

REAL-TIME PHYSICAL MODELLING FOR ANALOG TAPE MACHINES

Jatin Chowdhury

Center for Computer Research in Music and Acoustics
Stanford University
Palo Alto, CA
jatin@ccrma.stanford.edu

ABSTRACT

For decades, analog magnetic tape recording was the most popular method for recording music, but has been replaced over the past 30 years first by DAT tape, then by DAWs and audio interfaces [1]. Despite being replaced by higher quality technology, many have sought to recreate a "tape" sound through digital effects, despite the distortion, tape "hiss", and other oddities analog tape produced. The following paper describes the general process of creating a physical model of an analog tape machine starting from basic physical principles, then discusses in-depth a real-time implementation of a physical model of a Sony TC-260 tape machine.

"Whatever you now find weird, ugly, uncomfortable, and nasty about a new medium will surely become its signature. CD distortion, the jitteriness of digital video, the crap sound of 8-bit - all of these will be cherished and emulated as soon as they can be avoided." -Brian Eno [2].

1. INTRODUCTION

While analog magnetic tape recording (see fig. 1) is rarely used in modern recording studios, the sound of analog tape is still often sought after by mixing and mastering engineers. To that end, several prominent audio plugin manufacturers including Waves¹, Universal Audio², and U-He³ have created tape emulating plugins. Unfortunately, the existing literature on analog tape emulation is somewhat lacking. While Arnadottir et al. [3] and Valimaki et al. [4] describe the emulation of tape echo/delay devices, and Valimaki et al [5] describe the emulation of disk-based audio recording media, we were unable to locate any existing research directly discussing digital emulation of the magnetisation process, a gap in research that this publication intends to fill. That said, Kadis [1] and Camras [6] discuss musical use of analog tape recorders in a useful technical manner, and Bertram [7] gives a in-depth technical description of the physical underpinnings of analog magnetic recording; this work intends to build on their foundations. While tape machines also contain electronic circuits that contribute to the machine's characteristic sound, this publication only considers processes that relate directly to tape magnetisation. For readers wishing to emulate tape machine circuits, a good overview of circuit modelling techniques can be found in [8].

¹<https://www.waves.com/plugins/j37-tape>

²<https://www.uaudio.com/uad-plugins/plugin-bundles/magnetic-tape-bundle.html>

³<https://u-he.com/products/satin/>



Figure 1: An analog tape recorder (Sony TC 260)

2. CONTINUOUS TIME SYSTEM

Audio recorded to and played back from a tape machine can be thought of as going through three distinct processors: the record head, tape magnetisation, and the play head.

2.1. The Record Head

For an instantaneous input current $I(t)$, the magnetic field output of the record head is given as a function of distance along the tape (x), and depth into the tape (y). Using the Karlqvist medium field approximation, we find [7]:

$$H_x(x, y) = \frac{1}{\pi} H_0 \left(\tan^{-1} \left(\frac{(g/2) + x}{y} \right) + \tan^{-1} \left(\frac{(g/2) - x}{y} \right) \right) \quad (1)$$

$$H_y(x, y) = \frac{1}{2\pi} H_0 \ln \left(\frac{((g/2) - x)^2 + y^2}{((g/2) + x)^2 + y^2} \right) \quad (2)$$

where H_x and H_y are components of the magnetic field \vec{H} , g is the head gap, and H_0 is the deep gap field, given by:

$$H_0 = \frac{NEI}{g} \quad (3)$$

where N is the number of turns of wire around the head, and E is the head efficiency which can be calculated by:

$$E = \frac{1}{1 + \frac{lA_g}{\mu_r g} \int_{core} \frac{d\vec{l}}{A(l)}} \quad (4)$$

where A_g is the gap area, μ_r is the core permeability relative to free space (μ_0), and $A(l)$ is the cross-sectional area at a point l along its circumferential length.

2.2. Tape Magnetisation

The magnetisation recorded to tape from a magnetic field can be described using a hysteresis loop, as follows [7]:

$$\vec{M}(x, y) = F_{Loop}(\vec{H}(x, y)) \quad (5)$$

where F_{Loop} is a generalized hysteresis function.

Using the Jiles-Atherton magnetisation model, the following differential equation describes magnetisation in some direction (M) as a function of the magnetic field in that direction (H) [9]:

$$\frac{dM}{dH} = \frac{(1-c)\delta_M(M_{an}-M)}{(1-c)\delta_S k - \alpha(M_{an}-M)} + c \frac{dM_{an}}{dH} \quad (6)$$

where c is the ratio of normal and anhysteretic initial susceptibilities, k is a measure of the width of the hysteresis loop, α is a mean field parameter, representing inter-domain coupling, and δ_S and δ_M are given by:

$$\delta_S = \begin{cases} 1 & \text{if } H \text{ is increasing} \\ -1 & \text{if } H \text{ is decreasing} \end{cases} \quad (7)$$

$$\delta_M = \begin{cases} 1 & \text{if } \delta_S \text{ and } M_{an} - M \text{ have the same sign} \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

M_{an} is the anisotropic magnetisation given by:

$$M_{an} = M_s L\left(\frac{H + \alpha M}{a}\right) \quad (9)$$

where M_s is the magnetisation saturation, a characterizes the shape of the anhysteretic magnetisation and L is the Langevin function:

$$L(x) = \coth(x) - \frac{1}{x} \quad (10)$$

2.3. Play Head

2.3.1. Ideal Playback Voltage

The ideal playback voltage as a function of tape magnetisation at a point x along the tape is given by [7]:

$$V(x) = NWEv\mu_0 \int_{-\infty}^{\infty} dx' \int_{-\delta/2}^{\delta/2} dy' \vec{h}(x' + x, y') \cdot \frac{\vec{M}(x', y')}{dx} \quad (11)$$

where N is the number of turns of wire, W is the width of the playhead, E is the playhead efficiency, v is the tape speed, δ is the thickness of the tape, and μ_0 is the permeability of free space. Note that $V(x) = V(vt)$ for constant v . $\vec{h}(x, y)$ is defined as:

$$\vec{h}(x, y) \equiv \frac{\vec{H}(x, y)}{NIE} \quad (12)$$

where $\vec{H}(x, y)$ can be calculated by eqs. (1) and (2).

2.3.2. Loss Effects

There are several frequency-dependent loss effects associated with playback, described as follows [1]:

$$V(t) = V_0(t) [e^{-kd}] \left[\frac{1 - e^{-k\delta}}{k\delta} \right] \left[\frac{\sin(kg/2)}{kg/2} \right] \quad (13)$$

for sinusoidal input $V_0(t)$, where k is the wave number, d is the distance between the tape and the playhead, g is the gap width of the play head, and again δ is the thickness of the tape. The wave number is given by:

$$k = \frac{2\pi f}{v} \quad (14)$$

where f is the frequency and v is the tape speed.

3. DIGITIZING THE SYSTEM

3.1. Record Head

For simplicity, let us assume,

$$\vec{H}(x, y, t) = \vec{H}(0, 0, t) \quad (15)$$

In this case $H_y \equiv 0$, and $H_x \equiv H_0$. Thus,

$$H(t) = \frac{NEI(t)}{g} \quad (16)$$

or,

$$\hat{H}(n) = \frac{NE\hat{I}(n)}{g} \quad (17)$$

3.2. Hysteresis

Beginning from eq. (6), we can find the derivative of M w.r.t. time, as in [9]:

$$\frac{dM}{dt} = \frac{\frac{(1-c)\delta_M(M_s L(Q) - M)}{(1-c)\delta_S k - \alpha(M_s L(Q) - M)} \dot{H} + c \frac{M_s}{a} \dot{H} L'(Q)}{1 - c\alpha \frac{M_s}{a} L'(Q)} \quad (18)$$

where $Q = \frac{H + \alpha M}{a}$, and

$$L'(x) = \frac{1}{x^2} - \coth^2(x) + 1 \quad (19)$$

Note that eq. (18) can also be written in the general form for non-linear Ordinary Differential Equations:

$$\frac{dM}{dt} = f(t, M, \vec{u}) \quad (20)$$

where $\vec{u} = \begin{bmatrix} H \\ \dot{H} \end{bmatrix}$.

Using the trapezoidal rule for derivative approximation, we find:

$$\dot{H}(n) = 2 \frac{\hat{H}(n) - \hat{H}(n-1)}{T} - \dot{H}(n-1) \quad (21)$$

We can use the Runge-Kutta 4th-order method [8] to find an explicit solution for $\hat{M}(n)$:

$$\begin{aligned} k_1 &= Tf\left(n-1, \hat{M}(n-1), \hat{u}(n-1)\right) \\ k_2 &= Tf\left(n-\frac{1}{2}, \hat{M}(n-1) + \frac{k_1}{2}, \hat{u}\left(n-\frac{1}{2}\right)\right) \\ k_3 &= Tf\left(n-\frac{1}{2}, \hat{M}(n-1) + \frac{k_2}{2}, \hat{u}\left(n-\frac{1}{2}\right)\right) \\ k_4 &= Tf\left(n, \hat{M}(n-1) + k_3, \hat{u}(n)\right) \\ \hat{M}(n) &= \hat{M}(n-1) + \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} \end{aligned} \quad (22)$$

We use linear interpolation to find the half-sample values used to calculate k_2 and k_3 . Note that many audio DSP systems prefer lower-order implicit methods such as the trapezoidal rule to solve differential equations rather than a higher-order method like the Runge-Kutta method [8]. However in this case, it was found that the lower-order methods quickly became unstable for high-frequency input, particularly when the input is modulated by a bias signal (see Section 4.3).

3.2.1. Numerical Considerations

To account for rounding errors in the Langevin function for values close to zero, we use the following approximation about zero, as in [9]:

$$L(x) = \begin{cases} \coth(x) - \frac{1}{x} & \text{for } |x| > 10^{-4} \\ \frac{x}{3} & \text{otherwise} \end{cases} \quad (23)$$

$$L'(x) = \begin{cases} \frac{1}{x^2} - \coth^2(x) + 1 & \text{for } |x| > 10^{-4} \\ \frac{1}{3} & \text{otherwise} \end{cases} \quad (24)$$

Additionally, $\tanh(x)$, and by extension $\coth(x)$ is a rather computationally expensive operation. With this in mind, for real-time implementation, we approximate $\coth(x)$ as the reciprocal of a Gaussian continued fraction for $\tanh(x)$ [10], namely

$$\tanh(x) = \frac{x}{1 + \frac{x^2}{3 + \frac{x^2}{5 + \frac{x^2}{7}}}} \quad (25)$$

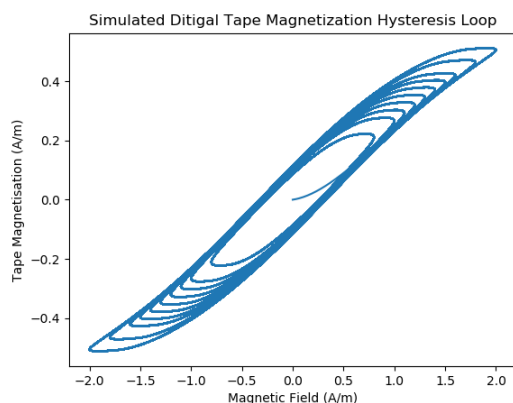


Figure 2: Digitized Hysteresis Loop Simulation

3.2.2. Simulation

The digitized hysteresis loop was implemented and tested offline in Python, using the constants M_s , a , α , k , and c from [11]. For a sinusoidal input signal with frequency 2kHz, and varying amplitude from 800 - 2000 Amperes per meter, fig. 2 shows the Magnetisation output.

3.3. Play Head

By combining eq. (11) with eqs. (12) and (16), we get:

$$V(t) = NWEv\mu_0gM(t) \quad (26)$$

or,

$$\hat{V}(n) = NWEv\mu_0g\hat{M}(n) \quad (27)$$

3.3.1. Loss Effects

In the real-time system, we model the playhead loss effects with an FIR filter, derived by taking the inverse DFT of the loss effects described in eq. (13). It is worth noting that as in eq. (14), the loss effects, and therefore the FIR filter are dependent on the tape speed.

The loss effects filter was implemented and tested offline in Python with tape-head spacing of 20 microns, head gap width of 5 microns, tape thickness of 35 microns, and tape speed of 15 ips. The following plot shows the results of the simulation, with a filter order of 100.

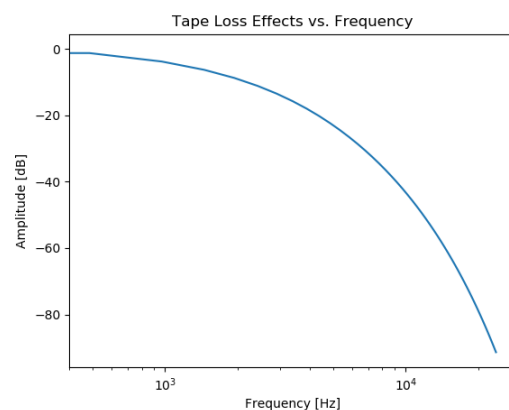


Figure 3: Frequency Response of Playhead Loss Effects

4. TAPE AND TAPE MACHINE PARAMETERS

In the following sections, we describe the implementation of a real-time model of a Sony TC-260 tape machine, while attempting to preserve generality so that the process can be repeated for any similar reel-to-reel tape machine.

4.1. Tape Parameters

A typical reel-to-reel tape machine such as the Sony TC-260 uses Ferric Oxide (γF_2O_3) magnetic tape. The following properties of the tape are necessary for the tape hysteresis process eq. (18):

- Magnetic Saturation (M_s): For Ferric Oxide tape the magnetic saturation is $3.5e5$ (A/m) [12].
- Hysteresis Loop Width (k): For soft materials, k can be approximated as the coercivity, H_c [13]. For Ferric Oxide, H_c is approximately 27 kA/m [12].
- Anhyseric magnetisatation (a): Knowing the coercivity and remnance magnetism of Ferric Oxide [12], we can calculate $a = 22$ kA/m by the method described in [13].
- Ratio of normal and hysteris initial susceptibilities (c): From [13], $c = 1.7e-1$.
- Mean field parameter (α): From [13], $\alpha = 1.6e-3$.

4.2. Tape Machine Parameters

4.2.1. Record Head

To determine the magnetic field output of the record head using eq. (17), the following parameters are necessary:

- Input Current ($\hat{I}(n)$): For the Sony TC-260 the input current to the record head is approximately 0.1 mA peak-to-peak [14].
- Gap Width (g): The gap width for recording heads can range from 2.5 to 12 microns [1].
- Turns of wire (N): The number of turns of wire is typically on the order of 100 [7].
- Head Efficiency (E): The head efficiency is typically on the order of 0.1 [7].

These values result in a peak-to-peak magnetic field of approximately $5e5$ A/m.

4.2.2. Play Head

Similar to the record head, the following parameters are needed to calculate the output voltage using eqs. (13) and (27) (note that values are only included here if notably different from the record head):

- Gap Width (g): The play head gap width ranges from 1.5 to 6 microns [1].
- Head Width (W): For the Sony TC-260, the play head width is 0.125 inches (note that this is the same as the width of one track on the quarter-inch tape used by the machine) [14].
- Tape Speed (v): The Sony TC-260 can run at 3.75 inches per second (ips), or 7.5 ips [14]. Note that many tape machines can run at 15 or 32 ips [1].
- Tape Thickness (δ): Typical tape that would be used with the TC-260 is on the order of 35 microns thick [14].
- Spacing (d): The spacing between the tape and the play head is highly variable between tape machines. For a typical tape machine spacing can be as high as 20 microns [1].

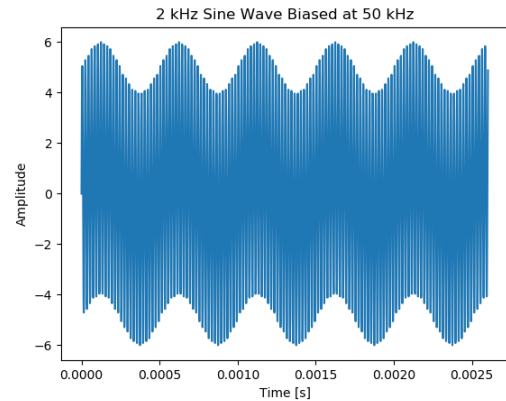


Figure 4: Example of a biased signal

4.3. Tape Bias

A typical analog recorder adds a high-frequency "bias" current to the signal to avoid the "deadzone" effect when the input signal crosses zero, as well as to linearize the output. The input current to the record head can be given by [6]:

$$\hat{I}_{head}(n) = \hat{I}_{in}(n) + B \cos(2\pi f_{bias}nT) \quad (28)$$

Where the amplitude of the bias current B is usually about one order of magnitude larger than the input, and the bias frequency f_{bias} is well above the audible range. Figure 4 shows a unit-amplitude, 2 kHz sine wave biased by a 50 kHz sine wave with amplitude 5. To recover the correct output signal, tape machines use a lowpass filter, with a cutoff frequency well below the bias frequency, though still above the audible range [1].

For the Sony TC-260, the bias frequency is 55 kHz, with a gain of 5 relative to the input signal. The lowpass filter used to recover the audible signal has a cutoff at 24 kHz, though note that due to the frequency response of the playhead loss effects, the effects of this filter may be essentially negligible to the real time system. [14]

4.4. Wow and Flutter

Each tape machine has characteristic timing imperfections known as "wow" and/or "flutter." These imperfections are caused by minor changes in speed from the motors driving the tape reels, and can cause fluctuations in the pitch of the output signal. To characterize these timing imperfections, we use a method similar to [3]: We recorded a pulse train of 1000 pulses through a TC-260, then recorded the pulses back from the tape. Figure 5 shows a section of a superimposed plot of the original pulse train against the pulse train recorded from the tape machine. From this data, we were able to generate a periodic function that accurately models the timing imperfections of the TC-260. The process was performed at both 7.5 ips and 3.75 ips. In the real-time system, the timing imperfection model is used to inform a modulating delay line, to achieve the signature "wow" effect of an analog tape machine.

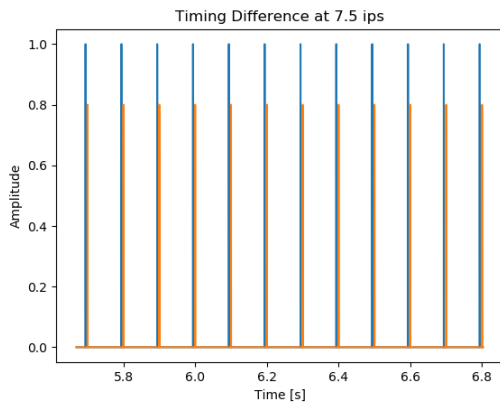


Figure 5: *Input pulse train superimposed with pulse train recorded from TC-260*

Record Head Constants	
Turns of wire (N)	100
Head Efficiency (E)	0.1
Record Head Gap (g)	6 (microns)
Tape Constants	
Magnetic Saturation (M_s)	3.5e5 (A/m)
Hysteresis Loop Width (k)	27 (kA/m)
Anhyseric Magnetisation (a)	22 (kA/m)
Ratio of magnetic susceptibilities (c)	1.7e-1
Play Head Constants	
Play Head Width (W)	0.125 (in)

Figure 6: *Constant values for model implementation*

5. REAL-TIME IMPLEMENTATION

We implemented our physical model of the Sony TC-260 as a VST audio plugin using the JUCE framework. Figure 7 shows the signal flow of the system in detail. We allow the user to control parameters in real-time including the tape speed, bias gain, gap width, tape thickness, tape spacing, and flutter depth.

Bias modulation is implemented using eq. (28), where bias gain B and bias frequency f_{bias} are controlled by the user.

The record head transfer function calculates the record head magnetic field H from the input current I , and is implemented using eq. (17), with constant values shown in fig. 6.

The hysteresis process calculates the tape magnetisation M from the record head magnetic field H , and is implemented using the Runge-Kutta method described in eq. (22), with constant value defined in fig. 6.

The flutter process is implemented using a modulated delay line as described in section 4.4, with user controlled modulation depth.

The play head transfer calculates the play head voltage V from the tape magnetisation M , using eq. (27) and the loss effects filter

described in section 3.3.1. The gap width g , tape speed v , tape thickness δ , and spacing d are controlled by the user, and other constant values are shown in fig. 6.

C/C++ code for the full real-time implementation is open-source and is available on GitHub.⁴

5.1. Oversampling

If no oversampling is used, the system will be unstable for input signal at the Nyquist frequency, due to limitations of the trapezoid rule derivative approximation used in eq. (21). To avoid this instability, early versions of the real-time implementation used a lowpass filter with cutoff frequency just below Nyquist. However, due to aliasing caused by the nonlinearity of the tape hysteresis model, oversampling is necessary to mitigate aliasing artifacts [8]. Further, the system must be able to faithfully recreate not only the frequencies in the audible range but the bias frequency as well. Since the TC-260 uses a bias frequency of 55 kHz [14] and the minimum standard audio sampling rate is 44.1 kHz, a minimum oversampling factor of 3x is required. However, since the biased signal is then fed into the hysteresis model, even more oversampling is required to avoid aliasing. With these considerations in mind, our system uses an oversampling factor of 16x.

5.2. Results

In subjective testing, our physical model sounds quite convincing, with warm, tape-like distortion, and realistic sounding flutter. The high-frequency loss and low-frequency “head bump” change correctly at different tape speeds, and are approximately within the frequency response specifications of the TC-260 service manual [14]. When the input to the plugin is silent, the hysteresis processing of the bias signal produces a very accurate “tape hiss” sound. The distortion and frequency response characteristics of our model are subjectively very close when compared to the output of an actual TC-260, though not nearly close enough to “fool” the listener. Additionally, as the bias gain is lowered, the “deadzone” effect appears exactly as expected [6]. The largest difference between the model and the physical machine is the subtle electrical and mechanical noises and dropouts present in the physical machine, presumably caused by the age and wear-and-tear of the machine, which we did not attempt to characterize in our model. Figure 8 shows the results of tests performed on the real-time system, including an example of the “deadzone” effect, and the timing irregularities or “flutter”. Figure 9 shows a comparison of hysteresis characteristics between the real-time software model and a physical Sony TC-260 tape machine. Note that some differences between the two hysteresis loops may be due to the circuitry of the tape machine that we did not attempt to model in the real-time system. Audio examples from the real-time system can be found online.⁵

5.3. Evaluation

While there is an audible difference between the real-time software model and a physical Sony TC-260, the most fundamental aspects of the tape machine sound including tape saturation, tape

⁴<https://github.com/jatinchowdhury18/AnalogTapeModel>

⁵<https://ccrma.stanford.edu/~jatin/420/tape/>

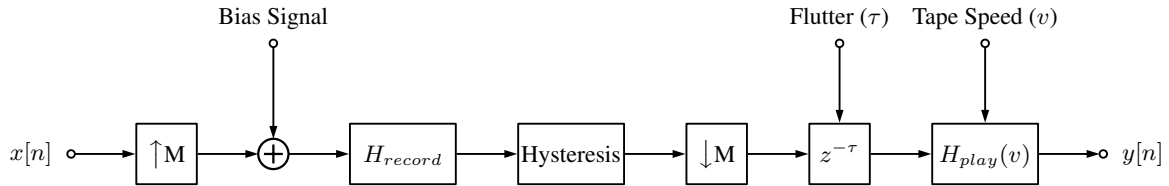


Figure 7: Flowchart of realtime system: M is the oversampling factor, H_{record} is the transfer function of the record head, and H_{play} is the play head transfer function including loss effects, and a de-biasing filter.

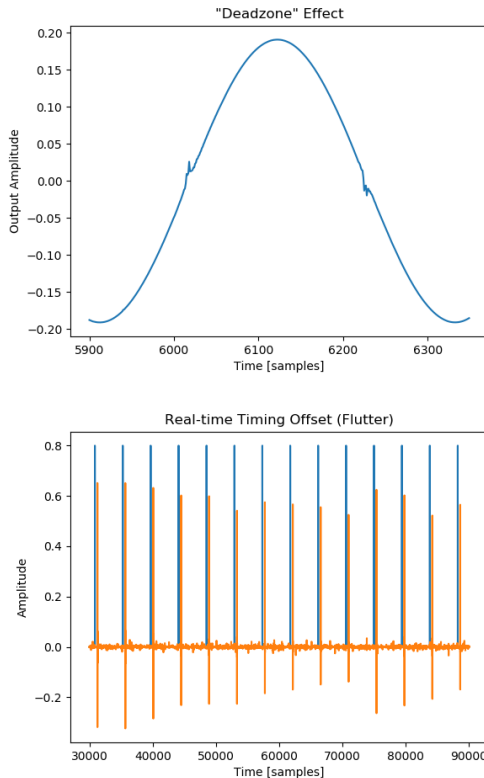


Figure 8: Testing results for real-time system: sine wave output with no biasing (above), input vs. output pulse train comparison (below).

hiss, flutter/wow, and frequency response, are clearly audible and sound very accurate. The main distinctions between the two systems can be attributed to the tape machine circuitry (in particular the TC-260 contains two shelving filters), as well as mechanical wear of the system, both elements that were not considered in our model.

In our opinion, the strongest proof of the efficacy of our model is that the model responds accurately to the adjustment of model parameters. In particular, the hysteresis process reacted correctly to changes in input gain (saturating for overdriven input, or fading into tape hiss for underdriven input), as well as bias gain (saturation for overbiasing, or “deadzone” effect for underbiasing). Additionally, adjusting the loss effect parameters correctly demonstrated known tape machine phenomena including head “bump”

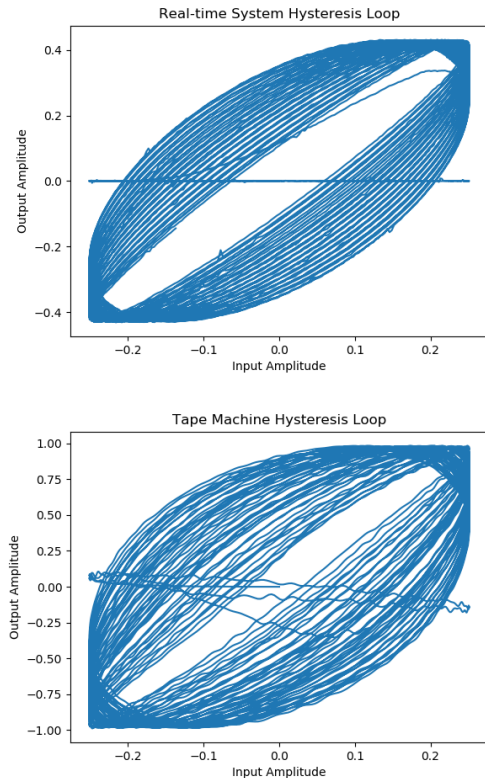


Figure 9: Test results comparing real-time system to Sony TC-260 physical unit: hysteresis loop for real-time system (above), hysteresis loop for TC-260 (below).

(a resonance at the wavelength of the play head gap width), and spacing loss (filtering due to the spacing between the the play head and tape). The reader is invited to download the plugin (available with the source code) and evaluate the model for themselves. In conclusion, we believe that our model successfully approximates the physical tape recording process, however for those wishing to model a full tape machine, we suggest using this model in combination with a model of the tape machine’s circuits.

6. FUTURE IMPROVEMENTS

6.1. Spatial Magnetic Effects

The most obvious improvement to be made for the physical model is the inclusion of spatial effects of the tape. In particular, the approximations made in eq. (15), negate any effects caused by mag-

netisation along the longitudinal length of the tape, and into the depth of the tape. Including spatial effects would involve deriving digital analogues for eqs. (1), (2) and (11), and re-deriving eq. (22) to take an 2-dimensional magnetic field input at every timestep, rather than the zero-dimensional input it currently takes. This change would greatly increase the computational complexity of the system. At an oversampling rate of 16x, using just 100 spatial samples would be 1600x more computationally complex than the current system.

7. ACKNOWLEDGEMENTS

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8. REFERENCES

- [1] Jay Kadis, *The Science of Sound Recording*, Focal Press, Waltham, MA, 1 edition, 2012.
- [2] Brian Eno, *A Year with Swollen Appendices*, Faber and Faber, 1996.
- [3] Steinunn Arnardottir, Jonathan S. Abel, and Julius O. Smith III, “A digital model of the echoplex tape delay,” in *Audio Engineering Society Convention 125*, 10 2008.
- [4] Vesa Valimaki, Stefan Bilbao, Julius Smith, J S. Abel, Jyri Pakarinen, and D Berners, *Virtual Analog Effects*, pp. 473 – 522, 03 2011.
- [5] Vesa Valimaki, Sira Gonzalez, Ossi Kimmelma, and Jukka Parviainen, “Digital audio antiquing - signal processing methods for imitating the sound quality of historical recordings,” *AES: Journal of the Audio Engineering Society*, vol. 56, pp. 115–139, 03 2008.
- [6] Marvin Camras, *Magnetic Recording Handbook*, Van Nostrand Reinhold Co., New York, NY, USA, 1987.
- [7] H. N. Bertram, *Theory of Magnetic Recording*, 04 1994.
- [8] D.T. Yeh, *Digital Implementation of Musical Distortion Circuits by Analysis and Simulation*, Ph.D. thesis, Stanford University, 06 2009.
- [9] Martin Holters and Udo Zolzer, “Circuit simulation with inductors and transformers based on the Jiles-Atherton model of magnetization,” 09 2016.
- [10] H. S. Wall, *Analytic Theory of Continued Fractions*, Chelsea, New York, 1948.
- [11] D. C. Jiles and D. L. Atherton, “Theory of ferromagnetic hysteresis,” *Journal of Magnetism and Magnetic Materials*, vol. 61, pp. 48–60, 09 1986.
- [12] D. Jiles, *Introduction to Magnetism and Magnetic Materials*, CRC Press, 2015.
- [13] D. C. Jiles, J. B. Thoeke, and M. K. Devine, “Numerical determination of hysteresis parameters for the modeling of magnetic properties using the theory of ferromagnetic hysteresis,” *IEEE Transactions on Magnetics*, vol. 28, pp. 27–35, 01 1992.
- [14] Sony Corporation, *Sony TC-260 Service Manual*, 1965.