№1.

$$\sqrt[5]{\frac{-3+2i}{1-5i}}$$

$$z = \frac{-3+2i}{1-5i} = \frac{(-3+2i)(1+5i)}{1+25} = \frac{-3+15i+2i-10}{26} = \frac{-13+17i}{26} = -\frac{1}{2} + \frac{13}{26}i =$$

$$= -\frac{1}{2} - \frac{1}{2}i$$

$$|z| = \sqrt{\frac{1}{4}+\frac{1}{4}} = \frac{\sqrt{2}}{2}$$

$$\arg\varphi = \pi + \operatorname{arctg} 1 = \frac{5\pi}{4}$$

$$z = \frac{\sqrt{2}}{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$$

$$z_k = \sqrt[5]{|z|}\cdot\left(\cos\left(\frac{\frac{5\pi}{4}+2\pi k}{5}\right) + i\sin\left(\frac{\frac{5\pi}{4}+2\pi k}{5}\right)\right) \quad k\in\{0,1,2,3,4\}$$

$$z_0 = \sqrt[5]{\frac{\sqrt{2}}{2}}\cdot\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = \frac{\sqrt[10]{512}}{2}\cdot\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right) = \frac{\sqrt[5]{4}}{2} + \frac{\sqrt[5]{4}}{2}i$$

№2.
$$\begin{cases} x_1 - 2x_3 - 2x_4 - 2x_5 = 0 \\ x_2 + x_3 - x_4 + x_5 = 0 \\ -2x_1 - 2x_2 + 3x_3 + 6x_4 + 2x_5 = 0 \\ -x_1 - 2x_2 + 4x_4 = 0 \\ -x_1 - x_2 + 8x_3 + 9x_4 + 7x_5 = 0 \end{cases}$$

$$\begin{pmatrix} 1 & 0 & -2 & -2 & -2 \\ 0 & 1 & 1 & -1 & 1 \\ -2 & -2 & 3 & 6 & 2 \\ -1 & -2 & 0 & 4 & 0 \\ -4 & -1 & 8 & 9 & 7 \end{pmatrix} \begin{matrix} \\ \\ +1 \\ \\ \cdot 4 \end{matrix} \begin{matrix} \\ \\ \cdot 2 \\ \\ \end{matrix} \sim \begin{pmatrix} 1 & 0 & -2 & -2 & -2 \\ 0 & 1 & 1 & -1 & 1 \\ 0 & -2 & -4 & 2 & -2 \\ 0 & -2 & -2 & 2 & -2 \\ 0 & -1 & 0 & -1 & -1 \end{pmatrix} \begin{matrix} \\ \\ \cdot 2 \\ \\ \end{matrix} \sim \begin{pmatrix} 1 & 0 & -2 & -2 & -2 \\ 0 & 1 & 1 & -1 & 1 \\ 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 0 & -2 & -2 & -2 \\ 0 & 1 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{cases} x_1 - 2x_3 - 2x_4 - 2x_5 = 0 \\ x_2 + x_3 - x_4 + x_5 = 0 \\ x_3 = 0 \\ x_4, x_5 - \text{св. переменные} \end{cases}$$

$$\begin{pmatrix} 2x_4 + 2x_5 \\ x_4 - x_5 \\ 0 \\ x_4 \\ x_5 \end{pmatrix}$$

$e_1 (1\ 0)^T$

$e_2 (0\ 1)^T$

ФСР: $c_1 \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 2 \\ -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$, $c_1, c_2 = const$

№3.

$$\left(\begin{array}{ccccc|c} 1 & 1 & -2 & -6 & 9 & b_1 \\ 0 & 1 & -2 & -4 & 4 & b_2 \\ 0 & 1 & -1 & -3 & 3 & b_3 \\ -2 & -4 & 6 & 18 & -24 & b_4 \\ 3 & 3 & -4 & -16 & 25 & b_5 \end{array}\right) \begin{array}{l} \\ \\ \cdot(-1) \\ \\ \cdot(-3) \end{array} \sim \left(\begin{array}{ccccc|c} 1 & 1 & -2 & -6 & 9 & b_1 \\ 0 & 1 & -2 & -4 & 4 & b_2 \\ 0 & 0 & 1 & 1 & -1 & b_3 - b_2 \\ 0 & -2 & 2 & 6 & -6 & b_4 + 2b_1 \\ 0 & 0 & 2 & 2 & -2 & b_5 - 3b_1 \end{array}\right) \begin{array}{l} \cdot(2) \\ \cdot(-2) \\ \sim \end{array}$$

$$\sim \left(\begin{array}{ccccc|c} 1 & 1 & -2 & -6 & 9 & b_1 \\ 0 & 1 & -2 & -4 & 4 & b_2 \\ 0 & 0 & 1 & 1 & -1 & b_3 - b_2 \\ 0 & 0 & 0 & 0 & 0 & b_5 - 3b_1 - 2b_3 + 2b_2 \\ 0 & 0 & -2 & -2 & 2 & b_4 + 2b_1 + 2b_2 \end{array}\right) \begin{array}{l} \\ \\ \\ \\ +(2) \end{array} \sim \left(\begin{array}{ccccc|c} 1 & 1 & -2 & -6 & 9 & b_1 \\ 0 & 1 & -2 & -4 & 4 & b_2 \\ 0 & 0 & 1 & 1 & -1 & b_3 - b_2 \\ 0 & 0 & 0 & 0 & 0 & b_5 - 3b_1 + 2b_2 - 2b_3 \\ 0 & 0 & 0 & 0 & 0 & b_4 + 2b_1 + 2b_3 \end{array}\right)$$

$$\begin{cases} -3b_1 + 2b_2 - 2b_3 + b_5 = 0 \\ 2b_1 + 2b_3 + b_4 = 0 \end{cases}$$

$b_{3,4,5}$ – св. пер. $\Rightarrow$

$e_1(1\ 0\ 0)^T$

$\Rightarrow e_2(0\ 1\ 0)^T$

$e_3(0\ 0\ 1)^T$

$$\left(\begin{array}{ccccc} -3 & 2 & -2 & 0 & 1 \\ 2 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccccc} -3 & 2 & -2 & 0 & 1 \\ -1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccccc} 1 & -2 & 0 & -1 & -1 \\ 0 & -4 & -2 & -3 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\Rightarrow \begin{cases} 2b_1 = -2b_3 - b_4 \\ 2b_2 = -b_3 + \frac{3}{2}b - b_5 \\ b_3 = b_3 \\ b_4 = b_4 \\ b_5 = b_5 \end{cases} \qquad \begin{cases} b_1 = -b_3 - \frac{1}{2}b_4 \\ b_2 = -\frac{1}{2}b_3 - \frac{3}{4}b_4 - \frac{1}{2}b_5 \\ b_3 = b_3 \\ b_4 = b_4 \\ b_5 = b_5 \end{cases}$$

(2)

№4.

$$\left(\begin{array}{ccccc|c} 1 & -1 & -2 & 3 & 0 & -53 \\ -2 & 3 & 4 & -6 & -2 & 157 \\ -3 & 4 & 7 & -11 & -2 & 223 \\ 7 & -10 & -15 & 23 & 6 & -537 \\ 9 & -13 & -20 & 31 & 8 & -707 \end{array}\right) \begin{matrix} \\ \downarrow (-2) \\ \\ \\ \\ \end{matrix} \sim \left(\begin{array}{ccccc|c} 1 & -1 & -2 & 3 & 0 & -53 \\ 0 & 1 & 0 & 0 & -2 & 51 \\ 0 & 0 & 1 & -2 & 0 & 13 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow$$

$\Rightarrow$
$$\begin{cases} x_1 = 2x_3 - 3x_4 + 2x_5 - 53 \\ x_2 = 2x_5 + 51 \\ x_3 = 2x_4 + 13 \\ x_4 = x_4 \\ x_5 = x_5 \end{cases}$$

x_4, x_5 – своб. переменные

$$\begin{pmatrix} 2x_3 - 3x_4 + 2x_5 - 53 \\ 2x_5 - 51 \\ 2x_4 - 13 \\ x_4 \\ x_5 \end{pmatrix}$$

частн. реш-е при $x_4 = x_5 = 0$:

$$\begin{pmatrix} -53 \\ +51 \\ +13 \\ 0 \\ 0 \end{pmatrix}$$

№5.

L_1: $v_1 = \begin{bmatrix} 3 \\ 0 \\ -8 \\ 20 \end{bmatrix}$ $v_2 = \begin{bmatrix} -3 \\ 2 \\ 4 \\ -14 \end{bmatrix}$ $v_3 = \begin{bmatrix} 3 \\ -3 \\ -2 \\ 11 \end{bmatrix}$

L_2: $u_1 = \begin{bmatrix} -2 \\ -2 \\ 9 \\ -18 \end{bmatrix}$ $u_2 = \begin{bmatrix} 1 \\ 1 \\ -4 \\ 8 \end{bmatrix}$

$$\left(\begin{array}{ccc|cc} 3 & -3 & 3 & -2 & 1 \\ 0 & 2 & -3 & -2 & 1 \\ -8 & 4 & -2 & 9 & -4 \\ 20 & -14 & 11 & -18 & 8 \end{array}\right) \sim \left(\begin{array}{ccc|cc} 3 & -3 & 3 & -2 & 1 \\ 0 & 2 & -3 & -2 & 1 \\ 0 & 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right) \Rightarrow$$

В качестве базиса можно выбрать $v_1; v_2; u_1; u_2$

$\dim(L_1 + L_2) = 4$