№1.

$$\left(\begin{array}{ccccc|c} 1 & 0 & -2 & 3 & -4 & -5 \\ -2 & 1 & 3 & -4 & 6 & 16 \\ -1 & 0 & 3 & -5 & 5 & -5 \\ 0 & 1 & -3 & 7 & -4 & 44 \\ 3 & -3 & 0 & -4 & -3 & -3 \end{array}\right) \sim \left(\begin{array}{ccccc|c} 1 & 0 & -2 & 3 & -4 & -5 \\ 0 & 1 & -1 & 2 & -2 & 6 \\ 0 & 0 & 1 & -2 & 1 & -10 \\ 0 & 1 & -3 & 7 & -4 & 44 \\ 0 & -3 & 6 & -13 & 5 & -48 \end{array}\right)$$

$$\sim \left(\begin{array}{ccccc|c} 1 & 0 & -2 & 3 & -4 & -5 \\ 0 & 1 & -1 & 2 & -2 & 6 \\ 0 & 0 & 1 & -2 & 1 & -10 \\ 0 & 0 & -2 & 5 & -2 & 38 \\ 0 & 0 & 3 & -7 & 3 & -48 \end{array}\right) \sim \left(\begin{array}{ccccc|c} 1 & 0 & -2 & 3 & -4 & -5 \\ 0 & 1 & -1 & 2 & -2 & 6 \\ 0 & 0 & 1 & -2 & 1 & -10 \\ 0 & 0 & 0 & 1 & 0 & 18 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

x_5 - своб. переменная

$$\begin{cases} x_5 = x_5 \\ x_4 = 18 \\ x_3 = 26 - x_5 \\ x_2 = x_5 - 4 \\ x_1 = 2x_5 - 7 \end{cases}$$

частн. р-е: $(-7, -4, 26, 18, 0)^T$

№2.

$$\left(\begin{array}{ccccc|c} 1 & 1 & -3 & -4 & 1 & 33 \\ -1 & 0 & 2 & 4 & -1 & -86 \\ 1 & 0 & -3 & -7 & 2 & 72 \\ -5 & -2 & 11 & 19 & -5 & -174 \\ 2 & 3 & -12 & -21 & 7 & 243 \end{array}\right) \sim \left(\begin{array}{ccccc|c} 1 & 1 & -3 & -4 & 1 & 33 \\ 0 & 1 & -1 & 0 & 0 & -3 \\ 0 & -2 & 3 & 1 & 0 & 6 \\ 0 & 3 & -4 & -1 & 0 & -9 \\ 0 & -4 & 4 & 0 & 0 & 12 \end{array}\right) \sim$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & -3 & -4 & 1 & 33 \\ 0 & 1 & -1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccccc|c} 1 & 1 & -3 & -4 & 1 & 33 \\ 0 & 1 & -1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \sim$$

$$\left(\begin{array}{ccccc|c} 1 & 0 & 0 & -2 & 1 & 36 \\ 0 & 1 & 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

x_4, x_5 – свободные пер-е

$$\begin{cases} x_1 = 2x_4 - x_5 + 36 \\ x_2 = -x_4 - 3 \\ x_3 = -x_4 \\ x_4 = x_4 \\ x_5 = x_5 \end{cases}$$

ФСР

частное реш. ($x_4 = x_5 = 0$)

$$\text{ОЛС: } C_1 \begin{pmatrix} 2 \\ -1 \\ -1 \\ 1 \\ 0 \end{pmatrix} + C_2 \begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 36 \\ -3 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$q = [2, 3, 1, -1, -2]$

$z = [2, 0, -2, 4, 1]$

№4.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ -2 & 1 & 0 & 1 & 1 \\ 3 & -2 & 1 & -3 & 0 \\ -6 & 3 & -1 & 4 & 2 \\ 13 & -6 & 1 & -7 & -6 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & -2 & 1 & -3 & 3 \\ 0 & 3 & -1 & 4 & -4 \\ 0 & -6 & 1 & -7 & 7 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & -1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{cases} x_1 = x_5 \\ x_2 = x_5 - x_4 \\ x_3 = x_4 - x_5 \\ x_4 = x_4 \\ x_5 = x_5 \end{cases}$$

$$\text{ОЛН: } C_1 \begin{pmatrix} 0 \\ -1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + C_2 \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

№3.

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & -3 & -7 & -128 \\ -1 & 0 & 0 & 2 & 4 & 83 \\ 0 & -1 & 1 & 1 & 2 & -5 \\ 0 & -2 & 1 & 0 & 3 & 40 \\ 2 & 3 & -1 & -5 & -10 & -251 \end{array}\right) \sim \left(\begin{array}{ccccc|c} 1 & 1 & 0 & -3 & -7 & -128 \\ 0 & 1 & 0 & -1 & -3 & -45 \\ 0 & -1 & 1 & -1 & 2 & -5 \\ 0 & 0 & -1 & 2 & 1 & 50 \\ 0 & 1 & -1 & 1 & -2 & 5 \end{array}\right) \sim$$

№5

$$\begin{pmatrix} 1 \\ -5 \\ -2 \\ 4 \\ 2 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & -3 & -7 & -129 \\ 0 & 1 & 0 & -1 & -3 & -45 \\ 0 & 0 & 1 & -2 & -1 & -50 \\ 0 & 0 & -1 & 2 & 1 & 50 \\ 0 & 0 & -1 & 2 & 1 & 50 \end{array}\right) \sim \left(\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & -2 & -4 & -83 \\ 0 & 1 & 0 & -3 & -7 & -129 \\ 0 & 1 & 0 & -1 & -3 & -45 \\ 0 & 0 & 1 & -2 & -1 & -50 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

x_4, x_5 – своб. пер-е

$x_1 = -x_2 + 3x_4 + 7x_5 - 83$

$x_2 = x_4 + 3x_5 - 45$

$x_3 = 2x_4 + x_5 - 50$

$x_4 = x_4$

$x_5 = x_5$

$$\text{ans: } C_1 \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \\ 1 \\ 0 \end{pmatrix} + C_2 \cdot \begin{pmatrix} 4 \\ 3 \\ 1 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} -83 \\ -45 \\ -50 \\ 0 \\ 0 \end{pmatrix}$$

ФСР — частн. реш-е

$C_1 = [2, 3, 1, -1, -2]$

$C_2 = [2, 0, -2, 4, 1]$

$$\begin{pmatrix} -x_2 + 3x_4 + 7x_5 = \\ = -x_1 + 3x_4 - 3x_5 + 9x_5 \\ x_2 + 3x_5 \\ 2x_4 + x_5 \\ x_4 \\ x_5 \end{pmatrix}$$

№5.

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & a_1 \\ -5 & -6 & -4 & -5 & a_2 \\ -2 & -2 & -3 & -2 & a_3 \\ 4 & 5 & 4 & 3 & a_4 \\ 2 & 0 & 3 & 4 & a_5 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & a_1 \\ 0 & 4 & 1 & 5 & a_2 + 5a_1 \\ 0 & 2 & -1 & 2 & a_3 + 2a_1 \\ 0 & -3 & 0 & -3 & a_4 - 3a_1 \\ 0 & -4 & 1 & 0 & a_5 - 2a_1 \end{array}\right) \sim$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & a_1 \\ 0 & 1 & 1 & 2 & a_2 + a_4 + 2a_1 \\ 0 & 2 & -1 & 2 & a_3 + 2a_1 \\ 0 & -3 & 0 & -3 & a_4 - 3a_1 \\ 0 & -1 & 1 & 0 & a_5 - a_4 + a_1 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & a_1 \\ 0 & 1 & 1 & 2 & a_2 + a_4 + 2a_1 \\ 0 & 0 & -3 & -2 & a_3 - 2a_2 + a_4 + 4a_1 \\ 0 & 0 & 3 & 3 & a_4 + a_2 - 3a_1 \\ 0 & 0 & 2 & 2 & \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & a_1 \\ -5 & -6 & -1 & -5 & a_2 \\ -2 & -2 & -3 & -2 & a_3 \\ 4 & 5 & 4 & 5 & a_4 \\ 2 & 0 & 3 & 4 & a_5 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & a_1 \\ 0 & 4 & 1 & 5 & a_2+5a_1 \\ 0 & 2 & -1 & 2 & a_3+2a_1 \\ 0 & -3 & 0 & -5 & a_4-4a_1 \\ 0 & -4 & 1 & 0 & a_5-2a_1 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & a_1 \\ 0 & 4 & 1 & 5 & a_2+5a_1 \\ 0 & 2 & -1 & 2 & a_3+2a_1 \\ 0 & 1 & 1 & 0 & a_4+a_2 \\ 0 & 0 & 2 & 5 & a_5+4a_1 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & a_1 \\ 0 & 1 & 1 & 0 & a_4+a_2+a_1 \\ 0 & 4 & 1 & 5 & a_2+5a_1 \\ 0 & 2 & -1 & 2 & a_3+2a_1 \\ 0 & 0 & 2 & 5 & a_5+a_2+3a_1 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & a_1 \\ 0 & 1 & 1 & 0 & a_4+a_2+a_1 \\ 0 & 0 & -3 & 5 & a_2+a_1-4a_4-4a_2 \\ 0 & 0 & -3 & 2 & a_3+2a_1-2a_1-2a_2-2a_4 \\ 0 & 0 & 2 & 5 & a_5+a_2+3a_1 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & a_1 \\ 0 & 1 & 1 & 0 & a_4+a_2+a_1 \\ 0 & 0 & 2 & 5 & a_2+a_1-4a_4-4a_2 \\ 0 & 0 & -3 & 2 & a_3-2a_4-2a_2 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & a_1 \\ 0 & 4 & 1 & 5 & 5a_1+a_2 \\ 0 & 0 & -6 & -2 & -2a_1-2a_2+4a_3 \\ 0 & 0 & 0 & 9 & 3a_1-3a_2-3a_3-6a_4 \\ 0 & 0 & 0 & 0 & 8a_1+16a_2+25a_3+26a_4+9a_5 \end{array}\right)$$

$8a_1 + 16a_2 + 25a_3 + 26a_4 + 9a_5 = 0$

$a_{4,5}$ — свободные переменные

$$\begin{cases} a_1 = -2a_1 - \frac{25}{8}a_3 - \frac{26}{8}a_4 - \frac{9}{8}a_5 \\ a_2 = a_2 \\ a_3 = a_3 \\ a_4 = a_4 \\ a_5 = a_5 \end{cases} \qquad e_1 = (8\,0\,0\,0\,0)^T \ \dots \ e_5 = (0\,0\,0\,0\,8)^T$$

ans: $C_1\begin{pmatrix} -16 \\ 8 \\ 0 \\ 0 \\ 0 \end{pmatrix} + C_2\begin{pmatrix} -25 \\ 0 \\ 8 \\ 0 \\ 0 \end{pmatrix} + C_3\begin{pmatrix} -26 \\ 0 \\ 0 \\ 8 \\ 0 \end{pmatrix} + C_4\begin{pmatrix} \\ \\ \\ \\ \end{pmatrix}$

общее решение

№6.

$$\left(\begin{array}{cc} 1 & 1 \\ 1 & 2 \\ -1 & -1 \\ -1 & 0 \\ 1 & 2 \end{array}\right) \sim \left(\begin{array}{cc} 1 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array}\right) \sim \left(\begin{array}{cc} 1 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array}\right)$$

$$\begin{cases} a_1 - a_2 \\ a_4 - a_2 \end{cases}$$

$$\sim \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$$

$$\begin{cases} x_1 = \\ x_2 = \\ x_3 = \\ x_4 = \\ x_5 = \end{cases}$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & -1 & 0 & a_1 \\ 1 & 2 & 0 & -1 & 0 & a_2 \\ -1 & -1 & 1 & 1 & 1 & a_3 \\ -1 & 0 & 1 & 1 & 1 & a_4 \\ 1 & 1 & -1 & -1 & -1 & a_5 \end{array}\right) \sim \left(\begin{array}{ccccc|c} 1 & 1 & 0 & -1 & 0 & a_1 \\ 0 & 1 & 0 & 0 & 0 & -a_1+a_2 \\ 0 & 0 & 1 & 0 & 1 & a_1+a_3 \\ 0 & 1 & 1 & 0 & 1 & a_1+a_4 \\ 0 & 1 & -1 & 0 & -1 & a_5-a_1 \end{array}\right) \sim$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & -1 & 0 & a_1 \\ 0 & 1 & 0 & 0 & 0 & a_2-a_1 \\ 0 & 0 & 1 & 0 & 1 & a_3+a_1 \\ 0 & 0 & 1 & 0 & 1 & 2a_1-a_2+a_4 \\ 0 & 0 & -1 & 0 & -1 & a_5-a_2 \end{array}\right) \sim \left(\begin{array}{ccccc|c} 1 & 1 & 0 & -1 & 0 & a_1 \\ 0 & 1 & 0 & 0 & 0 & a_2-a_1 \\ 0 & 0 & 1 & 0 & 1 & a_3+a_1 \\ 0 & 0 & 0 & 0 & 1 & a_4-a_3+a_1-a_2 \\ 0 & 0 & 0 & 0 & 0 & \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccccc|c} 1 & 1 & 0 & -1 & 0 & a_1 \\ 0 & 1 & 0 & 0 & 0 & -a_1+a_2 \\ 0 & 0 & 1 & 0 & 1 & a_1+a_3 \\ 0 & 0 & 0 & 0 & 0 & a_1-a_2-a_3+a_4 \\ 0 & 0 & 0 & 0 & 0 & a_1-a_2+a_3+a_5 \end{array}\right)$$

$$\begin{cases} a_1-a_2-a_3+a_4=0 \\ a_1-a_2+a_3+a_5=0 \end{cases} \qquad \left(\begin{array}{ccccc} 1 & -1 & -1 & 1 & 0 \\ 1 & -1 & 1 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccccc} 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 2 & -1 & 1 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccccc} 1 & -1 & -\frac{1}{2} & 0 & 1 \\ 0 & 0 & 2 & -1 & 1 \end{array}\right)$$

$$\begin{cases} x_1 = x_2 - x_3 - x_5 \\ x_2 = x_2 \\ x_3 = \frac{x_4 - x_5}{2} \\ x_4 = x_4 \\ x_5 = x_5 \end{cases} \qquad \left[\begin{array}{ccccc} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right] \leftarrow \text{задача 2}$$

$$x = c_1\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + c_2\begin{pmatrix} -1 \\ 0 \\ \frac{1}{2} \\ 1 \\ 0 \end{pmatrix} + c_3\begin{pmatrix} -1 \\ 0 \\ -\frac{1}{2} \\ 0 \\ 1 \end{pmatrix}$$

$e_1 = (100)^T$
$e_2 = (010)^T$
$e_3 = (001)^T$