Taller 1

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1. Desarrolle los siguientes ejercicios del libro:
   a) Ejercicio 3.1-2

      Show that for any real constants a and b, where b > 0,
      $(n+a)^b = \Theta(n^b)$

      (n + a) <= 2n, cuando |a| <= n
      y (n + a) >= n/2, cuando |a| <= n/2.
      Entonces: n >= 2a
      Tenemos:
      0 <= n/2 <= (n + a) <= 2n
      elevando todo a "b", obtenemos
      $0 <= (n/2)^b <= (n + a)^b <= (2n)^b$
      $0 <= (1/2)^b n^b <= (n + a)^b <= 2^b n^b$
      comparamos con:
      $0 <= c1n^b <= (n + a)^b <= c2n^b$,
      Observamos que:
      $c1 = (1/2)^b$, $c2 = 2^b$ y $n_0 = 2a$
      Entonces: $(n + a)^b = \Theta(n^b)$.

   b) Ejercicio 3.1-7

      Prove that o(g(n)) ∩ ω(g(n)) is the empty set.

      Suponiendo que o(g(n)) ∩ ω(g(n)) no es un conjunto vacío, es equivalente a decir que:
      Para todo c1, c2 > 0 tiene 0 <= c1g(n) < f(n) < c2g(n)
      donde n >= max (n1, n2).

      Hacemos c1 = c2, podemos es un absurdo, por lo que la suposición no es verdadera, por lo que o(g(n)) ∩ ω(g(n)) debe ser un conjunto vacío.

   c) Problema 3.3

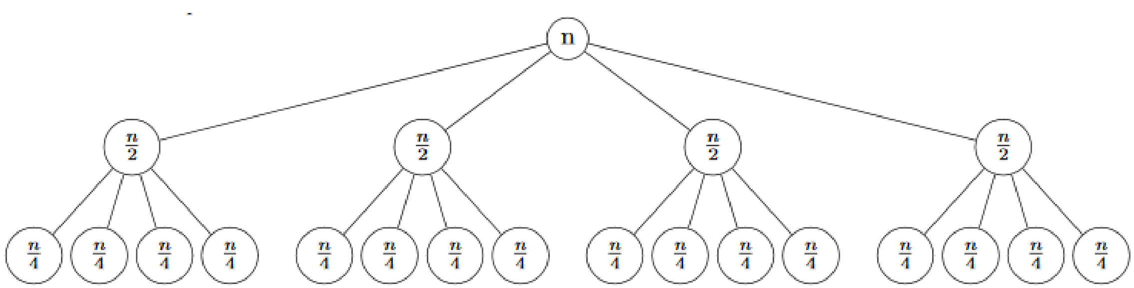
      Rank the following functions by order of growth;

$2^{2^{n+1}}$	
2^{2^n}	
$(n+1)!$	
$n!$	
$n2^n$	
e^n	
2^n	
$\left(\frac{3}{2}\right)^n$	
$(\lg(n))!$	
$n^{\lg(\lg(n))}$	$\lg(n)^{\lg(n)}$
n^3	
n^2	$4^{\lg(n)}$
$n\lg(n)$	$\lg(n!)$
$2^{\lg(n)}$	n
$(\sqrt{2})^{\lg(n)}$	
$2^{\sqrt{2\lg(n)}}$	
$\lg^2(n)$	
$\ln(n)$	
$\sqrt{\lg(n)}$	
$\ln(\ln(n))$	
$2^{\lg^*(n)}$	
$\lg^*(n)$	$\lg^*(\lg(n))$
$\lg(\lg^*(n)$	
1	$n^{1/\lg(n)}$

b- $2^{2^{Sen\, n}}$

d) Ejercicio 4.4-7

Draw the recursion tree for T .n/ D 4T .bn=2c/ C cn, where c is a constant, and provide a tight asymptotic bound on its solution. Verify your bound by the substitution method.



Sustituyendo se puede ver que es: $\Theta(n^2)$

Suponiendo $T(n) \leq c'n^2$ entonces:

$$T(n) = 4T\left(\left\lfloor\frac{n}{2}\right\rfloor\right) + cn \leq c'n^2 + cn$$

Lo cual es $\leq c'n^2$ siempre que tengamos que $c' + \frac{c}{n} \leq 1$que es cierto para un n lo suficientemente grande cuando $c' < 1$

e) Use

2. Dado el siguiente seudocódigo
   …

3. Ejercicio 22.3-1

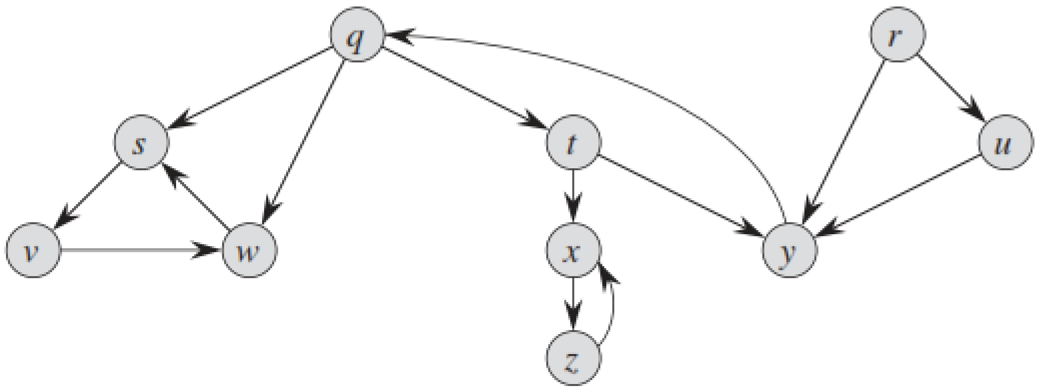
   Make a 3-by-3 chart with row and column labels WHITE, GRAY,andBLACK.In each cell .i; j /, indicate whether, at any point during a depth-first search of a directed graph, there can be an edge from a vertex of color i to a vertex of color j. For each possible edge, indicate what edge types it can be. Make a second such chart for depth-first search of an undirected graph.

from\to	BLACK	GRAY	WHITE
BLACK	All	Back, cross	Back, cross
GRAY	Tree, forward, cross	Tree, forward, cross	Back, cross
WHITE	Tree, forward, cross	Back, cross	All

Solo la diagonal superior

from\to	BLACK	GRAY	WHITE
BLACK	All	All	All
GRAY	-	Tree, forward, cross	All
WHITE	-	-	All

4. Ejercicio 22.3-2
   Show how depth-first search works on the graph of Figure 22.6. Assume that the **for** loop of lines 5–7 of the DFS procedure considers the vertices in alphabetical order, and assume that each adjacency list is ordered alphabetically. Show the discovery and finishing times for each vertex, and show the classification of each edge.

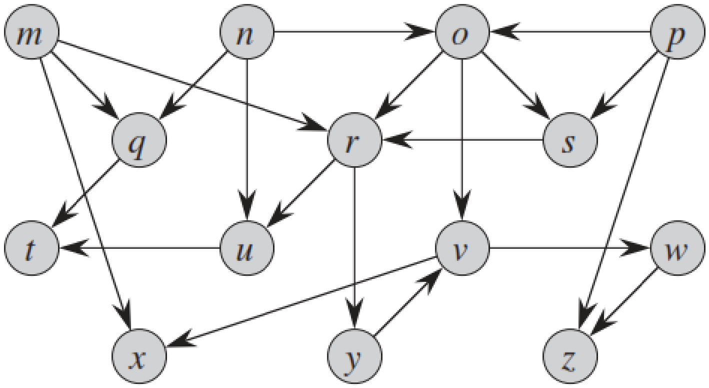


Vertice	Descubierto	Finalizado
Q	1	16
R	17	20
S	2	7
T	8	15
U	18	19
V	3	6
W	4	5
X	9	12
Y	13	14
Z	10	11

Arbol: (q,s), (s,v), (v,w), (q,t), (t,x), (x,z), (t,y), (r,u).
Atras: (w,s), (y,q), (z,x).
Adelante: (q,w).
cruce: (u,y),(r,y)

5. Ejercicio 22.4-2

Give a linear-time algorithm that takes as input a directed acyclic graph G D.V; E/ and two vertices and t , and returns the number of simple paths from s to t in G. For example, the directed acyclic graph of Figure 22.8 contains exactly four simple paths from vertex p to vertex _: po_, pory_, posry_,andpsry_(Your algorithm needs only to count the simple paths, not list them.)



6. e