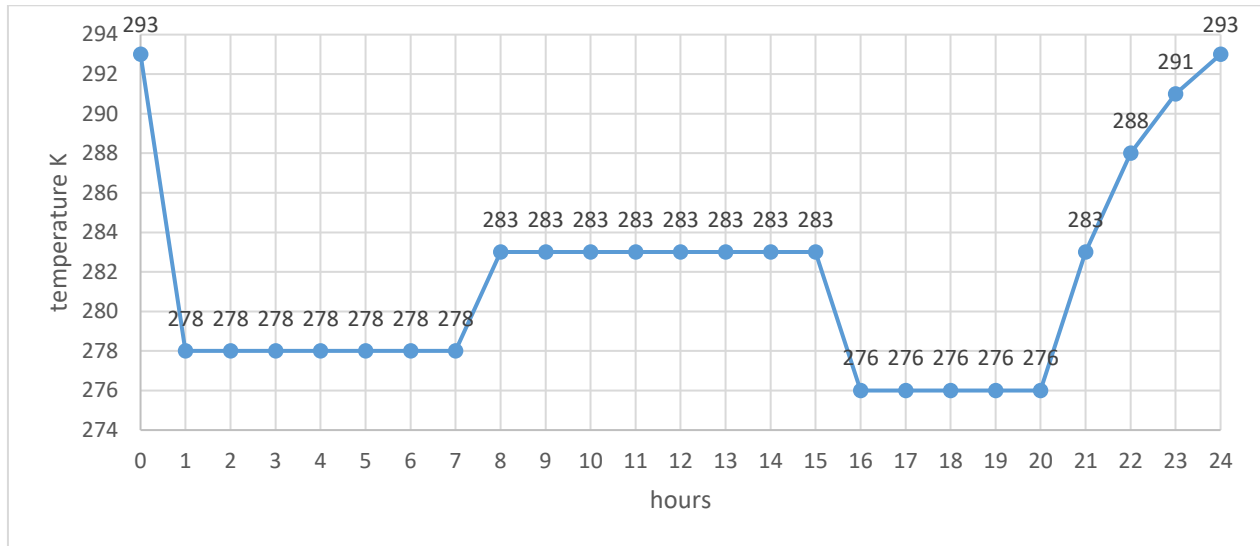


# The cooling cheese problem

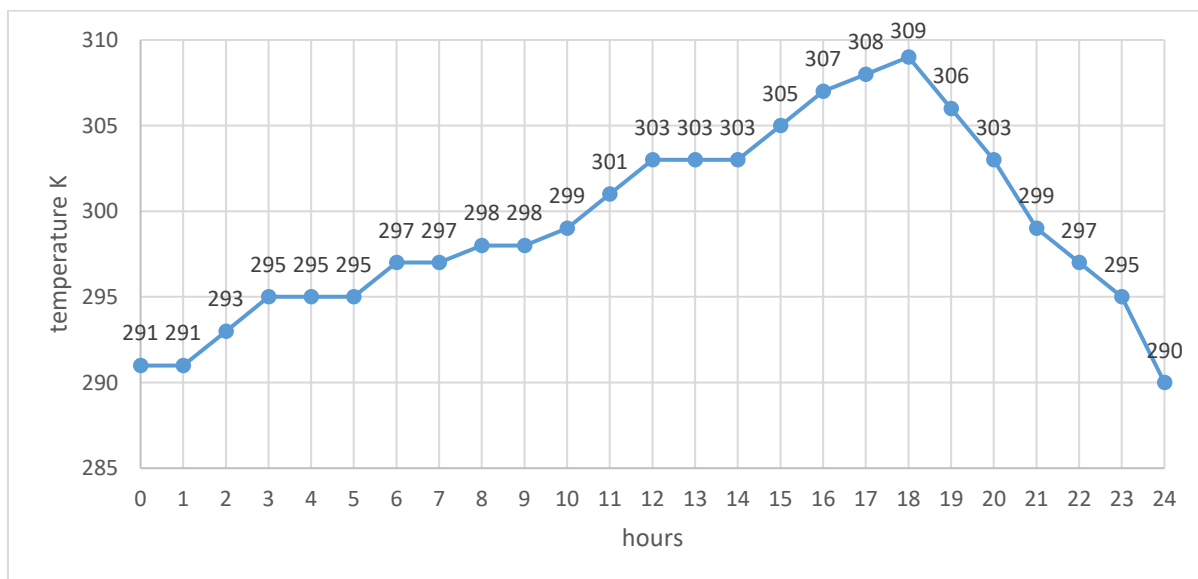
A manufacturing company producing cheese requires a specific process during its seasoning, as described below.

Let's the desired evolution of the temperature for the next 24 hours of a mass of cheese as the one given below.



**Figure 1.** Desired trend of the temperature  $\hat{\theta}_c(t)$  of the cheese mass in the next 24 hours (K)

Let the forecast of the environmental temperature for the next 24 hours be as shown below:



**Figure 2. Forecast of the external environmental temperature  $\theta_a(t)$  (K)**

Let's a cooling/heating cube box of  $1m^3$  be conditioned by heating air in the power range between -1kW (cooling) and 1 kW (heating).

We wish to insert a sphere of cheese of radius=10 cm ( $10^{-2}m$ !!) in the cube box to make the cheese follow such process.

Let the model of the system be described by the following equations:

$$C_c \dot{\theta}_c(t) = k_{fc}(\theta_f(t) - \theta_c(t)) + \omega_c(t)$$
$$C_f \dot{\theta}_f(t) = k_{fc}(\theta_c(t) - \theta_f(t)) + k_{af}(\theta_a(t) - \theta_f(t)) + q(t) + \omega_f(t)$$

Where:

$\theta_c(t)$  temperature of the cheese [K]

$\theta_f(t)$  temperature of the box [K]

$-1000 \leq q(t) \leq 1000$  heating/cooling power [W]

$C_c = c_c * m_c$  [kJ/K] average thermal capacity of the mass of cheese

$m_c = 1.3kg$  mass of the cheese (one sphere of cheese of radius 10cm)

$c_c = 2150$  [J/(kg K)]

$C_f = c_f * m_f$  [kJ/K] average thermal capacity of the air in the box

$m_f = 1.3 kg$  mass of the air in the box

$c_f = 1000$  [J/(kg K)]

$\bar{k}_{fc} = 100$  [W/(m<sup>2</sup> K)] average thermal transmittance x unit surface cheese/air

$k_{fc} = \bar{k}_{fc} * s_c$  [W/K] is the overall thermal transmittance between the air in the box and the cheese

$s_c = 0.12m^2$  surface of the cheese

$\bar{k}_{af} = 0.2$  [W/(m<sup>2</sup> K)] average thermal transmittances x unit surface air/external environment

$k_{af} = \bar{k}_{af} * s_f$  [W/K] is the overall thermal transmittance between the air in the box and the external environment

$s_f = 6m^2$  surface of the box

$\omega_c(t) \sim N(0,1)$

$\omega_f(t) \sim N(0,1)$

## Question

The output  $y$  given by the following:

$$y(t) = \theta_f(t)$$

- 1) Implement the system in simulink in continuous time.
- 2) Sample and hold the output with a sample time of 1 second.
- 3) Plot the evolution of the state variables with no control starting from  $\theta_c(t) = 25^\circ\text{C}$  and  $\theta_f(t) = 8^\circ\text{C}$
- 4) Implement a relay control on three states such that:
  - a.  $q(t) = -q^*$  if  $\theta_f(t) > \hat{\theta}_c(t) + 2$
  - b.  $q(t) = 0$  if  $\hat{\theta}_c(t) - 2 \leq \theta_f(t) \leq \hat{\theta}_c(t) + 2$
  - c.  $q(t) = q^*$  if  $\theta_f(t) < \hat{\theta}_c(t) - 2$Try the control with different values of  $q^* \in (0, 1000]$ . Which  $q^*$  would you suggest?
- 5) Implement a PID control to make the same control as in 4, limiting its values in the range  $q(t) \in [-1000, 1000]$
- 6) Then verify the performance of the controls at point 4 and 5 in terms of:
  - a. MSE of the cheese temperature with respect to the desired one
  - b. Maximum absolute deviation of the cheese temperature with respect to the desired one
  - c. Energy consumption in the 24 hours
  - d. Variance of the control in the 24 hours
- 7) We wish to add an additional laser sensor allowing to measure the external temperature of the cheese. Would this sensor allow to improve