Домашняя работа №7.

№1.

$$21x^2 + 25y^2 = 525 \;\Big|\; :525$$

$$\frac{x^2}{25} + \frac{y^2}{21} = 1 \Rightarrow \begin{aligned} &a = 5 \\ &b = \sqrt{21} \\ &c = 2 \Rightarrow 2c = 4 \\ &\varepsilon = \frac{c}{a} = 0{,}4 \end{aligned}$$

№2.

$$\frac{b}{a} = 1, \quad \varepsilon - ? \qquad \frac{b}{a} = 1 \Rightarrow a = b$$

$$\varepsilon = \frac{c}{a} = \frac{\sqrt{a^2+b^2}}{a} = \cancel{\sqrt{a+a} = 2a} = \frac{\sqrt{a^2+b^2}}{a} = \frac{a\sqrt{2}}{a} = \sqrt{2}$$

№3.

144 – директриса

$y^2 = 2px$

№5.

$A(5\sqrt{2}, 15)$ ши. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$y = \pm 3x$ $y = \pm \frac{b}{a} x$

$\frac{b}{a} = 3 \Rightarrow b = 3a$

$\frac{x^2}{a^2} - \frac{y^2}{9a^2} = 1 \quad | \cdot a^2$

$50 - \frac{15 \cdot 15}{9} = a^2$

$a = \pm 5 \Rightarrow b = \pm 15$

№4.

$F(1, 0)$ $y^2 = 2px$

$x = -5$ - дир. $F(1,0)$

$p = ?$ $x + 5 = 0$

$\rho(A, x=-5) = \rho(A, F)$ $A(x, y)$

$\frac{|x+5|}{1} = \sqrt{(x-1)^2 + y^2}$

$(x+5)^2 = (x-1)^2 + y^2$

$y^2 = (x+5-x+1)(x+5+x-1)$

$y^2 = 6 \cdot (2x+4) \Rightarrow y^2 = 12x + 24 \Rightarrow p = 6$

№6.

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$$\frac{xx_0}{a^2}-\frac{yy_0}{b^2}=1$$

$$y=\pm\frac{b}{a}x \;|\cdot a$$

$$ay\mp bx=0$$

$$l_1:\ ay+bx=0$$

$$l_1:\ -bx+ay=0$$

$$\rho(A,l_1)=\frac{|bx+ay|}{\sqrt{a^2+b^2}}=R_1$$

$$\rho(A,l_2)=\frac{|-bx+ay|}{\sqrt{a^2+b^2}}=R_2$$

$$R_1 \cdot R_2 = \frac{|bx + ay| \cdot |-bx + ay|}{a^2 + b^2} = \frac{|a^2y^2 - b^2x^2|}{a^2 + b^2} =$$

$$= \frac{\left|-a^2b^2\left(\frac{x^2}{a^2} - \frac{y^2}{b^2}\right)\right|}{a^2+b^2} = \frac{(ab)^2}{a^2+b^2} = \frac{a^2b^2}{a^2+b^2}$$

Для $\frac{x^2}{16} - \frac{y^2}{2} = 1$

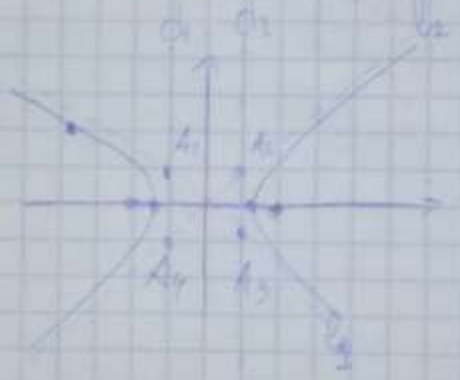
$a^2 = 16$ $\qquad \frac{16 \cdot 2}{16+2} = \frac{32}{18} = \frac{4 \cdot 8}{2 \cdot 9} = \frac{16}{9}$

$b^2 = 2$

№8

$V_1(a, 0)$

$V_2(-a, 0)$



$d_{1,2} = \pm \frac{a}{e}$

~~[illegible]~~ $\quad$ I $\quad A_1 = d_1 \cap l_1 \quad A_3 = d_2 \cap l_1$

$l_{1,2}: y = \pm \frac{b}{a} x \qquad A_2 = l_2 \cap d_2 \quad A_4 = l_2 \cap d_1$

$A_1: \begin{cases} x = \frac{a}{e} \\ y = -\frac{b}{a}x \end{cases} \Rightarrow \begin{cases} x = -\frac{a}{e} \\ y = +\frac{b}{e} \end{cases} \qquad A_3: \begin{cases} x = \frac{a}{e} \\ y = -\frac{b}{a}x \end{cases} \Rightarrow \begin{cases} x = \frac{a}{e} \\ y = -\frac{b}{e} \end{cases}$

$A_2: \begin{cases} x = \frac{a}{e} \\ y = \frac{b}{a}x \end{cases} \Rightarrow \begin{cases} x = \frac{a}{e} \\ y = \frac{b}{e} \end{cases} \qquad A_4: \begin{cases} x = -\frac{a}{e} \\ y = \frac{b}{a}x \end{cases} \Rightarrow \begin{cases} x = -\frac{a}{e} \\ y = -\frac{b}{e} \end{cases}$

№8(пр с)

$\varepsilon = \frac{c}{a}$

$A_1\left(-\frac{a^2}{c}, \frac{ab}{c}\right)$ $\quad V_1(-a, 0)$

$A_2\left(\frac{a^2}{c}, \frac{ab}{c}\right)$ $\quad V_2(a, 0)$

$A_3\left(\frac{a^2}{c}, -\frac{ab}{c}\right)$

$A_4\left(-\frac{a^2}{c}, -\frac{ab}{c}\right)$

$x^2 + y^2 = r^2$

V_1 и V_2 — $r^2 = a^2$

$A_1 \dots A_4 - \frac{a^4}{c^2} + \frac{a^2b^2}{c^2} = \frac{a^2(a^2+b^2)}{c^2} = \frac{a^2c^2}{c^2} = a^2 = r^2 \Rightarrow$

$\Rightarrow V_{1,2}$ и $A_{1..4}$ лежат на одной окр-ти, причем $r = \sqrt{a^2} = a$

$\frac{x^2}{16} - \frac{y^2}{2} = 1 \Rightarrow a^2 = 16 \Rightarrow \underline{\underline{r = 4}}$

№9. $C(1;0)$

$l_1: y = -\frac{b}{a}x$

$l_2: y = \frac{b}{a}x$

$k: \frac{xx_0}{a^2} - \frac{yy_0}{b^2} = 1$

$A = l_1 \cap k: \begin{cases} bx + ay = 0 \Rightarrow y = -\frac{b}{a}x \\ \frac{xx_0}{a^2} - \frac{yy_0}{b^2} = 1 \end{cases}$

$\frac{xx_0}{a^2} + \frac{xby_0}{ab^2} = 1$

$x\left(\frac{x_0b + y_0a}{a^2b}\right) = 1$

$\begin{cases} x = \frac{a^2b}{x_0b + y_0a} \\ y = -\frac{b^2a}{x_0b + y_0a} \end{cases}$

$$\begin{cases} y = kx + b \end{cases}$$

$$B = l_2 \cap k: \begin{cases} y = \frac{b}{a}x \\ \frac{xx_0}{a^2} - \frac{yy_0}{b^2} = 1 \end{cases} \qquad \begin{cases} y = \frac{b}{a}x \\ \frac{xx_0}{a^2} - \frac{by_0x}{ab^2} = 1 \end{cases} \to \begin{cases} x = \frac{a^2b}{x_0b - y_0a} \\ y = \frac{ab^2}{x_0b - y_0a} \end{cases}$$

$$x\left(\frac{x_0}{a^2} - \frac{y_0}{ab}\right) = 1$$

$$x = \frac{a^2b}{x_0b - y_0a}$$

$$A\left(\frac{a^2b}{x_0b + y_0a}; -\frac{b^2a}{x_0b + y_0a}\right)$$

$bx - ay = 0$ $\quad C(0,0)$

$$AC = \sqrt{\frac{a^4b^2 + b^4a^2}{(x_0b + y_0a)^2}}$$

$$\rho(B, l_1) = \frac{\left|\frac{a^2b^2}{x_0b - y_0a} + \frac{a^2b^2}{x_0b - y_0a}\right|}{\sqrt{a^2 + b^2}} = \frac{2a^2b^2}{\sqrt{a^2+b^2} \cdot |x_0b - y_0a|}$$

$$S = \rho(B, l_1) \cdot \frac{AC}{2} = \frac{a^2b^2 \cdot a \cdot b\sqrt{a^2+b^2}}{\sqrt{a^2+b^2} \cdot |x_0b - y_0a| \cdot |x_0b + y_0a|} =$$

из уравн. единств.

$$= \frac{a^3b^3}{x_0^2b^2 - y_0^2a^2} = \frac{ab}{\frac{x_0^2}{a^2} - \frac{y_0^2}{b^2}} = \underline{\underline{ab}} \quad (a, b > 0)$$

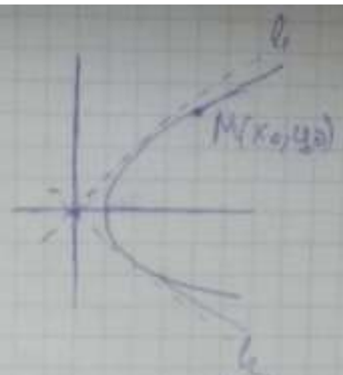
ч.т.д.

Для

$$\frac{x^2}{14} - \frac{y^2}{10} = 1 \qquad S = ab = \sqrt{140}$$

№1.

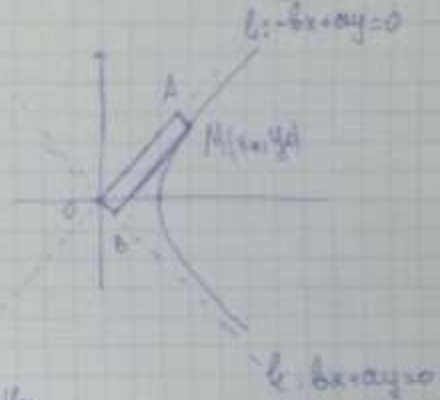
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad M(x_0, y_0)$$



$l_{1,2}: \ y = \pm\frac{b}{a}x \qquad bx \pm ay = 0$

Или:

$$l_1: \begin{cases} x = t \\ y = \frac{b}{a}t \end{cases} \qquad AM: \begin{cases} x = x_0 - t \\ y = y_0 + \frac{b}{a}t \end{cases}$$



$AM \cap l_1$

$$\begin{cases} y = \frac{b}{a}x \\ x_0 - x = \frac{a}{b}(y - y_0) \end{cases}$$

$$x_0 - x = x - \frac{a}{b}y_0 \Rightarrow \begin{cases} x = \frac{x_0}{2} + \frac{ay_0}{2b} \\ y = \frac{bx_0}{2a} + \frac{y_0}{2} \end{cases}$$

$$OA = \sqrt{\frac{x_0^2}{4} + \frac{y_0^2}{4} + \frac{a^2y_0^2}{4b^2} + \frac{b^2x_0^2}{4a^2} + \frac{ax_0y_0}{2b} + \frac{bx_0y_0}{2a}} =$$

$$= \sqrt{\frac{x_0^2}{4}\left(1 + \frac{b^2}{a^2}\right) + \frac{y_0^2}{4}\left(1 + \frac{a^2}{b^2}\right) + \frac{x_0y_0}{2}\left(\frac{a}{b} + \frac{b}{a}\right)}$$

$$\rho(M, l_1) = \frac{|-bx_0 + ay_0|}{\sqrt{a^2 + b^2}}$$

$$h \cdot OA = |ay_0 - bx_0| \cdot \sqrt{\frac{x_0^2}{4a^2} + \frac{y_0^2}{4b^2} + \frac{x_0y_0}{2ab}} = |ay_0 - bx_0| \cdot \sqrt{\left(\frac{x_0}{2a} + \frac{y_0}{2b}\right)^2} =$$

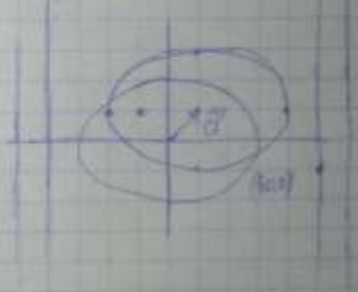
$$= \frac{|ay_0 - bx_0|(bx_0 + ay_0)}{2ab} = \frac{a^2y_0^2 - b^2x_0^2}{2ab} = \frac{1}{2}\left|\frac{ay_0^2}{b} - \frac{bx_0^2}{a}\right| = \frac{ab}{2}\left|\frac{y_0^2}{b^2} - \frac{x_0^2}{a^2}\right| =$$

$$= \frac{ab}{2} \Rightarrow \boxed{S = \frac{ab}{2}} \qquad \text{для } \frac{x^2}{5} - \frac{y^2}{14} = 1 \quad S = \frac{\sqrt{70}}{2}$$

№5. $F_1(-36, 4)$ $\quad x = \frac{1841}{40} = d$ $\quad a, b - ?$ $\quad$ (эллипс)

$F_2(44, 4)$

$d \qquad \rho(F_1, F_2) = \sqrt{(44+36)^2} = 80 \Rightarrow c = 40$



$(40, 0) = (44 + x', 4 + y')$

$\vec{a}(4, 4)$

$d_0 = \frac{1841}{40} - 4 = \frac{1681}{40}$

$d = \frac{a}{e} = \frac{a^2}{c} \qquad \frac{a^2}{40} = \frac{1681}{40} \qquad a = \sqrt{1681} = 41$

$c^2 = a^2 - b^2 \Rightarrow b^2 = 41^2 - 40^2 = 81 \Rightarrow b = 9$

№10. $y^2=2px \qquad y=kx+b$,

Пусть при b_0 парабола и прямая пересекаются в 2-х точках

$$\begin{cases} y=kx+b_0 \\ y^2=2px \end{cases}$$

$(kx+b_0)^2=2px$

$k^2x^2+2kb_0x-2px+b_0^2=0 \}\ k^2x^2+2(kb_0-p)x+b_0^2=0$

$\frac{D}{4}=(p-kb_0)^2-k^2b_0^2=p^2-2kb_0p$

$$x=\frac{p-kb_0\pm\sqrt{p^2-2kb_0p}}{k^2}$$

$$x_0=\frac{|x_2-x_1|}{2}=\frac{p-kb_0+\sqrt{\ldots}-p+kb_0+\sqrt{\ldots}}{2k^2}=\frac{\sqrt{p^2-2kb_0p}}{k^2}$$

$$y_0(x_0)=\frac{k\sqrt{p^2-2kb_0p}}{k^2}+b_0=\frac{kb_0+\sqrt{p^2-2kb_0p}}{k}=$$

$$=b_0+\sqrt{\left(\frac{p}{k}\right)^2-\frac{2b_0p}{k}}=kx_0+b_0$$

Аналогично можно получить для b_0':

$y_0'=kx_0'+b_0'$

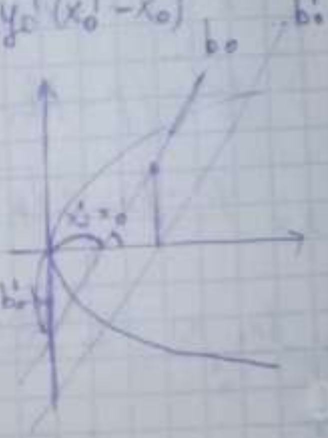
$y_0'-y_0=k(x_0'-x_0)+(b_0'-b_0)$

Ввиду того, что $b_0' - b_0 = const$ и $k(x_0' - x_0) = const$, то $y_0' - y_0 = const$ для заданных k, b, p.

$(x_0, y_0), (x_0', y_0')$

$$\frac{x - x_0}{x_0' - x_0} = \frac{y - y_0}{y_0' - y_0}$$

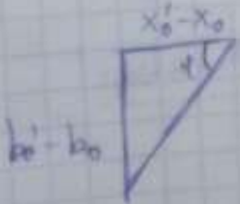
$$x(y_0' - y_0) - y(x_0' - x_0) = x_0(y_0' - y_0) - y_0'(x_0' - x_0)$$



$\vec{n}(y_0' - y_0, -x_0' + x_0)$

$$y_0' - y_0 = k(x_0' - x_0) + (b_0' - b_0)$$

$$x_0 - x_0' = \frac{\sqrt{p^2 - 2kb_0p} - \sqrt{p^2 - 2kb_0'p}}{k^2}$$



$tg\varphi = k \quad \left|\frac{b_0' - b_0}{x_0' - x_0}\right| = k \Rightarrow$

$\Rightarrow y_0' - y_0 = b_0' - b_0 - b_0' + b_0 = 0 \Rightarrow y_0' = y_0 \Rightarrow$

$\Rightarrow$ ГМТ середин хорд $\parallel y = kx + b \parallel y = 0$

ч.т.д.