Preliminary observations

June 8, 2020

1 Assumptions

Only public stations are considered, i.e., $K' = \emptyset$. Moreover, $cost(y^k) = 0$ in the leader's objective function.

We consider two cases based on the nature of the preference list v_{ik}^{tl}. The first one is when the preference list is a given data leading to formulation (1). The second case when it is variable leading to formulation (2).

2 Case 1: When the preference list is a given data

In this case, the problem of interest is (1)

$$
\begin{aligned}
\max_{p,x,y,z} \quad & \sum_{i\in I}\sum_{k\in K}\sum_{t\in T} p_k^t x_{ik}^t - x_{ik}^t && && \text{(1a)}\\
\text{s.t.} \quad & y^k \le \delta^k z^k, && \forall k \in K, && \text{(1b)}\\
& \sum_{k\in K} \mathcal{C}^k y^k \le \beta, && && \text{(1c)}\\
& y^k \in \mathbb{Z}, && \forall k \in K, && \text{(1d)}\\
& p_k^t \ge 0, && \forall k \in k, \forall t \in T, && \text{(1e)}\\
& \min_{x} \quad \sum_{t\in T}\sum_{i\in I}\sum_{k\in K\cup k_i'} (\lambda_i^k + \sum_{l=1}^{s} l v_{il}^{kt}) x_{ik}^t && && \\
& \quad \text{s.t.} \quad \sum_{i\in I} x_{ik}^t \le \gamma^k y^k, && \forall k \in K\cup k_i', \forall t \in T, && \text{(1f)}\\
& \quad \sum_{t\in T}\sum_{k\in K\cup k_i'} x_{ik}^t = 1, && \forall i \in I, && \text{(1g)}\\
& \quad x_{ik}^t \le \sum_{l=1}^{s} v_{il}^{kt}, && \forall i \in I, \forall t \in T, \forall k \in K\cup k_i', && \text{(1h)}\\
& \quad x_{ik}^t \in \{0,1\}, && \forall i \in I, \forall k \in K\cup k_i', \forall t \in T. && \text{(1i)}
\end{aligned}
$$

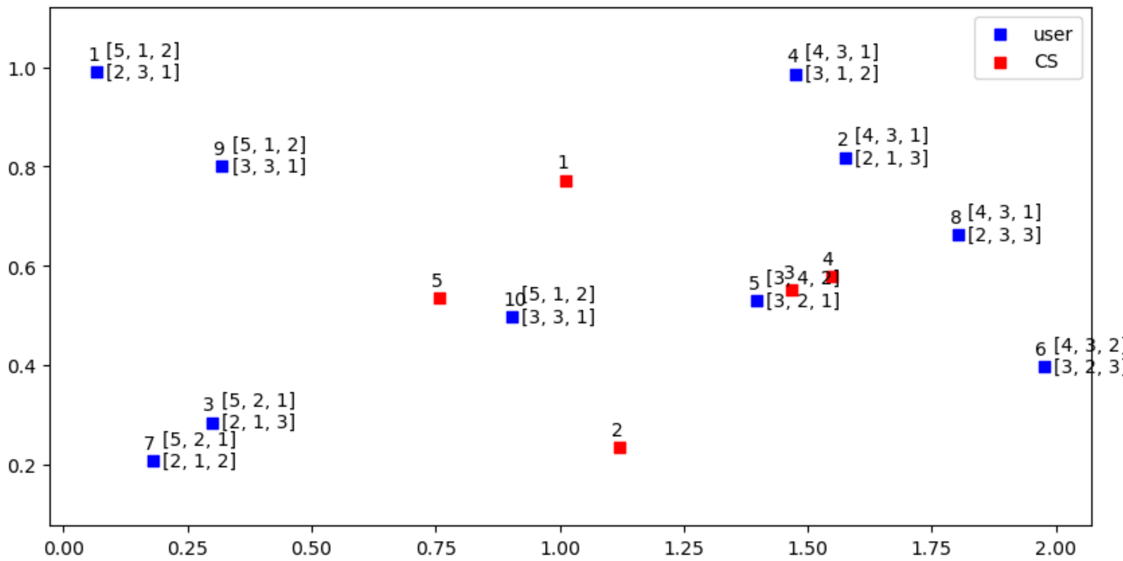


Figure 1: Shows locations of charging station candidates (red) and users (blues) and their preference list (2D array)

2.1 Data (v_{ik}^{tl})

- Give n users, m candidate charging stations, their location is randomly generated in a rectangle of dimension 1×2.
- The preference list is generated as follows:
  The number of elements in the preference list is first fixed to be s, $\forall i \in I$. Moreover, the number of time slats in a day is set to r.
  1. For each user $i \in I$, compute its distance from the candidate charging stations, and pick the closest (first) s candidate stations as its preference list
  2. For each charging station in the preference list, assign a random time slat from $\{1, ..., r\}$. For example, in Figure 1, user 9's preference list is the charging stations $5, 1, 2$ at times $t = 3, 3, 1$.
  3. Generate the tensor v_{ik}^{tl} for $i \in I$, $k \in K$, $t \in T$, and $l \in \{1, ..., s\}$

3 Case 2: When the preference list is variable

In this case, the problem of interest is (2)

$$
\begin{aligned}
\max_{p,x,y,z} \quad & \sum_{i\in I}\sum_{k\in K}\sum_{t\in T} p_k^t x_{ik}^t - cost(y^k) - energy\_cost(x_{ik}^t) && && \text{(2a)}\\
\text{s.t.} \quad & y^k \le \delta^k z^k, && \forall k\in K, && \text{(2b)}\\
& \sum_{k\in K} \mathcal{C}^k y^k \le \beta, && && \text{(2c)}\\
& y^k \in \mathbb{Z}, && \forall k\in K, && \text{(2d)}\\
& p_k^t \ge 0, && \forall k\in k, \forall t\in T, && \text{(2e)}\\
& \min_v \ \sum_{t\in T}(\sum_{k\in k}(\alpha_i p_k^t v_{ik}^{tl} + \beta_i d_{ik}^t v_{ik}^{tl} + \sum_{k\in k_i'} C_i^{kt} v_{ik}^{tl}), && \forall i\in I,\ \forall l\in\{1,..,m\}, && \\
& \quad \text{s.t.} \quad \sum_{k\in k}\sum_{t\in T} v_{ik}^{tl} = 1, && && \text{(2f)}\\
& \quad\quad v_{ik}^{tl} \le 1 - v_{ik}^{tl'}, && \forall k\in K\cup k_i', \forall t\in T, \forall l' < l, && \text{(2g)}\\
& \quad\quad v_{ik}^{tl} \in \{0,1\}, && \forall i\in I, \forall k\in K\cup k_i', \forall t\in T, && \text{(2h)}
\end{aligned}
$$

$$
\begin{aligned}
\min_x \quad & \sum_{t\in T}\sum_{i\in I}\sum_{k\in K\cup k_i'} (\lambda_i^k + \sum_{l=1}^{s} l v_{il}^{kt}) x_{ik}^t && && \\
\text{s.t.} \quad & \sum_{i\in I} x_{ik}^t \le \gamma^k y^k, && \forall k\in K\cup k_i', \forall t\in T, && \text{(2i)}\\
& \sum_{t\in T}\sum_{k\in K\cup k_i'} x_{ik}^t = 1, && \forall i\in I, && \text{(2j)}\\
& x_{ik}^t \le \sum_{l=1}^{m} v_{il}^{kt}, && \forall i\in I, \forall t\in T, \forall k\in K\cup k_i', && \text{(2k)}\\
& x_{ik}^t \in \{0,1\}, && \forall i\in I, \forall k\in K\cup k_i', \forall t\in T. && \text{(2l)}
\end{aligned}
$$

3.1 Data (d_{ik}^t)

- Give n users, m candidate charging stations, their location is randomly generated in a rectangle of dimension 1×2.
- d_{ik}^t is generated as follows:
  The number of elements in the preference list is first fixed to be s, $\forall i\in I$. Moreover, the number of time slats in a day is set to r.
  1. For each user $i\in I$, compute its distance from the candidate charging stations, and pick the closest (first) s candidate stations, i.e, $k_1,...,k_s$.
  2. For each picked charging station in step 1, assign a random time slat from $\{1,...,r\}$, and then for $k\in\{k_1,..,k_s\}$, d_{ik}^t is set to be the distance from user i to CS k. For $k\in K\setminus\{k_1,..,k_s\}$, d_{ik}^t is the distance from user i to CS k plus 1. This is to guide that the model returns $\{k_1,..,k_s\}$ as the preference list for user i.

4 Other data

- $c^k = 1, \forall k \in K$.
- $\delta_k = 2, \forall k \in K$.
- $\beta = m$.
- λ_i^k is the distance from user i to charging station k. (normalized so that $\max \lambda_i^k = 1$.)
- $\gamma_k = 2, \forall k \in K$.

5 Observations

- Problem (1) and (2) are unbounded. This could be because of the following. The constraint (1g) and (2j) imply every user has to use one of the charging stations, and this is regardless of the charging cost p_{kt}. If we assign an upper bound, say u^p, on p_{kt} the model produces optimal y_k^* and x_{ikt}^* but the charging cost is always $p_{kt}^* = u^p$. The objective function is $nu_p - n$. In general, this makes the models capable of optimizing the location and size of the charging stations but not pricing,
- Other minor observations and possible fixing techniques:
  - The models produce $z_k^* = 1, \forall k \in K$, and $p_{kt}^* = u^p, \forall k \in K, \forall t \in T$. However, this may be fixed by adding the following two constraints to the leader problem of (1)

    $$z^k \leq \sum_{i \in I} \sum_{t \in T} x_{ik}^t, \forall k \in K,$$

    $$p_{kt} \leq \sum_{i \in I} \sum_{t \in T} x_{ik}^t u^p, \forall k \in K, \forall t \in T$$

    (Tried and works)
    (Possible cons: The leader's constraint will have both leader and follower variables y and x . Check this !)
  - Actually, we may eliminate z from the formulation and replace constraints (1b) and 2b) with

    $$y_k \leq \sum_{i \in I} \sum_{t \in T} x_{ik}^t \delta^k, \forall k \in K$$

    (not tried yet)
  - Another minor suggestion: How about using $\sum_{l=1}^{m} l^2 v_{il}^{kt}$ (i.e., l^2 instead of l to increase the influence of picking the first available candidate in the preference list?
    (Not tried for large problems)
    (Possible cons: more optimal locations for CSs as users may be less flexible to go to other CSs than their first choice)

6 Toy problem for future discussion

We use $n = 10$ (expect in Figure where = 20), $m = 5$, $r = 3$, $s = 3$, $\delta_k = 2$, $\beta = 5$, $\gamma_k = 2$, λ_i^k normalized distance, and $u_p = 2$. Then, we generate the positions of the users and charging stations as described above and solve the problems (1) and (2). For problem (2), we consider the following two cases.

1 When $\alpha_i = 0$ and $\beta_i = 1$ to check if the results we get is those that we expect. Note that in this case cplex solves the problem very fast.

The preference list obtained is always those we expected. See the preference lists in the top and middle plots of Figures to .

This verifies the middle optimization problem of (2).

2 When $\alpha_i = 1/2$ and $\beta_i = 1/2$. In this case, cplex struggles (very slow) and sometimes fails to obtains the solution within t=500sec. (But, this needs to be checked for sensible data)

7 To do list and Qs

- $cost(y^k) - energy\_cost(x_{ik}^t)$
- Include K'. This may make the problem bounded
- How to set $C_{ik'}^t$?
- λ_i^k ?
- Is $v_{ik}^{tl'}$ variable or bound for the (i, l)th middle problem? Check the KKT if it is variable.
- Is $d_{ik'}^t = 0$? or this will implicitly considered in defining $C_{ik'}^t$
- sensible data (plus how big could n and m be)?

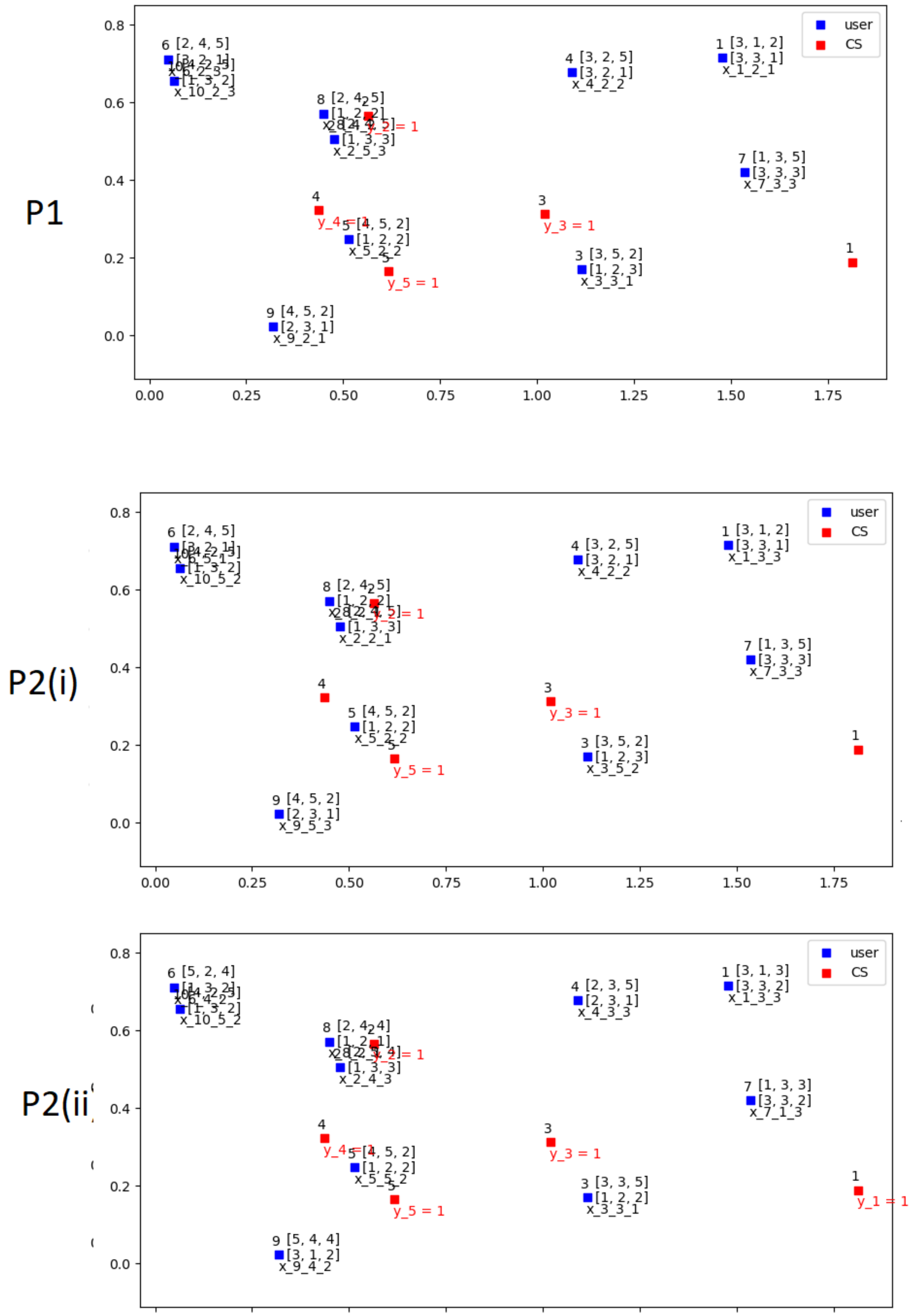
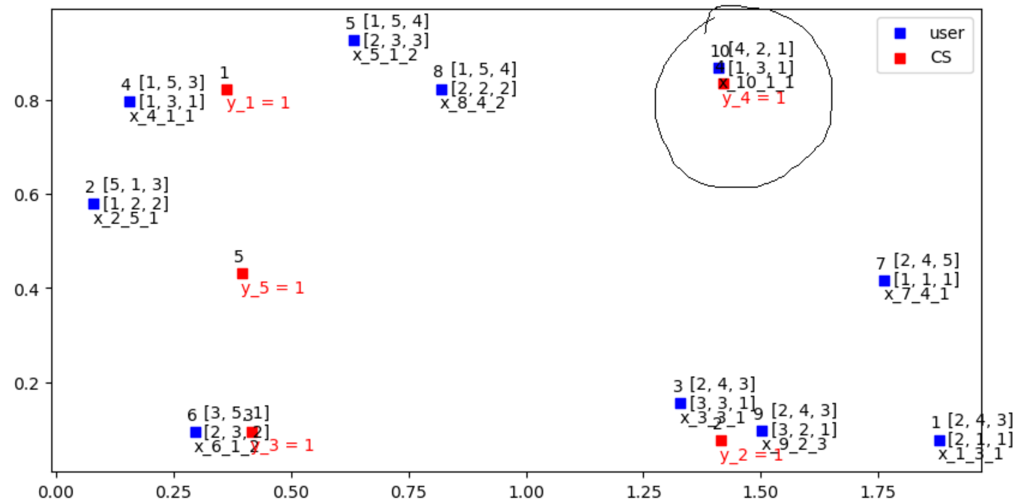
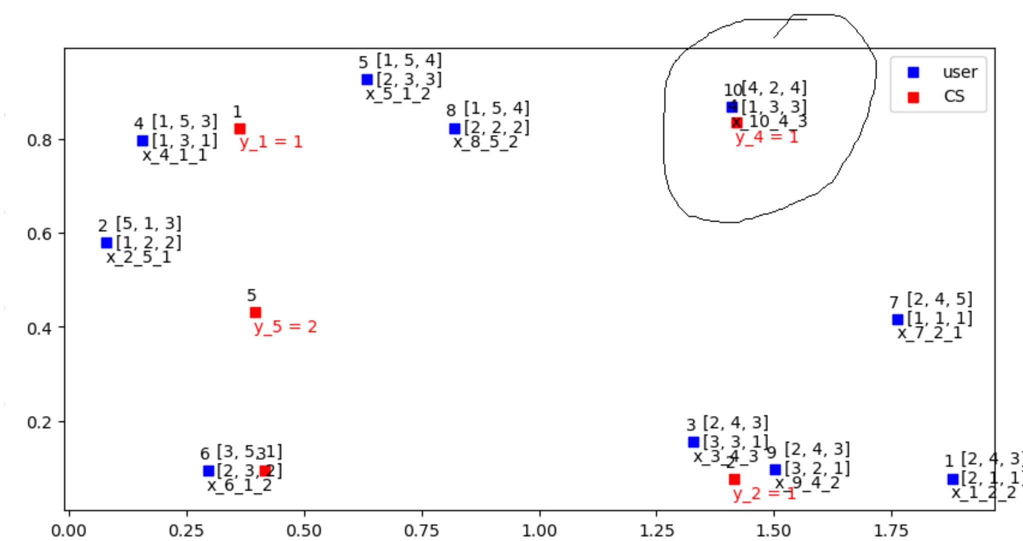


Figure 2:

P1



P2(i)



P2(ii)

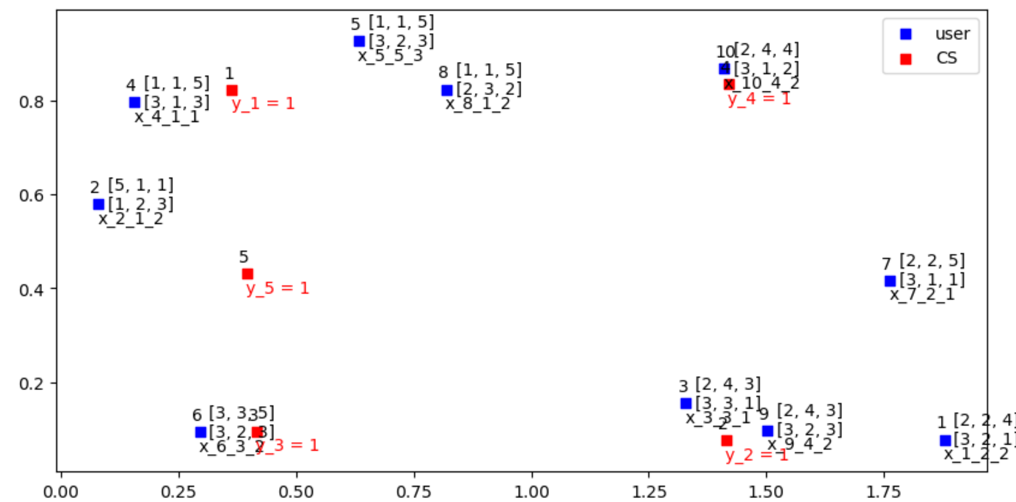
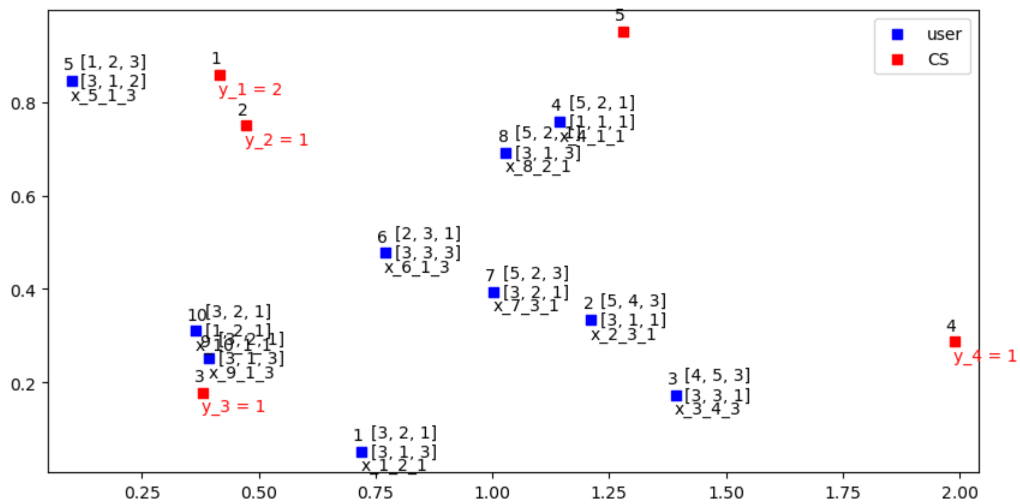
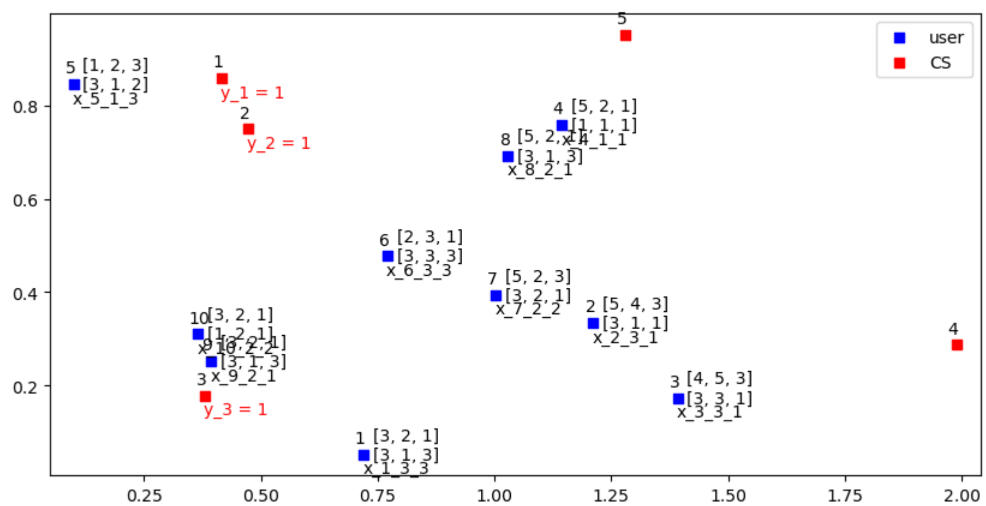


Figure 3:

P1



P2(i)



P2(ii)

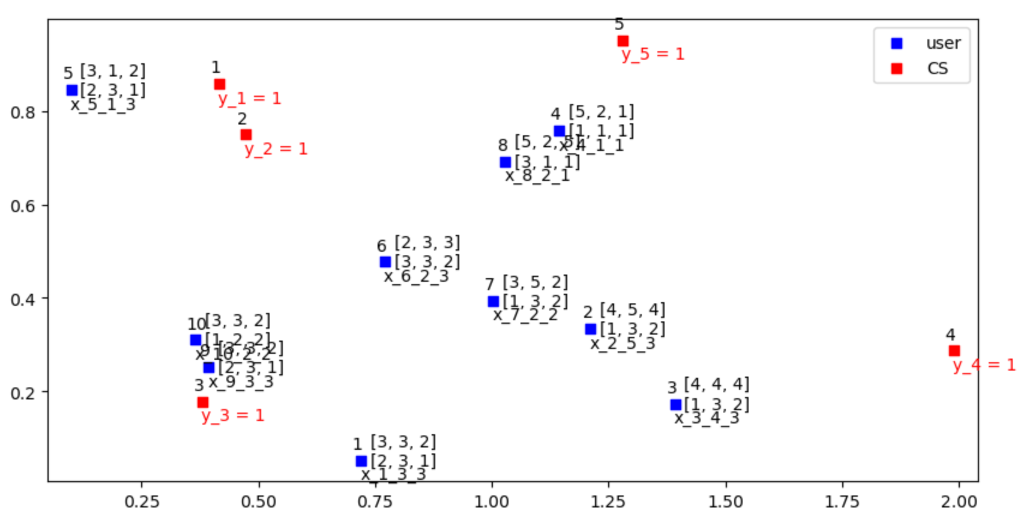


Figure 4:

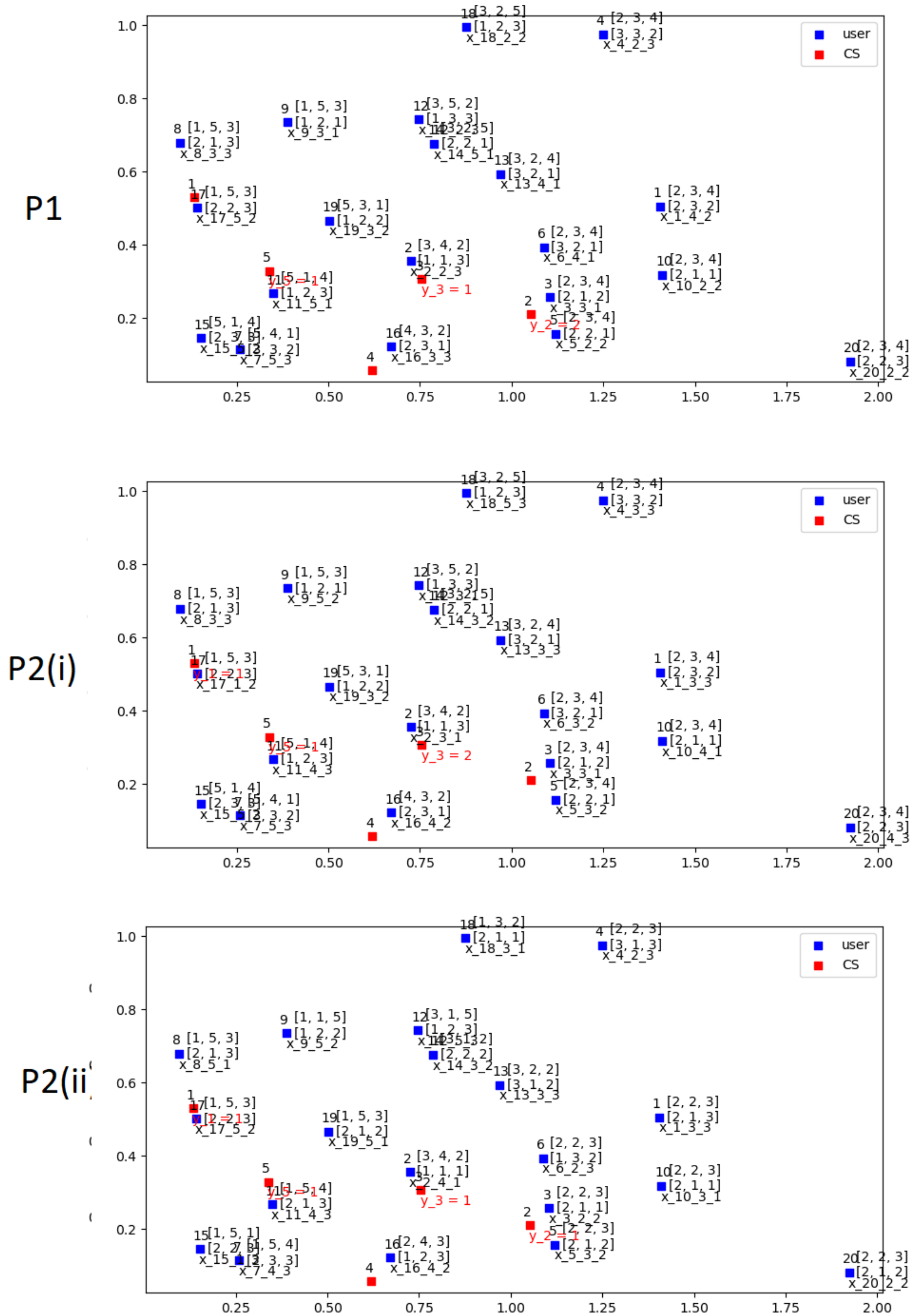


Figure 5: $n = 20$