Assignment 3 Alemayehu Bogale

1a. $V(x,y) = \frac{1}{2} m\omega^2(x^2+y^2)$, $K(\vec{p}) = \frac{p_x^2+p_y^2}{2m}$

$E = V + K$

$$E = \frac{\int E(\vec{p},\vec{r}) \exp(-\frac{E}{KT}) d\vec{p} d\vec{r}}{Z}$$

$$Z = \int_{-\infty}^{\infty} e^{-E/KT} dp_x dp_y dx dy = \int_{\infty}^{\infty} \exp\left(-\frac{1}{K_BT}\left(\frac{p_x^2+p_y^2}{2m} + \frac{1}{2}m\omega^2(x^2+y^2)\right)\right) dx dy dp_x dp_y$$

$$= \int_{-\infty}^{\infty} \exp\left(\frac{-p_x^2}{2mK_BT}\right) \exp\left(\frac{-p_y^2}{2mK_BT}\right) \exp\left(\frac{-m\omega^2 x^2}{2K_BT}\right) \exp\left(\frac{-m\omega^2}{2K_BT}\right) dx dy dp_y dx$$

$$= \sqrt{\pi 2mK_BT} \sqrt{\pi 2mK_BT} \sqrt{\frac{\pi 2K_BT}{m\omega^2}} \sqrt{\frac{\pi 2K_BT}{m\omega^2}} = 2\cancel{m}K_BT\pi \frac{\pi 2K_BT}{\cancel{m}\omega^2}$$

$$= \left(\frac{2\pi K_BT}{\omega}\right)^2$$

$$\int_{-\infty}^{\infty} E(\vec{p},\vec{r}) \exp\left(-\frac{1}{K_BT}\left((p_x^2+p_y^2)/2m + \frac{m\omega^2}{2}(x^2+y^2)\right)\right) dy dx dp_x dp_y$$

$$= \frac{4\pi^2K_B^3T^3}{2\omega^2} + \frac{2\pi^2K_B^3T^3}{\omega^2} + \frac{2\pi^2K^3T^3}{\omega^2} + \frac{2\pi^2K^3T^3}{\omega^2}$$

$$= \frac{8\pi^2K^3T^3}{\omega^2}$$

$$\frac{\int_{-\infty}^{\infty} E e^{-E/K_BT} dp_x dx dp_y dy}{Z} = \frac{8\pi^2K^3T^3}{\cancel{\omega^2}} \cdot \frac{\cancel{\omega^2}}{4\pi^2K_B^2T^2}$$

$$= 2K_BT$$

2a $\mathcal{E}(T) = \sum_{n=0} E_n P_n = \frac{\sum_{n=0} E_n e^{-\frac{1}{k_BT}E_n}}{\sum e^{-E_n/k_BT}} = \frac{\sum_n E_n e^{-E_n\beta}}{\sum_n e^{-E_n\beta}}$

$$\sum_{n=0} e^{-E_n/k_BT} = \sum_{n=0} e^{-(n+\frac{1}{2})\hbar\omega/k_BT} = e^{\frac{-\hbar\omega}{2k_BT}} \sum_{n=0} e^{-\frac{n\hbar\omega}{k_BT}}$$

$$= \frac{e^{-\hbar\omega/2k_BT}}{1 - \exp\left(\frac{-\hbar\omega}{k_BT}\right)} \#$$

$$\mathcal{E}(T) = \frac{-\sum_n \frac{\partial}{\partial\beta} e^{-E_n\beta}}{\sum_n e^{-E_n\beta}} = \frac{-\frac{\partial}{\partial\beta}\sum_n e^{-E_n\beta}}{\sum_n e^{-E_n\beta}} = \frac{-\frac{\partial}{\partial\beta} Z}{Z}$$

$$= -\frac{\partial}{\partial\beta}\ln(Z) = -\frac{\partial}{\partial\beta}\ln\left(\frac{e^{-\hbar\omega\beta/2}}{1-e^{(-\hbar\omega\beta)}}\right)$$

$$= -\frac{\partial}{\partial\beta}\left(-\frac{\beta\hbar\omega}{2} - \ln\left(1 - e^{-\hbar\omega\beta}\right)\right) = +\frac{\hbar\omega}{2} + \frac{\hbar\omega\exp(-\beta\hbar\omega)}{1-\exp(-\beta\hbar\omega)}$$

approximate

$$\hbar\omega\left(\frac{1}{2} + \frac{1}{-1-1+\beta\hbar\omega}\right) = \hbar\omega\left[\frac{1}{2} + \frac{1}{\beta\hbar\omega}\right]$$

$$\mathcal{E}(T) = \begin{cases} k_BT & k_BT \gg \hbar\omega \\ \frac{\hbar\omega}{2} & \hbar\omega \ll k_BT \end{cases}$$