1) $x = (1\ 1)^T$

$y = (-2\ -1)^T$

$G = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

$$\cos \underbrace{\widehat{x,y}}_{\alpha} = \frac{(x,y)}{|x|\,|y|} = \frac{(x,y)}{\sqrt{(x,x)}\sqrt{(y,y)}}$$

$\exists \{e_1; e_2\}$ – базис, тогда $(x,y) = (e_1+e_2, -2e_1-e_2) = -2\cdot 2 - 3\cdot 1 - 1\cdot 1 = -8$

$(x,x) = (e_1+e_2, e_1+e_2) = 1\cdot 2 + 2\cdot 1 + 1\cdot 1 = 5$

$(y,y) = (-2e_1-e_2, -2e_1-e_2) = 4\cdot 2 + 4\cdot 1 + 1 = 13$

Тогда $\cos\alpha = -\frac{8}{\sqrt{13}\cdot\sqrt{5}} = -\frac{8}{\sqrt{65}}$

2) $\lambda\ (4,3,2)$

$$A^J = \left[\begin{array}{ccc|c} \lambda & 1 & 0 & 0 \\ 0 & \lambda & 1 & 0 \\ 0 & 0 & \lambda & 0 \\ \hline 0 & 0 & 0 & \lambda \end{array}\right]$$

3) $p_1 = x^4 + 3x^3 - x^2 - 3x$

$p_1 = x(x^3 + 3x^2 - x - 3) = x\left(x^2(x+3) - (x+3)\right) =$

$= x(x+3)(x-1)(x+1)$

$p_2 = (x+3)^2(x+2)$

$p_2: \{-2^{(1)}, -3^{(2)}\}$ $\Rightarrow p_1 \cap p_2 = -3^{(1)} \Rightarrow p_{min} = \underline{\underline{x+3}}$

$p_1: \{-1^{(1)}, 1^{(1)}, 0^{(1)}, -3^{(1)}\}$

$$\det G = 1$$

4) $G = \begin{pmatrix} 5 & -7 \\ -7 & 10 \end{pmatrix} \quad G^{-1} = \begin{pmatrix} 10 & 7 \\ 7 & 5 \end{pmatrix}$

$\exists\, e_1 = (\alpha_1 \ \alpha_2)$

$e_2 = (\beta_1 \ \beta_2) \Rightarrow E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$F = G^{-1} E = \begin{pmatrix} 10 & 7 \\ 7 & 5 \end{pmatrix} \begin{matrix} f^1 \\ f^2 \end{matrix}$ +

5) $\sigma_A : \{ 4^{(3)} \}$

Найдем с.в.:

$$\lambda = 4: \left[\begin{array}{ccc|c} -3 & 1 & 3 & \eta^1 \\ -1 & 0 & 1 & \eta^2 \\ -3 & 1 & 3 & \eta^3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & -\eta^2 \\ 0 & 1 & 0 & \eta^1 - 3\eta^2 \\ 0 & 0 & 0 & \eta^3 - \eta^1 \end{array}\right]$$

$\exists\, \eta_1 = \eta_2 = \eta_3 = 0 \Rightarrow \xi_1 = [1 \ 0 \ 1]^T$

Найдем ξ_1': $(A-\lambda E)\xi_1' = \xi_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \xi_1' = [0\ -2\ 0]^T$$

Найдем ξ_1'': $(A-\lambda E)\xi_1'' = \xi_1'$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \xi_1'' = [2\ 6\ 0]^T$$

Значит $\{\xi_1, \xi_1', \xi_1''\}$ - базис

$$A^J = \begin{bmatrix} 4 & 1 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{bmatrix}$$

Тогда $\lambda = 4\ (3\ 3\ 1) \Rightarrow p_A = (\lambda - 4)^3$

б)

$$\begin{pmatrix} 1 & 1 & -2 & -1 \\ 1 & 2 & -4 & -2 \\ 1 & 1 & -1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -2 & -1 \\ 0 & 1 & -2 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \sim \begin{array}{c} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{array} \begin{array}{c} a_1 \\ a_2 \\ a_3 \end{array}$$

$$b_1 = a_1 = (1\ 0\ 0\ 0)^T$$

$$b_2 = a_2 - \frac{(b_1, a_2)}{(b_1, b_1)} b_1 = (0\ 1\ 0\ -1)^T$$

$$b_3 = a_3 - \frac{(a_3, b_2)}{(b_2, b_2)} b_2 - \frac{(a_3, b_1)}{(b_1, b_1)} b_1 = (0\ 0\ 1\ 0)^T$$

$$V^2 = |G| \quad N\,7$$

$$G = \begin{pmatrix} 19 & -23 & 14 \\ -23 & 30 & -18 \\ 14 & -18 & 11 \end{pmatrix} \Rightarrow \det G = 19 \cdot (330 - 18^2)$$

$$+ 23 \cdot (-253 + 18 \cdot 14)$$

$$+ 14 \cdot (23 \cdot 18 - 14 \cdot 30) = 7 \Rightarrow V = 2.65$$