Esame 18/09/2015

giovedì 3 giugno 2021 16:07

$$
\begin{array}{llllll}
\max & z = & \frac{16}{5} & -\frac{2}{5}x_3 & -\frac{4}{5}x_4 \\
 & x_1 = & \frac{2}{5} & -\frac{1}{5}x_3 & -\frac{3}{5}x_4 \\
 & x_2 = & \boxed{\frac{7}{5}} & \boxed{-\frac{1}{5}}x_3 & \boxed{+\frac{2}{5}}x_4
\end{array}
$$

$$\text{fract}\left(\frac{1}{5}\right)x_3 + \text{fract}\left(-\frac{1}{5}\right)x_4 \geq \text{fract}\left(\frac{7}{5}\right)$$

$$\frac{1}{5}x_3 + \frac{3}{5}x_4 \geq \frac{2}{5}$$

$$\frac{1}{5}x_3 + \frac{3}{5}x_4 - x_5 = \frac{2}{5} \rightarrow \boxed{x_5 = -\frac{2}{5} + \frac{1}{5}x_3 + \frac{3}{5}x_4}$$

$$(B \setminus \{x_5\}) \cup \{x_4\} \longrightarrow$$

$$x_4 = \frac{5}{3}\left(x_5 + \frac{2}{5} - \frac{1}{5}x_3\right) = $$
$$= \frac{2}{3} - \frac{1}{3}x_3 + \frac{5}{3}x_5$$

$$x_2 = \frac{7}{5} - \frac{1}{5}x_3 + \frac{2}{5}\left(\frac{2}{3} - \frac{1}{3}x_3 + \frac{5}{3}x_5\right) =$$
$$= \frac{5}{3} - \frac{1}{3}x_3 + \frac{2}{3}x_5$$

$$x_1 = \frac{2}{5} - \frac{1}{5}x_3 - \frac{3}{5}\left(\frac{2}{3} - \frac{1}{3}x_3 + \frac{5}{3}x_5\right) =$$
$$= 0 - x_5$$

$$z = \frac{16}{5} - \frac{2}{5}x_3 - \frac{4}{5}\left(\frac{2}{3} - \frac{1}{3}x_3 + \frac{5}{3}x_5\right) =$$
$$= \frac{8}{3} - \frac{2}{15}x_3 - \frac{4}{3}x_5$$

$$\max\ z = \frac{8}{3} - \frac{2}{15}x_3 - \frac{4}{3}x_5$$

$$x_1 = 0 - x_5$$
$$x_2 = \frac{5}{3} - \frac{1}{3}x_3 + \frac{2}{3}x_5$$
$$x_4 = 2/3 - 1/3\, x_3 + 5/3\, x_5$$

Esercizio 2. (5 punti) Per il seguente problema — noto come 2-knapsack — dove p_i, w_i, v_i e b_1, b_2 sono numeri interi positivi, proporre almeno due metodi di rilassamento, discutendone punti di forza e debolezze.

$$\max\ z = \sum_{j=1}^{n} p_j x_j$$

(1) soggetto a $\sum_{j=1}^{n} w_j x_j \leq b_1$ → limite peso

(2) $\sum_{j=1}^{n} v_j x_j \leq b_2$ → limite volume

$x_1, \ldots, x_n \in \{0, 1\}$.

1) Rilassamento continuo:

$$\max\ z = p^T x$$
$$w^T x \leq b_1 \quad (1)$$
$$v^T x \leq b_2 \quad (2)$$
$$x_1, \ldots, x_n \in [0, 1]$$

→ Efficiente: greedy $O(n)$

→ Efficacia:

GREEDY(I, p, w, v, b_1, b_2)

$v^T x \leq b_2$ (2)
$x_1, \dots, x_n \in [0,1]$

$n = 2$

i	1	2
p	1	M
w	1	2
v	1	2

GREEDY (I, p, w, v, b_1, b_2)
$\bar{w} = w$
$\bar{v} = v$
$i = 1$
while $(\bar{w} - w_i \geq 0$ && $\bar{v} - v_i \geq 0)$
$x_i = 1$
$\bar{w} = \bar{w} - w_i$
$\bar{v} = \bar{v} - v_i$
$i = i + 1$
// o $\bar{w} - w_i < 0$ o $\bar{v} - v_i < 0$
$f_1 = \infty, f_2 = \infty$
if $(\bar{w} - w_i < 0)$
$f_1 = \sum_{j=1}^{i-1} w_j / w_i$
if $(\bar{v} - v_i < 0)$
$f_2 = \sum_{j=1}^{i-1} v_j / v_i$
if $f_1 \leq f_2$:
$z = \sum_{j=1}^{i-1} p_j + f_1 \cdot p_i$
else:
$z = \sum_{j=1}^{i-1} p_i + f_2 \cdot p_i$

2) Elim. vincoli:

$\cancel{b_1}$ $\cancel{b_2}$ 2 problemi dello zaino
→ si prende la soluzione minore (UB migliore)

i	1	2	3	4
p	10	8	7	5
w	2	1	1	4
v	4	3	6	7

$b_1 = 5 = b_2$

$\rightarrow z^* = 25$

$z^* = 10$

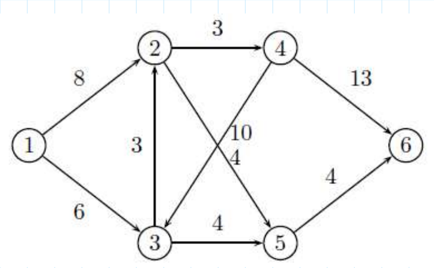
3) Ril. lagrangiano:

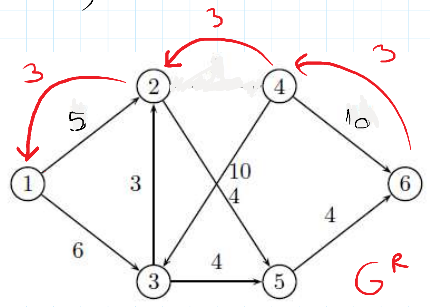
3) Ril. Lagrangiano:

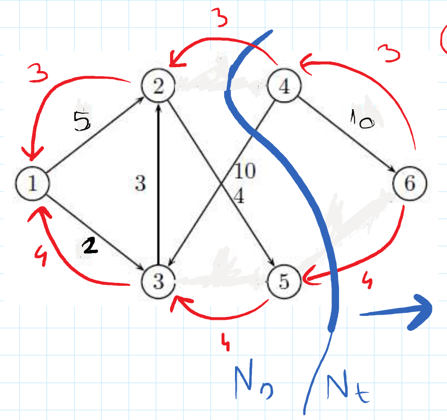
$$P(\lambda) = \max\left\{ \sum_{i=1}^{m} r_i x_i + \lambda\left(\sum_{i=1}^{m} w_i x_i - b_1\right) \,\middle|\, \sum_{i=1}^{m} v_i x_i \le b_2 \right\}$$

- è più facile da calcolare dato un λ, come KP-01
- si può trovare il λ migliore con il metodo del gradiente

3 Max Flow (Ford-Fulkerson)







G^R

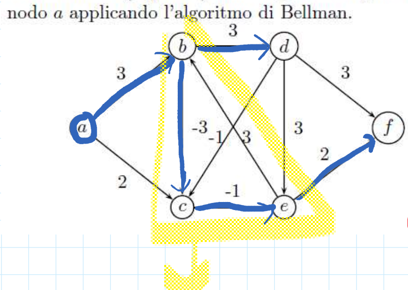
Flusso massimo

(6) $X = x_{12} + x_{13} =$
$= 3 + 4 = 7$

→ taglio di capacità minima
$U(\{N_s, N_t\}) = 3 + 4 = 7 = X$

$\Rightarrow$ Max flow 7

Esercizio 4. (5 punti) Dato il grafo in figura trovare l'albero dei cammini minimi al nodo *a* applicando l'algoritmo di Bellman.



ciclo di costo negativo!

d	a	b	c	d	e	f
	0	~~3~~	~~1~~	~~6~~	-1	~~9~~
		2	0	5		1

CODA: ~~a~~ ~~b~~ ~~c~~ ~~d~~ ~~e~~ ~~f~~ b c d

Esercizio 5. (7 punti) Risolvere il seguente problema dello zaino con il metodo del branch and bound.

$$\max\ z = 10x_1 + 8x_2 + 15x_3 + 7x_4 + 9x_5$$
$$\text{soggetto a } 4x_1 + 3x_2 + 7x_3 + 2x_4 + 5x_5 \le 8$$
$$x_1, \ldots, x_5 \in \{0, 1\}$$

soggetto a ...
$x_1, \ldots, x_5 \in \{0,1\}$

i	1	2	3	4	5
p	10	8	15	7	9
w	4	3	7	2	5
p/w	2,5	2,6	2,14	3,5	1,8

$[i]$	4	2	1	3	5
p	7	8	10	15	9
w	2	3	4	7	5

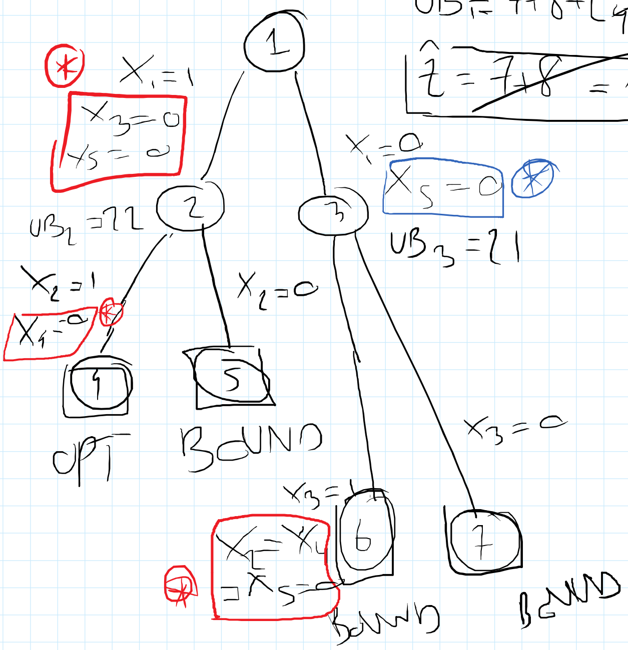
$b = 8$

$UB_1 = 7+8+\left\lfloor \frac{3}{4} \cdot 10 \right\rfloor = 22$

$\boxed{\cancel{\hat{z} = 7+8 = 15}}$

(*) posto a 0 per DOMINANZA $x_1 \to x_5$

(*) posto a 0 per VARIABLE FIXING



$UB_2 = 10+7+\left\lfloor \frac{2}{3} \cdot 8 \right\rfloor = 22$

$\boxed{\cancel{\hat{z} = 17}}$

$UB_3 = 7+8+\left\lfloor \frac{3}{7} \cdot 15 \right\rfloor = 21$

$\cancel{\hat{z} = 7+8 = 15}$

$UB_4 = \boxed{18 = \hat{z}}$

$UB_5 = 10+7 = 17$ BOUND

$UB_6 = 15$

$UB_7 = 7+8+\left\lfloor \frac{3}{5} \cdot 9 \right\rfloor = 18$ BOUND