REAL-TIME PHYSICAL MODEL FOR ANALOG TAPE MACHINES

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1. ABSTRACT

For decades, analog magnetic tape recording was the most popular method for recording music, but has been replaced over the past 30 years first by DAT tape, and then by DAWs [1]. Despite being replaced by higher quality technology, many have sought to recreate a "tape" sound through digital effects, despite the distortion, tape "hiss", and other oddities analog tape produced. The following describes a physical modelling process for an analog tape machine, starting from basic physical principles.

"Whatever you now find weird, ugly, uncomfortable, and nasty about a new medium will surely become its signature. CD distortion, the jitteriness of digital video, the crap sound of 8-bit - all of these will be cherished and emulated as soon as they can be avoided." -Brian Eno [2].

2. CONTINUOUS TIME SYSTEM

Audio recorded to and played back from a tape machine can be thought of as going through three distinct processors: the record head, tape magnetisation, and the play head.

2.1. The Record Head

For an instantaneous input current $I(t)$, the magnetic field output of the record head is given as a function of distance along the tape 'x', and depth into the tape 'y' (Karlqvist medium field approximation) [3].

$$H_x(x,y) = \frac{1}{\pi} H_0 \Big(\tan^{-1} \Big(\frac{(g/2)+x}{y} \Big) + \tan^{-1} \Big(\frac{(g/2)-x}{y} \Big) \Big) \quad (1)$$

$$H_y(x,y) = \frac{1}{2\pi} H_0 \ln \Big(\frac{((g/2)-x)^2 + y^2}{((g/2)+x)^2 + y^2} \Big) \quad (2)$$

where g = head gap, and H_0 = deep gap field, given by:

$$H_0 = \frac{NEI}{g} \quad (3)$$

where N = number of turns coils of wire around the head, and E = head efficiency which can be calculated by:

$$E = \frac{1}{1 + \frac{lA_g}{\mu_r g} \int_{core} \frac{d\vec{l}}{A(l)}} \quad (4)$$

where A_g is the gap area, μ_r is the core permeability relative to free space (μ_0), g if the gap width, and $A(l)$ is the cross-sectional area of the core as a function of length.

2.2. Tape Magnetisation

The magnetic field being recorded to tape can be described using a hysteresis loop, as follows [3]:

$$\vec{M}(x,y) = F_{Loop}(\vec{H}(x,y)) \quad (5)$$

where F_{Loop} is a generalized hysteresis function.

Using the Jiles-Atherton magnetisation model, the following differential equation describes magnetisation 'M' as a function of magnetic field 'H' [4]:

$$\frac{dM}{dH} = \frac{(1-c)\delta_M (M_{an} - M)}{(1-c)\delta k - \alpha (M_{an} - M)} + c \frac{dM_{an}}{dH} \quad (6)$$

where c is the ration of normal and anhysteric initial susceptibilities, k is a measure of the width of the hysteresis loop, α is a mean field parameter, representing inter-domain coupling, and

$$\delta = \begin{cases} 1 & \text{if H is increasing} \\ -1 & \text{if H is decreasing} \end{cases} \quad (7)$$

$$\delta_M = \begin{cases} 1 & \text{if } \delta \text{ and } M_{an} - M \text{ have the same sign} \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

M_{an} is the anisotropic magnetisation given by:

$$M_{an} = M_s L \Big(\frac{H + \alpha M}{a} \Big) \quad (9)$$

where M_s is the magnetisation saturation, a characterizes the shape of the anhysteric magnetisation and L is the Langevin function:

$$L(x) = \coth(x) - \frac{1}{x} \quad (10)$$

2.3. Play Head

2.3.1. *Ideal Playback Voltage*

The ideal playback voltage as a function of tape magnetisation is given by [3]:

$$V(x) = NWEv\mu_0 \int_{-\infty}^{\infty} dx' \int_{-\delta/2}^{\delta/2} dy' \vec{h}(x'+x, y') \cdot \frac{\vec{M}(x',y')}{dx} \quad (11)$$

where N = number of turns of wire, W = width of the playhead, E = playhead efficiency, v = tape speed, and μ_0 is the permeability of free space. Note that $V(x) = V(vt)$ for constant v. $\vec{h}(x,y)$ is defined as:

$$\vec{h}(x,y) \equiv \frac{\vec{H}(x,y)}{NIE} \tag{12}$$

where $\vec{H}(x,y)$ can be calculated by eqs. (1) and (2).

2.3.2. *Loss Effects*

There are several frequency-dependent loss effects associated with playback, described as follows [1]:

$$V(x) = V_0(x)[e^{-kd}]\left[\frac{1-e^{-k\delta}}{k\delta}\right]\left[\frac{\sin(kg/2)}{kg/2}\right] \tag{13}$$

for sinusoidal input, where k = wave number, d is the distance between the tape and the playhead, g is the gap width of the play head, and δ is the thickness of the tape.

3. DIGITIZING THE SYSTEM

3.1. Record Head

For simplicity, let us assume,

$$\vec{H}(x,y,t) = \vec{H}(0,0,t) \tag{14}$$

In this case $H_y \equiv 0$, and $H_x \equiv H_0$. Thus,

$$H(t) = \frac{NEI(t)}{g} \tag{15}$$

or,

$$\hat{H}(n) = \frac{NE\hat{I}(n)}{g} \tag{16}$$

3.2. Hysteresis

Beginning from eq. (6), we can find the derivative of M w.r.t. time, as in [4]:

$$\frac{dM}{dt} = \frac{\frac{(1-c)\delta_M(M_sL(Q)-M)}{(1-c)\delta k-\alpha(M_sL(Q)-M)}\frac{dH}{dt} + c\frac{M_s}{a}\frac{dH}{dt}L'(Q)}{1-c\alpha\frac{M_s}{a}L'(Q)} \tag{17}$$

where $Q = \frac{H+\alpha M}{a}$, and

$$L'(x) = \frac{1}{x^2} - \coth^2(x) + 1 \tag{18}$$

Note that eq. (17) can also be written in the general form for non-linear Ordinary Differential Equations:

$$\frac{dM}{dt} = f(t, M, \vec{u}) \tag{19}$$

where $\vec{u} = \begin{bmatrix} H \\ \dot{H} \end{bmatrix}$.

Using the trapezoidal rule for derivative approximation, we find:

$$\hat{\dot{H}}(n) = 2\frac{\hat{H}(n) - \hat{H}(n-1)}{T} - \hat{\dot{H}}(n-1) \tag{20}$$

We can use the Runge-Kutta 4th order method [5] to find an explicit solution for $\hat{M}(n)$:

$$\begin{aligned}
k_1 &= Tf\Big(n-1, \hat{M}(n-1), \hat{\vec{u}}(n-1)\Big) \\
k_2 &= Tf\Big(n-\frac{1}{2}, \hat{M}(n-1)+\frac{k_1}{2}, \hat{\vec{u}}\Big(n-\frac{1}{2}\Big)\Big) \\
k_3 &= Tf\Big(n-\frac{1}{2}, \hat{M}(n-1)+\frac{k_2}{2}, \hat{\vec{u}}\Big(n-\frac{1}{2}\Big)\Big) \\
k_4 &= Tf\Big(n, \hat{M}(n-1)+k_3, \hat{\vec{u}}(n)\Big) \\
\hat{M}(n) &= \hat{M}(n-1) + \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6}
\end{aligned} \tag{21}$$

We use linear interpolation to find the half-sample values used to calculate k_2 and k_3.

3.2.1. *Numerical Considerations*

To account for rounding errors in the Langevin function for values close to zero, we use the following approximation about zero, as in [4]:

$$L(x) = \begin{cases} \coth(x) - \frac{1}{x} & \text{for } |x| > 10^{-4} \\ \frac{x}{3} & \text{otherwise} \end{cases} \tag{22}$$

$$L'(x) = \begin{cases} \frac{1}{x^2} - \coth^2(x) + 1 & \text{for } |x| > 10^{-4} \\ \frac{1}{3} & \text{otherwise} \end{cases} \tag{23}$$

3.2.2. *Simulation*

The digitized hysteresis loop was implemented and tested offline in `Python`, using the constants M_s, a, α, k, and c from [6]. For a sinusoidal input signal with frequency 2kHz, and varying amplitude from 800 - 2000 Amperes per meter, the following plot shows the Magnetisation output.

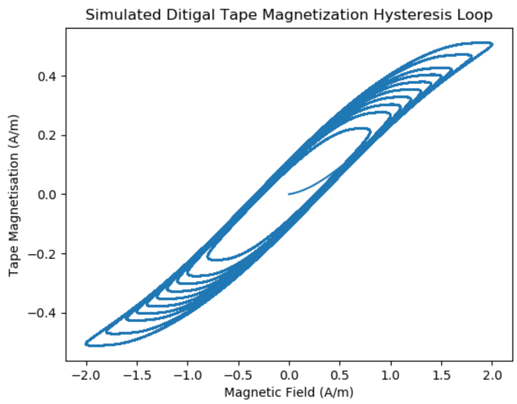

Figure 1: *Digitized Hysteresis Loop Simulation*

3.3. Play Head

By combining eq. (11) with eqs. (12) and (15), we get:

$$V(t) = NWEv\mu_0 gM(t) \tag{24}$$

or,

$$\hat{V}(n) = NWEv\mu_0 g\hat{M}(t) \tag{25}$$

The loss effects described in eq. (13) can be modelled as a series of digital filters.

3.4. Oversampling

To reduce frequency aliasing caused by the hysteresis non-linearity, we recommend oversampling by at least 4x.

4. SYSTEM PARAMETERS

4.1. Tape Parameters

For the tape hysteresis eq. (17), following parameters are necessary:

- Magnetic Saturation (M_s): For Ferric Oxide (γF_2O_3) magnetic tape as would typically be used for a reel-to-reel machine, the magnetic saturation is $3.5e5$ (A/m) [7]
- Hysteresis Loop Width (k): For soft materials, k can be approximated as the coercivity, H_c [8]. For Ferric Oxide, H_c is approximately 27 kA/m [7].
- a: Knowing the coercivity and remnance magnetism of Ferric Oxide [7], we can calculate a = 22 kA/m by the method described includegraphics [8]
- c: From [8], c = 1.7e-1.
- alpha: From [8], alpha = 1.6e-3

4.2. Tape Bias

A typical analog recorder adds a high-frequency "bias" current to the signal to avoid the "deadzone" effect when the input signal crosses zero, as well as to linearize the output. The input signal to the record head can be given by [9]:

$$\hat{V}_{head}(n) = \hat{V}_{in}(n) + B\cos(2\pi f_{bias} nT) \tag{26}$$

Where the amplitude of the bias current B is at least one order of magnitude larger than the input. The output signal can be recovered after the recording and playback processes by subtracting out the bias current.

4.3. Tape Speed

5. REFERENCES

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