$$\begin{pmatrix} 2a_1 & 2b_1 & 2c_1 & 1 \\ 2a_2 & 2b_2 & 2c_2 & 1 \\ 2a_3 & 2b_3 & 2c_3 & 1 \\ 2a_4 & 2b_4 & 2c_4 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ y_0 \\ z_0 \\ B \end{pmatrix} = \begin{pmatrix} \sum x^2 \\ \\ \\ \\ \end{pmatrix}$$

ДЗ 3.

N1

$$\begin{vmatrix} 1 & 2 & 3 & 4 & \dots & n \\ -1 & 0 & 3 & 4 & & n \\ -1 & -2 & 0 & 4 & & n \\ \dots \\ -1 & -2 & -3 & -4 & \dots & 0 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 & 4 & & n \\ 0 & 2 & 6 & 8 & \dots & 2n \\ 0 & 0 & 3 & 8 & & 2n \\ \dots \\ 0 & 0 & 0 & 0 & \dots & n \end{vmatrix} = \cancel{1\cdot 2\cdot(\dots)\cdot(n-1)\cdot n}$$

$= n!$

N2

$$\begin{vmatrix} \alpha+\beta & \alpha\beta & 0 & \dots & 0 \\ 1 & \alpha+\beta & \alpha\beta & \dots & 0 \\ 0 & 1 & \alpha+\beta & \dots & 0 \\ \dots \\ 0 & 0 & 0 & \dots & \alpha+\beta \end{vmatrix} = (\alpha+\beta)\cdot D_{n-1} - \begin{vmatrix} \alpha\beta & 0 & 0 & 0 \\ 1 & \alpha+\beta & \alpha\beta & 0 \\ 0 & 1 & \alpha+\beta & 0 \\ 0 & 0 & 0 & \alpha+\beta \end{vmatrix}$$

$= \boxed{(\alpha+\beta) D_{n-1} - \alpha\beta \cdot D_{n-2} = D_n}$

$D_1 = \alpha+\beta$

$D_2 = \alpha^2+\beta^2+\alpha\beta$

$D_n = \alpha(D_{n-1} - \beta D_{n-2}) + \beta D_{n-1}$

$D_n = \beta(D_{n-1} - \alpha D_{n-2}) + \alpha D_{n-1}$

$\mathcal{D}_n = (\alpha+\beta)\mathcal{D}_{n-1} - \alpha\beta\mathcal{D}_{n-2}$

1) $\mathcal{D}_n - \alpha\mathcal{D}_{n-1} = \beta(\mathcal{D}_{n-1} - \alpha\mathcal{D}_{n-2}) = \beta^{n-2}\cdot(\mathcal{D}_2 - \alpha\mathcal{D}_1)$.

2) $\mathcal{D}_n - \beta\mathcal{D}_{n-1} = \alpha^{n-2}(\mathcal{D}_2 - \beta\mathcal{D}_1)$

$= \beta^{n-2}(\alpha^2+\beta^2+\alpha\beta-\alpha^2-\alpha\beta) = \beta^n$

Аналогично $\mathcal{D}_n - \beta\mathcal{D}_{n-1} = \alpha^n$

$\mathcal{D}_n = \dfrac{\alpha^n - \beta^n + \mathcal{D}_{n-1}(\alpha+\beta)}{\alpha}$

сложим 1) − 2)

$\alpha\mathcal{D}_{n-1} + \beta^n = \beta\mathcal{D}_{n-1} + \alpha^n$

$$\mathcal{D}_{n-1} = \frac{\alpha^n - \beta^n}{\alpha-\beta} \Rightarrow \mathcal{D}_n = \frac{\alpha^{n+1} - \beta^{n+1}}{\alpha-\beta}$$

$\alpha = 3;\ \beta = 2;\ n = 18$

$$\mathcal{D}_{18} = \frac{3^{19} - 2^{19}}{1}$$

№3.

$$\begin{vmatrix} -x & a & b & c \\ a & -x & c & b \\ b & c & -x & a \\ c & b & a & -x \end{vmatrix} = \begin{vmatrix} a+b+c-x & a+b+c-x & a+b+c-x & a+b+c-x \\ a & -x & c & b \\ b & c & -x & a \\ c & b & a & -x \end{vmatrix} =$$

$$= (a+b+c-x)\underbrace{\begin{vmatrix} 1 & 1 & 1 & 1 \\ a & -x & c & b \\ b & c & -x & a \\ c & b & a & -x \end{vmatrix}}_{\mathcal{D}_1} = (a+b+c-x)\cdot\mathcal{D}_1$$

$$D_1 = \begin{vmatrix} 1 & 1 & 1 & 1 \\ a & -x & c & b \\ b & c & -x & a \\ c & b & a & -x \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 & 1 \\ a-x-b-c & -x & c & b \\ b+c+x-a & c & -x & a \\ c+b+x-a & b & a & -x \end{vmatrix} = (a-x-b-c) \underbrace{\begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & -x & c & b \\ -1 & c & -x & a \\ -1 & b & a & -x \end{vmatrix}}_{D_2}$$

$$D_2 = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & -x & c & b \\ -1 & c & -x & a \\ -1 & b & a & -x \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & -x & c & b \\ 0 & c-x & c-x & a+b \\ 0 & b-x & a+c & b-x \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & -x & c & b \\ 0 & c-x-b-a & c-x-b-a & 0 \\ 0 & b-x-c-a & 0 & b-x-c-a \end{vmatrix} =$$

$$= (c-x-b-a)(b-x-c-a) \underbrace{\begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & -x & c & b \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{vmatrix}}_{D_3}$$

$$D_3 = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & -x & c & b \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 1 & -x & c & b \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 1 \Rightarrow$$

$\Rightarrow D = (a-b-c-x)(a-x-b-c)(c-x-b-a)(b-x-c-a) =$

$= (a+b+c-x)(a+b-c+x)(a-b-c-x)(a-b+c+x)$

~~$x=7$~~ ; $a=8$; $b=3$; $c=4$; $x=7$

$D = (8+3+4-7)(8+3-4+7)(8-3-4-7)(8-3+4+7) =$

$= 8 \cdot 14 \cdot (-6) \cdot 16 = -10752$

№4.

$$D_n = \begin{vmatrix} x_1 & a_1b_2 & a_1b_3 & \dots & a_1b_n \\ a_2b_1 & x_2 & a_2b_3 & \dots & a_2b_n \\ a_3b_1 & a_3b_2 & x_3 & \dots & a_3b_n \\ \dots & \dots & \dots & \dots & \dots \\ a_nb_1 & a_nb_2 & a_nb_3 & \dots & x_n \end{vmatrix} =$$

$$= \prod_{i=1}^{n} a_i \cdot \underbrace{\begin{vmatrix} \frac{x_1}{a_1} & b_2 & b_3 & \dots & b_n \\ b_1 & \frac{x_2}{a_2} & b_3 & \dots & b_n \\ b_1 & b_2 & \frac{x_3}{a_3} & \dots & b_n \\ \dots & \dots & \dots & \dots & \dots \\ b_1 & b_2 & b_3 & \dots & \frac{x_n}{a_n} \end{vmatrix}}_{D_n^*}$$

$$] \ \frac{x_i}{a_i} = \left(\frac{x_i}{a_i} - b_i\right) + b_i$$

Тогда по th:

$$D^* = D^{*\prime} + x_i \sum_{i,j} A_{ij}, \text{тогда}$$

$$D_n = a_1 \cdot \ldots \cdot a_n \cdot \left(\frac{x_1}{a_1} - b_1\right) \cdot \ldots \cdot \left(\frac{x_n}{a_n} - b_n\right) \cdot \left(1 + \frac{a_1b_1}{x_1} + \ldots + \frac{a_nb_n}{x_n}\right)$$

~~$D_n = a_1 \cdot \ldots \cdot a_n \cdot \left(\frac{x_1}{a_1} - b_1\right) \cdot \ldots \cdot \left(\frac{x_n}{a_n} - b_n\right) \cdot$~~