Кр.

$$G=\begin{pmatrix}5 & -2\\ -2 & 1\end{pmatrix} \qquad x=(1\,1)^T \qquad y=(0\,1)^T$$

1) $\cos\widehat{\vec{x},\vec{y}}=?$

$$\cos\widehat{\vec{x},\vec{y}}=\frac{(x,y)}{|x|\cdot|y|}=\frac{(x,y)}{\sqrt{(x,x)}\cdot\sqrt{(y,y)}}$$

$$(x,y)=(1,1)\begin{pmatrix}5 & -2\\ -2 & 1\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix}=(3\ \ -1)\begin{pmatrix}0\\ 1\end{pmatrix}=-1$$

$\cancel{(x,x)=5(e_1+e_2,e_2)=2(e_1,e_1)+(e_2,e_2)=-2+1}$

$$(x,x)=(e_1+e_2,e_1+e_2)=(e_1,e_1)+2(e_1,e_2)+(e_2,e_2)=5+1-4=2$$

$$(y,y)=(e_2,e_2)=1$$

Тогда $\cos\widehat{\vec{x},\vec{y}}=-\frac{1}{\sqrt{2}}\approx-0.71$

2) $$\begin{pmatrix}1 & -2 & -1 & 3\\ 0 & 1 & 0 & 0\\ 1 & -1 & 0 & 1\end{pmatrix}\sim\begin{pmatrix}0 & 0 & -1 & 2\\ 0 & 1 & 0 & 0\\ 1 & 0 & 0 & 1\end{pmatrix}$$

$$b_1=(0\ 0\ -1\ 2)^T$$

$$b_2=a_2-\frac{(b_1,a_2)}{(b_1,b_1)}b_1=(0\,1\,0\,0)^T-0=(0\,1\,0\,0)^T$$

$$b_3=a_3-\frac{(b_1,a_3)}{(b_1,b_1)}b_1-\frac{(b_2,a_3)}{(b_2,b_2)}b_2=(1\,0\,0\,1)^T-\frac{2}{5}(0\,0\,-1\,2)^T-0=$$
$$=(1\,0\,0\,1)^T-(0\ 0\ -\tfrac{2}{5}\ \tfrac{4}{5})^T=(1\ 0\ \tfrac{2}{5}\ \tfrac{9}{5})^T$$

$$b_1=(0\ 0\ -1\ 2)^T$$

$$b_2=(0\,1\,0\,0)^T$$

$$b_3=(1\,0\,0\,1)^T-\frac{2}{5}(0\ 0\ -1\ 2)^T=\begin{matrix}(1\,0\,0\,1)^T+\\ +(0\ 0\ \tfrac{2}{5}\ -\tfrac{4}{5})^T\end{matrix}=(1\ 0\ \tfrac{2}{5}\ \tfrac{1}{5})^T$$

3) $\begin{array}{c} e_1 \\ e_2 \end{array}\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$ $x = (-2\ -1\ -2\ -2)^T$ $\quad Pr_L - ?$

$(-2\ -1\ -2\ -2)^T\ x$

$(1\ 0\ 1\ 1)^T\ b_1$

$b_1 = e_1 = (1\ 0\ 1\ 1)^T$

$$b_2 = e_2 - \frac{(b_1, e_2)}{(b_1, b_1)} b_1 = (0\ 1\ 1\ 0)^T - \frac{1}{3}(1\ 0\ 1\ 1)^T = \left(-\frac{1}{3}\ 1\ \frac{2}{3}\ -\frac{1}{3}\right)^T b_2$$

$$Pr(x)_L = Pr(x)_{b_1} + Pr(x)_{b_2} = \frac{(x, b_1)}{(b_1, b_1)} b_1 + \frac{(x, b_2)}{(b_2, b_2)} b_2 =$$

$\frac{1}{9} + \frac{4}{9} + \frac{1}{9}$ $\quad \frac{6}{5} + 1$ $\quad \frac{2}{3} - 1$ $\quad \frac{5}{3}$

$\frac{2}{3} - 1 - \frac{4}{3} + \frac{1}{3}$

$-1 \quad \frac{6}{5} - \frac{2}{3}$

$$= \frac{-6}{3}(1\ 0\ 1\ 1)^T + \frac{\left(\frac{2}{3} - 1 - \frac{4}{3} + \frac{2}{3}\right)}{\left(\frac{1}{9} + 1 + \frac{4}{9} + \frac{1}{9}\right)} \cdot \left(\frac{1}{3}\ 1\ \frac{2}{3}\ \frac{1}{3}\right)^T = -2 \cdot b_1 + \left(-\frac{3}{5}\right) b_2$$

$$\cancel{= ((-2\ 0\ -2\ -2)^T + \left(\frac{1}{5}\ \frac{3}{5}\ \frac{2}{5}\ -\frac{1}{5}\right)^T = (-2{,}2\ -0{,}6\ -2{,}4\ -1{,}8)^T}$$

6) $\lambda : (4\ 2\ 2)$ $\left(\begin{array}{cc|cc} \lambda & 1 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ \hline 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & \lambda \end{array}\right)$

7) $\begin{pmatrix} 6 & 1 & 1 \\ 2 & 4 & 1 \\ -4 & -2 & 2 \end{pmatrix}$ $\chi_A(x) = (x-4)^3$

$\lambda = 4$ $\left(\begin{array}{ccc|c} 2 & 1 & 1 & \eta^1 \\ 2 & 0 & 1 & \eta^2 \\ -4 & -2 & -2 & \eta^3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 0 & 1 & \eta^2 \\ 0 & 1 & 0 & \eta^1-\eta^2 \\ 0 & 0 & 0 & \eta^3+2\eta^1 \end{array}\right)$

$\exists\ \eta^1 = \eta^2 = \eta^3 = 0 \Rightarrow \xi_1 = (1\ 0\ {-2})^T$

$\xi_1' : \left(\begin{array}{ccc|c} 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \xi_1' = (0\ 1\ 0)^T$

$\xi_1'' : \left(\begin{array}{ccc|c} 2 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \xi_1'' (1\ 0\ {-1})^T$

тогда базис: $\{\xi_1, \xi_1', \xi_1''\}$ $A^J = \begin{pmatrix} 4 & 1 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{pmatrix} \Rightarrow p_{min} = \chi_A$

$\lambda = 4\ (3\,3\,1)$

5) $A = \begin{pmatrix} 6 & 0 & -2 \\ 0 & 4 & 0 \\ -2 & 0 & 6 \end{pmatrix}$ $\sigma_A - ?$

$|A - \lambda E| = \det\begin{pmatrix} 6-\lambda & 0 & -2 \\ 0 & 4-\lambda & 0 \\ -2 & 0 & 6-\lambda \end{pmatrix} = (4-\lambda)\left((6-\lambda)^2 - 4\right) = (4-\lambda)(4-\lambda)(8-\lambda) \Rightarrow$

$\Rightarrow \sigma_A = \{4^{(2)}, 8^{(1)}\}$

4) $A = \begin{pmatrix} 8 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 8 \end{pmatrix}$ $\quad \sigma_A - ?$

$|A - \lambda E| = \det \begin{pmatrix} 8-\lambda & 0 & -4 \\ 0 & 4-\lambda & 0 \\ -4 & 0 & 8-\lambda \end{pmatrix} = (4-\lambda)\left((8-\lambda)^2 - 16\right) = (4-\lambda)(4-\lambda)(12-\lambda)$

$\sigma_A = \{4^{(2)}, 12^{(1)}\}$

тогда

$\lambda = 4$: $\begin{pmatrix} 4 & 0 & -4 \\ 0 & 0 & 0 \\ -4 & 0 & 4 \end{pmatrix} \begin{pmatrix} \xi^1 \\ \xi^2 \\ \xi^3 \end{pmatrix} = 0 \begin{cases} \xi^1 = \xi^3 \\ 0 = 0 \\ \xi^1 = \xi^3 \end{cases} \Rightarrow x_1 = (1\ 0\ 1)^T$

$x_{1.2} = (2\ 0\ 2)^T$

$\lambda = 12$: $\begin{pmatrix} -4 & 0 & -4 \\ 0 & -8 & 0 \\ -4 & 0 & -4 \end{pmatrix} \begin{pmatrix} \xi^1 \\ \xi^2 \\ \xi^3 \end{pmatrix} = 0 \Leftrightarrow \begin{cases} \xi^1 = -\xi^3 \\ \xi^2 = 0 \\ \xi^1 = -\xi^3 \end{cases} \Rightarrow x_2 = (1\ 0\ -1)^T$