REAL-TIME PHYSICAL MODEL FOR ANALOG TAPE MACHINES

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1. ABSTRACT

For decades, analog magnetic tape recording was the most popular method for recording music, but has been replaced over the past 30 years first by DAT tape, then by DAWs and audio interfaces [1]. Despite being replaced by higher quality technology, many have sought to recreate a "tape" sound through digital effects, despite the distortion, tape "hiss", and other oddities analog tape produced. The following describes a physical modelling process for an analog tape machine, starting from basic physical principles.

"Whatever you now find weird, ugly, uncomfortable, and nasty about a new medium will surely become its signature. CD distortion, the jitteriness of digital video, the crap sound of 8-bit - all of these will be cherished and emulated as soon as they can be avoided." -Brian Eno [2].

2. CONTINUOUS TIME SYSTEM

Audio recorded to and played back from a tape machine can be thought of as going through three distinct processors: the record head, tape magnetisation, and the play head.

2.1. The Record Head

For an instantaneous input current $I(t)$, the magnetic field output of the record head is given as a function of distance along the tape 'x', and depth into the tape 'y' (Karlqvist medium field approximation) [3].

$$H_x(x,y) = \frac{1}{\pi} H_0 \Big(\tan^{-1}\Big(\frac{(g/2)+x}{y}\Big) + \tan^{-1}\Big(\frac{(g/2)-x}{y}\Big)\Big) \quad (1)$$

$$H_y(x,y) = \frac{1}{2\pi} H_0 \ln\Big(\frac{((g/2)-x)^2+y^2}{((g/2)+x)^2+y^2}\Big) \quad (2)$$

where g is the head gap, and H_0 isthe deep gap field, given by:

$$H_0 = \frac{NEI}{g} \quad (3)$$

where N is the number of turns coils of wire around the head, and E is the head efficiency which can be calculated by:

$$E = \frac{1}{1 + \frac{lA_g}{\mu_r g}\int_{core}\frac{d\vec{l}}{A(l)}} \quad (4)$$

where A_g is the gap area, μ_r is the core permeability relative to free space (μ_0), and $A(l)$ is the cross-sectional area of the core as a function of length.

2.2. Tape Magnetisation

The magnetic field being recorded to tape can be described using a hysteresis loop, as follows [3]:

$$\vec{M}(x,y) = F_{Loop}(\vec{H}(x,y)) \quad (5)$$

where F_{Loop} is a generalized hysteresis function.

Using the Jiles-Atherton magnetisation model, the following differential equation describes magnetisation 'M' as a function of magnetic field 'H' [4]:

$$\frac{dM}{dH} = \frac{(1-c)\delta_M(M_{an}-M)}{(1-c)\delta k - \alpha(M_{an}-M)} + c\frac{dM_{an}}{dH} \quad (6)$$

where c is the ratio of normal and anhysteric initial susceptibilities, k is a measure of the width of the hysteresis loop, α is a mean field parameter, representing inter-domain coupling, and

$$\delta = \begin{cases} 1 & \text{if H is increasing} \\ -1 & \text{if H is decreasing} \end{cases} \quad (7)$$

$$\delta_M = \begin{cases} 1 & \text{if } \delta \text{ and } M_{an}-M \text{ have the same sign} \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

M_{an} is the anisotropic magnetisation given by:

$$M_{an} = M_s L\Big(\frac{H+\alpha M}{a}\Big) \quad (9)$$

where M_s is the magnetisation saturation, a characterizes the shape of the anhysteric magnetisation and L is the Langevin function:

$$L(x) = \coth(x) - \frac{1}{x} \quad (10)$$

2.3. Play Head

2.3.1. Ideal Playback Voltage

The ideal playback voltage as a function of tape magnetisation is given by [3]:

$$V(x) = NWEv\mu_0 \int_{-\infty}^{\infty} dx' \int_{-\delta/2}^{\delta/2} dy' \vec{h}(x'+x, y') \cdot \frac{\vec{M}(x',y')}{dx} \quad (11)$$

where N is the number of turns of wire, W is the width of the playhead, E is the playhead efficiency, v is the tape speed, and μ_0 is the permeability of free space. Note that $V(x) = V(vt)$ for constant v. $\vec{h}(x,y)$ is defined as:

$$\vec{h}(x,y) \equiv \frac{\vec{H}(x,y)}{NIE} \tag{12}$$

where $\vec{H}(x,y)$ can be calculated by eqs. (1) and (2).

2.3.2. *Loss Effects*

There are several frequency-dependent loss effects associated with playback, described as follows [1]:

$$V(x) = V_0(x)[e^{-kd}]\Big[\frac{1-e^{-k\delta}}{k\delta}\Big]\Big[\frac{\sin(kg/2)}{kg/2}\Big] \tag{13}$$

for sinusoidal input, where k is the wave number, d is the distance between the tape and the playhead, g is the gap width of the play head, and δ is the thickness of the tape. The wave number is given by:

$$k = \frac{2\pi f}{v} \tag{14}$$

where f is the frequency and v is the tape speed.

3. DIGITIZING THE SYSTEM

3.1. Record Head

For simplicity, let us assume,

$$\vec{H}(x,y,t) = \vec{H}(0,0,t) \tag{15}$$

In this case $H_y \equiv 0$, and $H_x \equiv H_0$. Thus,

$$H(t) = \frac{NEI(t)}{g} \tag{16}$$

or,

$$\hat{H}(n) = \frac{NE\hat{I}(n)}{g} \tag{17}$$

3.2. Hysteresis

Beginning from eq. (6), we can find the derivative of M w.r.t. time, as in [4]:

$$\frac{dM}{dt} = \frac{\frac{(1-c)\delta_M(M_sL(Q)-M)}{(1-c)\delta k-\alpha(M_sL(Q)-M)}\frac{dH}{dt} + c\frac{M_s}{a}\frac{dH}{dt}L'(Q)}{1-c\alpha\frac{M_s}{a}L'(Q)} \tag{18}$$

where $Q = \frac{H+\alpha M}{a}$, and

$$L'(x) = \frac{1}{x^2} - \coth^2(x) + 1 \tag{19}$$

Note that eq. (18) can also be written in the general form for non-linear Ordinary Differential Equations:

$$\frac{dM}{dt} = f(t, M, \vec{u}) \tag{20}$$

where $\vec{u} = \begin{bmatrix} H \\ \dot{H} \end{bmatrix}$.

Using the trapezoidal rule for derivative approximation, we find:

$$\dot{\hat{H}}(n) = 2\frac{\hat{H}(n)-\hat{H}(n-1)}{T} - \dot{\hat{H}}(n-1) \tag{21}$$

We can use the Runge-Kutta 4th order method [5] to find an explicit solution for $\hat{M}(n)$:

$$\begin{aligned} k_1 &= Tf\Big(n-1, \hat{M}(n-1), \hat{\vec{u}}(n-1)\Big) \\ k_2 &= Tf\Big(n-\frac{1}{2}, \hat{M}(n-1)+\frac{k_1}{2}, \hat{\vec{u}}\Big(n-\frac{1}{2}\Big)\Big) \\ k_3 &= Tf\Big(n-\frac{1}{2}, \hat{M}(n-1)+\frac{k_2}{2}, \hat{\vec{u}}\Big(n-\frac{1}{2}\Big)\Big) \\ k_4 &= Tf\Big(n, \hat{M}(n-1)+k_3, \hat{\vec{u}}(n)\Big) \\ \hat{M}(n) &= \hat{M}(n-1) + \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} \end{aligned} \tag{22}$$

We use linear interpolation to find the half-sample values used to calculate k_2 and k_3.

3.2.1. *Numerical Considerations*

To account for rounding errors in the Langevin function for values close to zero, we use the following approximation about zero, as in [4]:

$$L(x) = \begin{cases} \coth(x) - \frac{1}{x} & \text{for } |x| > 10^{-4} \\ \frac{x}{3} & \text{otherwise} \end{cases} \tag{23}$$

$$L'(x) = \begin{cases} \frac{1}{x^2} - \coth^2(x) + 1 & \text{for } |x| > 10^{-4} \\ \frac{1}{3} & \text{otherwise} \end{cases} \tag{24}$$

3.2.2. *Simulation*

The digitized hysteresis loop was implemented and tested offline in `Python`, using the constants M_s, a, α, k, and c from [6]. For a sinusoidal input signal with frequency 2kHz, and varying amplitude from 800 - 2000 Amperes per meter, the following plot shows the Magnetisation output.

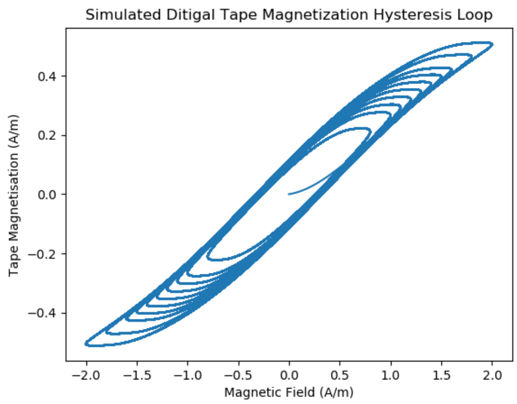

Figure 1: *Digitized Hysteresis Loop Simulation*

3.3. Play Head

By combining eq. (11) with eqs. (12) and (16), we get:

$$V(t) = NWEv\mu_0 gM(t) \tag{25}$$

or,

$$\hat{V}(n) = NWEv\mu_0 g\hat{M}(t) \tag{26}$$

3.3.1. *Loss Effects*

In the real-time system, we model the playhead loss effects with an FIR filter, derived by taking the inverse DFT of the loss effects described in eq. (13). It is worth noting that as in eq. (14), the loss effects, and therefore the FIR filter as well are dependent on the tape speed.

The loss effects filter was implemented and tested offline in `Python` with tape-head spacing of 20 microns, head gap width of 5 microns, tape thickness of 35 microns, and tape speed of 15 ips. The following plot shows the results of the simulation, with a filter order of 100.

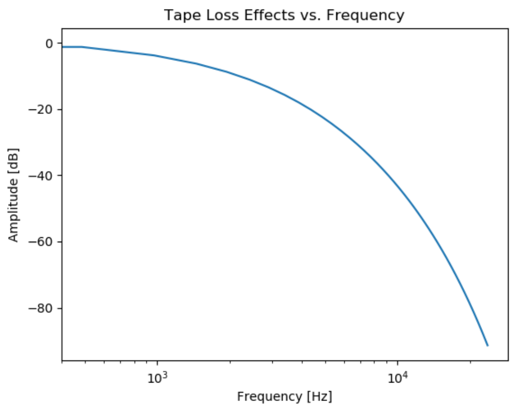

Figure 2: *Frequency Response of Playhead Loss Effects*

3.4. Oversampling

If no oversampling is used, the system will be unstable for input signal at the Nyquist frequency, due to limitations of the trapezoid rule derivate approximation used in eq. (21). To avoid this, a lowpass filter with cutoff frequency below Nyquist should suffice. However, due to aliasing caused by the nonlinearity of the tape hysteresis model, oversampling is necessary to mitigate aliasing artifacts [5]. Further, the system must be able to faithfully recreate not only the frequencies in the audible range but the bias frequencies as well. Since the minimum standard audio sampling rate is 44.1 kHz, and the minimum standard biasing frequency is approximately 50 kHz [7], a minimum oversampling factor of 3x is required. In the real-time implementation, we use a minimum oversampling factor of 4x, with options to oversample at 8x or 16x if the user desires.

4. SYSTEM PARAMETERS

4.1. Tape Parameters

For the tape hysteresis eq. (18), following parameters are necessary:

- Magnetic Saturation (M_s): For Ferric Oxide (γF_2O_3) magnetic tape as would typically be used for a reel-to-reel machine, the magnetic saturation is $3.5e5$ (A/m) [8]
- Hysteresis Loop Width (k): For soft materials, k can be approximated as the coercivity, H_c [9]. For Ferric Oxide, H_c is approximately 27 kA/m [8].
- Anhysteric magnetisataion (a): Knowing the coercivity and remnance magnetism of Ferric Oxide [8], we can calculate a = 22 kA/m by the method described includegraphics [9]
- Ratio of normal and hysteris initial susceptibilities (c): From [9], c = 1.7e-1.
- Mean field parameter (α): From [9], alpha = 1.6e-3

4.2. Tape Bias

A typical analog recorder adds a high-frequency "bias" current to the signal to avoid the "deadzone" effect when the input signal crosses zero, as well as to linearize the output. The input current to the record head can be given by [7]:

$$\hat{I}_{head}(n) = \hat{I}_{in}(n) + B\cos(2\pi f_{bias} nT) \qquad (27)$$

Where the amplitude of the bias current B is usually about one order of magnitude larger than the input, and the bias frequency f_{bias} is well above the audible range. The plot below shows a unit-amplitude, 2 kHz sine wave biased by a 50 kHz sine wave with amplitude 5. To recover the correct output signal, tape machines use a lowpass filter, with a cutoff frequency well below the bias frequency, thought still above the audible range [1].

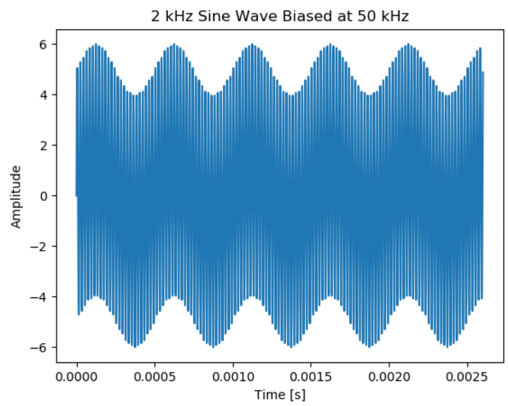

Figure 3: *Example of a biased signal*

4.3. Wow and Flutter

@TODO

5. RESULTS OF REAL-TIME SYSTEM

@TODO

6. REFERENCES

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