \frac{AE'}{E'C} \quad \text{(Why ?)}$$

Adding 1 to both sides of above, you can see that E and E′ must coincide. (Why ?)

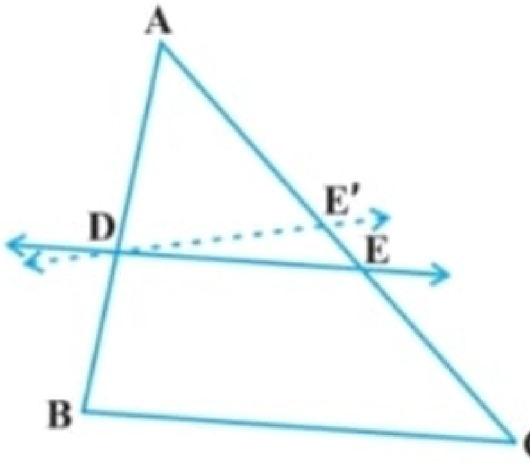

Fig. 6.12

Let us take some examples to illustrate the use of the above theorems.

Example 1 : If a line intersects sides AB and AC of a △ABC at D and E respectively and is parallel to BC, prove that

$$\frac{AD}{AB} = \frac{AE}{AC} \quad \text{(see Fig. 6.13).}$$

Solution : DE || BC (Given)

So,

$$\frac{AD}{DB} = \frac{AE}{EC} \text{(Theorem 6.1)}$$

or,

$$\frac{DB}{AD} = \frac{EC}{AE}$$

or,

$$\frac{DB}{AD} + 1 = \frac{EC}{AE} + 1$$

or,

$$\frac{AB}{AD} = \frac{AC}{AE}$$

So,

$$\frac{AD}{AB} = \frac{AE}{AC}$$

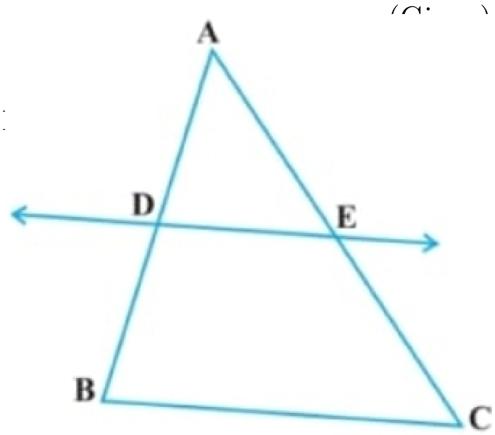

Fig. 6.13

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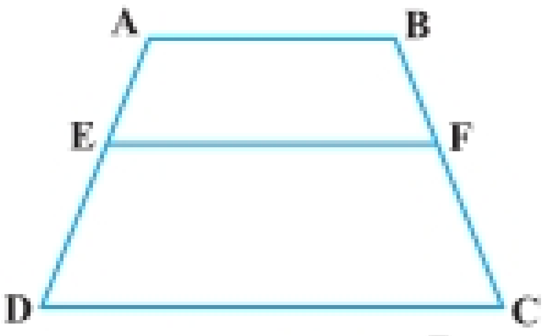
Example 2 : ABCD is a trapezium with AB || DC. E and F are points on non-parallel sides AD and BC respectively such that EF is parallel to AB (see Fig. 6.14). Show that

$$\frac{AE}{ED} = \frac{BF}{FC}$$



Solution : Let us join AC to intersect EF at G (see Fig. 6.15).

AB || DC and EF || AB (Given)
So, EF || DC (Lines parallel to the same line are parallel to each other)
Now, in $\triangle ADC$,
EG || DC (As EF || DC)

So,

$$\frac{AE}{ED} = \frac{AG}{GC} \qquad (1)$$

(Theorem 6.1)

Similarly, from $\triangle CAB$,

$$\frac{CG}{AG} = \frac{CF}{BF} \Rightarrow \frac{AG}{GC} = \frac{BF}{FC}$$

Therefore, from (1) and (2),

$$\frac{AE}{ED} = \frac{BF}{FC}$$

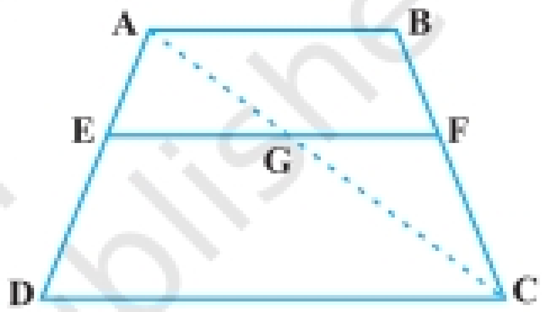


Fig. 6.15

Example 3 : In Fig. 6.16, $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that $\triangle PQR$ is an isosceles triangle.

Solution : It is given that

$$\frac{PS}{SQ} = \frac{PT}{TR}$$

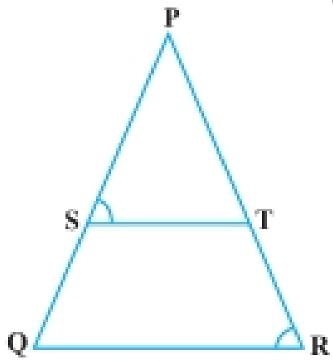


Fig. 6.16

So, ST || QR (Theorem 6.2)
Therefore, $\angle PST = \angle PQR$ (Corresponding angles)
Now,

$$\angle PST = \angle PRQ \Rightarrow \angle PRQ = \angle PQR \Rightarrow PQ = PR$$

i.e., $\triangle PQR$ is an isosceles triangle.