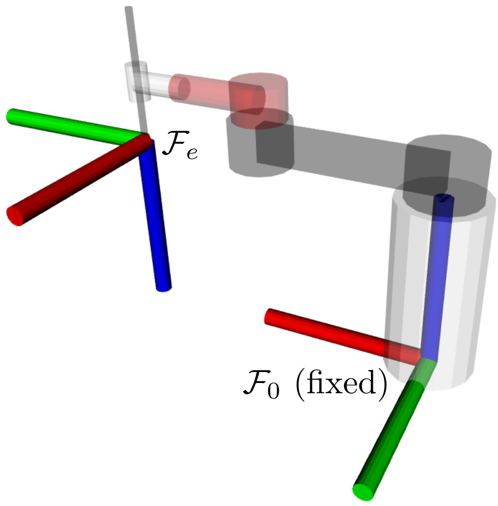
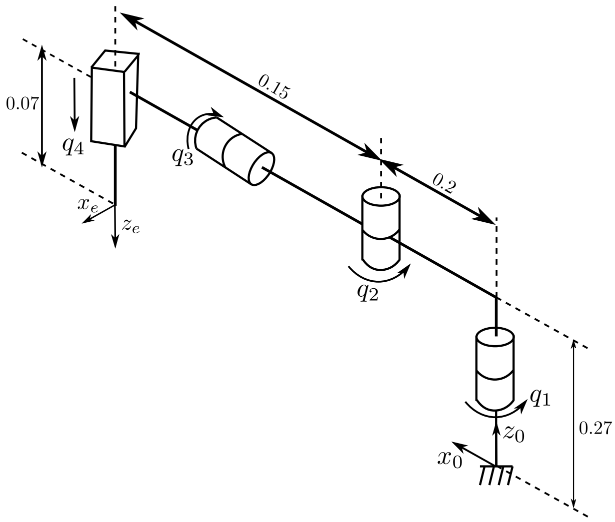
Manipulator Modeling & Control

Example for the RRRP robot

Description of the robot

The considered robot is the one seen during the lectures:





As we saw in class, the MDH table is as follows:

Joint	α_i	a_i	θ_i	r_i
1	0	0	q_1	r_1
2	0	a_2	$q_2 + \pi/2$	0
3	$\pi/2$	0	q_3	r_3
4	$\pi/2$	0	0	q_4
e	0	0	0	r_4

with values:

r_1	0.27
a_2	0.2
r_3	0.15
r_4	0.07

The wrist-to-end effector transform is:

$$^w\mathbf{M}_e = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & r_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

And the Direct Geometric model is:

$$^0\mathbf{M}_4 = {}^f\mathbf{M}_w = \begin{bmatrix} -s_{12}c_3 & c_{12} & -s_3 s_{12} & a_2 c_1 - q_4 s_3 s_{12} + r_3 c_{12} \\ c_3 c_{12} & s_{12} & s_3 c_{12} & a_2 s_1 + q_4 s_3 c_{12} + r_3 s_{12} \\ s_3 & 0 & -c_3 & -q_4 c_3 + r_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We of course have ${}^f\mathbf{M}_e = {}^f\mathbf{M}_w {}^w\mathbf{M}_e$

Solving the inverse model

When solving the inverse model, we want to find (q_1, q_2, q_3, q_4) that solve:

$$\begin{bmatrix} -s_{12}c_3 & c_{12} & -s_3s_{12} & a_2c_1 - q_4s_3s_{12} + r_3c_{12} \\ c_3c_{12} & s_{12} & s_3c_{12} & a_2s_1 + q_4s_3c_{12} + r_3s_{12} \\ s_3 & 0 & -c_3 & -q_4c_3 + r_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} x_x & y_x & z_x & t_x \\ x_y & y_y & z_y & t_y \\ x_z & y_z & z_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}^* = {}^f\mathbf{M}_w \tag{1}$$

Assuming that the right-hand side matrix is given with numerical values.
In order to do so, we have to identify remarquable equations types.

Solving for $\mathbf{q_3}$

In (1) we notice: $\left\{\begin{array}{ll} s_3 & = x_z \\ -c_3 & = z_z \end{array}\right.$

This can be written as a Type 3 equation:

$$\left\{\begin{array}{l} X_1s_3 + Y_1c_3 = Z_1 \\ X_2s_3 + Y_2c_3 = Z_2 \end{array}\right. \quad \text{with} \left\{\begin{array}{lll} X_1 = 1, & Y_1 = 0, & Z_1 = x_z = {}^f\mathbf{M}_w[2][0] \\ X_2 = 0, & Y_2 = -1, & Z_2 = z_z = {}^f\mathbf{M}_w[2][2] \end{array}\right.$$

We can thus solve it for q_3 with the following syntax:

```
for(auto q3: solveType3(1, 0, fMw[2][0], 0, -1, fMw[2][2]))
{
    // q3 is a valid solution, can be used to find other joints

}
```

Solving for $\mathbf{q_1}$ and $\mathbf{q_4}$

From a valid value for q_3, it is tempting to use t_z to solve q_4. Indeed it would write:

$$q_4 = \frac{r_1 - t_z}{\cos(q_3)}$$

Unfortunately this only works if $\cos(q_3) \neq 0$.
On the opposite, we notice that (t_x, t_y) form a system of two unkowns (q_1, q_4):

$$\left\{\begin{array}{ll} a_2c_1 - q_4s_3s_{12} + r_3c_{12} & = t_x \\ a_2s_1 + q_4s_3c_{12} + r_3s_{12} & = t_y \end{array}\right. \tag{2}$$

While we do not know the values of q_1 and q_2, we know from (1) the values of s_{12} and c_{12}.
This makes (2) a Type 5 equation:

$$\left\{\begin{array}{l} X_1s_1 = Y_1 + Z_1q_4 \\ X_2c_1 = Y_2 + Z_2q_4 \end{array}\right. \quad \text{with} \left\{\begin{array}{lll} X_1 = a_2, & Y_1 = t_y - r_3s_{12}, & Z_1 = -c_{12}s_3 \\ X_2 = a_2, & Y_2 = t_x - r_3c_{12}, & Z_2 = s_{12}s_3 \end{array}\right.$$

This is solved in practice with:

```
    const auto c12 = fMw[0][1];
    const auto s12 = fMw[1][1];
    const auto tx = fMw[0][3];
    const auto ty = fMw[1][3];
    const auto s3 = sin(q3);    // we are inside the q3 loop

    for(auto q14: solveType5(a2, ty-r3*yy, -yx*s3, a2, tx-r3*yx, yy*s3))
    {
        auto q1 = q14.qi;   // extract joint i = 1
        auto q4 = q14.qj;   // extract joint j = 4

    }
```

Solving for q_2 and adding the candidate solution

The system (y_x, y_y) will give the solutions of $q_1 + q_2$ from a Type 3 equation. Then q_2 can easily be computed as we now have q_1. As we have a full candidate, we can add it to the potential solutions.

```
    for(auto q12: solveType3(0, 1, c12, 1, 0, s12)
    {
        auto q2 = q12 - q1;
        addCandidate({q1, q2, q3, q4});
    }
```