Microsoft Quantum Challenge

Australian National University

Simulating Dynamical Input-Output Quantum Systems with LIQ$Ui|\rangle$

Bridging Analog and Digital Quantum Representations

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Introduction

Quantum computation is usually described by unitary operations which is based on closed quantum system dynamics. In contrast, **open quantum dynamical systems** are often modeled by the master equation which takes into account the environmental interaction. Using the complementary Heisenberg-picture approach, we can derive the quantum stochastic differential equations that describe the dynamics of system operators of an open quantum plant. This is commonly referred to as the quantum input-output formalism, which underpins the burgeoning field of quantum feedback control.

The inclusion of dissipation processes, allows us to passively and autonomously perform a wide variety of quantum operations, such as enhancing adiabatic quantum computation, and state preparation/stabilization. More importantly, fully coherent feedback loops mediated by field coupling without measurement have been shown to have major advantages such as high operating speed for quantum optics applications and the non-demolition nature of coherent operations.

Quantum network theory has been developed based on the quantum stochastic differential equation model. In particular, a modular and system-theoretic parametrization scheme which is commonly referred to as the SLH formalism[1] provides the mathematical framework for quantum network description and analysis. The SLH-based **Q**uantum **H**ardware **D**escription **L**anguage (QHDL),[2] which is illustrated in Figure 1, has been successfully deployed to design a number of quantum optics devices and applications such as quantum NAND logic gate and latch,[3] optical Set-Reset flip-flop,[4] and a coherent error correction loop.[5] Backend differential equation solver based tools such as QuTiP[6] and XMDS[7] are often used to simulate the system dynamics described by its SLH parameters.

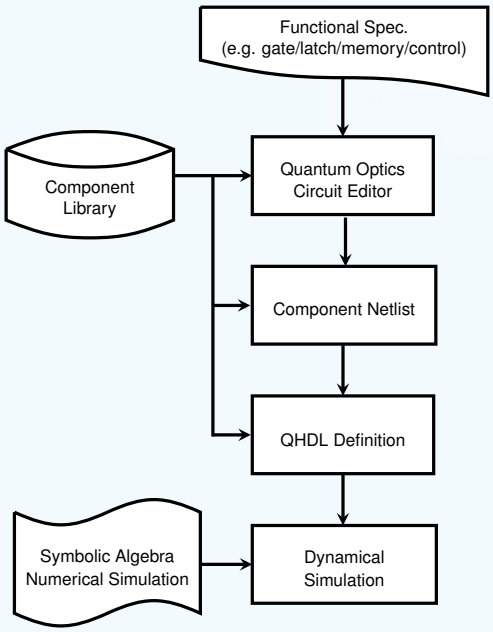

Figure 1: QHDL design flow which incorporates quantum optics schematic editor, netlisting tool, Hardware Description Language (HDL) parser, and a full quantum dynamical simulation backend.

The Microsoft Language-Integrated Quantum Operations framework (LIQ$Ui|\rangle$[8]), which is based on quantum gate formalism, is not specifically designed for this type of low level dynamical simulation. However, since LIQ$Ui|\rangle$ is capable of faithfully simulating a general purpose quantum computer, porting the dynamical SLH model onto LIQ$Ui|\rangle$ will pave the way for the implementation of discrete/digital quantum emulators of open quantum systems.

In this competition entry, we describe the use of LIQ$Ui|\rangle$ in simulating complex engineered **open** quantum systems by discretizing the physical system composition, and introducing a dissipation emulator gadget. The Hamiltonian simulator module of LIQ$Ui|\rangle$ is used for time evolution simulation of the Trotter decomposed unitary. In particular, we will apply this method to compound gradient echo quantum memory which is capable of efficiently capturing and storing a quantum state of light. In these experiments, memory hardware consists of a continuous solid-state material (rare-earth-ion-doped crystal) which will be represented in simulation by a network of qubits. This not only provides analysis results for such a discrete quantum memory model, but can also potentially be used as a blueprint for a quantum digital signal processor capable of processing incoming continuous variable quantum information.

The report is organized as follows. In the next two pages, the theoretical foundations of this project are briefly summarized. Then, the LIQ$Ui|\rangle$ simulation set-up is discussed. A single qubit memory is simulated first to correlate LIQ$Ui|\rangle$ with established theoretical results. Then more extensive two- and four-qubit programs are examined which offer insights about the performance of such a discrete memory scheme. Lastly, some conclusions and suggestions for future development are given.

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[1] J. Gough et al. In: *IEEE Trans. Autom. Control* 54.11 (2009), [2] N. Tezak et al. In: *Philos Trans A Math Phys Eng Sci* 370.1979 (2012), [3] H. Mabuchi. In: *Appl. Phys. Lett.* 99.15 (2011), [4] H. Mabuchi. In: *Phys. Rev. A* 80 (4 2009), [5] J. Kerckhoff et al. In: *New J. Phys* 13.5 (2011), [6] J. Johansson et al. In: *Comput Phys Commun* 183.8 (2012), [7] G. R. Dennis et al. In: *Comput Phys Commun* 184.1 (2013), [8] D. Wecker et al. In: *arXiv:1402.4467* (2014).

THEORETICAL APPROACH

"The Devil is in the details, but so is salvation."
–Hyman G. Rickover

SLH Quantum Network Formalism

The stochastic evolution of an open Markov quantum system driven by vacuum noise inputs is given by the Hudson-Parthasarathy Quantum Stochastic Differential Equation[9] or also known as the quantum input-output formalism:[10]

$$\begin{aligned} dU(t) &= \{-iHdt + (S-I)d\Lambda(t) \\ &+ \sum_i \left(dA_i^\dagger(t) L_i - L_i^\dagger S dA_i(t) - \frac{1}{2} L_i^\dagger L_i dt \right) \Big\} U(t), \end{aligned} \tag{1}$$

in which, the unitary evolution $U(t)$ is defined on the combined space of the system plant and coupling fields. Any system operator dynamics can be derived from that using the relation $X(t) = U(t)^\dagger X U(t)$. The resulting operator-based differential equations are referred to as *Heisenberg-Langevin equations*.

This Heisenberg-picture dynamical model can be parametrized conveniently by a triple $G = (S, L, H)$, where H is the internal Hamiltonian, $L = \{L_i\}$ is a set of couping operators (e.g. annihilation operators for amplitude damping), and S is a unitary input-to-output scattering matrix (e.g. beam-splitters in quantum optics or quantum point contacts in solid-state). This parametrization scheme is often referred to as the SLH quantum network theory.

The network part of this model is what really important since from the three parameters and the given network topology, we can compute the equivalent SLH model of the entire network thanks to the two basic rules as shown in Figure 2.

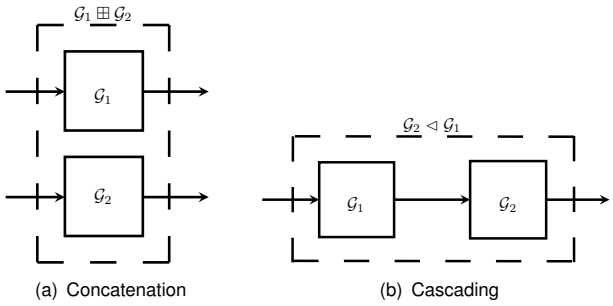


Figure 2: SLH network connections

Mathematically, these two connection rules can be expressed as:

$$G_2 \boxplus G_1 = \left(\begin{pmatrix} S_1 & 0 \\ 0 & S_2 \end{pmatrix}, \begin{pmatrix} L_1 \\ L_2 \end{pmatrix}, H_1 + H_2 \right), \tag{2}$$

$$G_2 \lhd G_1 = (S_2 S_1, L_2 + S_2 L_1, H_1 + H_2 + \mathrm{Im}\{L_2^\dagger S_2 L_1\}). \tag{3}$$

This is the backbone of the QHDL toolbox, whereby network topology (can be in the form of schematics or Verilog-style inputs) is processed symbolically to derive the overall network model. The system dynamics can then be simulated by differential equation solvers.

Besides the bottom-up approach, we can also perform a top-down decomposition in the SLH framework by the network synthesis theory.[11] Given an arbitrary SLH model which may contain a large number of internal dynamical variables (optical modes or qubits) and inputs/outputs, one can always faithfully identify suitable collection of one degree of freedom oscillator components and to connect them serially with proper Hamiltonian interaction to build up the prescribed system model.

One recent development of the SLH modeling approach is the effort to extend its application to a wide variety of input states besides the conventional vacuum inputs, such as thermal field, single-photon and two-photon states. Next, we will introduce the SLH model for a system driven by a continuous-mode single-photon wave packet.

Single-photon State

Depending on its source, a photon can be single or multi-modal. In this work, continuous-mode single photon state is concerned. We can decomposed such state in the time and frequency domains as followings,

$$|1_\xi\rangle = \int_0^{+\infty} \xi(t) dB^\dagger(t) |0\rangle, \quad |1_\xi\rangle = \int_{-\infty}^{+\infty} d\omega \hat{\xi}(\omega) a^\dagger(\omega) |0\rangle. \tag{4}$$

The function $\xi(t)$ is the time-domain wave packet shape of the photon, which can be interpreted as the probability amplitude of detecting the photon at time t. $\hat{\xi}(\omega)$ is the Fourier transform of $\xi(t)$.

In the following simulation, we will consider incoming single-photon wave packets which have Gaussian shape parametrized by:

$$\xi(t) = \left(\frac{\Omega^2}{2\pi} \right)^{1/4} \exp\left[-\frac{\Omega^2}{4} (t - t_c)^2 \right], \tag{5}$$

where t_c specifies the peak arrival time and Ω is the frequency bandwidth of the pulse.

Using a pre-"signal-generating filter" which is driven by vacuum, we can derive the SLH model for the augmented system. Equations (6) and (7) show the SLH parameters of the pre-filter ancilla and the overall network, respectively.

$$G_{anc} = (I, \lambda(t)\sigma_-, 0), \quad \lambda(t) = \frac{\xi(t)}{\sqrt{\int_t^\infty |\xi(s)|^2 ds}}, \tag{6}$$

$$G_T = \left(S, \quad L + \lambda(t) S \sigma_-, \quad H + \lambda(t) \mathrm{Im}(L^\dagger S \sigma_-) \right), \tag{7}$$

in which $G_s = (S, L, H)$ is the original system parameters.

The signal-generating filter (G_{anc}) is a vacuum-driven two-level atom initially prepared in the excited state which will decay in time to the ground state via spontaneous emission of a photon into the light field. That output field is then channeled to the system of interest as shown in Figure 3.

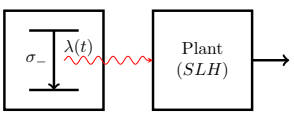


Figure 3: Single-photon generating filter cascading realization.

[9] R. L. Hudson et al. In: *Commun. Math. Phys.* 93.3 (1984), [10] C. W. Gardiner et al. In: *Phys. Rev. A* 31 (6 1985), [11] H. I. Nurdin et al. In: *SIAM J Control Optim* 48.4 (2009),

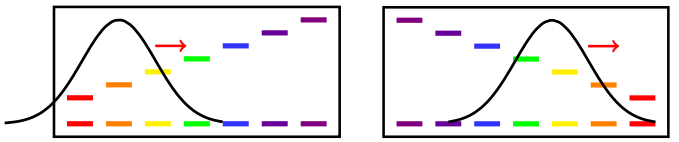
THEORETICAL APPROACH

"Patience is bitter, but its fruit is sweet."
–Aristotle

Compound Gradient Echo Memory

In the conventional quantum computation paradigm, there is little or no use of a "memory" device since in most cases the problems are formulated as isolated computing tasks with classical information inputs, e.g. factoring, optimization, etc. Nevertheless, if networking between distant quantum computers are required, photons are the best candidate for quantum information carriers. In that scenario, a quantum memory is needed to capture and release photons on-demand.

One of the most promising schemes for quantum memory is the so-called Controlled Reversible Inhomogeneous Broadening (CRIB). CRIB can be implemented on any "controllable" absorbing physical system including NV centers, quantum dots, atomic vapors and rare-earth-metal-ion-doped solids. Gradient echo memories (GEM) fall under the CRIB category, in which an electric field induced Stark shift is deployed for broadening. Two different linear gradients are applied for writing and reading phases that causes the rephasing effect as shown in Figure 4.



(a) Stark shift in gradient echo memory during writing stage

(b) Stark shift in gradient echo memory during reading stage

Figure 4: Gradient settings for GEM operations

While semi-classical approaches[12] can be used to simulate the memory operations, no direct access to individual quantum state of each ion can be attained that way. On the other hand, using the SLH formalism, Hush *et al.*[13] modeled the memory as a series of optical cavities. Each cavity represents a collection of atomic absorbers at the corresponding location in the memory, which all absorb light of a single frequency. The number of excitations within a given cavity may be thought of as the number of excited atoms in the ensemble at the corresponding location. In this framework, the SLH model of a gradient echo memory is given by

$$\begin{aligned} G_{GEM} &= G_1 \rhd G_2 ... \rhd G_n \\ &= \left(S_{GEM} = I, \quad L_{GEM} = \sum_k \sqrt{\beta} a_k, \right. \\ &\qquad \left. H_{GEM} = \sum_k \xi_k a_k^\dagger a_k + \frac{\beta}{2i} \sum_{j=2}^{N} \sum_{k}^{j-1} \left(a_j^\dagger a_k - h.c \right) \right). \end{aligned} \tag{8}$$

The above model is a vacuum input model which has not taken into account the source term. To simulate non-classical input such as the single photon state, the generating filter approach mentioned previously becomes handy and can be apply directly to get the complete model as followings,

$$\begin{aligned} G_T &= G_a \rhd G_{GEM} \\ &= \left(I, L_{GEM} + \lambda(t)\sigma_-, H_{GEM} + \tfrac{1}{2i} \left(\lambda^*(t)\sigma_+ L_{GEM} - h.c \right) \right), \end{aligned} \tag{9}$$

in which $\lambda(t)$ term dictates the photon wave packet shape as given by Eq. (6).

The cascading cavity approach is extremely accurate in the case that there are high concentrations of light absorbers (ions). What we want to investigate is the performance of such memory protocols in the discrete settings where only a countable number of two-level qubits involved as absorbers. More importantly, the dynamics is simulated on a discrete-time basis which is executable on a general-purpose quantum computer. The analogy could be using a Analog-to-Digital converter (ADC) with limited resolution and sampling rate to digitize continuous signals for further processing. Not only will this provide new insights into such discrete memory operations, with a synthesis flow to a target quantum hardware, we can implement such protocol on a real quantum computer.

The last piece of this simulation/emulation approach is a method of implementing engineering dissipation which is given by the L operator in the SLH model. This will be introduced in the next section.

Engineering Dissipation Control

At first glance, the input-output open quantum system approach can be easily implemented on LIQ$Ui|\rangle$ by a set of suitably designed Kraus super-operators. While being ideal for simulation purposes, hardware synthesis to real physical systems will be extremely challenging. An alternative approach could be implementing a universal dissipative gadget which is just an amplitude-damped qubit as proposed by Verstraete *et al.*[14] The damped ancillary qubit is directly coupled to the other qubits (such as the memory qubits) to provide the desired dissipation. The engineering dissipation configuration is illustrated in Figure 5.

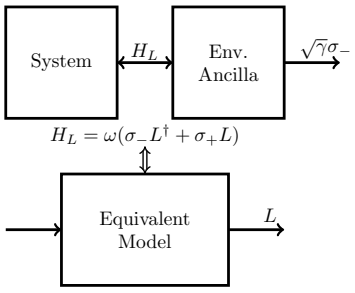


Figure 5: Engineered dissipation by direct coupling to an amplitude damping ancillary system.

The coupling Hamiltonian between the ancilla and the system has the form $H_c = \omega(\sigma_- L^\dagger + \sigma_+ L)$. In the limit that the damping rate, γ, of this ancilla is much greater than the strength ω of the coupling Hamiltonian, one can adiabatically eliminate the excited level of the ancilla, thus reconstructing the desired dissipation L for the system. The upper bound for error of this adiabatic approximation is $\|\epsilon\| \sim \frac{1}{\gamma^2}$.

The coupling Hamiltonian can be easily transformed into a sequence of discrete quantum gates executable on a quantum computer while there are many physical systems that may act as a dissipative gadget, such as low-Q optical or superconducting cavities. Therefore, a LIQ$Ui|\rangle$ program written in this way is more likely to be ready for hardware synthesis.

[12] M. Fleischhauer et al. In: *Phys. Rev. Lett.* 84 (22 2000), [13] M. Hush et al. In: *New J. Phys* 15.8 (2013), [14] F. Verstraete et al. In: *Nat. Phys.* 5.9 (2009),

LIQ$Ui|\rangle$ SET-UP

"Be willing to be a beginner every single morning."
–Meister Eckhart

Trotter Decomposition

One key aspect of simulating quantum dynamics is the time evolution simulation, i.e. applying $\exp(-iHt)$ to the quantum state. Unlike differential equation solvers which directly solve for this time evolution, LIQ$Ui|\rangle$ uses the quantum gate model where the complex exponential term can be approximated by:[15]

$$\exp(-iHt) = \left(\prod_k \exp(-ih_k\Delta t)\right)^{t/\Delta t} + \mathcal{O}(\Delta t), \quad (10)$$

in which, the original Hamiltonian has the summation form $H = \sum_k h_k$ and Δt is the Trotter step size.

The term $\exp(-ih_k\Delta t)$ is in turn implemented by a sequence of quantum gates. This is known as first-order Trotter decomposition which is often used in quantum chemistry simulation.[16]

For open quantum systems with engineered dissipation coupling, we can use the "dissipative gadget" that was introduced in the previous section to adiabatically emulate the environmental coupling. As shown in Eq. (9), the GEM Hamiltonian consists of two classes of terms. The first one is the number operator term $(a^\dagger a)$ which can be implemented directly by a $T(\theta)$ gate. The coupling Hamiltonian H_c required for engineering dissipation, $H_c = \omega(\sigma_- L^\dagger + \sigma_+ L)$, has the form of the excitation operator $(h_{pq}(a_p^\dagger a_q + a_q^\dagger a_p))$ which can be realized by the sequence of gates as shown in FIG. 5 (ii) in Wecker *et al.*[17]

Last but not least, the cascading term, i.e. $\frac{\beta}{2i}\sum_{j=2}^{N}\sum_{k}^{j-1}\left(a_j^\dagger a_k - h.c\right)$, which is not commonly seen in the quantum chemistry model, can also be decomposed in the same manner, i.e.

$$\begin{aligned} a_j^\dagger a_k - h.c. &\equiv \frac{\sigma_x^{(j)} - i\sigma_y^{(j)}}{2}\frac{\sigma_x^{(k)} + i\sigma_y^{(k)}}{2} - h.c. \\ &= i\frac{\sigma_y^{(k)}\sigma_x^{(j)} - \sigma_x^{(k)}\sigma_y^{(j)}}{2}. \end{aligned} \quad (11)$$

The resulting sequence of quantum gates (rendered by LIQ$Ui|\rangle$) which can be used to implement complex exponential of the terms in Eq. (11) is shown in Figure 6 which involves change-of-basis gates (H and Y) and the standard unitary $\exp[-i(\theta/2)(\sigma_z \otimes \sigma_z)]$.

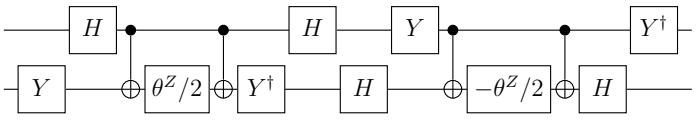


Figure 6: Quantum circuit that represents the unitary generated by the cascading Hamiltonian term.

The LIQ$Ui|\rangle$ code snippet that is used to run this Trotter decomposed term is shown as below.

```
//Trotterization of Cascading Hamiltonian
let HamiltonianCasOp(theta:float, p:int, q:int)
    (qs:Qubits) =
   let a_qubit = qs.[p..]
   let s_qubit = qs.[q..]
   Ybasis s_qubit
   H a_qubit
   CNOT !! (qs,q,p)
   Rz(-theta) 1.0 "" a_qubit
   CNOT !! (qs,q,p)
   YbasisAdj s_qubit
   H a_qubit
   H s_qubit
   Ybasis a_qubit
   CNOT !! (qs,q,p)
   Rz(theta) 1.0 "" a_qubit
   CNOT !! (qs,q,p)
   H s_qubit
   YbasisAdj a_qubit
```

Amplitude Damping Set-up

Using the dissipation gadget, the discretized gradient echo memory model used for LIQ$Ui|\rangle$ simulation is shown in Figure 7 below. The dissipation ancilla at the bottom is a qubit which is subjected to general amplitude damping. This is done by running idle gate at each Trotter step with the error probability of $(1 - \exp(-\gamma\Delta t))$. A LIQ$Ui|\rangle$ code snapshot for the amplitude damping set-up is also shown below.

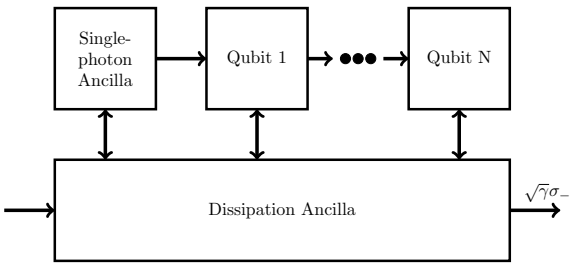


Figure 7: Discretized GEM model driven by single-photon source used for LIQ$Ui|\rangle$ simulation.

```
//Probability of amplitude damping
let probDamp_E = 1.0 - Math.Exp(-gamma*dt)
let noise = Noise(circ,ket,models)
ket.TraceRun <- 0
noise.LogGates <- false
noise.TraceWrap <- false
noise.TraceNoise <- false
noise.DampProb(0) <- 0.0 //single-photon ancilla
noise.DampProb(1) <- 0.0 //qubit 1
noise.DampProb(2) <- 0.0 //qubit 2
...
noise.DampProb(Nq+1) <- probDamp_E //dissipation
    ancilla
```

The LIQ$Ui|\rangle$ simulation is performed on a Windows 7 desktop machine using Intel Core i7-4770 3.4 GHz (Haswell) with 16 GB of main memory. In the case of running 4-qubit gradient echo memory model (6 qubits in total), the runtime for 1,000 trajectories each with 10,000 Trotter steps is 3 hours.

[15] J. D. Whitfield et al. In: *Mol. Phys.* 109.5 (2011), [16] D. Wecker et al. In: *Phys. Rev. A* 90 (2 2014), [17] Ibid.

SIMULATION RESULTS

"A great man is always willing to be little."
–Ralph Waldo Emerson

Toy Example: Single-Atom Memory

Before delving into the discretized GEM simulation, we will test the LIQ$Ui|\rangle$ set-up in a much simpler situation, namely a single-atom memory. Here we consider a single two-level atom driven by a Gaussian continuous-mode single photon as given by Eq. (5). This problem has been investigated thoroughly by both conventional and quantum network approaches.

The coupling operator of the atom is $L = \sqrt{\Gamma}\sigma_-$. The atom is assumed to be free of internal Hamiltonian dynamics and field scattering for simplicity, i.e., $H = 0$ and $S = I$. Here $\Gamma > 0$ is the normalized coupling rate ($\Gamma = 1$).

By using Trotter decomposition technique and LIQ$Ui|\rangle$, we were able to simulate the dynamics of this open quantum system. In our simulation, dissipation is induced by an ancilla which undergoes amplitude damping. In Figure 8, we plot the "detection time" histogram and the excited-state population for the case of $\Omega = 1.5\Gamma$, which is known to be the optimal Gaussian wave packet for atomic excitation. The detection time is triggered by the state-collapse event of amplitude damping noise and histogram is plotted from 10,000 LIQ$Ui|\rangle$ runs.

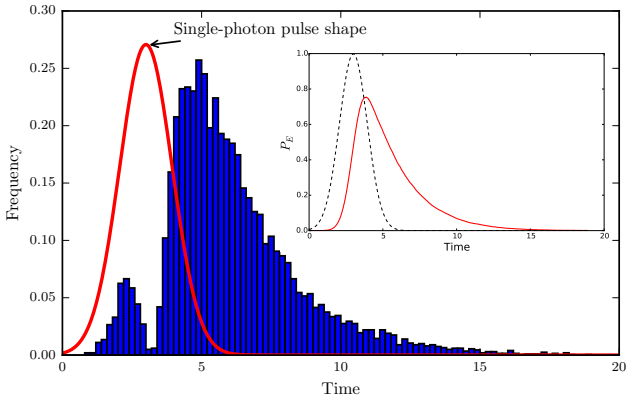


Figure 8: Histogram of photon detection events from a two-level atom driven by a Gaussian-shape single photon state. Inset: the excited state population of the atom. The photon wave-packet is included for reference (red curve in main and dotted curve in inset). Trotter step is chosen to be $2\pi \times 10^{-4}$, and the number of trajectories (realizations) is 10,000.

This is consistent with theoretical analysis which shows that that the maximum excitation probability of an atom driven by a Gaussian-shaped single photon is about 80%.[18] Since we are essentially running a stochastic simulation on the input-output model, we have the benefit of being able to simulate the conditional dynamics and output observable statistics (photon detection in this case) if we choose to observe the output.

By sweeping the Gaussian bandwidth (larger Ω means broader spectrum in frequency domain and vice versa), we can get the maximum excitation (absorption) rate of our single atom model, as shown in Figure 9 (black curve). There is a very narrow range of bandwidth where the absorption peaks. The optical "depth" created by cascading absorbers of gradient echo memory can enhance the photon absorption as shown in the below section.

[18] Y. Wang et al. In: *Phys. Rev. A* 83 (6 2011),

Discretized Gradient Echo Memory

Using the full GEM model with signal generating filter as in Eq. (10) and Figure 7, we can investigate the performance of the discrete memory in terms of photon absorption (efficiency). Due to computing resource and runtime constraints, only models of two- and four-atom GEM are considered in this report (4 and 6 qubits in total, respectively). Despite minimal system size, the performance enhancement is significant as shown in Figure 9.

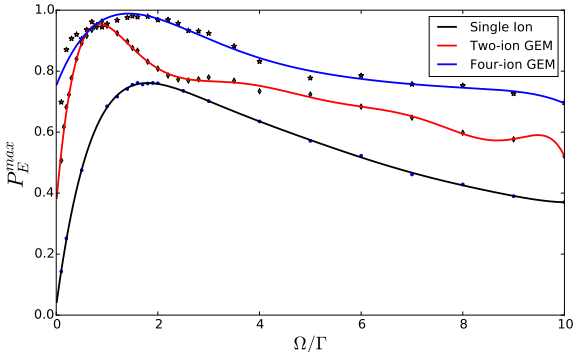


Figure 9: Single photon absorption probability as a function of photon bandwidth. The baseline (black) is the single atom model which has the max absorption of 80% for a very narrow range of bandwidth. The more atoms involve, the better absorption and also wider range of bandwidth can be achieved. Each data point is a LIQ$Ui|\rangle$ run of 1,000 trajectories. Solid line is polynomial fit.

Almost perfect absorption (100% excitation) can be achieved for some Gaussian wave packet shapes by using just 4 atoms in series. More importantly, a monotonic improvement over the full range of bandwidth is achieved by introducing more qubit resources. Notice that in the case of the multi-atom memory, the single excitation from the photon is distributed among all memory qubits in a Fourier-transformed (frequency) basis. Therefore, the choice of gradient is of importance and can be optimized for a given target input wave packet. Since we use a linear symmetric gradient and the number of atoms are even (thus missing a zero point), the absorption rate has bi-modal behavior, as is clearly seen in the four-atom case. If, as for continuous solid-state GEM, the gradient is a smooth continuous function, the memory can effectively capture the full Fourier spectrum of incoming photons.

Besides the fact that this discrete memory model can be implemented on a general purpose quantum computer, one major advantage is that we can potentially extend the range of gradient. Inhomogeneous broadening (by the electric field induced Stark shift, for instance) has difficulty in achieving the bandwidth required for telecommunication wavelength photon absorption. If implemented on a digital quantum computer, the gradient term is controlled by the Trotter decomposed unitary thus can be changed freely provided that the time step can be reduced for sufficient error bound.

This discretized memory model has close analogy to classical digital signal processing technology. The gradient quantum memory program running on the a quantum computer is equivalent to an Analog-to-Digital Converter (ADC), where (i) the Trotter step serves as the sampling rate; (ii) the resolution is dictated by the number of qubits; and (iii) the sampling method is frequency gradient mapping (classical ADC's have a wide variety of sampling methods such as integrating and sigma-delta.)

SUMMARY & OUTLOOK

"Knowing too much of your future is never a good thing."
–Rick Riordan

Conclusions

In this competition entry, we have adopted LIQ$Ui|\rangle$ to simulate quantum open systems using the input-output formalism. A combination of theoretical (Trotter decomposition, dissipation gadgets) and practical (Hamiltonian simulator and the noise models available in LIQ$Ui|\rangle$) solutions can provide accurate and consistent simulation results with other tool suites as demonstrated in the "single atom driven by single photon" case.

Simulation-wise, the use of LIQ$Ui|\rangle$ for this sort of numerical simulation struggles to compete with more specialized direct differential solver based tools such as XMDS or QuTiP due to (i) extra efforts needed to perform the Trotter decomposition and set-up the dissipation ancilla; (ii) simulation errors associated with these approximations (could be trade-off by additional runtime). However, if we look at the grand scheme of quantum computation with reference to the classical computing paradigm, this approach is essentially bridging the analog and digital worlds of the quantum framework. The fact that LIQ$Ui|\rangle$ is a quantum simulator which guarantees a direct "plug-and-play" on a future digital quantum computer. Being able to simulate this system on LIQ$Ui|\rangle$ shows that we can readily emulate this sort of quantum dynamics on a general purpose quantum computer. This is what we have shown in the discrete quantum memory example where a small collection of qubits running synchronously to perform photon quantum state storage. An initial dry run of this hypothetical memory demonstrates great improvement in terms of the absorption rate even with an unoptimized gradient scheme.

Using this discretized method to demonstrate open quantum system approach, we can get new insights about hardware requirements and potential errors associated with emulating continuous "analog" quantum functions and inputs. This is somewhat missing in the current quantum computing picture where computing problems are often formulated as an algorithm started on a given initial state without the need for external quantum inputs. The SLH formalism provides a modular and systematic approach toward this modeling problem thus worthwhile integrating to the LIQ$Ui|\rangle$ framework, especially when quantum hardware synthesis is taken into account since the SLH framework has native models for a lot of quantum optics, opto-mechanical and cavity QED devices.

The Path Forward

In the discretized GEM simulation, we have introduced an analogy between a GEM with a finite number of qubits and a classical A/D converter. There are some immediate considerations that warrant further investigation such as error bound estimation regarding the system size and Trotter step size; whether aliasing effect exists in this context; some optimization techniques that we can do to improve the performance.

As this approach can be used to treat a large variety of quantum input-output dynamical models besides the GEM, some future research avenues which can be considered are listed below.

- **Non-Markovian quantum noise model**: Recent work[19] has shown that the amplitude damping dissipation ancilla, when treated non-adiabatically, is equivalent to a non-Markovian (colored) Lorentzian noise source. By combining multiple ancillas, we can construct an arbitrary noise spectrum. We can thus use this model to evaluate the performance of a quantum computer under influence of colored noise, especially regarding the performance of quantum error correction codes. The advantage of this dynamical approach is that an experimentally measured noise spectrum can be fed directly into the model by adjusting the parameters of the dissipation ancillas. For instance, quantum dot - cavity QED is a good candidate, since the noise in this platform is intrinsically colored.

- **Coherent digital quantum controller**: Another direction is to look at the use of a quantum computer in quantum control settings which usually involve continuous input-output models. Especially in the case of coherent feedback control whereby control objectives are achieved by field coupling quantum systems together. This removes the measurement step and can perform control actions at the intrinsic system speed (e.g. for quantum optics systems). A coherent digital controller can perform equally well in this setting if equipped with appropriate quantum coherent A/D and D/A converters at its input and output ports. By allowing quantum programing, a coherent digital quantum controller can potentially achieve much better performance than just wiring up quantum components (e.g. cavities, beamsplitters, Kerr non-linear media.)[20]

- **Quantum synthesis**: As LIQ$Ui|\rangle$ is part of a larger effort to create a full quantum design flow from software to target hardware implementation, it is worthwhile considering the integration of the SLH network model at the low-level physical description. It is one of the most accurate and systematic dynamical descriptions for quantum optics devices and circuits.

  In comparison, a classical VLSI design flow always has a so-called "*parasitic extraction and analysis*" step after "place and route" during which the full RLC model of the entire circuit plus routing is compiled and simulated. Besides simulations at the logic and transistor levels, it is deemed necessary to perform such low level verification especially for complex and high speed designs. The approach taken by classical integrated circuit design is by no mean the absolute solution for quantum computer design. However, by incorporating this "analog" model of quantum devices, we can definitely improve the versatility of the synthesis process especially in cases where continuous time is the most natural description (e.g. continuous variable quantum computation).

Given the "*Big Hairy Audacious Goal*" of creating a functional quantum computer in the next one or two decades, a solid foundation for quantum system design is imperative. And LIQ$Ui|\rangle$ created by Microsoft is the first bold step in that direction. Augmenting the quantum gate model of LIQ$Ui|\rangle$ with the SLH dynamical model either by emulating or intrinsic integration will open up new applications as well as cater to a larger research community.

[19] S. Xue et al. In: *arXiv:1503.07999* (2015). [20] H. Mabuchi. In: *Phys. Rev. A* 78 (3 2008),