The cooling cheese problem

A manufacturing company producing cheese requires a specific process during its seasoning, as described below.

Let's the desired evolution of the temperature for the next 24 hours of a mass of cheese as the one given below.

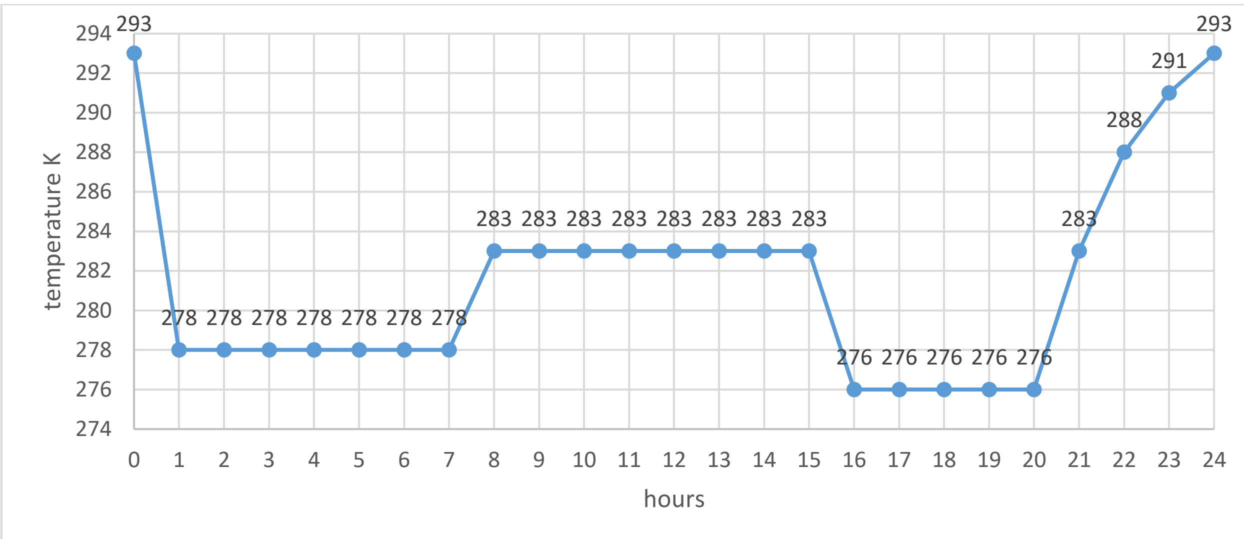

Figure 1. Desired trend of the temperature $\hat{\theta}_c(t)$of the cheese mass in the next 24 hours (K)

Let the forecast of the environmental temperature for the next 24 hours be as shown below:

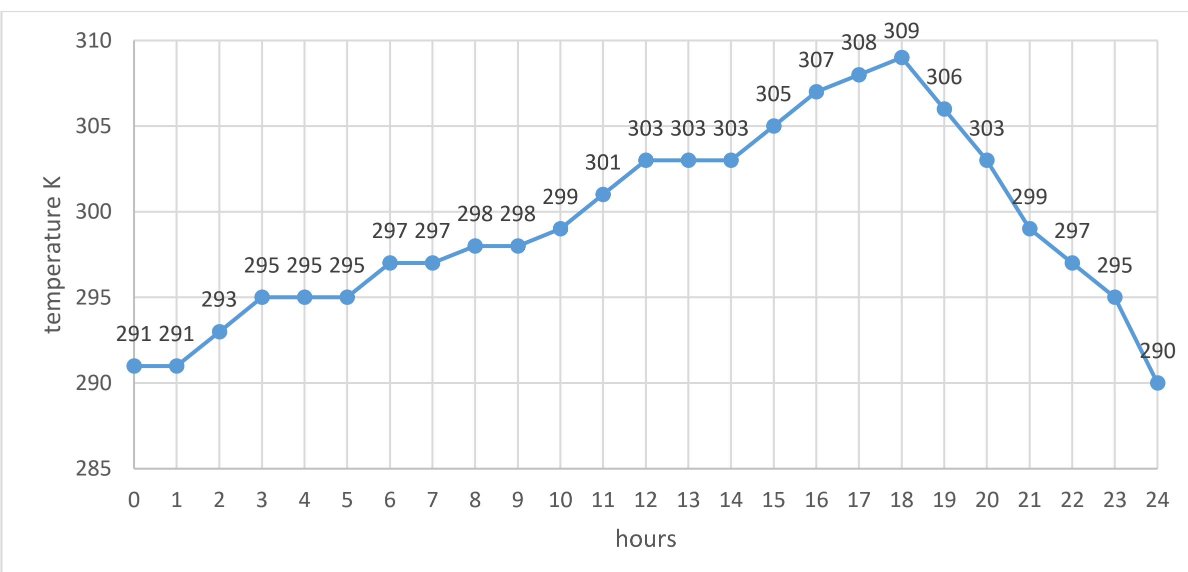

Figure 2. Forecast of the external environmental temperature $\theta_a(t)$ (K)

Let's a cooling/heating cube box of $1m^3$ be conditioned by heating air in the power range between -1kW (cooling) and 1 kW (heating).

We wish to insert a sphere of cheese of radius=10 cm ($10^{-2}m$!!) in the cube box to make the cheese follow such process.

Let the model of the system be described by the following equations:

$$C_c\dot{\theta}_c(t) = k_{fc}\big(\theta_f(t) - \theta_c(t)\big) + \omega_c(t)$$

$$C_f\dot{\theta}_f(t) = k_{fc}\big(\theta_c(t) - \theta_f(t)\big) + k_{af}\big(\theta_a(t) - \theta_f(t)\big) + q(t) + \omega_f(t)$$

Where:

$\theta_c(t)$ temperature of the cheese [K]

$\theta_f(t)$ temperature of the box [K]

$-1000 \le q(t) \le 1000$ heating/cooling power [W]

$C_c = c_c * m_c$ [kJ/K] average thermal capacity of the mass of cheese

$m_c = 1.3kg$ mass of the cheese (one sphere of cheese of radius 10cm)

$c_c = 2150$ [J/(kg K)]

$C_f = c_f * m_f$ [kJ/K] average thermal capacity of the air in the box

$m_f = 1.3\ kg$ mass of the air in the box

$c_f = 1000$ [J/(kg K)]

$\bar{k}_{fc} = 100$ [W/(m^2 K)] average thermal transmittance x unit surface cheese/air

$k_{fc} = \bar{k}_{fc} * s_c$ [W/K] is the overall thermal transmittance between the air in the box and the cheese

s_c = 0.12m^2 surface of the cheese

$\bar{k}_{af} = 0.2$ [W/(m^2 K)] average thermal transmittances x unit surface air/external environment

$k_{af} = \bar{k}_{af} * s_f$ [W/K] is the overall thermal transmittance between the air in the box and the external environment

$s_f = 6m^2$ surface of the box

$\omega_c(t) \sim N(0,1)$

$\omega_f(t) \sim N(0,1)$

Question

The output y given by the following:

$$y(t) = \theta_f(t)$$

1) Implement the system in simulink in continuous time.
2) Sample and hold the output with a sample time of 1 second.
3) Plot the evolution of the state variables with no control starting from $\theta_c(t) = 25°C$ and $\theta_f(t) = 8°C$
4) Implement a relay control on three states such that:
   a. $q(t) = -q^* \quad if\ \theta_f(t) > \hat{\theta}_c(t) + 2$
   b. $q(t) = 0 \quad if\ \hat{\theta}_c(t) - 2 \leq \theta_f(t) \leq \hat{\theta}_c(t) + 2$
   c. $q(t) = q^* \quad if\ \theta_f(t) < \hat{\theta}_c(t) - 2$

   Try the control with different values of $q^* \in (0, 1000]$. Which q^* would you suggest?
5) Implement a PID control to make the same control as in 4, limiting its values in the range $q(t) \in [-1000, 1000]$
6) Then verify the performance of the controls at point 4 and 5 in terms of:
   a. MSE of the cheese temperature with respect to the desired one
   b. Maximum absolute deviation of the cheese temperature with respect to the desired one
   c. Energy consumption in the 24 hours
   d. Variance of the control in the 24 hours
7) We wish to add an additional laser sensor allowing to measure the external temperature of the cheese. Would this sensor allow to improve