Домашнее задание 3.

№2

$$A=\begin{pmatrix} 0 & -2 & 4 & -3 & -1 \\ 0 & 8 & 2 & -2 & -1 \\ 0 & 8 & 0 & 7 & 2 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

т.к. $\det A = 0$, то не существует A^{-1}

№2

$$A=\begin{pmatrix} 4x-1 & 0 & 0 \\ 0 & x-1 & -2x+2 \\ -4x-1 & 8x+2 & -20x-5 \end{pmatrix} \qquad A^{-1}$$

$$\det A = (4x-1)\cdot(-1)^{1+1}\cdot\begin{vmatrix} x-1 & -2x+2 \\ 8x+2 & -20x-5 \end{vmatrix} = (4x-1)\left((x-1)(20x+5)+(2x-2)(8x+2)\right)=$$

$$=(4x-1)\left(20x^2+5x-20x-5+20x^2-5x+20x+5\right)=(4x-1)\left(-4x^2+3x+1\right)=$$

$$=-(4x-1)(4x^2-3x-1)$$

$\det A = 0$

$-(4x-1)(4x^2-3x-1)=0$

$$\begin{cases} x=\frac{1}{4} \\ x=\frac{3\pm\sqrt{9+16}}{8}=\frac{3\pm5}{8} \end{cases} \qquad \begin{cases} x=0{,}25 \\ x=1 \\ x=-0{,}25 \end{cases}$$

№3.

$A=ABC$

$$\begin{pmatrix} -x-1 & x+1 \\ -5x-1 & 0 \end{pmatrix}\begin{pmatrix} -4x+0 & 0 \\ 4x-1 & 4x+4 \end{pmatrix}\begin{pmatrix} 2x-1 & -4x+2 \\ 8x+2 & -20x-5 \end{pmatrix}=$$

$$=\begin{pmatrix} (x+1)(4x+4)+(x+1)(4x-1) & (x+1)(4x-1) \\ (5x+1)(4x+4) & 0 \end{pmatrix}\begin{pmatrix} 2x-1 & -4x+2 \\ 8x+2 & -20x-5 \end{pmatrix}=$$

$$=\begin{pmatrix} 8x(x+1) & (x+1)(8x-1) \\ (5x+1)(4x+4) & 0 \end{pmatrix}\begin{pmatrix} 2x-1 & -4x+2 \\ 8x+2 & -20x-5 \end{pmatrix}=$$

$$8x(x+1)(8x-1)+(x+1)(4x+1)(8x+2)$$

№5. ? $AXB=C$

$\begin{pmatrix}1 & 1\\ -1 & 0\end{pmatrix} X \begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix} = \begin{pmatrix}1 & 0\\ -1 & 1\end{pmatrix}$

$AXB=C$

$XB=A^{-1}C$

$X=A^{-1}CB^{-1}$

$A^{-1} = \begin{pmatrix}0 & -1\\ 1 & 1\end{pmatrix}$

$X\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix} = \begin{pmatrix}1 & -1\\ 0 & 1\end{pmatrix}$

$B^{-1} = \begin{pmatrix}1 & 0\\ -1 & 1\end{pmatrix}$

$X = \begin{pmatrix}2 & -1\\ -1 & 1\end{pmatrix}$

№6.

$(ABA^{-1})^2 = ?$

$A = \begin{pmatrix}1 & -1\\ -1 & 2\end{pmatrix}$ $B = \begin{pmatrix}1 & 1\\ 1 & 2\end{pmatrix}$ $A^{-1} = 1\begin{pmatrix}2 & 1\\ 1 & 1\end{pmatrix} = \begin{pmatrix}6 & 3\\ 3 & 3\end{pmatrix}$

$AB = \begin{pmatrix}0 & -1\\ 1 & 3\end{pmatrix}$

$ABA^{-1} = \begin{pmatrix}6 & -3\\ 15 & 12\end{pmatrix}\begin{pmatrix}-1 & -1\\ 5 & 4\end{pmatrix}$

$(ABA^{-1})^2 = \begin{pmatrix}126 & 27\\ 225 & 135\end{pmatrix}\begin{pmatrix}-6 & -3\\ 15 & 15\end{pmatrix}\begin{pmatrix}-1 & -1\\ 5 & 4\end{pmatrix}\begin{pmatrix}-1 & -1\\ 5 & 4\end{pmatrix} = \begin{pmatrix}-4 & -3\\ 15 & 11\end{pmatrix}$

№7.

$R = \begin{pmatrix}1 & -1\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ -1 & 1\end{pmatrix}\begin{pmatrix}1 & -1\\ 0 & 1\end{pmatrix}^{-1} = \begin{pmatrix}2 & -1\\ -1 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix} =$

$= \begin{pmatrix}2 & 1\\ -1 & 0\end{pmatrix}$

№ $AX + XB = C$

$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} X + X \begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} 8 & -5 \\ -8 & -1 \end{pmatrix}$$

пусть $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} 8 & -5 \\ -8 & -1 \end{pmatrix}$$

$$\begin{pmatrix} a-2c & b-2d \\ c & d \end{pmatrix} + \begin{pmatrix} a-2b & a-b \\ c-2d & c-d \end{pmatrix} = \begin{pmatrix} 8 & -5 \\ -8 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 2a-2b-2c & a-2d \\ 2c-2d & c \end{pmatrix} = \begin{pmatrix} 8 & -5 \\ -8 & -1 \end{pmatrix}$$

$$\begin{cases} 2a-2b-2c=8 \\ a-2d=-5 \\ 2c-2d=-8 \\ c=-1 \end{cases} \quad \begin{cases} b=a-c-4 \\ a=2d-5 \\ d=c+4 \\ c=-1 \end{cases} \quad \begin{cases} a=1 \\ b=-2 \\ c=-1 \\ d=3 \end{cases}$$

$$X = \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}$$

№3.

$R = ABC$

$\det R = 0$

$\det R = \det(ABC) = \det A \cdot \det B \cdot \det C$

$$\begin{cases} \det A = 0 \\ \det B = 0 \\ \det C = 0 \end{cases}$$

$\det A = (x+1)(5x+1) = 0 \Leftrightarrow \begin{bmatrix} x=-1 \\ x=-0{,}2 \end{bmatrix}$

$\det B = -(4x-1)(4x+1) = 0 \Leftrightarrow \begin{bmatrix} x=0{,}25 \\ x=-0{,}25 \end{bmatrix}$

$\det C = -(20x-5)(8x-4) + (8x+2)(4x-2) = 32x^2 - 16x + 8x - 4 - 40x^2 + 20x - 16x + 10 = -8x^2 + 6$

$\Leftrightarrow -8x^2 + 4x + 1 = 0$

$x = \frac{-4 \pm \sqrt{16+32}}{-8} = \frac{-4 \pm 8}{-8}$ $\quad \begin{bmatrix} x=0{,}5 \\ x=-0{,}25 \end{bmatrix}$

Ответ: $\begin{bmatrix} x=-1 \\ x=-0{,}2 \\ x=0{,}25 \\ x=-0{,}25 \\ x=0{,}5 \end{bmatrix}$

№4

$R = AFG + BFG + CFG + DFG + EFG$

$FG = \begin{pmatrix} -4x-1 & 8x+2 \\ 2x+2 & -5x-5 \end{pmatrix}\begin{pmatrix} 3x-1 & -3x+1 \\ 4x-1 & 0 \end{pmatrix} =$

$= \begin{pmatrix} -(4x+1)(3x-1) + (8x+2)(4x-1) & (4x+1)(3x-1) \\ (2x+2)(3x-1) - (5x+5)(4x-1) & -(2x+2)(3x-1) \end{pmatrix} =$

$= \begin{pmatrix} -12x^2 - 4x + 3x + 1 + 32x^2 + 8x - 8x + 2 & 12x^2 - x - 1 \\ 6x^2 + 2x - 6x - 2 - 20x^2 + 5x - 20x + 5 & -6x^2 - 4x + 2 \end{pmatrix} =$

$= \begin{pmatrix} 20x^2 + 4x + 3 & 12x^2 - x - 1 \\ -14x^2 - 35x + 3 & -6x^2 - 4x + 2 \end{pmatrix}$

$R = AFG + BFG + CFG + DFG + EFG = (A+B+C+D+E)\cdot FG =$

$= \begin{pmatrix} 3x-1 & -6x+2 \\ 5x+1 & -15x-3 \end{pmatrix}\begin{pmatrix} -4x-1 & 8x+2 \\ 2x+2 & -5x-5 \end{pmatrix}\begin{pmatrix} 3x-1 & -3x+1 \\ 4x-1 & 0 \end{pmatrix}$

$\det R = \det(kFG) = \det K \det F \det G$

$\det K = (6x+2)(5x+1) - (3x-1)(15x+3) = 30x^2 + 16x + 2 - 45x^2 + 6x + 3 =$

$\Leftrightarrow -15x^2 + 22x + 5 = 0$

$x = \frac{-11 \pm \sqrt{121+75}}{-15} = \frac{-11 \pm 14}{-15}$ $\quad \begin{bmatrix} x=-0{,}2 \\ x=\frac{5}{3} \end{bmatrix}$

$\Leftrightarrow -15x^2 + 2x + 1 = 0$

$x = \frac{-1 \pm \sqrt{1+15}}{-15} = \frac{-1 \pm 4}{-15}$ $\quad \begin{bmatrix} x=\frac{1}{3} \\ x=-0{,}2 \end{bmatrix}$

$A = \begin{pmatrix} 5 & 4 \\ -2 & 3 \end{pmatrix}$ $B = \begin{pmatrix} 2 & 6 \\ 7 & 8 \end{pmatrix}$

$A \cdot X = B$

$X = A^{-1} \cdot B = \frac{1}{23} \begin{pmatrix} 3 & -4 \\ 2 & 5 \end{pmatrix} \cdot \begin{pmatrix} 2 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

Домашняя работа №3 (пр-е)

№10

$\det F = (4x+1)(5x+5) - (2x+2)(8x+2) = 20x^2 + 25x + 5 - 16x^2 - 20x - 4 =$

$= 4x^2 + 5x + 1 = 0$

$x = \frac{-5 \pm \sqrt{25-16}}{8} = \frac{-5 \pm 3}{8}$ $\begin{cases} x = -1 \\ x = -0{,}25 \end{cases}$

$\det G = (3x-1)(4x-1) = 0$ $\begin{cases} x = \frac{1}{3} \\ x = 0{,}25 \end{cases}$

Ответ: $\begin{cases} x = -0{,}2 \\ x = \frac{5}{2} \\ x = \frac{1}{3} \\ x = -1 \\ x = \pm 0{,}25 \end{cases}$