HW 10.

1 $\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \cdot X = \begin{pmatrix} 1 & -1 \\ -2 & 2 \end{pmatrix}$ $AX = B$

$X = A^{-1}B$

$X = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -2 & 2 \end{pmatrix}$

$X = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$

2.

$$\begin{pmatrix} 1 & -2 & 3 & 5 & -7 \\ -2 & 5 & -7 & -12 & 16 \\ -3 & 7 & -9 & -17 & 24 \\ 2 & -5 & 8 & 13 & -17 \\ 6 & -15 & 20 & 37 & -51 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 3 & 5 & -7 \\ 0 & 1 & -1 & -2 & 2 \\ 0 & 1 & 0 & -2 & 3 \\ 0 & -1 & 2 & 3 & -3 \\ 0 & -3 & 2 & 7 & -9 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & -2 & 3 & 5 & -7 \\ 0 & 1 & -1 & -2 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & -1 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 3 & 5 & -7 \\ 0 & 1 & -1 & -2 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Ответ 4

5.

$$\mathcal{A}(x) = \begin{pmatrix} \xi^1 - \xi^2 \\ -2\xi^1 + 3\xi^2 \end{pmatrix}, \quad X = \begin{pmatrix} \xi^1 \\ \xi^2 \end{pmatrix}$$

$e_1(1,0)^T$; $e_2(0,1)^T$

$$\mathcal{A}(e_1) = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \quad \mathcal{A}(e_2) = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$A_{\mathcal{A}}: \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$$

6.

$$A = \begin{pmatrix} 6 & 0 & -2 \\ 0 & 2 & 0 \\ -2 & 0 & 6 \end{pmatrix}$$

$$\chi_A(\lambda) = \begin{vmatrix} 6-\lambda & 0 & -2 \\ 0 & 2-\lambda & 0 \\ -2 & 0 & 6-\lambda \end{vmatrix} = (2-\lambda)\left((6-\lambda)^2 - 4\right) =$$

$$= (2-\lambda)(8-\lambda)(8-\lambda)$$

$$\chi_A(\lambda) = 0 \Leftrightarrow \begin{bmatrix} \lambda = 2 \\ \lambda = 4 \\ \lambda = 8 \end{bmatrix}$$

7. $T = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \Rightarrow S = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

$$SX = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

3.

$$A = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 2 \\ 0 & 2 & 6 \end{pmatrix}$$

$$\chi_A(\lambda) = \begin{vmatrix} 6-\lambda & 0 & 0 \\ 0 & 6-\lambda & 2 \\ 0 & 2 & 6-\lambda \end{vmatrix} = (6-\lambda)\left((6-\lambda)^2 - 4\right) =$$

$$= (6-\lambda)(4-\lambda)(8-\lambda)$$

$$\chi_A(\lambda) = 0 \Leftrightarrow \begin{bmatrix} \lambda = 6 \\ \lambda = 4 \\ \lambda = 8 \end{bmatrix}$$

$\lambda = 4$: $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 2 & 2 \end{pmatrix} \begin{pmatrix} \xi_1^1 \\ \xi_1^2 \\ \xi_1^3 \end{pmatrix} \Rightarrow$

$$2\xi_1^1 = 0$$
$$2\xi_1^2 + 2\xi_1^3 = 0$$
$$2\xi_1^2 + 2\xi_1^3 = 0$$

$$x_1 = (0\ 1\ -1)^T$$

$\lambda = 6$: $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix} \begin{pmatrix} \xi_2^1 \\ \xi_2^2 \\ \xi_2^3 \end{pmatrix} \Rightarrow$

$$2\xi_2^3 = 0$$
$$2\xi_2^2 = 0$$

$$x_2 = (1\ 0\ 0)^T$$

$$T = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

$\lambda = 8$: $\begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 2 \\ 0 & 2 & -2 \end{pmatrix} \begin{pmatrix} \xi_3^1 \\ \xi_3^2 \\ \xi_3^3 \end{pmatrix} \Rightarrow$

$$-2\xi_3^1 = 0$$
$$-2\xi_3^2 + 2\xi_3^3 = 0$$
$$2\xi_3^2 - 2\xi_3^3 = 0$$

$$x_3 = (0\ 1\ 1)^T$$

4. $A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -2 & 0 & 2 \end{pmatrix}$ $\ker - ?$

$$\begin{cases} \xi^1 - 2\xi^3 = 0 \\ 0 = 0 \\ -\xi^1 + 2\xi^3 = 0 \end{cases} \qquad \begin{matrix} \xi^1 - 2\xi^3 = 0 \\ 0 = 0 \\ 0 = 0 \end{matrix}$$

ФСР: $\xi^1 = 2\xi^3$

ξ^2, ξ^3 - своб. пер.

$$\begin{pmatrix} 2\xi^3 \\ \xi^2 \\ \xi^3 \end{pmatrix}$$

общий вид: $X = C_1 \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + C_2 \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -2 & 0 & 2 \end{pmatrix} \begin{pmatrix} \xi^1 \\ \xi^2 \\ \xi^3 \end{pmatrix} = 0 \Rightarrow \begin{cases} \xi^1 - \xi^3 = 0 \\ 0 = 0 \\ -2\xi^1 + 2\xi^3 = 0 \end{cases}$$

ξ^2, ξ^3 - св. пер-е; $\begin{pmatrix} \xi^3 \\ \xi^2 \\ \xi^3 \end{pmatrix}$

$$X = C_1 \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + C_2 \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$