

Given Equations:

$$\frac{d^2\theta}{dt^2} = \frac{1}{J} \left( K_t i - b \frac{d\theta}{dt} \right)$$
$$\frac{di}{dt} = \frac{1}{L} \left( -Ri + v - K_e \frac{d\theta}{dt} \right)$$

My Garbage:

Applying Laplace to motion equation

$$s^2\theta(s) = \frac{1}{J} (K_t I(s) - bs\theta(s))$$
$$\left( s^2 + \frac{b}{J}s \right) \theta(s) = \frac{K_t}{J} I(s)$$

Applying Laplace to voltage equation

$$sI(s) = \frac{1}{L} (-RI(s) + V(s) - K_e s\theta(s))$$
$$(sL + R)I(s) = V(s) - K_e s\theta(s)$$
$$I(s) = \frac{V(s) - K_e s\theta(s)}{R + sL}$$

Combining the equations

$$\left( s^2 + \frac{b}{J}s \right) \theta(s) = \frac{K_t}{J} I(s) = \frac{K_t}{J} \left( \frac{V(s) - K_e s\theta(s)}{R + sL} \right)$$
$$\left( s^2 + \frac{b}{J}s \right) \theta(s) = \frac{K_t}{J(R + sL)} V(s) - \frac{K_t K_e}{J(R + sL)} s\theta(s)$$
$$\left( s^2 + \frac{1}{J} \left( b + \frac{K_t K_e}{R + sL} \right) s \right) \theta(s) = \frac{K_t}{J(R + sL)} V(s)$$
$$G(s) = \frac{\theta(s)}{V(s)} = \frac{\frac{K_t}{J(R + sL)}}{s^2 + \frac{1}{J} \left( b + \frac{K_t K_e}{R + sL} \right) s}$$