Задача 3

Вычислить с $a^{(ij)}_{ki}$:

$$a^{ij}_{kl} \sim \left\| \begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ -1 & 0 & -1 & 1 \\ \hline 1 & 1 & 1 & 1 \\ 0 & 1 & -2 & -1 \end{array} \right\| .$$

$a^{ij}_{kl} \sim$

$$a^{ji}_{kl} \sim \left\| \begin{array}{cc|cc} a^{11}_{11} & a^{21}_{11} & a^{11}_{12} & a^{21}_{12} \\ a^{12}_{11} & a^{22}_{11} & a^{12}_{12} & a^{22}_{12} \\ \hline a^{11}_{21} & a^{21}_{21} & a^{11}_{22} & a^{21}_{22} \\ a^{12}_{21} & a^{22}_{21} & a^{12}_{22} & a^{22}_{22} \end{array} \right\|$$

$$a^{(ij)}_{kl} = \frac{1}{2}\left(a^{ij}_{kl} + a^{ji}_{kl}\right) \sim \left\| \begin{array}{cc|cc} a^{11}_{11} & \frac{a^{12}_{11}+a^{21}_{11}}{2} & a^{11}_{12} & \frac{a^{12}_{12}+a^{21}_{12}}{2} \\ \frac{a^{12}_{11}+a^{21}_{11}}{2} & a^{22}_{11} & \frac{a^{12}_{12}+a^{21}_{12}}{2} & a^{22}_{12} \\ \hline a^{11}_{21} & \frac{a^{12}_{21}+a^{21}_{21}}{2} & a^{11}_{22} & \frac{a^{12}_{22}+a^{21}_{22}}{2} \\ \frac{a^{12}_{21}+a^{21}_{21}}{2} & a^{22}_{21} & \frac{a^{12}_{22}+a^{21}_{22}}{2} & a^{22}_{22} \end{array} \right\|$$

$a^{ij}_{ki} = a^{ij}_{k1} + a^{ij}_{k2}$

$$a^{(ij)}_{kl} \sim \left\| \begin{array}{cc|cc} 1 & 0 & 1 & -0.5 \\ 0 & 0 & -0.5 & 1 \\ \hline 1 & 0.5 & 1 & -0.5 \\ 0.5 & 1 & -0.5 & -1 \end{array} \right\|$$

$$a^{(ij)}_{ki} \sim \begin{pmatrix} 0.5 & 1 \\ 0.5 & -0.5 \end{pmatrix}^T$$

Используем полученные результаты так же для решения задачи 6

Для заданий 1, 2, 4, 5 будем использовать следующие соотношения

$$a_{\underset{123}{ijk}} \sim \left\| \begin{matrix} a_{111} & a_{121} \\ a_{211} & a_{221} \\ \hline a_{112} & a_{122} \\ a_{212} & a_{222} \end{matrix} \right\| \quad (-1)^0=1$$

$$a_{\underset{231}{jki}} \sim \left\| \begin{matrix} a_{111} & a_{112} \\ a_{121} & a_{122} \\ \hline a_{211} & a_{212} \\ a_{221} & a_{222} \end{matrix} \right\| \quad (-1)^2=1$$

$$a_{\underset{312}{kij}} \sim \left\| \begin{matrix} a_{111} & a_{211} \\ a_{112} & a_{212} \\ \hline a_{121} & a_{221} \\ a_{122} & a_{222} \end{matrix} \right\| \quad (-1)^2=1$$

$$a_{\underset{213}{jik}} \sim \left\| \begin{matrix} a_{111} & a_{211} \\ a_{121} & a_{221} \\ \hline a_{112} & a_{212} \\ a_{122} & a_{222} \end{matrix} \right\| \quad (-1)^1=-1$$

$$a_{\underset{132}{ikj}} \sim \left\| \begin{matrix} a_{111} & a_{112} \\ a_{211} & a_{212} \\ \hline a_{121} & a_{122} \\ a_{221} & a_{222} \end{matrix} \right\| \quad (-1)^1=-1$$

$$a_{\underset{321}{kji}} \sim \left\| \begin{matrix} a_{111} & a_{121} \\ a_{112} & a_{122} \\ \hline a_{211} & a_{221} \\ a_{212} & a_{222} \end{matrix} \right\| \quad (-1)^3=-1$$

1) $a_{ijk} \sim \left\| \begin{matrix} 1 & -2 \\ 1 & -1 \end{matrix} \middle| \begin{matrix} 1 & -2 \\ 1 & -1 \end{matrix} \right\|$

$$a_{i(jk)} = \frac{1}{2}(a_{ijk}+a_{ikj}) = \cancel{\frac{1}{2}\left\| \begin{matrix} 2 & -4 \\ 2 & -3 \end{matrix} \middle| \begin{matrix} 2 & -3 \\ 2 & -2 \end{matrix} \right\| = \left\| \begin{matrix} 1 & -2 \\ 1 & -1.5 \end{matrix} \middle| \begin{matrix} 1 & -1.5 \\ 1 & -1 \end{matrix} \right\|}$$

$$= \frac{1}{2}\left\| \begin{matrix} 2 & -1 \\ 2 & 0 \end{matrix} \middle| \begin{matrix} -1 & -4 \\ 0 & -2 \end{matrix} \right\| = \left\| \begin{matrix} 1 & -0.5 \\ 1 & 0 \end{matrix} \middle| \begin{matrix} -0.5 & -2 \\ 0 & -1 \end{matrix} \right\|$$

$b_{ijk} = a_{ikj}$

$$\left.\begin{matrix} a_{1jk} = \begin{pmatrix} 1 & 1 \\ -2 & -2 \end{pmatrix} \\ a_{2jk} = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} \end{matrix}\right\} \Rightarrow \begin{matrix} a_{1kj} = \begin{pmatrix} 1 & -2 \\ 1 & -2 \end{pmatrix} \\ a_{2kj} = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \end{matrix} \Rightarrow a_{ikj} \sim \cancel{\left\| \begin{matrix} 1 & -2 \\ 1 & 1 \end{matrix} \middle| \begin{matrix} 1 & 2 \\ 1 & -1 \end{matrix} \right\|} \; \left\| \begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix} \middle| \begin{matrix} -2 & -2 \\ -1 & -1 \end{matrix} \right\|$$

$$\left\| \begin{matrix} 1 & -2 \\ 1 & -1 \end{matrix} \middle| \begin{matrix} 1 & -2 \\ 1 & -1 \end{matrix} \right\|$$

Задача 2

Вычислить симметризацию $a_{(ijk)}$:

$$a_{ijk} \sim \left\| \begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 1 & 2 & -2 & 1 \end{array} \right\|.$$

$$\frac{1}{6}\left\| \begin{array}{cc} 6a_{111} & 2(a_{121}+a_{211}+a_{112}) \\ 2(a_{211}+a_{121}+a_{112}) & 2(a_{221}+a_{212}+a_{122}) \\ \hline 2(a_{112}+a_{121}+a_{211}) & 2(a_{122}+a_{212}+a_{221}) \\ 2(a_{212}+a_{122}+a_{221}) & 6a_{222} \end{array} \right\| = \left\| \begin{array}{cc} a_{111} & \frac{a_{121}+a_{211}+a_{112}}{3} \\ \frac{a_{211}+a_{121}+a_{112}}{3} & \frac{a_{221}+a_{212}+a_{122}}{3} \\ \hline \frac{a_{112}+a_{121}+a_{211}}{3} & \frac{a_{122}+a_{212}+a_{221}}{3} \\ \frac{a_{212}+a_{122}+a_{221}}{3} & a_{222} \end{array} \right\| \quad (*)$$

$$\left\| \begin{array}{cc} a_{111} & a_{121} \\ a_{211} & a_{221} \\ \hline a_{112} & a_{122} \\ a_{212} & a_{222} \end{array} \right\| \sim a_{ijk}$$

Значит согласно результату (*) для $a_{ijk} \sim \left\| \begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 1 & -2 & -2 & -1 \end{array} \right\|$

$$a_{(ijk)} \sim \left\| \begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \end{array} \right\|$$

Задача 4

Вычислить асимметризацию $a_{i[jk]}$:

$$a_{ijk} \sim \left\| \begin{array}{cc|cc} 1 & 1 & 1 & -2 \\ -2 & -1 & -2 & 5 \end{array} \right\|.$$

$$a_{i[jk]} = \frac{1}{2}\left(a_{ijk} - a_{ikj}\right) = \frac{1}{2}\left(\left\| \begin{array}{cc} a_{111} & a_{121} \\ a_{211} & a_{221} \\ \hline a_{112} & a_{122} \\ a_{212} & a_{222} \end{array} \right\| - \left\| \begin{array}{cc} a_{111} & a_{112} \\ a_{211} & a_{212} \\ \hline a_{121} & a_{122} \\ a_{221} & a_{222} \end{array} \right\| \right)$$

$$= \left\| \begin{array}{cc} 0 & \frac{a_{121} - a_{112}}{2} \\ 0 & \frac{a_{221} - a_{212}}{2} \\ \hline \frac{a_{112} - a_{121}}{2} & 0 \\ \frac{a_{212} - a_{221}}{2} & 0 \end{array} \right\| = \left\| \begin{array}{cc} 0 & 0 \\ 0 & 0,5 \\ \hline 0 & 0 \\ -0,5 & 0 \end{array} \right\|$$

Задача 5

Вычислить асимметризацию $a_{[ijk]}$:

$$a_{ijk} \sim \left\| \begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ -2 & 5 & -2 & 1 \end{array} \right\|.$$

$$a_{[ijk]} = \left\| \begin{array}{cc} 0 & 0 \\ 0 & 0 \\ \hline 0 & 0 \\ 0 & 0 \end{array} \right\|$$

Задача 6

Вычислить $a_{ki}^{[ij]}$:

$$a_{kl}^{ij} \sim \left\| \begin{array}{cc|cc} 1 & 0 & 1 & -2 \\ -2 & 1 & 1 & -1 \\ \hline 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & 0 \end{array} \right\| .$$

$$a_{kl}^{[ij]} = \frac{1}{2}\left(a_{kl}^{ij} - a_{kl}^{ji}\right) \sim \left\| \begin{array}{cc|cc} 0 & \frac{a_{11}^{12} - a_{11}^{11}}{2} & 0 & \frac{a_{12}^{12} - a_{12}^{21}}{2} \\ \frac{a_{11}^{21} - a_{11}^{12}}{2} & 0 & \frac{a_{12}^{21} - a_{12}^{12}}{2} & 0 \\ \hline 0 & \frac{a_{21}^{12} - a_{21}^{21}}{2} & 0 & \frac{a_{21}^{12} - a_{12}^{21}}{2} \\ \frac{a_{21}^{21} - a_{21}^{12}}{2} & 0 & \frac{a_{22}^{21} - a_{22}^{12}}{2} & 0 \end{array} \right\|$$

$$\begin{pmatrix} 1,5 & 1 \\ -1 & 0 \end{pmatrix}^T$$