Diophantine Approximation with Farey Sequences

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1 Objectives

Describe the objective(s) of the project and how these will be accomplished. You must give the necessary context to make the document self-contained, i.e., explain the problem or domain of application considered, the algorithm(s) that will be analyzed, which particular algorithmic issue(s) will be subject to scrutiny, etc.

2 Experimental Setup

Describe the configuration used in the experiments. This implies the following: (1) indicate what kind of experiments will be conducted (i.e., indicate in which way the algorithm will be run and what will be measured) and what will be the particular parameters that will be used in those experiments (i.e., their numerical values); (2) provide a description of the computational environment in which the experiments are run (see Table 1).

Table 1: Computational environment considered.

CPU	Intel(R) Core(TM) i5-6200U CPU @ 2.30GHz 2.40 GHz 8,00 GB
OS	x64 bits, Windows 10 Pro 22H2
Java	java 22.0.2 2024-07-16

3 Empirical Results

A summary of the experimental results is provided in Tables 2-2 in the Appendix, along with the statistical fitting of the data to different growth models.

Describe the results, in particular Figure 1.

4 Discussion

Provide your interpretation of the results: discuss whether the results match the theoretical predictions, whether some algorithm is better in practice than others, etc.

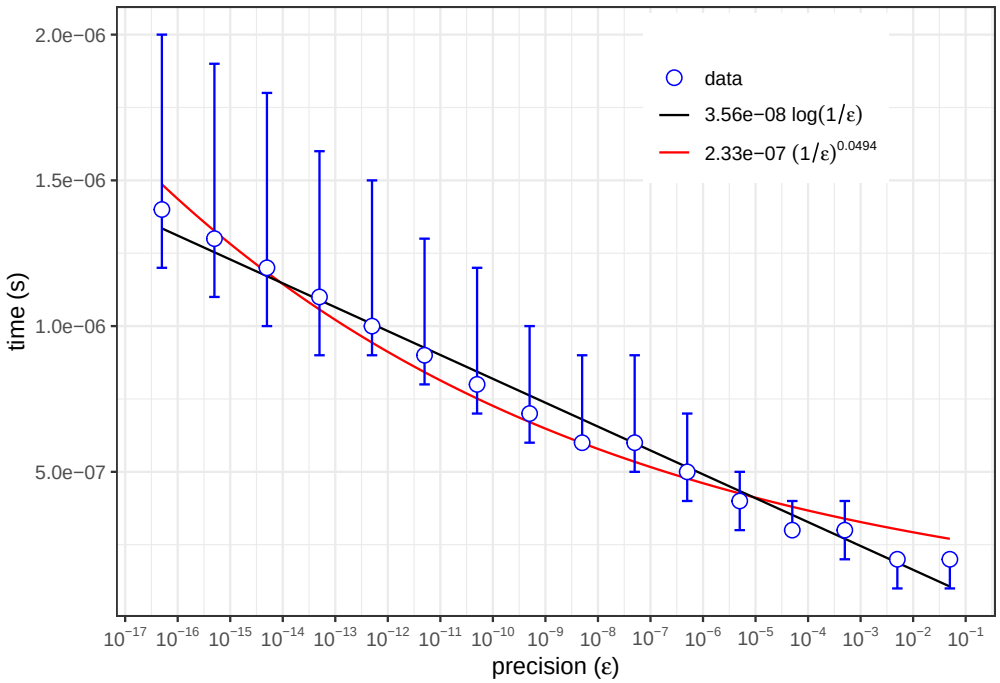

Figure 1: Time required for finding the rational approximation of a number for increasing precision

A Appendix

A.1 Data Summary

Table 2: Summary of the experimental results. Q1, Q2 and Q3 represent the 1st, 2nd (i.e., median) and 3rd quartile respectively. All times are in seconds.

ϵ	**time (Q1)**	**time (Q2)**	**time (Q3)**
5e-02	1.0e-07	2.0e-07	2.0e-07
5e-03	1.0e-07	2.0e-07	2.0e-07
5e-04	2.0e-07	3.0e-07	4.0e-07
5e-05	3.0e-07	3.0e-07	4.0e-07
5e-06	3.0e-07	4.0e-07	5.0e-07
5e-07	4.0e-07	5.0e-07	7.0e-07
5e-08	5.0e-07	6.0e-07	9.0e-07
5e-09	6.0e-07	6.0e-07	9.0e-07
5e-10	6.0e-07	7.0e-07	1.0e-06
5e-11	7.0e-07	8.0e-07	1.2e-06
5e-12	8.0e-07	9.0e-07	1.3e-06
5e-13	9.0e-07	1.0e-06	1.5e-06
5e-14	9.0e-07	1.1e-06	1.6e-06
5e-15	1.0e-06	1.2e-06	1.8e-06
5e-16	1.1e-06	1.3e-06	1.9e-06
5e-17	1.2e-06	1.4e-06	2.0e-06

A.2 Model Fitting

A.2.1 Power-Law Fit

```
## Power law fit
## --------------------


##
## Formula: time ~ a * (1/precision)^b
##
## Parameters:
##    Estimate Std. Error t value Pr(>|t|)
## a 2.331e-07  1.614e-08   14.44 8.39e-10 ***
## b 4.935e-02  2.264e-03   21.80 3.34e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.914e-08 on 14 degrees of freedom
##
## Number of iterations to convergence: 6
## Achieved convergence tolerance: 1.849e-06
```

A.2.2 Logarithmic Fit

```
## Logarithmic fit
## ------------------


##
## Formula: time ~ a * log(1/precision)
##
## Parameters:
##    Estimate Std. Error t value Pr(>|t|)
## a 3.557e-08  5.207e-10   68.31   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.765e-08 on 15 degrees of freedom
##
## Number of iterations to convergence: 1
## Achieved convergence tolerance: 1.379e-09
```