

Домашняя работа № 2

№1.

$$\begin{pmatrix} 0 & 18 & 11 & 14 & 13 \\ 0 & 0 & 18 & 18 & 13 \\ 0 & 0 & 0 & 18 & 17 \\ 0 & 0 & 0 & 0 & 16 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}^6$$

= $O_{5 \times 5}$, т.к. матрица nilpotentная (на $k=5$)

№3.

$$\begin{vmatrix} -2 & 0 & 0 & -1 \\ 4 & 0 & 2 & -4 \\ -2 & -2 & 2 & -1 \\ -5 & -3 & 2 & -4 \end{vmatrix} = -4 \cdot (-1)^5 \cdot \begin{vmatrix} -2 & 0 & -1 \\ 4 & 2 & -4 \\ -5 & 2 & -4 \end{vmatrix} + (-3) \cdot (-1)^6 \cdot \begin{vmatrix} -2 & 0 & -1 \\ 4 & 2 & -4 \\ -2 & 2 & -1 \end{vmatrix}$$

$$= 4 \cdot \left(2 \begin{vmatrix} -2 & -1 \\ -5 & -4 \end{vmatrix} - 2 \begin{vmatrix} -2 & -1 \\ 4 & -4 \end{vmatrix} \right) - 3 \cdot \left(2 \begin{vmatrix} -2 & -1 \\ -2 & -1 \end{vmatrix} - 2 \begin{vmatrix} -2 & -1 \\ 4 & -4 \end{vmatrix} \right) =$$

$$= 4 \cdot (2(8-5) - 2(8+4)) - 3 \cdot (2(2+2) - 2(8-4)) =$$

$$= -22 - 48 = -70$$

№4.

$$\begin{vmatrix} 0 & 3 & 4 & -4 \\ -5 & 0 & -3 & -3 \\ 3 & -3 & 1 & 2 \\ -5 & 0 & -5 & 0 \end{vmatrix} = -3 \begin{vmatrix} -5 & -3 & -3 \\ 3 & 1 & 2 \\ -5 & -5 & 0 \end{vmatrix} + 3 \begin{vmatrix} 0 & 4 & -4 \\ -5 & -3 & -3 \\ -5 & -5 & 0 \end{vmatrix} =$$

$$= -3 \cdot (30 + 45 - 15 - 50) + 3 \cdot (60 - 100 + 60) = -30 + 60 = 30$$

N5.

$$\begin{vmatrix} 4 & -4 & 4 \\ -3 & X & -2 \\ X & -3 & -7 \end{vmatrix} = 0$$

$$-24X + 8X + 36 - 4X^2 + 28X - 24 = 0$$

$$-4X^2 - 20X + 60 + 36 = 0 \quad | :(-4)$$

$$X^2 + 5X - 15 - 9 = 0$$

$$X^2 + 5X - 24 = 0$$

$$X = 3$$

$$X = -8$$

Ответ: $X = -8$

$$X = 3$$

N6.

$$\begin{vmatrix} -2 & 2 & 0 & 2 & 0 \\ 0 & 1 & -1 & -1 & -1 \\ 0 & 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 & -1 \end{vmatrix}$$

$$= -2 \cdot 1 \cdot 1 \cdot (-2) \cdot (-1) = -4, \text{ т.к. 2mo BTM}$$

N7.

$$\begin{vmatrix} 2 & -3 & -1 & 0 & 1 & 1 \\ 4 & -7 & 0 & 0 & 2 & -1 \\ 12 & -20 & -2 & -3 & 7 & 1 \\ 60 & -100 & -10 & -12 & 34 & -1 \\ 0 & 0 & 0 & 0 & -2 & -1 \\ 0 & 0 & 0 & 0 & 4 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 2 & -3 & -1 & 0 & 1 & 1 \\ 0 & -1 & 2 & 0 & 0 & -3 \\ 12 & -20 & -2 & -3 & 7 & 1 \\ 0 & 0 & 0 & 3 & -1 & -6 \\ 0 & 0 & 0 & 0 & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix} = 0$$

М.

$$\sum_{i=1}^n x_i = 8$$

$$\begin{cases} \sum_{i=1}^n x_i - \sum_{i=2}^n x_i = -3 \\ -2\sum_{i=1}^n x_i + 3\sum_{i=2}^n x_i + \sum_{i=3}^n x_i = 1 \\ 4\sum_{i=1}^n x_i - 6\sum_{i=2}^n x_i - \sum_{i=3}^n x_i = 4 \end{cases}$$

$$-2\sum_{i=1}^n x_i + 3\sum_{i=2}^n x_i + \sum_{i=3}^n x_i = 1$$

$$4\sum_{i=1}^n x_i - 6\sum_{i=2}^n x_i - \sum_{i=3}^n x_i = 4$$

Решим систему методом Крамера

$$\Delta = \begin{vmatrix} 1 & -1 & 0 \\ -2 & 3 & 1 \\ 4 & -6 & -1 \end{vmatrix} = -3 - 4 + 2 + 6 = 1$$

$$\Delta_1 = \begin{vmatrix} -3 & -1 & 0 \\ 1 & 3 & 1 \\ 4 & -6 & -1 \end{vmatrix} = 3 - 4 - 18 - 1 = -14 \Rightarrow \sum_{i=1}^n x_i = \frac{\Delta_1}{\Delta} = -14$$

$$\Delta_2 = \begin{vmatrix} 1 & -3 & 0 \\ -2 & 1 & 1 \\ 4 & 4 & -1 \end{vmatrix} = -1 - 12 + 6 - 4 = -11 \Rightarrow \sum_{i=2}^n x_i = \frac{\Delta_2}{\Delta} = -11$$

$$\Delta_3 = \begin{vmatrix} 1 & -1 & -3 \\ -2 & 3 & 1 \\ 4 & -6 & 4 \end{vmatrix} = 12 - 4 - 36 + 36 - 8 + 6 = 6 \Rightarrow \sum_{i=3}^n x_i = \frac{\Delta_3}{\Delta} = 6$$

Проверка:

$$-14 + 11 = -3 \text{ (верно)}$$

$$+28 - 33 + 6 = 1 \text{ (верно)}$$

$$\text{Ответ: } (-14; -11; 6)$$

№9.

$$\begin{vmatrix} -3 & -4 & 4 & 0 \\ -4 & 0 & 2 & 0 \\ 0 & -4 & 1 & -1 \\ -5 & 0 & 1 & -2 \end{vmatrix} = \begin{vmatrix} -3 & -4 & 4 & 0 \\ -4 & 0 & 2 & 0 \\ 0 & -4 & 1 & -1 \\ -5 & 8 & -1 & 0 \end{vmatrix} = \begin{vmatrix} 5 & -4 & 0 & 0 \\ -4 & 0 & 2 & 0 \\ 0 & -4 & 1 & -1 \\ -5 & 8 & -1 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 5 & -4 & 0 \\ -4 & 0 & 2 \\ -5 & 8 & -1 \end{vmatrix} = \begin{vmatrix} 5 & -4 & 0 \\ -4 & 0 & 2 \\ 0 & -4 & -1 \end{vmatrix} = \underline{\underline{-24}}$$

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Основная задача №2
№3.

$$\begin{vmatrix} -2 & 0 & 0 & -1 \\ 1 & 0 & 2 & -4 \\ -2 & -4 & 2 & -1 \\ -5 & -3 & 2 & -4 \end{vmatrix} = -2 \cdot \begin{vmatrix} 0 & 2 & -4 \\ -4 & 2 & -1 \\ -3 & 2 & -4 \end{vmatrix} + 1 \cdot \begin{vmatrix} 4 & 0 & 2 \\ -2 & -4 & 2 \\ -5 & -3 & 2 \end{vmatrix} = -2(6 + 22 - 12 - 24) + (-32 + 12 - 40 + 24) = 24 \cdot 2 - 12 + 12 + 24 - 72 = 0$$

№2.

$$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} \equiv \begin{vmatrix} 1 & a & bc \\ 0 & b-a & ca-bc \\ 0 & b-c & a(b-c) \end{vmatrix} = \begin{vmatrix} 1 & a & bc \\ 0 & b-a & c(a-b) \\ 0 & b-c & a(b-c) \end{vmatrix} =$$

$$= a(b-a)(b-c) - c(b-c)(a-b) = (b-c)(ab - a^2 - ac + bc)$$

$$\equiv ab^2 - a^2c + bc^2 - b^2c - a^2b - c^2a = \cancel{b^2(a-c)} = k$$

$$\Delta \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = bc^2 + ab^2 + a^2c - a^2b - b^2c - ac^2 = k$$

умг.