

# Computer Vision

A.A. 2023-2024

Lecture 03: Image Pyramids



# References

Basic reading:

- Szeliski textbook, Sections 3.4, 3.5

Additional reading:

The original Laplacian pyramid paper

- Burt and Adelson, “The Laplacian Pyramid as a Compact Image Code,” IEEE ToC 1983.

Gaussian image pyramid

# Gaussian image pyramid



The name of this sequence of subsampled images



# Idea for Today:

## Form a Multi-Resolution Representation



**original**



**sigma = 1**



**sigma = 3**



**sigma = 10**

# Pyramid representation

Because a large amount of smoothing limits the frequency of features in the image, we do not need to keep all the pixels around!

Strategy: progressively reduce the number of pixels as we smooth more and more. Leads to a “pyramid” representation if we subsample at each level.

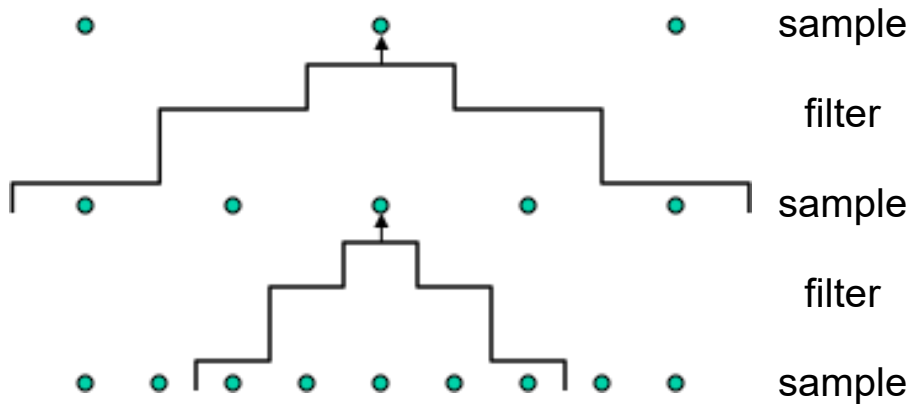
Synthesis: Smooth image with a Gaussian and downsample. Repeat.

- Gaussian is used because it is self-reproducing (enables incremental smoothing).

# Constructing a Gaussian pyramid

## Algorithm

```
repeat:  
    filter  
    subsample  
until min resolution reached
```

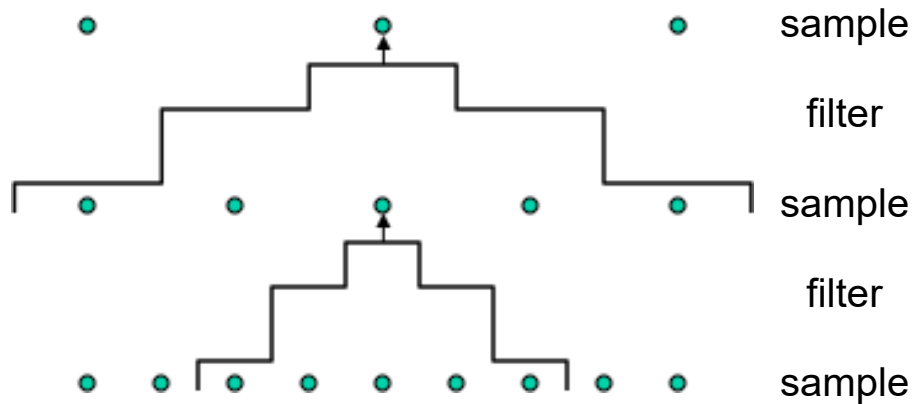


Question: How much bigger than the original image is the whole pyramid?

# Constructing a Gaussian pyramid

## Algorithm

```
repeat:  
    filter  
    subsample  
until min resolution reached
```

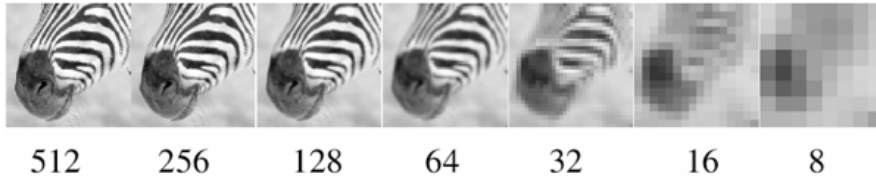


Question: How much bigger than the original image is the whole pyramid?

Answer: Just  $\frac{4}{3}$  times the size of the original image!



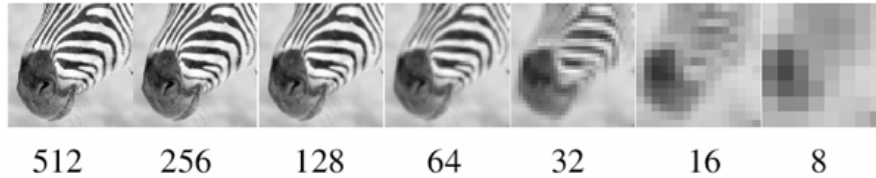
# Some properties of the Gaussian pyramid



What happens to the details of the image?



# Some properties of the Gaussian pyramid

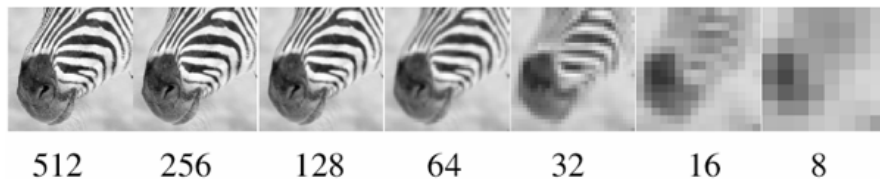


What happens to the details of the image?

- They get smoothed out as we move to higher levels.

What is preserved at the higher levels?

# Some properties of the Gaussian pyramid



What happens to the details of the image?

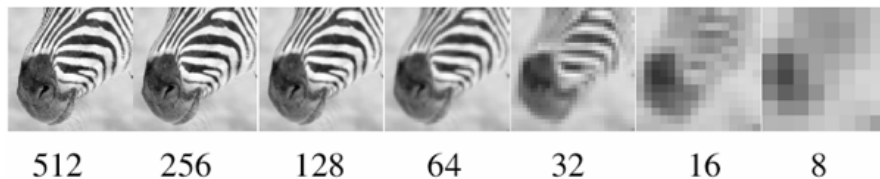
- They get smoothed out as we move to higher levels.

What is preserved at the higher levels?

- Mostly large uniform regions in the original image.

How would you reconstruct the original image from the image at the upper level?

# Some properties of the Gaussian pyramid



What happens to the details of the image?

- They get smoothed out as we move to higher levels.

What is preserved at the higher levels?

- Mostly large uniform regions in the original image.

How would you reconstruct the original image from the image at the upper level?

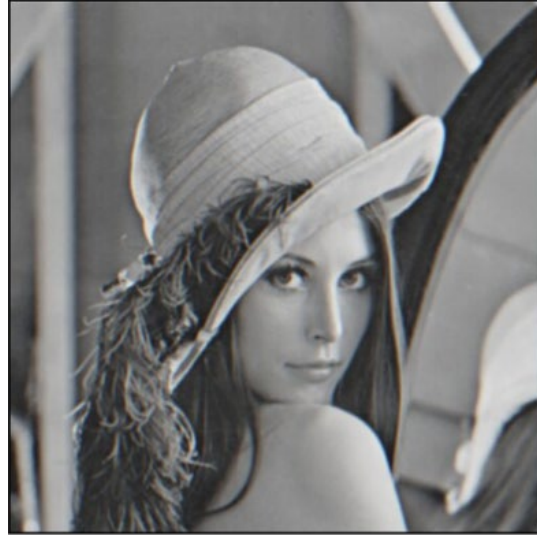
- That's not possible.

# Blurring is lossy



level 0

-



level 1 (before downsampling)

=



residual

What does the residual look like?

# Blurring is lossy



level 0

-



level 1 (before downsampling)

=

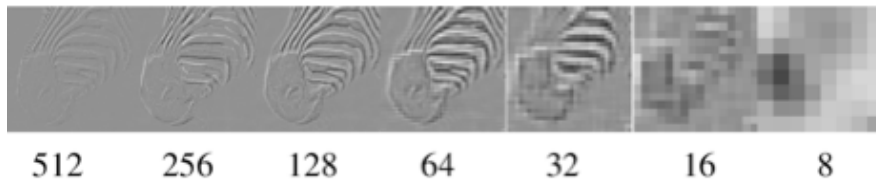


residual

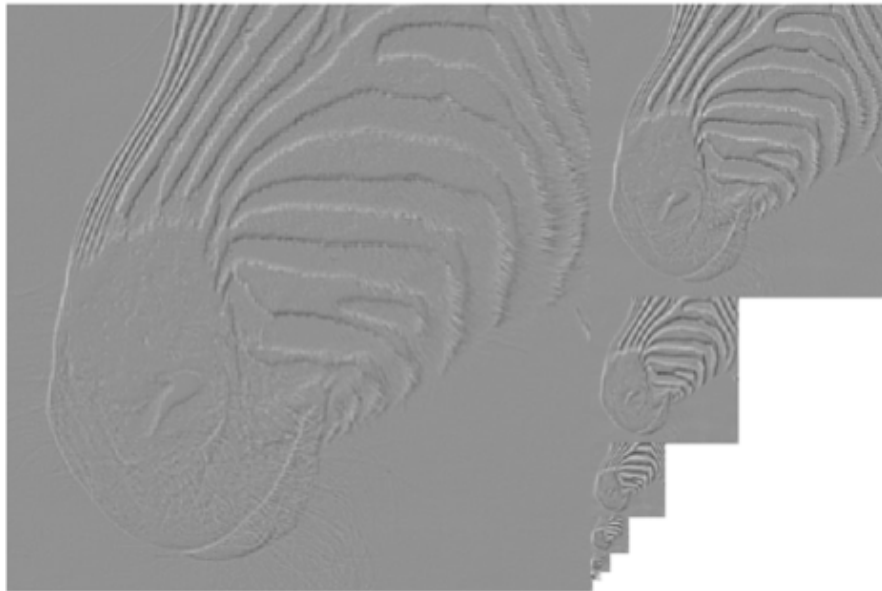
Can we make a pyramid that is lossless?

# Laplacian image pyramid

# Laplacian image pyramid



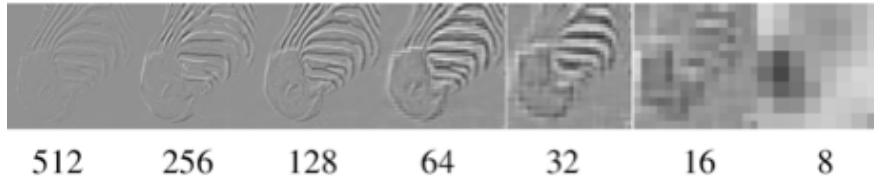
At each level, retain the residuals instead of the blurred images themselves.



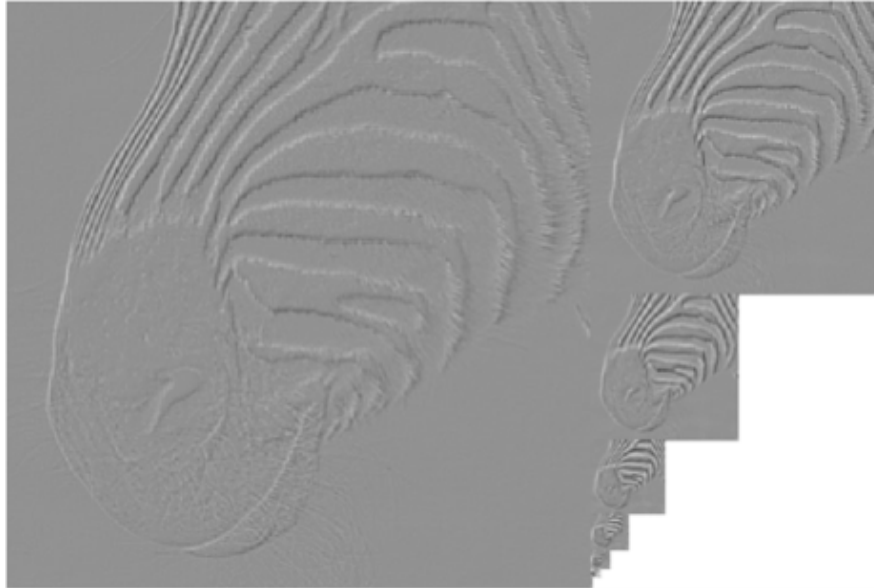
Can we reconstruct the original image using the pyramid?



# Laplacian image pyramid



At each level, retain the residuals instead of the blurred images themselves.



Can we reconstruct the original image using the pyramid?

- Yes we can!

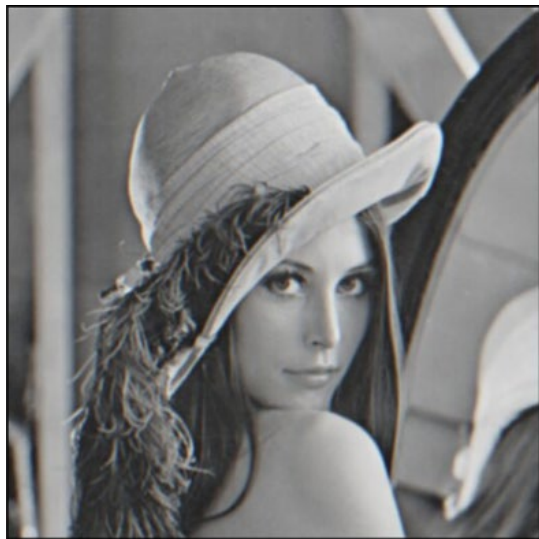
What do we need to store to be able to reconstruct the original image?

# Let's start by looking at just one level



level 0

=



level 1 (upsampled)

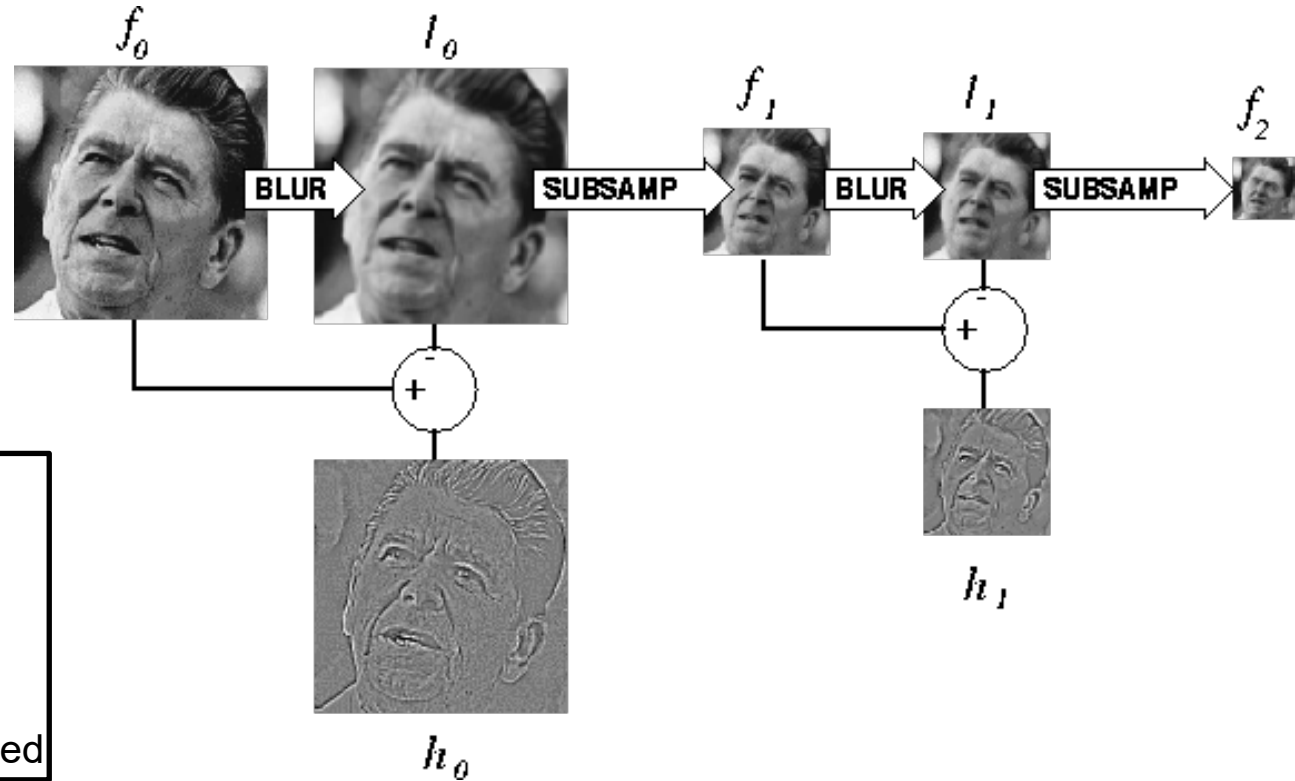
+



residual

Does this mean we need to store both residuals and the blurred copies of the original?

# Constructing a Laplacian pyramid



## Algorithm

repeat:

filter

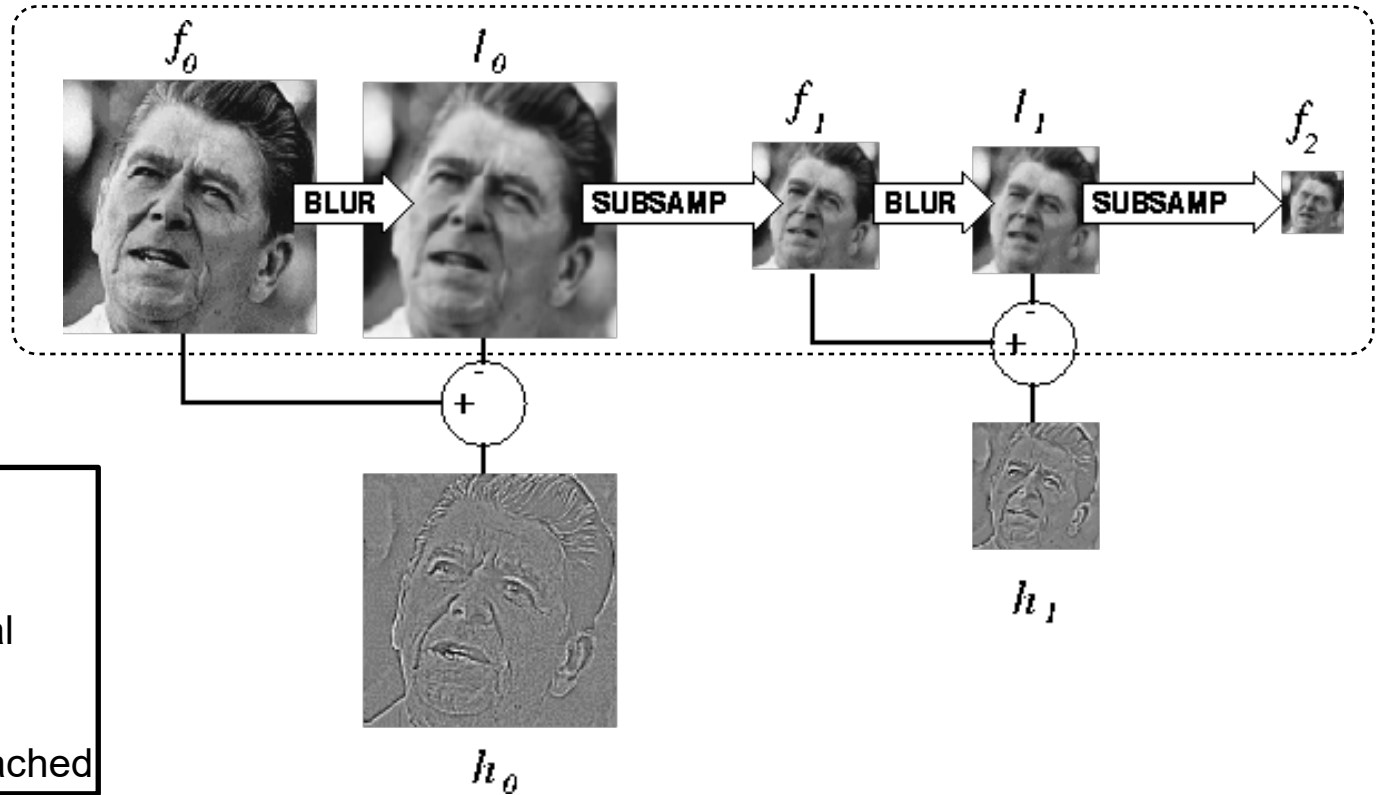
compute residual

subsample

until min resolution reached

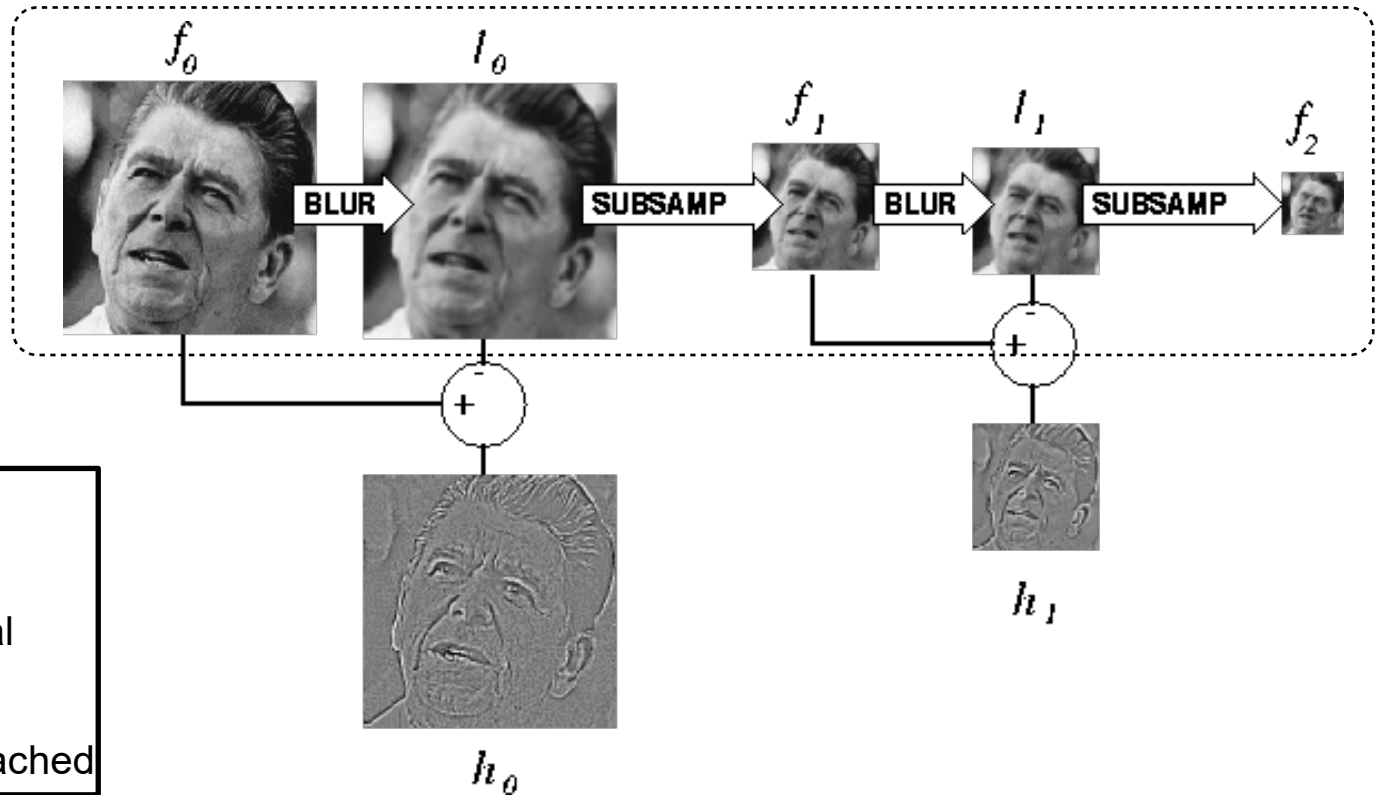
# Constructing a Laplacian pyramid

What is this part?



# Constructing a Laplacian pyramid

It's a Gaussian pyramid.



## Algorithm

repeat:

filter

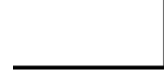
compute residual

subsample

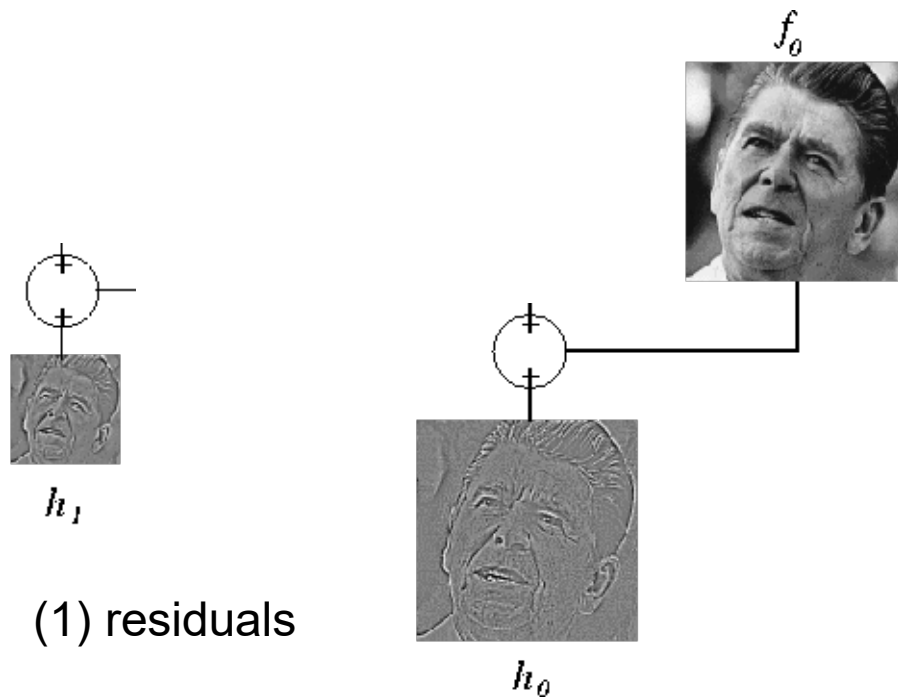
until min resolution reached

# What do we need to construct the original image?

$f_0$



# What do we need to construct the original image?



(1) residuals

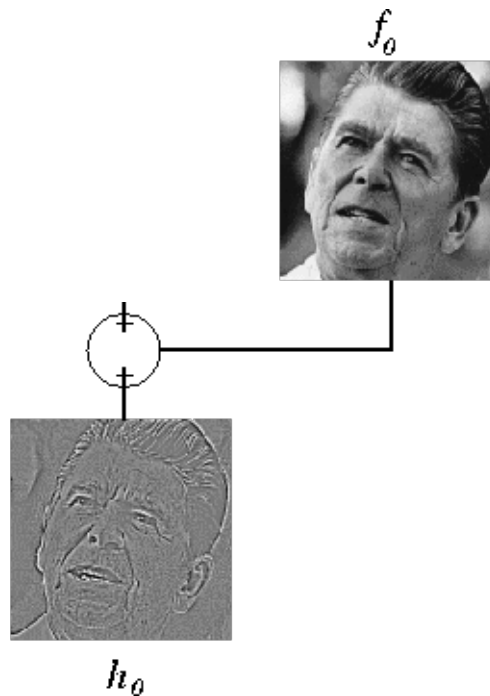
# What do we need to construct the original image?

(2) smallest  
image  $f_2$



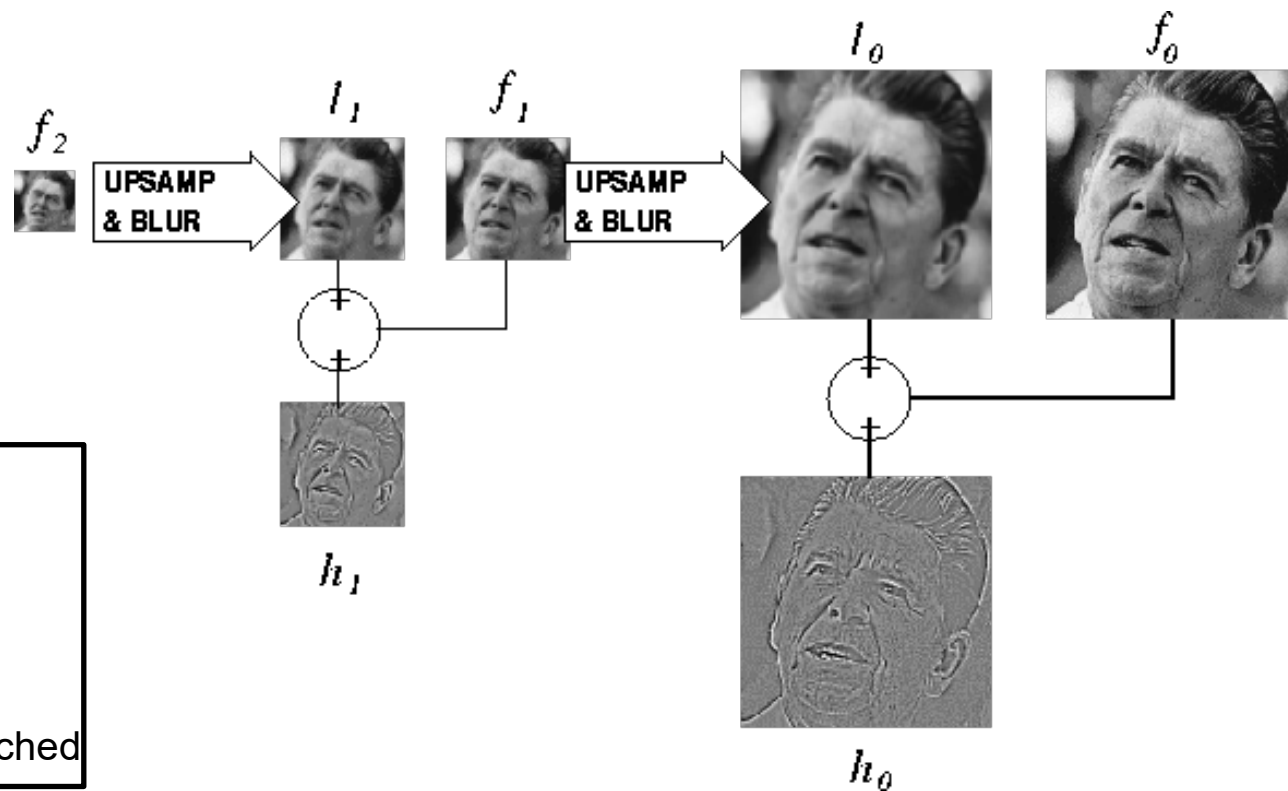
$h_1$

(1) residuals





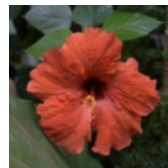
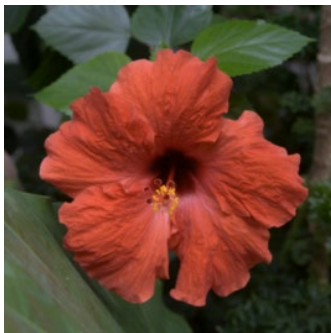
# Reconstructing the original image



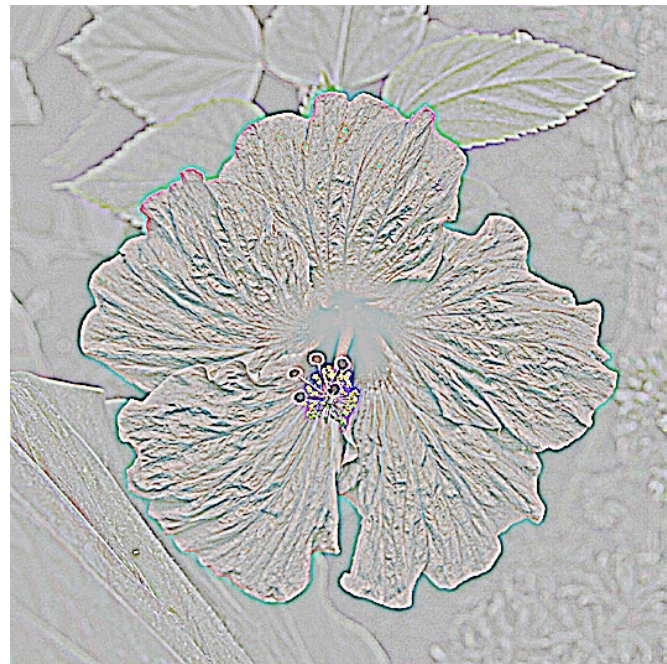
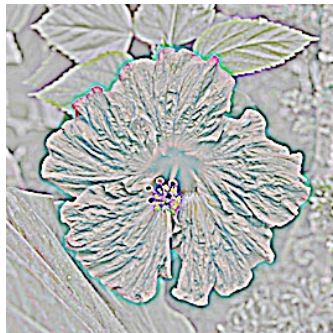
## Algorithm

repeat:  
    upsample  
    sum with residual  
until orig resolution reached

# Gaussian vs Laplacian Pyramid



Shown in opposite  
order for space.

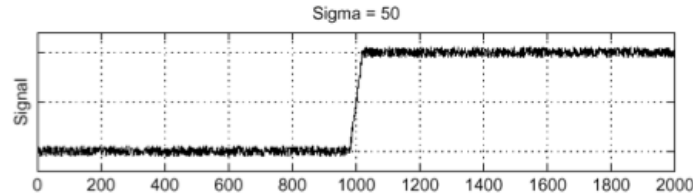


Why is it called a Laplacian pyramid?

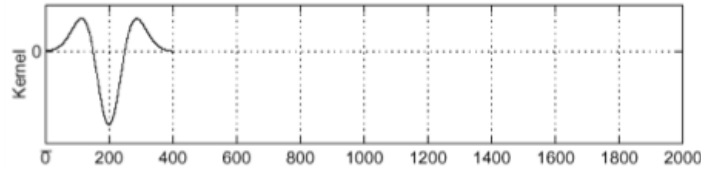
# Reminder: Laplacian of Gaussian (LoG) filter

As with derivative, we can combine Laplace filtering with Gaussian filtering

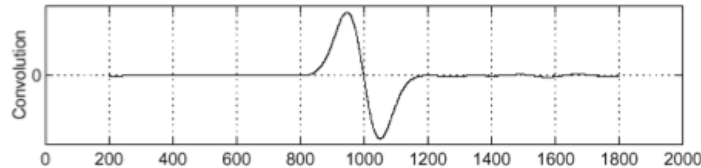
input



Laplacian of  
Gaussian

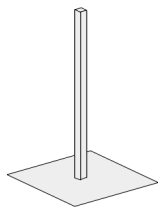
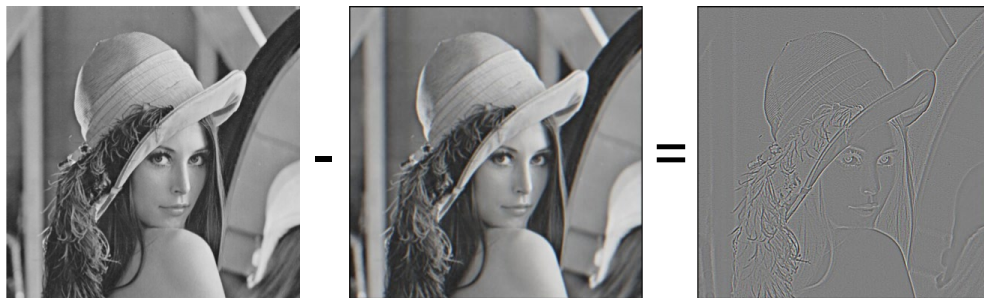


output

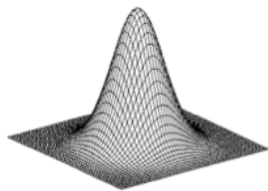


“zero crossings” at edges

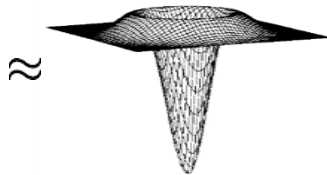
# Why is it called a Laplacian pyramid?



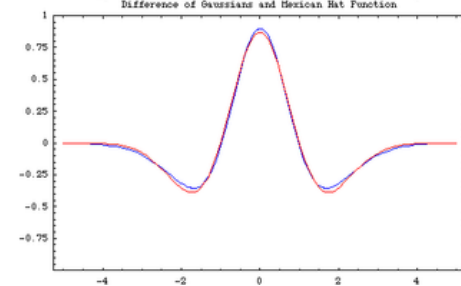
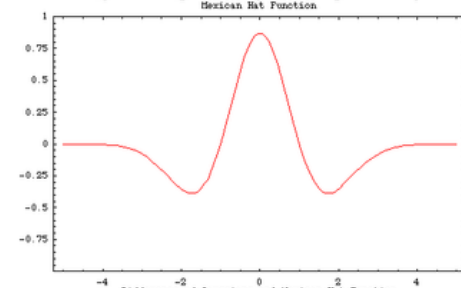
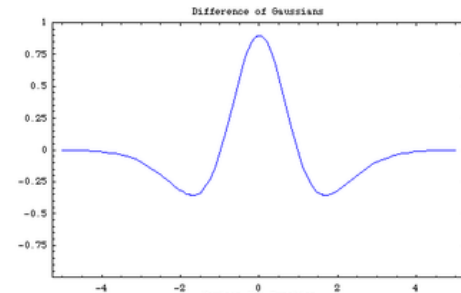
unit



Gaussian



Laplacian



Difference of Gaussians approximates the Laplacian

# What are image pyramids used for?

image compression



image blending



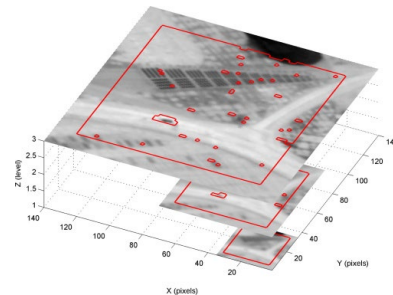
denoising



multi-scale detection

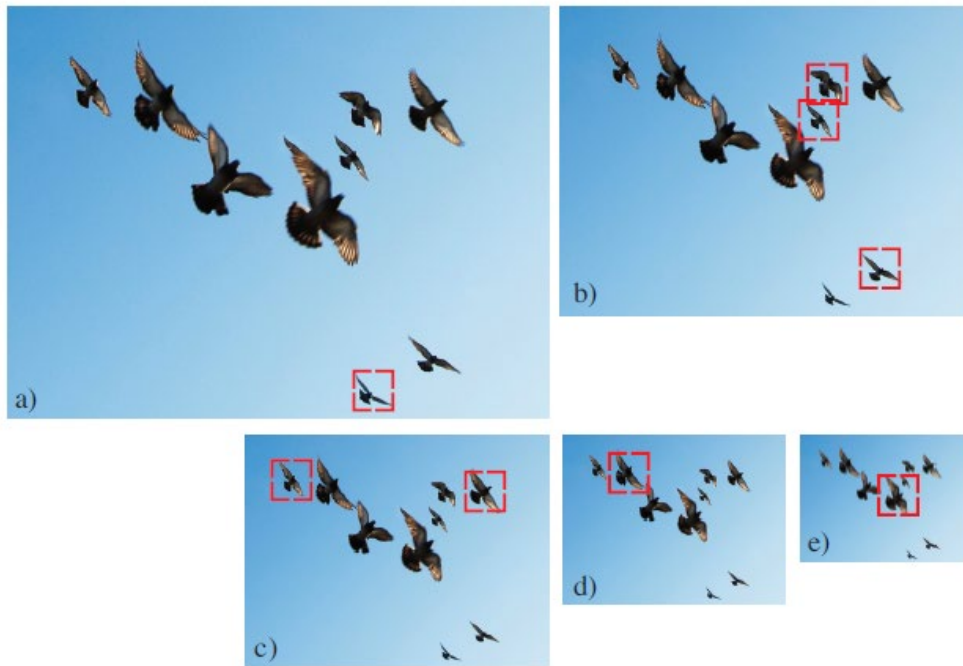


multi-scale registration





# Multi-scale image analysis



Multiscale image pyramid.

Each image is 25% smaller than the previous one.

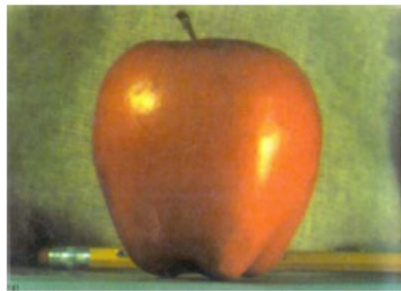
The red box indicates the size of a template used for detecting flying birds.

As the size of the template is fixed, it will only be able to detect the birds that tightly fit inside the box.

Birds that are smaller or larger will not be detected within a single scale.

By running the same template across many levels in this pyramid, different birds instances are detected at different scales.

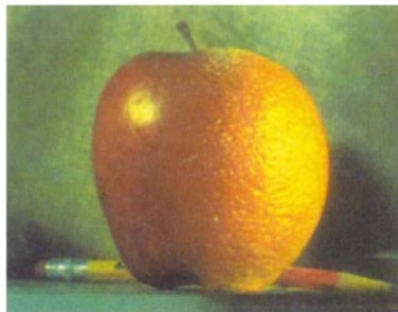
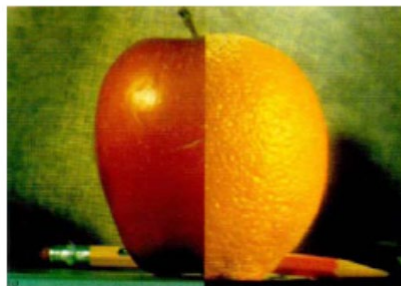
# Image blending



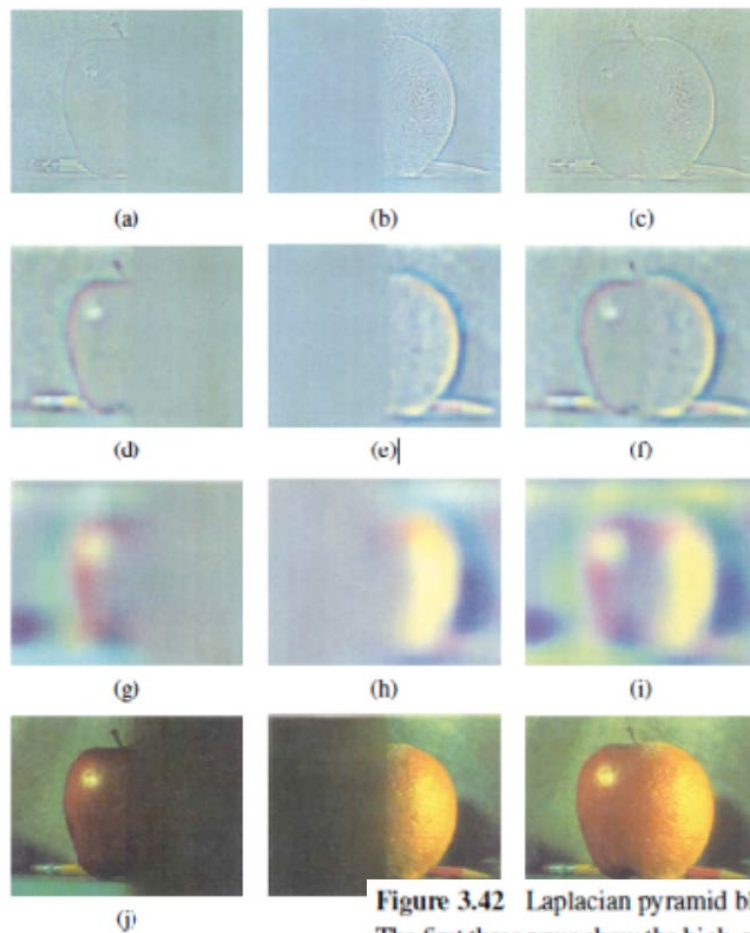
(a)



(b)





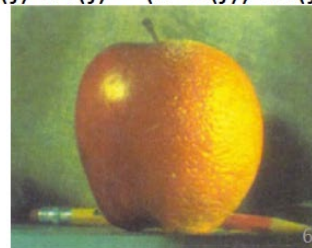


**Figure 3.42** Laplacian pyramid blending details (Burt and Adelson 1983b) © 1983 ACM. The first three rows show the high, medium, and low frequency parts of the Laplacian pyramid (taken from levels 0, 2, and 4). The left and middle columns show the original apple and orange images weighted by the smooth interpolation functions, while the right column shows the averaged contributions.

# Image blending



- Build Laplacian pyramid for both images:  $LA, LB$
- Build Gaussian pyramid for mask:  $G$
- Build a combined Laplacian pyramid:  $L(j) = G(j) LA(j) + (1-G(j)) LB(j)$
- Collapse  $L$  to obtain the blended image

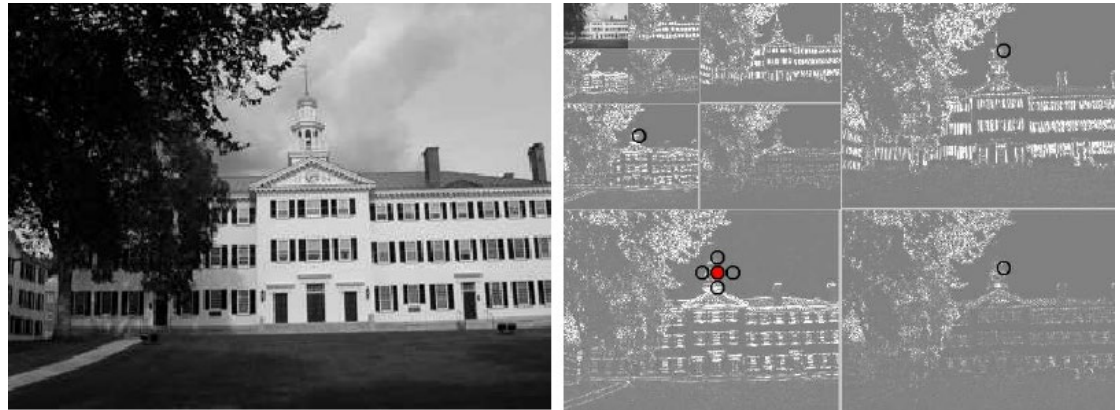


# Other types of pyramids

Steerable pyramid: at each level keep multiple versions, one for each direction.

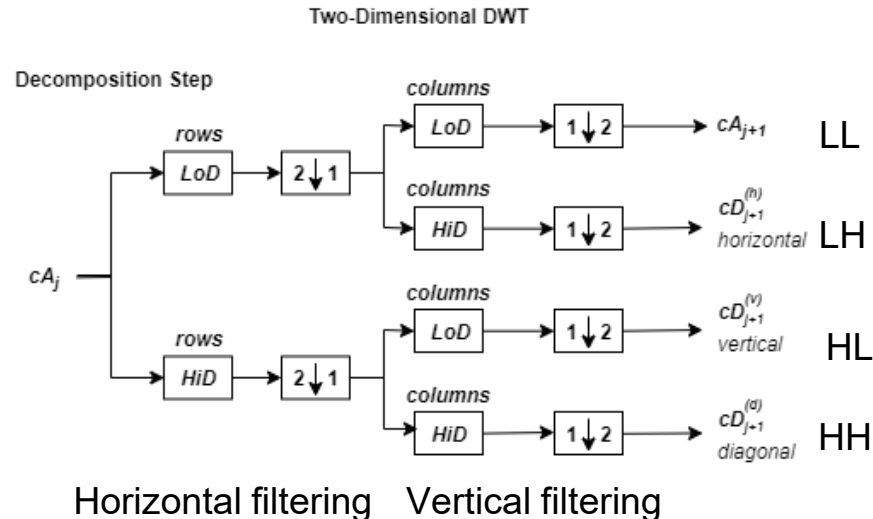


Wavelets: huge area in image processing



# Wavelets

- Wavelet analysis is used to divide information present on an image (signals) into two discrete components: approximations and details (sub-signals).
- A signal is passed through two filters, high pass and low pass filters. The image is then decomposed into high frequency (details) and low frequency components (approximation). At every level, we get 4 sub-signals. The approximation shows an overall trend of pixel values and the details as the horizontal, vertical and diagonal components.
- Haar Wavelets



where

- $\begin{bmatrix} 2 & \downarrow & 1 \end{bmatrix}$  – Downsample columns: keep the even-indexed columns
- $\begin{bmatrix} 1 & \downarrow & 2 \end{bmatrix}$  – Downsample rows: keep the even-indexed rows
- $\begin{bmatrix} \text{rows} \\ \boxed{X} \end{bmatrix}$  – Convolves with filter  $X$  the rows of the entry
- $\begin{bmatrix} \text{columns} \\ \boxed{X} \end{bmatrix}$  – Convolves with filter  $X$  the columns of the entry

# Wavelets example

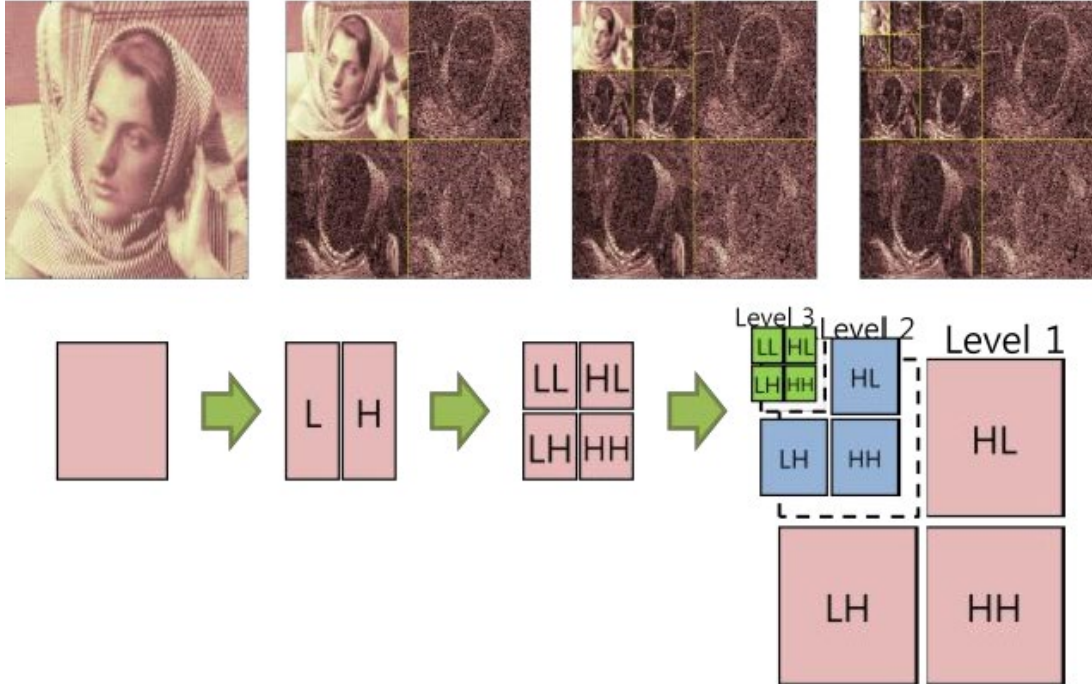


Fig. 3. Process of 3-Level 2D-DWT.

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