

$$1) \begin{aligned} Q &= aR + bS \\ R &= cQ + bS + \varepsilon \\ S &= cS + bQ \end{aligned} \quad S = c * bQ$$

$$Q = aR + bc * bQ$$

$$R = cQ + bc * bQ + \varepsilon$$

$$Q = a \cdot (cQ + bc * bQ + \varepsilon) + bc * bQ$$

$$Q = acQ + abc * bQ + a + bc * bQ$$

$$Q = Q \cdot (ac + abc * b + bc * b) + a$$

$$Q = (ac + abc * b + bc * b)^* \cdot a$$

$$2) \text{ } L_1 = \{w \mid w \in \{a, b, c\}^* \wedge \#_a(w) > \#_b(w) \wedge \#_c(w) > 2\}$$

predpokládám $L_1 \in \mathcal{L}_3$

$$w = b^{p-1} c^2 a^p \quad \begin{aligned} p-1+2+p &> p \\ p-1 &< p \end{aligned}$$

$$xyz = w$$

$$x = b^d$$

$$y = b^B$$

$$z = b^{p-1-d-B} c^2 a^p$$

$$p = d + B + (p - d - B)$$

$$d \geq 0$$

$$B \geq 1$$

$$p - 1 - d - B < p$$

$$i = 3$$

$$xy^iz = b^d b^{3 \cdot B} b^{p-1-d-B} c^2 a^p$$

jelikož $B \geq 1$, pak $d + 3 \cdot B + (p - d - B - 1) > p$, dochází ke spotu $\Rightarrow L_1 \notin \mathcal{L}_3$

$$3) L_2 = \{xw \mid w \in \{a, b, c\}^* \wedge (\#_a(w) \bmod 2 = x)\}$$

$$4) L_3 = \{ w_1 \# w_2 \mid w_1, w_2 \in \{a, b, c\}^* \wedge (\#_a(w_1) = \#_b(w_2) \vee \#_a(w_1) = \#_c(w_2)) \}$$

$$S \rightarrow A \mid B$$

$$A \rightarrow aAb \mid bA \mid cA \mid Aa \mid \#$$

$$B \rightarrow aBc \mid bB \mid cB \mid Bb \mid Ba \mid \#$$

$$P = (\{q\}, \{a, b, c, \#\}, \{z_0, S, A, B, a, b, c, \#\}, q, z_0, \emptyset)$$

$$\delta(q, S, z_0) = \{(q, A), (q, B)\}$$

$$\delta(q, A, A) = \{(q, aAb), (q, bA), (q, cA), (q, Aa), (q, \#)\}$$

$$\delta(q, B, B) = \{(q, aBc), (q, bB), (q, cB), (q, Bb), (q, Ba), (q, \#)\}$$

$$\delta(q, a, a) = \{(q, \epsilon)\}$$

$$\delta(q, b, b) = \{(q, \epsilon)\}$$

$$\delta(q, c, c) = \{(q, \epsilon)\}$$

$$\delta(q, \#, \#) = \{(q, \#)\}$$

$$5) L_4 = \{ w_1 \# w_2 x \mid w_1, w_2 \in \{a, b\}^* \wedge (w_1 = w_2^R \wedge x = \epsilon) \vee (|w_1| < |w_2| \wedge x = 1) \}$$

