№1.

$$23x^2+2\sqrt{3}xy+21y^2+(8\sqrt{3}+138)x+(6\sqrt{3}+168)y+24\sqrt{3}+539=0$$

$$B^2-4AC=4\cdot3-21\cdot23\cdot4<0 \Rightarrow \text{эллипт. тип}$$

$$tg2\varphi=\frac{2\sqrt{3}}{23-21}=\sqrt{3} \Rightarrow \varphi=\frac{\pi}{6}$$

$$\begin{cases} x=x'\cos\varphi-y'\sin\varphi \\ y=x'\sin\varphi+y'\cos\varphi \end{cases} \quad \begin{cases} x=\frac{\sqrt{3}}{2}x'-\frac{1}{2}y' \\ y=\frac{1}{2}x'+\frac{\sqrt{3}}{2}y' \end{cases}$$

$$23\left(\frac{\sqrt{3}}{2}x'-\frac{1}{2}y'\right)^2=23\cdot\left(\frac{3}{4}(x')^2-\frac{\sqrt{3}}{2}x'y'+\frac{1}{4}(y')^2\right)$$

$$2\sqrt{3}\left(\frac{\sqrt{3}x'}{2}-\frac{y'}{2}\right)\left(\frac{x'}{2}+\frac{\sqrt{3}y'}{2}\right)=2\sqrt{3}\left(\frac{\sqrt{3}(x')^2}{4}+\frac{3x'y'}{4}-\frac{x'y'}{4}-\frac{\sqrt{3}(y')^2}{4}\right)$$

$$21\left(\frac{1}{2}x'+\frac{\sqrt{3}}{2}y'\right)^2=21\cdot\left(\frac{(x')^2}{4}+\cancel{\frac{2\sqrt{3}x'y'}{2\cdot4}}+\frac{3(y')^2}{4}\right)$$

$$\frac{69}{4}(x')^2-\cancel{\frac{23\sqrt{3}x'y'}{2}}+\frac{23(y')^2}{4}+\frac{6(x')^2}{4}+\cancel{\frac{2\sqrt{3}x'y'}{2}}-\frac{6(y')^2}{4}+\frac{21(x')^2}{4}+$$

$$+\cancel{\frac{21\sqrt{3}x'y'}{2}}+\frac{63(y')^2}{4}=\frac{\overset{96}{\cancel{119}}(x')^2}{4}+20(y')^2=\underline{24(x')^2+20(y')^2}$$

$$(8\sqrt{3}+138)\left(\frac{\sqrt{3}x'}{2}-\frac{y'}{2}\right)=12x'-4\sqrt{3}y'+69\sqrt{3}x'-69y'$$

$$+\ (6\sqrt{3}+168)\left(\frac{x'}{2}+\frac{\sqrt{3}y'}{2}\right)=3\sqrt{3}x'+9y'+84x'+84\sqrt{3}y'$$

$$96x'+72\sqrt{3}x'-60y'+80\sqrt{3}y'$$

№1.

$$24(x')^2+20(y')^2+96x'+72\sqrt{3}x'-60y'+80\sqrt{3}y'+24\sqrt{3}+559=0$$

$$\begin{cases} x'=x_0+x'' \\ y'=y_0+y'' \end{cases}$$

$$24(x'')^2+20(y'')^2+2\cdot 24x''\cdot x_0+24x_0^2+20\cdot 2y''y_0+20y_0^2+$$
$$+96x''+96x_0+72\sqrt{3}x''+72\sqrt{3}x_0+80\sqrt{3}y_0+80\sqrt{3}y''-60y_0-60y''+$$
$$+24\sqrt{3}+559=0$$

$$\begin{cases} 48x_0+96+72\sqrt{3}=0 \\ 40y_0+80\sqrt{3}-60=0 \end{cases} \qquad \begin{cases} x_0=-2-\frac{3\sqrt{3}}{2} \\ y_0=-2\sqrt{3}+\frac{3}{2} \end{cases}$$

$$24(x'')^2+20(y'')^2+258+144\sqrt{3}+285-120\sqrt{3}+24\sqrt{3}+559+$$
$$+120\sqrt{3}-90-480+120\sqrt{3}-192-144\sqrt{3}-144\sqrt{3}-324=0$$

$24(x'')^2+20(y'')^2=-16 \Rightarrow$ мнимый эллипс!

№2. $3x^2+y^2-6x-4y-1=0$

$$\begin{cases} x = x_0 + x' \\ y = y_0 + y' \end{cases}$$

$$3(x_0+x')^2+(y_0+y')^2-6(x_0+x')-4(y_0+y')-1=0$$

$3(x')^2+(y')^2$

$$3x^2-2\cdot 3x+3-3+y^2-2\cdot 2y+4-4-1=0$$

$$(\sqrt{3}x-\sqrt{3})^2+(y-2)^2-8=0 \;|:8$$

$$\frac{3}{8}(x-1)^2+\frac{1}{8}(y-2)^2=1$$

№3

$$x^2-2x+4y^2-3=0$$

$$(x-1)^2+4y^2=4 \;|:4$$

$$\frac{(x-1)^2}{4}+y^2=1$$

№4. $c^2=a^2+b^2$

$$2x^2-10x-y^2+4y-5=0$$

$$2x^2-10x-(y^2-4y+4)-1=0$$

$$2(x^2-5x+5)-(y-2)^2=10 \qquad \frac{(x-\sqrt{5})^2}{5}-\frac{(y-2)^2}{10}=1$$

1/2

№1. $A=61$ $\quad B=2\sqrt{3}$ $\quad C=63$ $\quad D=182\sqrt{3}-366$ $\quad E=-822-6\sqrt{3}$

$$61x^2+2\sqrt{3}xy+63y^2+x(182\sqrt{3}-366)+y(-822-6\sqrt{3})-546\sqrt{3}=-3084$$

$$\begin{cases} x=x'+x_0 \\ y=y'+y_0 \end{cases}$$

$$61(x')^2+\underline{61\cdot 2x'x_0}+61x_0^2+2\sqrt{3}x'y'+\underline{2\sqrt{3}x'y_0}+\underline{2\sqrt{3}x_0y'}+2\sqrt{3}x_0y_0+$$
$$+63(y')^2+\underline{63\cdot 2y'y_0}+63y_0^2+\underline{(182\sqrt{3}-366)x'}+(182\sqrt{3}-366)x_0+$$
$$+(-822-6\sqrt{3})y_0+\underline{(-822-6\sqrt{3})y'}-546\sqrt{3}+3084=0$$

$$\cancel{x'(2\cdot 61x_0+2\sqrt{3}y_0+182\sqrt{3}-366)=0}$$
$$\cancel{y'(2\sqrt{3}x_0+63\cdot 2y_0-822-6\sqrt{3})=0}$$

$$\begin{cases} x_0=\dfrac{2CD-BE}{B^2-4AC} \\ y_0=\dfrac{2AE-BD}{B^2-4AC} \end{cases}$$

$$\cancel{\begin{cases} x_0=3-\dfrac{815\sqrt{3}}{512} \\ y_0=\dfrac{3379}{512} \end{cases}}$$

$$\begin{cases} x_0=\dfrac{15-8\sqrt{3}}{5} \\ y_0=\dfrac{39}{5} \end{cases}$$

$$\cancel{61(x')^2+2\sqrt{3}x'y'+63(y')^2}$$

$$61(x')^2+2\sqrt{3}x'y'+63(y')^2-\frac{3072}{5}=0$$

$$tg\,2\varphi=\frac{2\sqrt{3}}{61-63}=-\sqrt{3}\Rightarrow \varphi=-\frac{\pi}{6}\text{, тогда}$$

$$60(x'')^2+64(y'')^2=\frac{3072}{5}\;\Big|\cdot\frac{5}{3072}$$

$$\frac{(x'')^2}{\frac{3072}{3600}} + \frac{(y'')^2}{\frac{3072}{300}} = 1$$

$$\varepsilon = \frac{c}{a} \qquad \cancel{c = \sqrt{a^2 + b^2} = \frac{\sqrt{496}}{5}} \quad c = \frac{4}{5}$$

$$\cancel{a = \sqrt{\frac{3072}{300}}} \qquad a = \frac{16}{5}$$

$$\cancel{\varepsilon = \sqrt{\frac{496 \cdot 300}{25 \cdot 3072}} = \frac{\sqrt{31}}{4} \approx 1.39}$$

$$\varepsilon = \frac{c}{a} = \frac{4 \cdot 5}{5 \cdot 16} = 0,25$$

$$d = \rho(F, d)$$

$$p = \varepsilon d = a - \varepsilon c = \frac{16}{5} - \frac{1}{4} \cdot \frac{4}{5} = 3$$

№5.

$-4x+4y^2-10y-7=0 \mid (-1)$

$4x-4y^2+10y+7=0 \qquad \left]\begin{array}{l} x=x_1+y_1 \\ y=x_1-y_1 \end{array}\right.$

$4(x_1+y_1)-4(x_1-y_1)^2+10(x_1-y_1)+7=0$

$-4x_1^2-4y_1^2+8x_1y_1+4x_1+4y_1+10x_1-10y_1+7=0$

$4x_1^2+4y_1^2-8x_1y_1-14x_1+6y_1-7=0$

$\begin{cases} \cancel{x=x'+y_0}, \; x_0=-\frac{}{20} \\ y=y'+y_0 \end{cases}$

$-4x+4\left(y^2-2\cdot\frac{5}{4}y+\left(\frac{5}{4}\right)^2\right)-\left(\frac{5}{4}\right)^2 4-7=0$

$4\left(y-\frac{5}{4}\right)^2-4x=7+\frac{25}{4}$

$\left(y-\frac{5}{4}\right)^2-x=\frac{53}{16}$

$\left]\begin{cases} x'=x+\frac{53}{16} \\ y'=y-\frac{5}{4} \end{cases}\right. \Rightarrow (x_0,y_0)=\left(-\frac{53}{16},\frac{5}{4}\right)$ и $p=\frac{1}{2}$

$(y')^2=2\cdot\frac{1}{2}x'$

№6.

$9x^2-6\sqrt{3}xy+3y^2-2=0$

$\begin{cases} x=x'\cos\varphi+y'\sin\varphi \\ y=-x'\sin\varphi+y'\cos\varphi \end{cases}$

$9\left((x')^2\cos^2\varphi+x'y'\sin 2\varphi+(y')^2\sin^2\varphi\right)-6\sqrt{3}\left(-(x')^2\sin\varphi\cos\varphi+y'x'\cos^2\varphi-x'y'\sin^2\varphi+\right.$
$\left.+(y')^2\cos\varphi\sin\varphi\right)+3\left((x')^2\sin^2\varphi-2x'y'\sin\varphi\cos\varphi+(y')^2\cos^2\varphi\right)-2=0$

$$(x')^2\left(9\cos^2\varphi + 6\sqrt{3}\sin\varphi\cos\varphi + 3\sin^2\varphi\right) + x'y'\left(9\sin 2\varphi - 6\sqrt{3}\cos 2\varphi + 3\sin 2\varphi\right) +$$
$$+ (y')^2\left(9\sin^2\varphi - 6\sqrt{3}\cos\varphi\sin\varphi + 3\cos^2\varphi\right) - 2 = 0$$

$$12\sin 2\varphi = 6\sqrt{3}\cos 2\varphi$$
$$2\sin 2\varphi = \sqrt{3}\cos 2\varphi \mid \cos 2\varphi \neq 0$$
$$2\,tg\,2\varphi = \sqrt{3}$$

$$tg\,2\varphi = \frac{-6\sqrt{3}}{9-3} = -\sqrt{3} \Rightarrow \varphi = \frac{\pi}{3}\ (60^\circ)$$

$$(3x)^2 - 2\cdot 3x\cdot\sqrt{3}y + (\sqrt{3}y)^2 - 2 = 0$$
$$(3x - \sqrt{3}y)^2 = 2$$
$$\left[\begin{array}{l} 3x - \sqrt{3}y = \sqrt{2} \\ 3x - \sqrt{3}y = -\sqrt{2} \end{array}\right.$$
$$\left[\begin{array}{l} y = \sqrt{3}x - \sqrt{\frac{2}{3}} \\ y = \sqrt{3}x + \sqrt{\frac{2}{3}} \end{array}\right. \quad tg\,\alpha = \sqrt{3} \Rightarrow$$
$$\Rightarrow \alpha = 60^\circ \ ?$$

№7.

$$3x^2 - 18\sqrt{3}xy + 24x - 15y^2 - 72\sqrt{3}y + 56 = 0$$

$$tg\,2\varphi = \frac{-18\sqrt{3}}{3+15} = -\sqrt{3} \Rightarrow \varphi = \frac{\pi}{3}$$

$$\begin{cases} x = x'\cos\frac{\pi}{3} - y'\sin\frac{\pi}{3} \\ y = x'\sin\frac{\pi}{3} + y'\cos\frac{\pi}{3} \end{cases}$$

$$3\left(\frac{x'}{2} - \frac{\sqrt{3}y'}{2}\right)^2 - 15\left(\frac{\sqrt{3}x'}{2} + \frac{y'}{2}\right)^2 - 18\sqrt{3}\left(\frac{x'}{2} - \frac{\sqrt{3}y'}{2}\right)\left(\frac{\sqrt{3}x'}{2} + \frac{y'}{2}\right) + 24\left(\frac{x'}{2} - \frac{\sqrt{3}y'}{2}\right) - 72\sqrt{3}\left(\frac{\sqrt{3}x'}{2} + \frac{y'}{2}\right) + 56 = 0$$

$$\frac{3}{4}(x')^2$$

N7.

$$3x^2 - 18xy\sqrt{3} + 24x - 15y^2 - 72\sqrt{3}y + 56 = 0$$

$$\begin{cases} x = x' + x_0 \\ y = y' + y_0 \end{cases}$$

$$3(x'+x_0)^2 - 18\sqrt{3}(x'+x_0)(y'+y_0) - 15(y'+y_0)^2 + 24(x'+x_0) - 72\sqrt{3}(y'+y_0) + 56 = 0$$

$$3(x')^2 + 6x'x_0 + 3x_0^2 - 18\sqrt{3}x'y' - 18\sqrt{3}x'y_0 - 18\sqrt{3}x_0y' - 18\sqrt{3}x_0y_0 - 15(y')^2 - 30y'y_0 - 15y_0^2 + 24x' + 24x_0 - 72\sqrt{3}y_0 - 72\sqrt{3}y' + 56 = 0$$

$$x'(6x_0 - 18\sqrt{3}y_0 + 24) = 0$$

$$y'(-18\sqrt{3}x_0 - 30y_0 - 72\sqrt{3}) = 0$$

$$\begin{cases} x_0 - 3\sqrt{3}y_0 + 4 = 0 \\ 3\sqrt{3}x_0 + 5y_0 + 12\sqrt{3} = 0 \end{cases}$$

$$3\sqrt{3}(3\sqrt{3}y_0 - 4) + 5y_0 + 12\sqrt{3} = 0$$

$(-4,0)$ центр

$$32y_0 = 0 \Rightarrow y_0 = 0 \Rightarrow x_0 = -4$$

$$3(x')^2 - 18\sqrt{3}x'y' - 15(y')^2 + 3\cdot 16 - 24\cdot 4 + 56 = 0$$

$$3(x')^2 - 18\sqrt{3}x'y' - 15(y')^2 + 8 = 0$$

$$tg2\varphi = \frac{-18\sqrt{3}}{3+15} = -\sqrt{3} \Rightarrow \varphi = \frac{5\pi}{6}$$

№7.

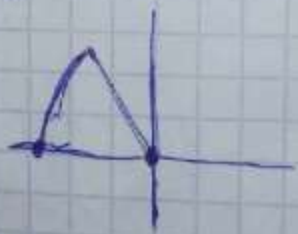
$$-24(x'')^2 - 48x''x_0 - 24x_0^2 + 12(y'')^2 + 24y''y_0 + 12y_0^2 -$$
$$- 96x_0 - 96x'' - 48\sqrt{3}y_0 - 48\sqrt{3}y'' + 56 = 0$$

$$\begin{cases} -48x_0 - 96 = 0 \\ 24y_0 - 48\sqrt{3} = 0 \end{cases} \quad \begin{cases} x_0 = -2 \\ y_0 = 2\sqrt{3} \end{cases}$$

$$\cancel{-24(x'')^2 - 24\cdot 4 + 12(y'')^2 + 12\cdot 6 - 96\cdot 2 - 48\sqrt{3}\cdot 2\sqrt{3} + 56 = 0}$$

$$\cancel{-24(x'')^2 + 12(y'')^2 = 376}$$

$$\cancel{-\frac{(x'')^2}{\frac{56}{3}} + \frac{(y'')^2}{\frac{112}{3}} = 1}$$



$$6x^2 - 3y^2 = 2 \quad |:2$$
$$3x^2 - \frac{3}{2}y^2 = 2$$

$$\begin{pmatrix} -2 \\ 2\sqrt{3} \end{pmatrix} T = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$$

$$T = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \quad \begin{cases} -2\cos\alpha - 2\sin\alpha = -4 \\ \end{cases}$$

$(-4, 0, 150°)$!

$+yx'\cos$

$\cos^2\varphi)$

№9.

$(27L^2+6)x^2 - 12\sqrt{3}xy + (27L^2-6)y^2 + (-126L^2-32)x + (-18\sqrt{3}L^2+16\sqrt{3})y + \ldots$

$B^2 - 4AC = 0 \Rightarrow$ парабола

$144\cdot 3 - 4\cdot(27^2L^4 - 36) = 0 \;|:4$

$36\cdot 3 - 27^2L^4 - 36 = 0$

$27^2L^4 = 36\cdot 2$

$8\cdot 9\cdot 9\, L^4 = 8\cdot 4\cdot 2$

$L^4 = \frac{8}{81}$

$L = \pm\frac{\sqrt[4]{8}}{3}$

$144\cdot 3 - 4(t^2-36) = 0$

$108 = t^2 - 36$

$t^2 = 144$

$t = \pm 12 \; (t \geq 0)$

$27L^2 = 12 \qquad L^2 = \frac{3\cdot 4}{3\cdot 9} \Rightarrow L = \pm\frac{2}{3}$

$L = \pm\frac{2}{3}$

$L = \frac{2}{3} \qquad L^2 = L_1^2 = L_2^2 = \frac{4}{9}$

№10.

$(2-36L^2)x^2 - 72L^2xy + (2-36L^2)y^2 + (144L^2-12)x + (144L^2+4)y + 20 - 144L^2 = 0$

$B^2 - 4AC > 0 \Rightarrow$ гиперб. тип

$] \; t = 36L^2$

$4t^2 - 4(2-t)^2 = 4t^2 - 4(t^2-4t+4) = 16t - 16 > 0$

$t > 1$

$36L^2 > 1 \Rightarrow L \in (-\infty, -\frac{1}{6}) \cup (\frac{1}{6}, +\infty)$

$e = 6 = \ldots = \frac{c}{a}$

$A_1 = A\cos^2\varphi + B\cos\varphi\sin\varphi + C\sin^2\varphi$

$C_1 = A\sin^2\varphi - B\sin\varphi\cos\varphi + C\cos^2\varphi$

Т.к. $A = C$, то $\varphi = \frac{\pi}{4} \Rightarrow$

$\Rightarrow A_1 = \frac{A}{2} + \frac{B}{2} + \frac{C}{2} \overset{A=C}{=} A + \frac{B}{2} = 2 - 36L^2 - 36L^2$

$C_1 = \frac{A}{2} - \frac{B}{2} + \frac{C}{2} \overset{A=C}{=} A - \frac{B}{2} = \cancel{36L^2}$.

$A_1 = 2 - 72x^2$

$C_1 = 2$

$A = C = 2 - 36L^2$

$B = -72L^2$

$A_1 (x'')^2 + C_1 (y'')^2 = + F_2$

$\frac{(x'')^2}{\frac{F_2}{A_1}} + \frac{(y'')^2}{\frac{F_2}{C_1}} = 1$

-1 и 1

$\varepsilon = 6 = \frac{c}{a} = \frac{\sqrt{a^2+b^2}}{a} = \sqrt{1 + \left(\frac{b}{a}\right)^2}$

$\left(\frac{b}{a}\right)^2 = \frac{F_2 \cdot A_1}{C_1 \cdot F_2} = \frac{A_1}{C_1} \cancel{\ldots} = 1 - 36L^2$

$\cancel{7L^2 = t}$

$\cancel{\sqrt{1+k^2} = 6}$

$\cancel{k^2 = 35, \; k = \frac{2(1-36L^2)}{36L^2} = \frac{1-36t}{18t}}$

$\begin{cases} 36b^2 = 1 + \sqrt{35} \\ 36L^2 = e^2 \end{cases}$