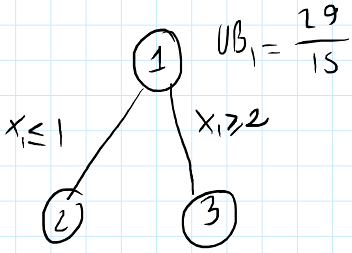
$$\begin{aligned} \max \quad z &= \tfrac{29}{15} - \tfrac{2}{5}x_3 - \tfrac{1}{15}x_4 \\ x_1 &= \tfrac{17}{15} - \tfrac{1}{5}x_3 + \tfrac{2}{15}x_4 \\ x_2 &= \tfrac{4}{5} - \tfrac{1}{5}x_3 - \tfrac{1}{5}x_4 \end{aligned}$$

a) Branch and Bound



(2) $x_1 \le 1 \rightarrow x_1 + x_5 = 1$

$$\boxed{x_5} = 1 - \frac{17}{15} + \frac{1}{5}x_3 - \frac{2}{15}x_4 = -\frac{2}{15} + \frac{1}{5}\boxed{x_3} - \frac{2}{15}x_4$$

ESCE ENTRA

$$x_3 = 5\left(x_5 + \frac{2}{15} + \frac{2}{15}x_4\right) = \frac{2}{3} + \frac{2}{3}x_4 + 5x_5$$

$$x_1 = \frac{17}{15} - \frac{1}{5}\left(\frac{2}{3} + \frac{2}{3}x_4 + 5x_5\right) + \frac{2}{15}x_4 = 1 - x_5$$

$$x_2 = \frac{4}{5} - \frac{1}{5}\left(\frac{2}{3} + \frac{2}{3}x_4 + 5x_5\right) - \frac{1}{5}x_4 = \frac{2}{3} - \frac{1}{3}x_4 - x_5$$

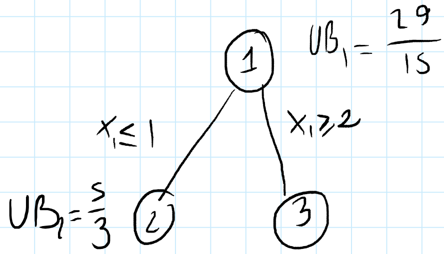
$$\max z = \frac{29}{15} - \frac{2}{5}\left(\frac{2}{3} + \frac{1}{3}x_4 + 5x_5\right) - \frac{1}{15}x_4 = \frac{5}{3} - \frac{1}{3}x_4 - 2x_5$$

$$\rightarrow \max z = \frac{5}{3} - \frac{1}{3}x_4 - 2x_5$$

$$x_1 = 1 - x_5$$

$$x_2 = \frac{2}{3} - \frac{1}{3}x_4 - x_5$$

$$x_3 = \frac{2}{3} + \frac{2}{3}x_4 + 5x_5$$



(3) $x_1 \ge 2 \rightarrow x_1 - x_5 = 2 \rightarrow x_5 = x_1 - 2$

$$\left(\begin{aligned} \max \quad z &= \tfrac{29}{15} - \tfrac{2}{5}x_3 - \tfrac{1}{15}x_4 \\ x_1 &= \tfrac{17}{15} - \tfrac{1}{5}x_3 + \tfrac{2}{15}x_4 \\ x_2 &= \tfrac{4}{5} - \tfrac{1}{5}x_3 - \tfrac{1}{5}x_4 \end{aligned}\right)$$

$$\boxed{x_5} = -\frac{13}{15} - \frac{1}{5}x_3 + \frac{2}{15}\boxed{x_4}$$

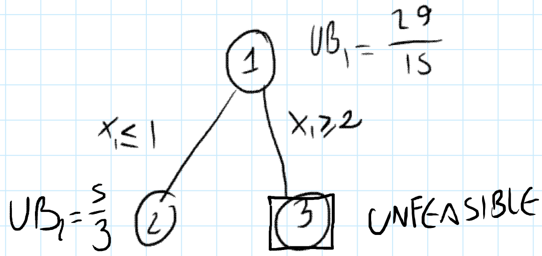
ESCE ENTRA

$$x_4 = \frac{15}{2}\left(x_5 + \frac{13}{15} + \frac{1}{5}x_3\right) = \frac{13}{2} + \frac{3}{2}x_3 + \frac{15}{2}x_5$$

$$x_1 = \frac{17}{15} - \frac{1}{5}x_3 + \frac{2}{15}\left(\frac{13}{2} + \frac{3}{2}x_3 + \frac{15}{2}x_5\right) = 2 + x_5$$

$$x_2 = \frac{4}{5} - \frac{1}{5}x_3 - \frac{1}{5}\left(\frac{13}{2} + \frac{3}{2}x_3 + \frac{15}{2}x_5\right) = \boxed{-\frac{1}{2}} \boxed{-\frac{1}{2}}x_3 - \boxed{\frac{3}{2}}x_5$$

DUALE ILLIMITATO



b) Taglio di Gomory

$$\begin{array}{llll} \max & z = & \frac{29}{15} & -\frac{2}{5}x_3 & -\frac{1}{15}x_4 \\ & x_1 = & \frac{17}{15} & -\frac{1}{5}x_3 & +\frac{2}{15}x_4 \\ & x_2 = & \frac{4}{5} & -\frac{1}{5}x_3 & -\frac{1}{5}x_4 \end{array}$$

$\rightarrow frac\left(\frac{1}{5}\right)x_3 + frac\left(-\frac{2}{15}\right)x_4 \geq frac\left(\frac{17}{15}\right)$

$\frac{1}{5}x_3 + \frac{\cancel{12}^{4}}{\cancel{15}_{5}}x_4 \geq \frac{2}{15} \rightarrow x_3 + 4x_4 \geq \frac{2}{3}$

ERRORE! 13/15, quindi i calcoli più avanti non sono corretti

$\xrightarrow{STD} x_3 + 4x_4 - x_5 = \frac{2}{3} \rightarrow \boxed{x_5} = -\frac{2}{3} + x_3 + 4\boxed{x_4}$

ESCE ENTRA

$\frac{\frac{2}{5}}{1} > \frac{\frac{1}{15}}{4}$

$x_4 = \frac{1}{4}\left(x_5 + \frac{2}{3} - x_3\right) = \frac{1}{6} - \frac{1}{4}x_3 + \frac{1}{4}x_5$

$x_1 = \frac{17}{15} - \frac{1}{5}x_3 + \frac{2}{15}\left(\frac{1}{6} - \frac{1}{4}x_3 + \frac{1}{4}x_5\right) = \frac{51+2}{45} - \frac{7}{30}x_3 + \frac{1}{30}x_5$

$x_2 = \frac{4}{5} - \frac{1}{5}x_3 - \frac{1}{5}\left(\frac{1}{6} - \frac{1}{4}x_3 + \frac{1}{4}x_5\right) = \frac{23}{30} - \frac{3}{20}x_3 - \frac{1}{20}x_5$

$\max z = \frac{29}{15} - \frac{2}{5}x_3 - \frac{1}{15}\left(\frac{1}{6} - \frac{1}{4}x_3 + \frac{1}{4}x_5\right) = \frac{29 \cdot 6 - 1}{15 \cdot 6} + \frac{-12+1}{60}x_3 - \frac{1}{60}x_5 =$

$= \frac{173}{90} - \frac{11}{60}x_3 - \frac{1}{60}x_5$

$\rightarrow \max z = \frac{173}{90} - \frac{11}{60}x_3 - \frac{1}{60}x_5$

$x_1 = \frac{53}{45} - \frac{7}{30}x_3 + \frac{1}{30}x_5$

$x_2 = \frac{23}{30} - \frac{3}{20}x_3 - \frac{1}{20}x_5$

$x_4 = \frac{1}{6} - \frac{1}{4}x_3 + \frac{1}{4}x_5$

Esercizio 2. (7 punti) Dato un grafo $G = (N, A)$ con un peso $c_{ij} \geq 0$ ed una capacità massima u_{ij} per ogni arco $(i, j) \in A$, ed un insieme di K commodity, si consideri il seguente modello di flusso di costo minimo multicommodity:

$$(1) \quad \min z = \sum_{(i,j)\in A} \sum_{k\in K} c_{ij} x_{ij}^k$$

$$(2) \quad \sum_{(j,i)\in BS(i)} x_{ji}^k - \sum_{(i,j)\in FS(i)} x_{ij}^k = b_i^k \qquad \forall k \in K,\ \forall i \in N$$

$$(3) \quad \sum_{k\in K} x_{ij}^k \leq u_{ij} \qquad \forall (i,j) \in A$$

$$(4) \quad x_{ij}^k \geq 0 \qquad \forall (i,j) \in A.$$

Si discuta un rilassamento del problema dato trattandone l'efficienza e l'efficacia.

a) Rilassamento lagrangiano:

$\min z = \sum_{(i,j)\in A} \sum_{k\in K} c_{ij} x_{ij}^k + \sum_{(i,j)\in A} \lambda_{ij}\left(\sum_{k\in K} x_{ij}^k - u_{ij}\right)$

$\sum_{(j,i)\in BS(i)} x_{ji}^k - \sum_{(i,j)\in FS(i)} x_{ij}^k = b_i^k, \ \forall k \in K, \forall i \in N$

$x_{ij}^k \geq 0$

$\min z = \sum \left(\sum c_{ij} x_{ij}^k + \lambda_{ij} \sum x_{ij}^k - \lambda_{ij} u_{ij}\right) =$

$$\min z = \sum_{(i,j)\in A} \left(\sum_{k\in K} c_{ij} x_{ij}^k + \lambda_{ij} \cdot \sum_{k\in K} x_{ij}^k - \lambda_{ij} u_{ij} \right) =$$

$$= \sum_{(i,j)\in A} \left(\sum_{k\in K} (c_{ij}+\lambda_{ij}) x_{ij}^k - \lambda_{ij} u_{ij} \right) =$$

$$= \underbrace{\sum_{k\in K} \sum_{(i,j)\in A} \left[(c_{ij}+\lambda_{ij}) x_{ij}^k \right]} - \sum_{(i,j)\in A} \lambda_{ij} u_{ij}$$

minimo se $\forall k \in K$. $\sum_{(i,j)\in A} (c_{ij}+\lambda_{ij}) x_{ij}^k$ è minimo.

→ Si risolvono K MCF single commodity data la matrice λ

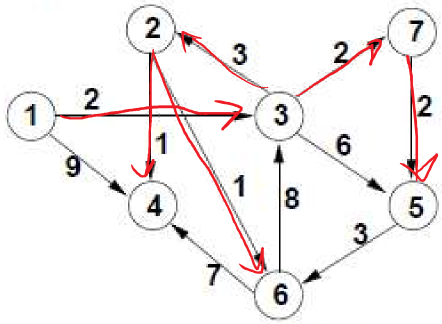
→ Si può trovare λ migliore con il gradiente coniugato

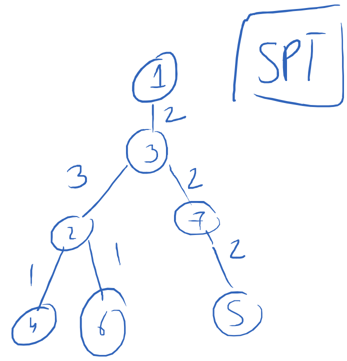
Esercizio 3. (7 punti) Dato il grafo in figura

VALORI

1	2	3	4	5	6	7
0	5	2	~~9~~ 6	~~8~~ 6	6	4

CODA : ~~1~~ ~~3~~ ~~4~~ ~~2~~ ~~7~~ ~~5~~ ~~4~~ ~~6~~

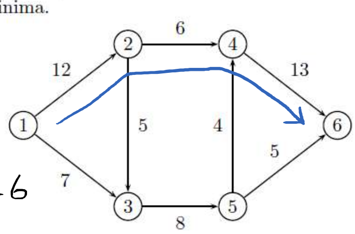




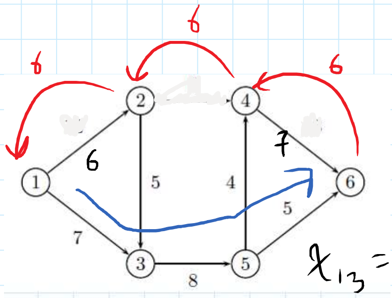
trovare l'albero dei cammini minimi a partire dal nodo 1 applicando l'algoritmo di Bellman-Ford-Moore (ovvero quello che estrae ed inserisce nodi in una coda).

Esercizio 4. (7 punti) Si consideri il seguente problema di flusso di massimo, dove per ogni arco sono riportate le capacità. Trovare la soluzione ottima (valore totale del flusso e flussi sui singoli archi), illustrando i passi dell'algoritmo applicato. Si individui il taglio di capacità minima.

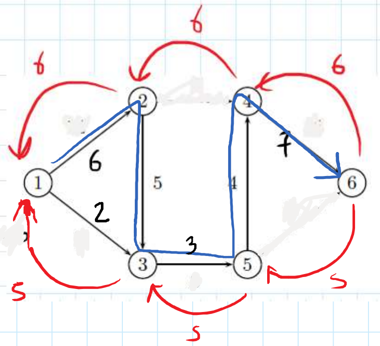
FORD FULKERSON

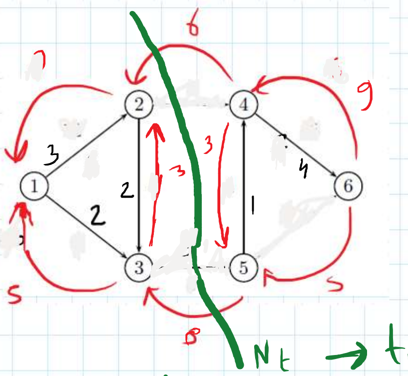


$x_{12} = x_{24} = x_{46} = 6$



$x_{13} = x_{35} = x_{56} = 5$





N_s N_t → taglio di capacità minima = 14

$x_{12} \leftarrow x_{12} + 3$ $\quad x_{54} = 3$

$x_{12} \leftarrow x_{12} + 3$ $x_{54} = 3$

$x_{23} = 3$ $x_{46} \leftarrow x_{46} + 3$

$x_{35} \leftarrow x_{35} + 3$

N_s $N_t \rightarrow$

minimo $= 14$

$\equiv$ flusso $x_{12} + x_{13} = 14$