

DYNAMIC SIMULATION OF WHOLE BODY MOTION: AN APPLICATION TO HORSE VAULTING.

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#### Abstract

This paper deals with the dynamics analysis of horse vaulting performances. Particular attention is paid to the last phase of the exercise when, during the jump, the athlete executes acrobatic feats creating inertia actions by moving his limbs. The paper presents the first steps of a study whose final aim is the analysis of the effects of slight variations in the limb movements. In particular the features of a program for the direct dynamic analysis of the whole body motion during a jump are shown. This program predicts the trajectory and the orientation of the whole body which is determined by its initial position and velocity and by the limb movements.

#### Sommario

Questa nota tratta dell'analisi dinamica del "volteggio al cavallo". Particolare attenzione e' posta all'ultima fase dell'esercizio quando, durante il "volo", l'atleta comple alcune acrobazie creando azioni d'inerzia per mezzo di movimenti degli arti. Vengono presentati i primi risultati di uno studio il cui scopo finale e' quello di analizzare l'effetto di piccole variazioni nelle leggi di moto degli arti. In particolare vengono presentate le caratteristiche di un programma per una completa simulazione dinamica diretta dell'intero corpo durante il volo. Questo programma e' in grado di predire la traiettoria e l'orientamento del corpo che e' determinata dalle sue condizioni iniziali (posizione e velocita') e dal movimento degli arti.

### 1. - INTRODUCTION.

The evolution of horse vaulting performance (and more generally of gymnastic) is especially due to new figures introduced year after year by some athletes. Innovations are encouraged by the sports regulations which assign an additive score to the performances which contain original exercises [1]. The planning of new exercises is however obstructed by several facts ranging from the limitation of the coaches' fantasy to the fear of possible injuries.

The most interesting phase of many gymnastic exercises is the "flight" (the jump) during which the athlete performs acrobatics feats creating inertia actions by moving his limbs (arms, legs and head). The resulting performances are widely affected by many factors such as the coordination of the different movements, their amplitude and their speed of execution. Generally the athlete learns the "art of moving" in a non rational way. He listens to the coach's suggestions, he studies what his colleagues and adversaries do and tries the exercise many times.

Although the personal experience of coaches and athletes cannot be substituted with a computer simulation tool, it can be very useful. In fact a computer program can help to make the study of an exercise more rational, it can quantify the effects of many parameters (e.g. speed, weight, ...), and it can simulate many different athletes' movements in a brief time. For these reasons, computers are widely applied to the study of gymnastic performances. Typical subjects of these studies (e.g. [2-6]) are the improvement of the exercises (increasing of the score), the prevention of injuries and the discovery of intrinsic opportunities and limits due to a certain athlete's peculiarities (e.g height and weight).

However, despite the fact that the dynamics of many gymnastic specialties (e.g. horse vaulting) cannot be studied as two-dimensional problems, few authors [7] have developed software (like ours) for a full three-dimensional simulation of the human motion.

The aim of our work is an attempt to rationalize the analysis of the influence of the limbs laws of motion. In particular we want to verify the effects on the trunk trajectory and orientation of slight variations in the laws of motion of the limbs. Our work follows two different directions: the analysis of data obtained from actual exercises and the development of a computer program for the dynamics simulation of the whole body motion. This program is able to predict the whole body trajectory and orientation of a man during a jump simply by knowing the initial position and speed of his trunk and the limb movements during the whole jump.

The analysis of the actual data is performed by using traditional approaches taken from the literature [7-10], while the dynamic simulation is approached with an original matrix method previously developed by the authors [11-15].

The whole study is still in progress and the paper presents the plane of the research and the first results.

Notes on terminology.

\* The expression "limb movements" will be used incorrectly to indicate either the movements of the limbs (arms and legs) as well the movements of the head.

\* Sometimes the expression "trunk trajectory" is used incorrectly in order to indicate both the linear and the angular movements.

# 2. - THE HORSE VAULTING.

The horse vaulting performance (Fig.1) which lasts from 1.25 to 1.40 seconds approximately [6], is made up of the following parts

- a) THE RUN UP during which the athlete accumulates velocity and so kinetic energy and momentum.
- b) THE BOARD CONTACT during which the athlete partially transforms his velocity (and momentum) from horizontal to vertical by exploiting the elastic characteristics of the springboard. There are also changes in the

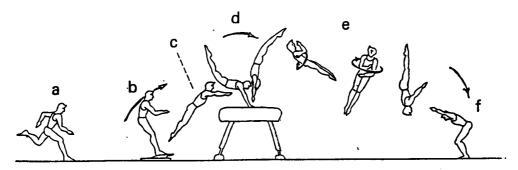


Fig. 1 - The phases of horse vaulting.

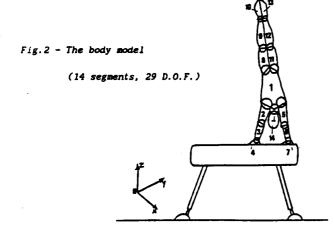
angular momentum. This phase lasts for a minimum of 0.08 to a maximum of 0.12 seconds approximately.

- c) THE FIRST FLIGHT (preflight) during which the athlete reaches the horse at the desired point with the desired angle. Approx. length: 0.1 to 0.25 seconds.
- d) THE HORSE CONTACT during which the athlete performs the last variations in the horizontal and the vertical velocity as well as in the linear and the angular momentum. Approx. length: 0.15 to 0.25 seconds.
- e) THE SECOND FLIGHT (postflight) during which the athlete performs the major acrobatic feats creating inertia actions by moving his limbs. These movements change the total inertia moments influencing the trunk crientation (the total angular momentum is preserved). However, since during this phase there is no force exchange between the athlete and any other external body, the gymnast cannot change the trajectory of his center of mass which is necessarily a parabola. Approx. length: 0.75 to 1.0 seconds.
- f) THE LANDING is the phase which terminates the performance.

### 3. - BODY MODEL.

The body of the athlete (Fig.2) is simulated by a five-chain system (the trunk, the head, the two arms and the two legs) of rigid segments connected by revolute or spherical pairs. The total number of the segments is 14: the head, the trunk and three parts for each limb.

This partition of the body is detailed enough for our aims except for the trunk which will probably be split into two parts (chest and lower-trunk) in the future. The preliminary assumption of considering the trunk as a unique rigid body was made for two reasons. The first reason is that many other authors use such a model and the adoption of the same conventions simplify the comparison of the results. The second reason is that a lot of experimental data obtained from recorded actual data (films of gymnastic championships) does not supply enough information to reconstruct the exact behaviour of the two parts. In fact, generally only the trajectory of the two shoulders and of the two hips are available.



On the other hand, the reduction of the trunk to a unique rigid body does not allow the simulation of some figures.

The knees and the elbows are considered simple hinges, while the wrists and the ankles are double revolute pairs (two degrees of freedom). The neck, the shoulders and the hips are considered ball joints (three degrees of freedom). Finally, two further degrees of freedom have been included in order to take into account the "torsion" of the forearms (i.e. intra/extra rotation) but they rarely have been utilized due to the impossibility of obtaining their values from filmed recorded data.

To summarize the model has 29 degrees of freedom.

#### 4. - ESTIMATION OF THE INERTIA DATA OF THE BODY SEGMENTS.

A practical development of the dynamic analysis or simulation of the whole body motion requires knowledge of the mass distribution (i.e. masses, centers of mass and inertia moments). As is well known, for many reasons, this information cannot be easily obtained.

First of all, the body is not rigid and then the boundaries between the different segments are not very clear. Moreover the body is just one entire "entity" which cannot be "dismounted" to make measurements.

Since 1979 when Borelli published a famous study [16] on this subject, many other researchers have considered this problem. Sometimes data has been collected by measuring the anatomic parts of dissected cadavers. Other methods making use of multiple measures obtained by particular scales have also been studied. The latter methods are based on the determination of the position of the center of mass of the whole body when the limbs assume different positions (e.g. [17]). This methodology has the advantage of being applicable also to living subjects. Recently more modern techniques based on stereo-photogrammetry have also been applied.

Each of these methodologies has bad and good points but all of them share the disadvantage of being very complicated and time consuming. For this reason these procedures cannot always be applied in many situations.

These difficulties have encouraged the development of statistical research in order to relate the mass distribution data with a small number of easily measurable parameters like height and weight [18-21]. A remarkable study is reported in [18], it contains several tables which relate each inertial parameter of the body with a few characteristic body dimensions. From that work we have borrowed a number of data which allows us to estimate each inertial data from equations of this type:

DATUM = 
$$K_1 + K_2 + \text{Height} + K_3 + \text{Weight}$$

Although more precise procedures are suggested, we chose this one because it is very simple and requires only two simple measurements for each subject.

Finally the assumption of considering the human body as being composed of <u>rigid</u> segments is obviously approximated as well as the experimental data on the movements. So the attempt to estimate the inertial data in an extremely precise way does not make sense.

### 5. - ANALYSIS OF ACTUAL DATA.

In order to plan new exercises it is essential, first of all, to analyze actual gymnastic performances. This kind of study is helped by the fact that the recording of

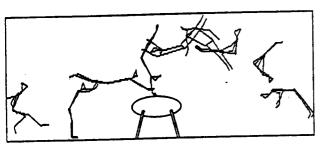


Fig. 3.a - The performance: a "piked cuervo" (handspring plus full twist).

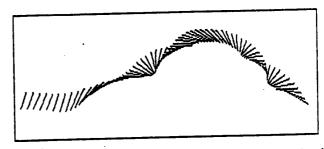


Fig. 3.b - Example of peculiarity magnification: the trunk trajectory.

Fig. 3 - Replay of an actual horse vaulting performance.

actual performances is sometimes allowed during official international meetings.

A number of commercial systems allows the collection of 3D-data of athletic performances starting with two or more different films taken from different views. They generally supply a file which contains 3D-coordinates of the body landmarks.

Some systems can automatically digitize some reference points placed on the subject, but it is generally not possible to apply markers on the athletes during competitions. Therefore, a tedious hand digitizing phase is needed. The "raw data" obtained thus must be "smoothed" in order to reduce the effects of systematic is needed. and statistical errors [7,22-25].

We have obtained some recorded data from the last Olympic Games (Seoul 1988) during a research cooperation with the Biom. Lab. of Pennsylvania State University USA. From this data, with a rather simple dedicated program, we can display actual horse vault performances (Fig. 3a) looking at them from chosen points of view; we can also analyze the relative motion between the body segments and we can magnify a number of their peculiarities (Fig. 3b). In the future we will also be able to calculate inertial parameters (moment, inertial actions and joint torques). These results will be extremely useful for both checking the data (e.g. the angular moment must be constant during a jump) and for the analysis of the performances. In particular, this program allows us to extract the joint laws of motion really used by the athletes. These data will be possibly modified and eventually introduced in the program for the dynamic simulation (see §6.3) for two purposes: the test of the program and the study of the effects of slight variations in the laws of motion.

# 6. - WHOLE BODY MOTION SIMULATION.

# 6.1 - Theoretical Background.

The mathematical approach used here is based on a matrix method developed by the authors and described in detail in [11-15]. However it is briefly summarized here as an introduction to §6.2 where it will be applied in depth to the analysis of horse vaulting.

The method is a generalization of the homogeneous matrix approach, and it makes a coherent use of 4x4 matrices in order to obtain a kinematic and dynamic model of systems of rigid bodies.

According to our notation, the kinematic behaviour of each rigid body is described by the following matrices

M Position matrix (the usual transformation matrix). It describes the position and the orientation of a body.

W Velocity matrix. It describes both the angular and the linear velocity of a body.

H Acceleration matrix which contains the angular and the linear acceleration of a body.

All these matrices generally have three subscripts ij(k) in order to specify the bodies to which the quantity are relative and the frame assumed as reference. For instance W is the projection onto the frame (k) of the

relative speed of body ; with respect to the body i. W and H matrices are related to the time derivatives of the position matrix by the simple relations

$$W = W, W_{1}$$
  $H = W, W_{1}$  (1)

where ' and " mark the first and second time derivatives. The usual relative kinematic theorems can be easily written in matrix notation in order to find the "absolute" motion between bodies (say i and k) as a result of the "drag motion" (between bodies i and j) and the "relative motion" (between j and k). The matrix form of the position and the velocity composition and of the Coriolis' theorem are

$$\begin{aligned} & \mathbf{H}_{ik} = \mathbf{H}_{ij} & \mathbf{H}_{jk} & \mathbf{W}_{ik(r)} = \mathbf{W}_{ij(r)} + \mathbf{W}_{jk(r)} \\ & \mathbf{H}_{ik(r)} = \mathbf{H}_{ij(r)} + \mathbf{H}_{jk(r)} + 2 & \mathbf{W}_{ij(r)} & \mathbf{W}_{jk(r)} \end{aligned} \tag{2}$$

where (r) can be any reference frame.

The dynamics requires the use of the further matrices Φ Action matrix. It describes the systems of forces and torques applied to a body (wrench);

describes J Pseudo-inertial matrix. It distribution of a body.

These matrices generally have a two index subscript ``k(s)'' in order to specify the body to which the quantities are related and the reference utilized. For example  $\Phi_{\text{k(s)}}$  is the matrix describing in frame (s), the wrench (force and torque) applied to the body k.

In this paper we always use as reference an inertial frame labelled with (0). However, generally we will omit to write it for the sake of simplicity.

We recall that the evaluation of the action  $\Phi$  that must be applied to the body k in order to force an absolute acceleration H on it, can be easily calculated by introducing the akers operator (akers(X) :=  $X - X^{L}$ ):

$$\phi_{i} = \text{akeus}\{H_{0k} J_{k}\}$$
 (3)

where J is the mass distribution of the body and 0 is an inertial frame.

# 6.2 - The Trunk Trajectory and Orientation Prediction.

The most interesting exercises performed by an athlete during the horse vaulting are executed in the last phase of the jump when the body is free in space and there is no contact with the horse or the ground. In such a situation the gymnast changes the orientation of his trunk by creating inertial actions by moving his limbs.

In this paragraph we show how the trajectory and the orientation of the body can be predicted just by knowing its initial position and velocity as well as its limb movements for the whole period. From a mathematical point of view this problem will be solved by integrating a set of six second-order differential equations which relate the acceleration of the trunk with its position, its velocity and with the limb movements. The integration will be performed starting from known initial conditions (trunk position and valority), this is and velocity): this is a typical Cauchy's problem. This task can be easily executed using the matrix approach above presented. One frame must be fixed on each segment of the body and their relative motions will be described by means of the M, W and H matrices. The determination of the trunk trajectory and orientation is thus reduced to the evaluation of the position matrix of the trunk at each instant of time. A solution to this problem is presented in the following.

Let us label as 0 (zero) the fixed (inertial) frame and as 1 the trunk and the frame posed on it. The other frames will be labelled with the same numbers of the segments they are fixed on (see Fig.2). The position, velocity and acceleration of the trunk are then described by the three matrices  $M_{01}$ ,  $W_{01}$  and  $H_{01}$ . Remembering the properties of H [4], it can be considered the sum of the square of W and its time derivative

$$H_{01} = W_{01}' + W_{01}^2 \tag{4}$$

Since at each instant the position and the velocity (but not the acceleration) of the trunk are known, it is possible to evaluate the matrices H and W immediately while matrix  $\mathbf{W}'_{01}$  is unknown and it will be evaluated taking into account the athlete's movements. Since the limb movements are known it is possible to calculate the relative position, velocity and acceleration matrices between any couple of contiguous segments. Then eq.s (2) allow us to evaluate the absolute position N , speed W oi and acceleration H<sub>OI</sub> of each part i of the body starting from the trunk and "visiting" one by one all the pieces of the five chains of our body model. According to this procedure, the absolute position of all the segments are

The same methodology can be applied in order to evaluate the absolute velocities and accelerations

But while all the terms of the position and of the velocity equations are known, this is not true for the acceleration equations. In fact we recall that the acceleration of each segment is evaluated starting from the trunk acceleration which is composed of two terms (see eq. (4)) one of which (i.e. W oi) is the unknown. For this reason, the acceleration of each part of the body can also be written as the sum of two terms: W and

 $\widetilde{\boldsymbol{H}}_{01}$  . While the former addendum is unknown, the latter is the sum of all the known terms of eq. (6)

$$\begin{cases} \tilde{H}_{02} = W_{01}^2 + 2 W_{01} W_{12} + H_{12} \\ \tilde{H}_{03} = \tilde{H}_{02} + 2 W_{02} W_{23} + H_{23} \\ \dots \end{cases}$$
(7)

The knowledge of the acceleration of each body segment allows the use of the dynamic equilibrium equation of the whole body which can be written simply by summing up all the forces applied to the body with the inertia actions and making the total equal to zero. If the body is free in space and subjected only to the weight force, the dynamic equilibrium equation is

$$\sum_{i} \hat{\Phi}_{i} + \sum_{i} \Phi_{i} = [0] \tag{8}$$

where  $\hat{\boldsymbol{\varphi}}_i$  and  $\boldsymbol{\varphi}_i$  are the weight and the inertia actions applied to segment i and [0] is the null matrix. If the inertia matrix of the body i is  $J_i$ , its inertia force is

$$\Phi_{i} = -akeus\{H_{0i} J_{i}\} \tag{9}$$

(see eq. (3)). The weight action can be evaluated by

means of the gravity acceleration matrix H

So, remembering eq. (6), eq. (8) can be reduced to

$$akeus\left(\sum_{i} \begin{bmatrix} H_{i}J_{i} \end{bmatrix}\right) - akeus\left(\sum_{i} \begin{bmatrix} W_{i}J_{i} \end{bmatrix} + \sum_{i} \begin{bmatrix} \widetilde{H}_{0i}J_{i} \end{bmatrix}\right) = [0]$$
 (11)

Re-arranging the terms of eq. (11), it becomes

$$\phi^* = \text{akeus} \left\{ W_{01}^* J^* \right\} \tag{12}$$

with

$$\mathbf{p}^{\circ} = \text{skew}\left\{\mathbf{H}_{\mathbf{g}} \mathbf{J}^{\circ} - \sum_{i} \left(\widetilde{\mathbf{H}}_{0i} \mathbf{J}_{i}\right)\right\} \qquad \mathbf{J}^{\circ} = \sum_{i} \mathbf{J}_{i} \qquad (13)$$

where  $W_{01}$  is the only unknown which depends on  $\Phi$  and Jand it can be evaluated by solving a six-order linear system [15] that is obtained by handling the matrix equation (3). For this reason, eq. (12) can be formally rewritten in the following way

 $W'_{01} = F(\Phi^{\bullet}, J^{\bullet}) = F(H_{ij}, W_{ij}, J_{ij})$ 

o depends on the velocity and the position of the body and M, W and J matrices depend on the limbs movements. So eq. (12) (or (14)) is the sought twelve order differential equation. This can be integrated numerically in order to find the trunk trajectory and orientation. If the module of the angular velocity and acceleration of the body are not too high, the following simple numerical algorithm can be used

$$\begin{cases} M_{\text{O1}<\text{today}} = M_{\text{O1}<\text{to}} + M_{\text{O1}<\text{to}} + \frac{1}{2} M^{\text{m}} dt^2 = \Delta M_{\text{e1}} M_{\text{O1}<\text{to}} \\ M_{\text{O1}<\text{today}} = M_{\text{O1}<\text{to}} + M_{\text{O1}<\text{to}} dt \end{cases}$$
(15)

with

$$\begin{cases} M'_{01} = W_{0} H_{01} & \Delta M = [1] + W_{01} dt + \frac{1}{2} H'_{01} dt^{2} \\ M''_{01} = H_{01} M_{01} = (W'_{01} + W'_{01}) M_{01} & (16) \end{cases}$$

where [1] is the identity matrix and M is the value of the position matrix of the trunk at the time t.

NOTE: All the calculations must be executed in a inertial frame but W, H and J matrices are known in local frames and so they must be transformed to frame (0) [4,5]. This operation must be repeated at each time [14].

# 6.3 - The programs for the dynamic simulation.

Beside the programs for the estimation of, the inertial data (see §4) and for the analysis of the recorded data (see §5), two other programs have been realized for the dynamic simulations analysis (trajectory prediction) and for the choice of the limbs motion. These programs have been called DY\_MAN (DYnamic analysis of a MAN) and LOM (Laws Of Motion) respectively. Both of these programs will be described briefly in the following paragraphs.

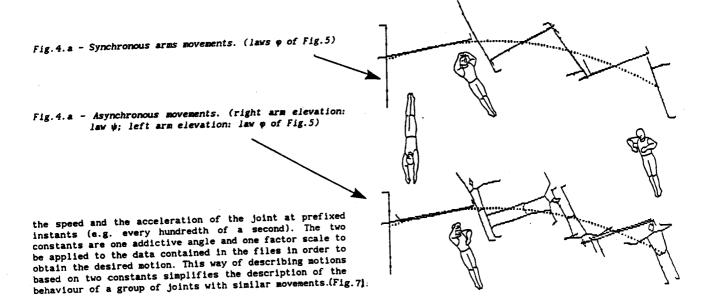
# 6.3.1 - The program DY\_MAN.

DY\_MAN which is based on the matrix method described above, allows the prediction of the trunk trajectory when its initial conditions (position and speed) and the limb movements are chosen. The program also requires the input of the geometrical and dynamic description of the athlete. For each simulation, all these inputs must be supplied by means of a few files.

A file (called DYN.PAR) is used to store the geometrical and the dynamic parameters of the athlete. These data are the geometrical dimensions (e.g. the length of the limbs) and the mass, the inertia moments and the center of mass position of each part of the body.

Another file (MOTION.INI) stores the body's initial condition (i.e. the position and the velocity matrices of the trunk at the beginning of the flight). It also contains the angle values of the joints which do not move

The third file (MOTION.LOM) defines which joints move and specifies their laws of motion. The assignment of the motion to each joint is performed by means of a file-name and two constant values. The file-name is the name of a file which contains the values of the angular position,



The program reads the input data from the above described files and then evaluates the body motion displaying in real time the performance (Fig. 4). The body motion can be displayed in different views (top, lateral or frontal).

The spherical joints are simulated by a series of three revolute pairs.

The program is quite general and allows the study of any type of body motion.

# 6.3.2 - The program lom.

LOM is a general purpose program which allows the choice and the memorization of laws of motion.

Each law of motion may be composed of several parts. Each of them can be either a "motion part" or a "stop (e.g. Fig.5). The motion parts of a joint law of motion are the periods during which the variable which describes an angle changes its value, while the stop parts are periods during which the joint does not move. length of each part and the shape (acceleration profile) of the motion parts can be fixed freely. Each motion part is built up by seven intervals (fig.6) whose length can be freely chosen in order to assign the acceleration profile. This kind of law is currently used in mechanical design [26] because it is very easy to handle and is able to generate a large variety of movements simply by acting on the size of the seven intervals. However we plan to use the program of §5 in order to discover the peculiarities of the "natural" human laws of motion.

With LON, a few different laws of motion can be displayed at the same time in order to verify the sequence of movements of different joints (Fig.5).

The movement of each human joint is chosen by fixing a number of laws of motion equal to the number of its degrees of freedom. Each law of motion is stored by LOM in a file in order to be read and utilized by DY\_MAN.

# 6.4 - Examples.

A typical use of DY\_MAN is the analysis of the effects of similar limb movements on the trunk trajectory.

Fig. 4.a and 4.b display two sample simulations of the last part of a sham horse vaulting. We pretended that, in both cases, the athlete had just finished contact with the horse and he was performing a somersault. During the jump he was moving his arms from the vertical position to the front of his chest. If the movements of the two arms are executed simultaneously (Fig. 4.a) they have not particular effect but if they are executed at different speeds (Fig. 4.b) the asymmetry creates a spin around the main body axis (head-to-feet axis). In this example, the asymmetry of the movements (see Fig.5) was purposely exaggerated in order to magnify its effects.

Fig. 4 - Two simulations of the postflight with the center of mass trajectory. The horizontal displacement is amplified to avoid overlap between the figures

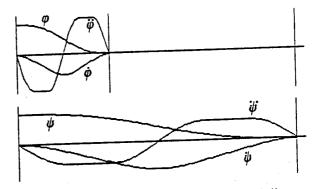


Fig. 5 - The laws of motion of the elevation of the arms during the vaulting of Fig. 4b.

Law  $\phi$  is composed of a motion part plus a stop part. Law # is composed of a single motion part.

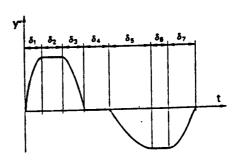
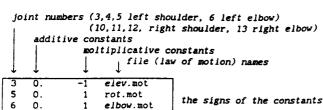


Fig. 6 - A motion-part of a law of motion: the 7 acceleration segments.



ь	U.	1	elbow.mot	1.
10	٥.	1	elev.mot	
12	٥.	-1	rot.mot	'
13	0.	-1	elbow.mot	

depend on the chosen conventions.

synchronous Fig. 7 - The file MOTION.LOM for the movements of Fig. 4a.

#### 7 - PRESENT STAGE AND FUTURE DEVELOPMENTS OF THE RESEARCH.

At the present stage much of the preparatory work necessary for the whole research has been done. This preparation phase concerned the two trends of the research: the analysis of actual data and the dynamic simulation of the whole body. The main activities carried out and the main results achieved are briefly summarized in the following.

## BODY MODEL.

A body model (14 segments, 29 degrees of freedom) has

been developed and data about geometrical and inertial parameters has been taken from literature. A methodology for the data estimation from the height and the weight of the athletes was also chosen.

# ANALYSIS OF ACTUAL DATA.

Information about the 3D-reconstruction of actual data and its smoothing has been taken from the literature. Actual data of elite performances (Seoul Olympic Games 1988) was found and initially used for the test of computer programs especially written for the analysis of horse vaulting performances.

### DYNAMIC SIMULATION

A program for the analysis simulation of the whole body motion has been realized. This program, based on a particular matrix approach, is able to predict the whole body trajectory and orientation starting from the knowledge of its initial condition (position and velocity) and the limb movements.

The future developments of the activity will concern the use of the programs already developed. The joint laws of motion of the athletes performing actual exercises will be extracted from the recorded data. These laws will be analyzed and slightly modified in order to find (and quantify) the effects on the trunk trajectory and orientation. In this phase a remarkable importance will be taken by the program for the dynamic simulation.

For the moment, the graphic output of DY\_MAN is very simple (just a stick figure) but we plan to interface it with a proper graphic package in order to obtain a three--dimensional solid representation of the performances.

### ACKNOVLEDGEMENT.

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