Вар 10. Ермов А.Р. Системы

| Y \ X | 0 | 1 | 2 | 3 | |
|---|---|---|---|---|---|
| -1 | 0.15 | 0.2 | 0.15 | 0.1 | 0.6 |
| 2 | 0.2 | 0 | 0.15 | 0.05 | 0.4 |
| | 0.35 | 0.2 | 0.3 | 0.15 | 1 |

$E(X|Y) = ?$

$E(Y|X) = ?$

$EX, EY = ?$

п.м.о. - ?

мин. регр. - ?

$EX = 0,2 + 0,6 + 0,45 = 1,25$

$EY = -0,6 + 0,8 = 0,2$

| $x_i$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $p_i$ | 0.35 | 0.2 | 0.3 | 0.15 |

| $y_i$ | -1 | 2 |
|---|---|---|
| $p_i$ | 0.6 | 0.4 |

| Y | -1 | 2 |
|---|---|---|
| P(Y\|X=0) | $\frac{3}{7}$ | $\frac{4}{7}$ |
| P(Y\|X=1) | $\frac{1}{3}$ 1 | 0 |
| P(Y\|X=2) | $\frac{1}{2}$ | $\frac{1}{2}$ |
| P(Y\|X=3) | $\frac{2}{3}$ | $\frac{1}{3}$ |

] $m_{Y|X}(x)$ - регр X на Y, тогда

$m_{Y|X}(0) = -\frac{3}{7} + \frac{8}{7} = \frac{5}{7}$

$m_{Y|X}(1) = -1 = -1$

$m_{Y|X}(2) = -\frac{1}{2} + 1 = \frac{1}{2}$

$m_{Y|X}(3) = -\frac{2}{3} + \frac{2}{3} = 0$

| $E(Y\|X)_i$ | -1 | 0 | $\frac{1}{2}$ | $\frac{5}{7}$ |
|---|---|---|---|---|
| $p_i$ | 0.2 | 0.15 | 0.3 | 0.35 |

$\Rightarrow E(E(Y|X)) = 0.2 = EY$ (верно)

Теперь проверим формулу п.м.о для $E(E(X|Y))$

| X | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| P(X\|Y=-1) | $\frac{1}{4}$ | $\frac{1}{3}$ | $\frac{1}{4}$ | $\frac{1}{6}$ |
| P(X\|Y=2) | $\frac{1}{2}$ | 0 | $\frac{3}{8}$ | $\frac{1}{8}$ |

] $m_{X|Y}(y)$ - регр X на X, тогда

$m_{X|Y}(-1) = \frac{1}{3} + \frac{1}{2} + \frac{1}{2} = \frac{4}{3}$

$m_{X|Y}(2) = \frac{3}{4} + \frac{1}{8} = \frac{3}{2}\ \frac{9}{8}$

| $E(X\|Y)_j$ | $\frac{9}{8}$ | $\frac{4}{3}$ |
|---|---|---|
| $P_j$ | 0.4 | 0.6 |

$\Rightarrow E(E(X|Y)) = 1.25 = EX$ (верно)

Перейдем к лин. регр. $y = g(x) = ax + b$, где $a = \frac{K(X,Y)}{D(X)}$

$b = EY - a \cdot EX$

$E(XY) = 0.1$

$K(X,Y) = E(XY) - EX \cdot EY = -0.15$

$E(X^2) = 0,2 + 4 \cdot 0,3 + 9 \cdot 0,15 = 2,75$

$DX = (EX^2) - (EX)^2 = 1.1875$

$E(Y^2) = 0.6 + 4 \cdot 0,4 = 2.2$

$DY = EY^2 - (EY)^2 = 2.16$

$\sigma_X = \sqrt{DX} \approx 1.0897$

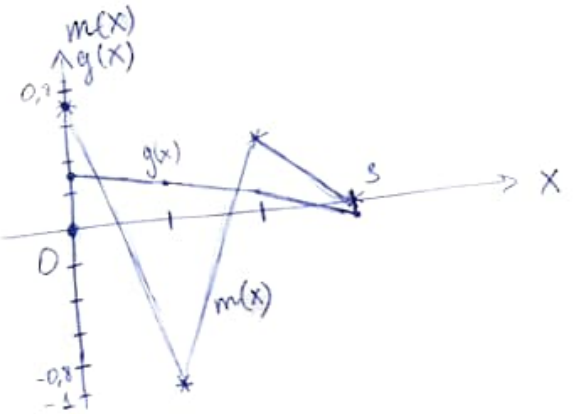
$\sigma_Y = \sqrt{DY} \approx 1.4697$

$a_1 = \frac{K(X,Y)}{DX} \approx -0,1263$

$b_1 = EY - a_1 \cdot EX \approx 0,3579$

$g(x) = -0,1263x + 0,3579$

| X | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| g(x) | 0.3579 | 0.2316 | 0.1053 | -0.0211 |
| $m_{Y\|X}(x)$ | 0.7143 | -1 | 0.5 | 0 |

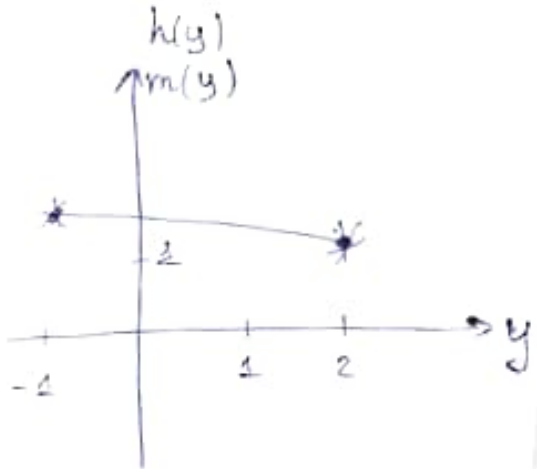


$h(y) = a_2 y + b_2$, где $a_2 = \frac{K(X,Y)}{DY}$ и $b_2 = EX - a_2 \cdot EY$

$a_2 \approx -0.0694$

$b_2 \approx 1.2639$

| Y | -1 | 2 |
|---|---|---|
| $h(y)$ | 1.333 | 1.125 |
| $m_{X\|Y}(y)$ | 1.333 | 1.125 |



```
x = [0,1,2,3]
y = [-1,2]
p = [0.15,0.2,0.15,0.1;0.2,0,0.15,0.05]
px = sum(p)
py = sum(p')
test = sum(py)
Ex = x * px'
Ey = y * py'
Exy = y * p * x'
k = Exy - Ex*Ey
Ex2 = x.^2*px'
Dx = Ex2 - Ex^2
sx = sqrt(Dx)
Ey2 = y.^2*py'
Dy = Ey2 - Ey^2
sy = sqrt(Dy)
r = k/(sx*sy)
mx = y*p./px
my = x*p'./py
Ey1 = sum(mx.*px)
Ex1 = sum(my.*py)
k1 = k / Dx
b1 = Ey - k1*Ex
g1 = k1*x + b1
a2 = k / Dy
b2 = Ex - a2*Ey
g2 = a2*y + b2
```

Так же я проверил вычисления в Octave:

Ex =  1.2500 (мат. ожидание X)
Ey =  0.20000 (мат. ожидание Y)
Exy =  0.10000 (мат. ожидание XY)

k = -0.15000

Ex2 =  2.7500 (мат. ожидание $X^2$)
Dx =  1.1875 (дисперсия X)
sx =  1.0897 (СКО X)

Ey2 =  2.2000 (мат. ожидание $Y^2$)
Dy =  2.1600 (дисперсия Y)
sy =  1.4697 (СКО Y)

```
Проверяю регрессию:
mx =

   0.71429  -1.00000   0.50000   0.00000

my =

   1.3333   1.1250

Ey1 =  0.20000
Ex1 =  1.2500
a1 = -0.12632
b1 =  0.35789
g1 =

   0.357895   0.231579   0.105263  -0.021053 (значения g(x) в точках x)

a2 = -0.069444
b2 =  1.2639
g2 =

   1.3333   1.1250 (значения h(y) в точках y)
```

А вот карие получились графики: