REAL-TIME PHYSICAL MODEL FOR SONY TC-260 TAPE MACHINE

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1. ABSTRACT

For decades, analog magnetic tape recording was the most popular method for recording music, but has been replaced over the past 30 years first by DAT tape, and then by DAWs [1]. Despite being replaced by higher quality technology, many have sought to recreate a "tape" sound through digital effects, despite the distortion, tape "hiss", and other oddities analog tape produced. To quote Brian Eno, "Whatever you now find weird, ugly, uncomfortable, and nasty about a new medium will surely become its signature. CD distortion, the jitteriness of digital video, the crap sound of 8-bit - all of these will be cherished and emulated as soon as they can be avoided." [2]. The following describes a physical model of an analog tape machine, starting from basic physical principles.

2. CONTINUOUS TIME SYSTEM

Audio recorded to and played back from a tape machine can be thought of as going through three distinct processors: the record head, tape magnetisation, and the play head.

2.1. The Record Head

For an instantaneous input current $I(t)$, the magnetic field output of the record head is given as a function of distance along the tape 'x', and depth into the tape 'y' (Karlqvist medium field approximation) [3].

$$H_x(x,y) = \frac{1}{\pi} H_0 \Big(\tan^{-1}\Big(\frac{(g/2)+x}{y}\Big) + \tan^{-1}\Big(\frac{(g/2)-x}{y}\Big)\Big) \quad (1)$$

$$H_y(x,y) = \frac{1}{2\pi} H_0 \ln\Big(\frac{((g/2)-x)^2+y^2}{((g/2)+x)^2+y^2}\Big) \quad (2)$$

where g = head gap, and H_0 = deep gap field, given by:

$$H_0 = \frac{NEI}{g} \quad (3)$$

where N = number of turns coils of wire around the head, and E = head efficiency given by:

$$E = \frac{1}{1 + \frac{lA_g}{\mu_r g}\int_{core}\frac{d\vec{l}}{A(l)}} \quad (4)$$

where A_g is the gap area, μ_r is the core permeability relative to free space (μ_0), g if the gap width, and $A(l)$ is the cross-sectional area of the core as a function of length.

2.2. Tape Magnetisation

The magnetic field being recorded to tape can be described using a hysteresis loop, as follows [3]:

$$\vec{M}(x,y) = F_{Loop}(\vec{H}(x,y)) \quad (5)$$

where F_{Loop} is a generalized hysteresis function.

Using the Jiles-Atherton magnetisation model, the following differential equation describes magnetisation 'M' as a function of magnetic field 'H' [4]:

$$\frac{dM}{dH} = \frac{(1-c)\delta_M(M_{an}-M)}{(1-c)\delta k - \alpha(M_{an}-M)} + c\frac{dM_{an}}{dH} \quad (6)$$

where c is the ration of normal and anhysteric initial susceptibilities, k is a measure of the width of the hysteresis loop, α is a mean field parameter, representing inter-domain coupling, and

$$\delta = \begin{cases} 1 & \text{if H is increasing} \\ -1 & \text{if H is decreasing} \end{cases} \quad (7)$$

$$\delta_M = \begin{cases} 1 & \text{if } \delta \text{ and } M_{an}-M \text{ have the same sign} \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

M_{an} is the anisotropic magnetisation given by:

$$M_{an} = M_s L\Big(\frac{H+\alpha M}{a}\Big) \quad (9)$$

where M_s is the magnetisation saturation, a characterizes the shape of the anhysteric magnetisation and L is the Langevin function:

$$L(x) = \coth(x) - \frac{1}{x} \quad (10)$$

2.3. Play Head

2.3.1. *Ideal Playback Voltage*

The ideal playback voltage as a function of tape magnetisation is given by [3]:

$$V(x) = NWEv\mu_0 \int_{-\infty}^{\infty} dx' \int_{-\delta/2}^{\delta/2} dy' \vec{h}(x'+x, y') \cdot \frac{\vec{M}(x',y')}{dx} \quad (11)$$

where N = number of turns of wire, W = width of the playhead, E = playhead efficiency, v = tape speed, and μ_0 is the permeability of free space. Note that $V(x) = V(vt)$ for constant v. $\vec{h}(x,y)$ is defined as:

$$\vec{h}(x,y) \equiv \frac{\vec{H}(x,y)}{NIE} \tag{12}$$

where $\vec{H}(x,y)$ can be calculated by eqs. (1) and (2).

2.3.2. Loss Effects

There are several frequency-dependent loss effects associated with playback, described as follows [1]:

$$V(x) = V_0(x)[e^{-kd}]\Big[\frac{1-e^{-k\delta}}{k\delta}\Big]\Big[\frac{\sin(kg/2)}{kg/2}\Big] \tag{13}$$

where k = wave number, d is the distance between the tape and the playhead, g is the gap width of the play head, and δ is the thickness of the tape.

3. DIGITIZING THE SYSTEM

3.1. Record Head

For simplicity, let us assume,

$$\vec{H}(x,y,t) = \vec{H}(0,0,t) \tag{14}$$

In this case $H_y \equiv 0$, and $H_x \equiv H_0$. Thus,

$$H(t) = \frac{NEI(t)}{g} \tag{15}$$

or,

$$\hat{H}(n) = \frac{NE\hat{I}(n)}{g} \tag{16}$$

3.2. Hysteresis

Beginning from eq. (6), we can find the derivative of M w.r.t. time, as in [4]:

$$\frac{dM}{dt} = \frac{(1-c)\delta_M(M_sL(Q)-M)}{(1-c)\delta k - \alpha(M_sL(Q)-M)}\frac{dH}{dt} + c\frac{M_s}{a}\Big(\frac{dH}{dt}+\alpha\frac{dM}{dt}\Big)L'(Q) \tag{17}$$

where $Q = \frac{H+\alpha M}{a}$, and

$$L'(x) = \frac{1}{x^2} - \coth^2(x) + 1 \tag{18}$$

Taking the second derivative, we find:

$$\frac{d^2M}{dt^2} = \frac{(1-c)\delta_M(M_sL(Q)-M)}{(1-c)\delta k - \alpha(M_sL(Q)-M)}\frac{d^2H}{dt^2} + \frac{(1-c)\delta_M(c\frac{M_s}{a}(\frac{dH}{dt}+\alpha\frac{dM}{dt})L'(Q)-\dot{M})}{(1-c)\delta k - \alpha(M_sL(Q)-M)}\frac{dH}{dt} + \frac{(1-c)\delta_M(M_sL(Q)-M)(-\alpha(c\frac{M_s}{a}(\frac{dH}{dt}+\alpha\frac{dM}{dt})L'(Q)-\dot{M}))}{((1-c)\delta k - \alpha(M_sL(Q)-M))^2}\frac{dH}{dt} + c\frac{M_s}{a}\Big(\frac{dH}{dt}c\frac{M_s}{a}(\frac{dH}{dt}+\alpha\frac{dM}{dt})L''(Q)+\frac{d^2H}{dt^2}L'(Q)+\alpha\frac{dM}{dt}c\frac{M_s}{a}(\frac{dH}{dt}+\alpha\frac{dM}{dt})L''(Q)+\alpha\frac{d^2M}{dt^2}L'(Q)\Big) \tag{19}$$

where

$$L''(x) = 2\coth(x)\cdot(\coth^2(x)-1) - \frac{2}{x^3} \tag{20}$$

Solving for $\frac{dM}{dt}$ and $\frac{d^2M}{dt^2}$, we get:

$$\frac{dM}{dt} = \frac{\frac{(1-c)\delta_M(M_sL(Q)-M)}{(1-c)\delta k-\alpha(M_sL(Q)-M)}\frac{dH}{dt} + c\frac{M_s}{a}\frac{dH}{dt}L'(Q)}{1-c\alpha\frac{M_s}{a}L'(Q)} = f(t,M,\vec{u}) \tag{21}$$

$$\frac{d^2M}{dt^2} = \frac{1}{1-c\alpha\frac{M_s}{a}L'(Q)}\Bigg[\frac{(1-c)\delta_M(M_sL(Q)-M)}{(1-c)\delta k - \alpha(M_sL(Q)-M)}\frac{d^2H}{dt^2} + \frac{(1-c)\delta_M(c\frac{M_s}{a}(\frac{dH}{dt}+\alpha\frac{dM}{dt})L'(Q)-\dot{M})}{(1-c)\delta k - \alpha(M_sL(Q)-M)}\frac{dH}{dt} + \frac{(1-c)\delta_M(M_sL(Q)-M)(-\alpha(c\frac{M_s}{a}(\frac{dH}{dt}+\alpha\frac{dM}{dt})L'(Q)-\dot{M}))}{((1-c)\delta k - \alpha(M_sL(Q)-M))^2}\frac{dH}{dt} + c\frac{M_s}{a}\Big(\frac{d^2H}{dt^2}L'(Q) + c\frac{M_s}{a}L''(Q)(\dot{H}+\alpha\dot{M})^2\Big)\Bigg] \tag{22}$$

where $\vec{u} = \begin{bmatrix} H \\ \dot{H} \end{bmatrix}$.

Using the trapezoidal rule:

$$\dot{\hat{H}}(n) = 2\frac{\hat{H}(n)-\hat{H}(n-1)}{T} - \dot{\hat{H}}(n-1) \tag{23}$$

$$\ddot{\hat{H}}(n) = 2\frac{\dot{\hat{H}}(n)-\dot{\hat{H}}(n-1)}{T} - \ddot{\hat{H}}(n-1) \tag{24}$$

Then, using the semi-implicit trapezoidal rule for non-linear differential equations [5].

$$\hat{M}(n) = \hat{M}(n-1) + \frac{T}{2}\frac{f[n,\hat{M}(n-1),\hat{\vec{u}}(n)] + f[n-1,\hat{M}(n-1),\hat{\vec{u}}(n-1)]}{1-\frac{T}{2}\ddot{\hat{M}}(n-1)} \tag{25}$$

3.3. Play Head

By combining eq. (11) with eqs. (12) and (15), we get:

$$V(t) = NWEv\mu_0 gM(t) \tag{26}$$

or,

$$\hat{V}(n) = NWEv\mu_0 g\hat{M}(t) \tag{27}$$

4. REFERENCES

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