Group 15: Alessandro Caggia, Edoardo Calleri di Sala, Emanuele Cinti, Stefano De Bianchi

Task 1: Time Series Decomposition

Figure 1 presents an analysis of the STL trend component, with the raw data overlaid and trendlines fitted to the previous 30 years of data at multiple cutoffs. First, the STL-extracted trend suggests a long-term warming pattern from the 1950s that has accelerated in recent decades. Second, there is no evidence of a progressively increasing seasonal variation over time, ruling out the need for a multiplicative decomposition.

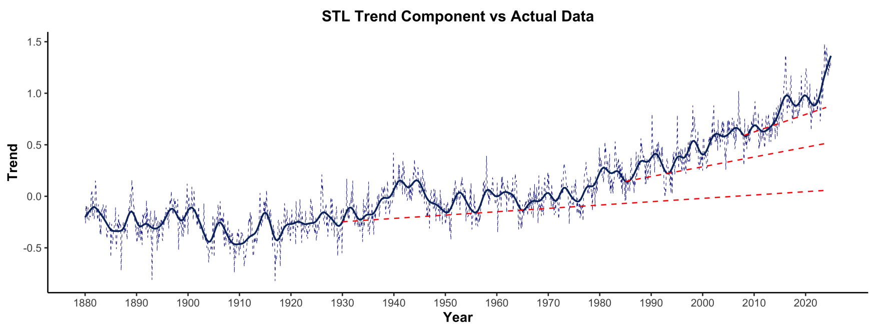

Figure 1: The time series with the STL trend component and trendlines

Third, we performed seasonal decomposition using both the additive decompose and the STL functions. STL decomposition accommodates time-varying seasonal patterns, whereas the additive decomposition assumes a fixed seasonal component (figure 2). Our plot also shows that the difference in magnitude of the estimated seasonal components is significant: from -0.25 to 0.25 for the STL decomposition to -0.02 to 0.02 for the additive. This leads to lower residuals for the STL decomposition (more infra).

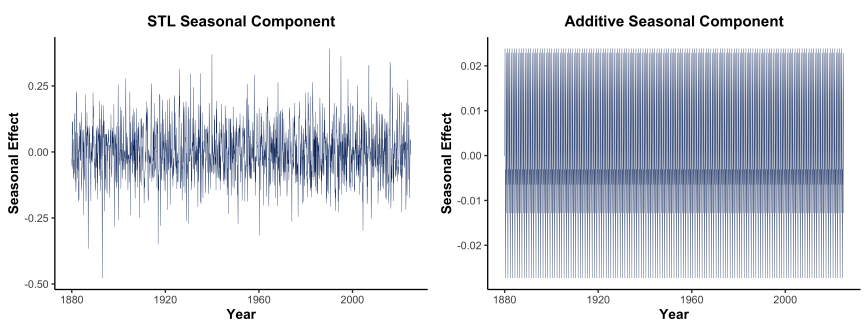

Figure 2: Figure 2a plots the seasonal component estimated through the STL decomposition on the left, figure 2b plots the seasonal component estimated through the additive decomposition

We found no clear evidence of strong seasonal patterns. Figure 3a plots the overlaid raw time series for one year out of every ten and does not reveal systematic spikes. A similar conclusion is reached by observing the polar plot in Figure 3b, constructed using the seasonal components extracted from the STL decomposition.

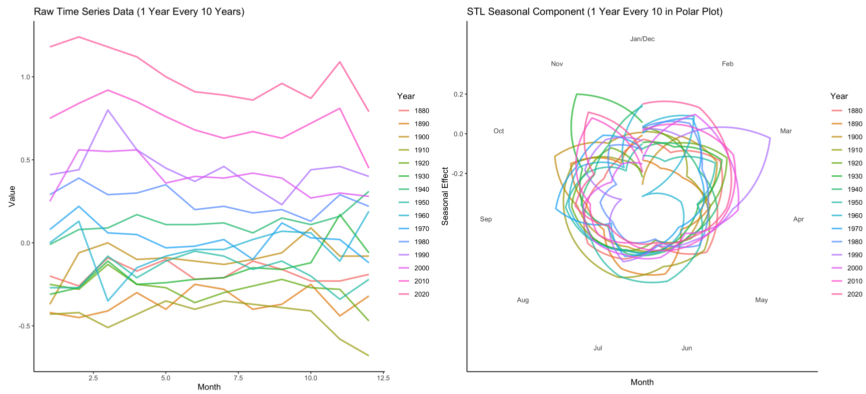

Figure 3: Figure 3a plots the time series for one year out of every 10, while figure 3b plots the seasonal component extracted from the STL decomposition in a 12-period polar plot

Lastly, we analyze residuals by plotting them as a share of the total time series (figure 4). The residual component appears largely random but it is possible to identify a series of occasional anomalies and a weaker model fit during the 50s. STL decomposition provides lower residuals, due to the adaptability of its short seasonal window, whereas the additive decomposition exhibits a significantly higher level of error. However, overfitting to noise is a crucial issue that must be considered in this comparison.

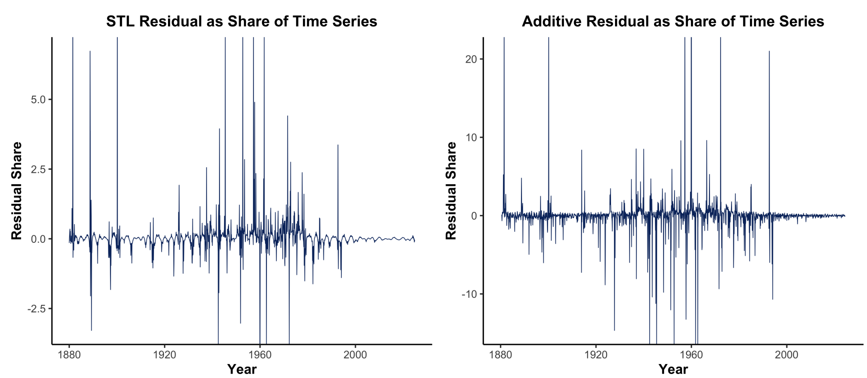

Figure 4: Figure 4 plots on the left the STL residuals as a share of the total, on the right the additive residuals as a share of the total

Task 2: Exponential Smoothing and Forecasting

1. Exponential Smoothing Model

The α parameter determines how much weight is given to recent and past observations. The Holt-Winters function automatically optimizes α by minimizing the sum of squared errors (SSE). To simulate real-time data arrival, we fit a simple exponential smoothing model on the 1880–1930 subset using the Holt-Winters method without a trend or seasonal component. The smoothing parameter was estimated as $\alpha = 0.52898$, striking a balance between short-term adaptation and long-term stability. To assess the impact of α, we tested different values:

- $\alpha = 0.2$: A smoother response with less adaptability.
- $\alpha = 0.52898$ (default): A balanced response to changes.
- $\alpha = 0.9$: A highly reactive model that closely follows fluctuations but is sensitive to noise.

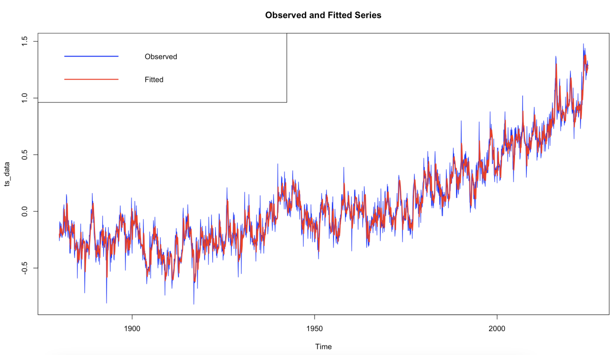

Figure 5: Observed and Fitted Series

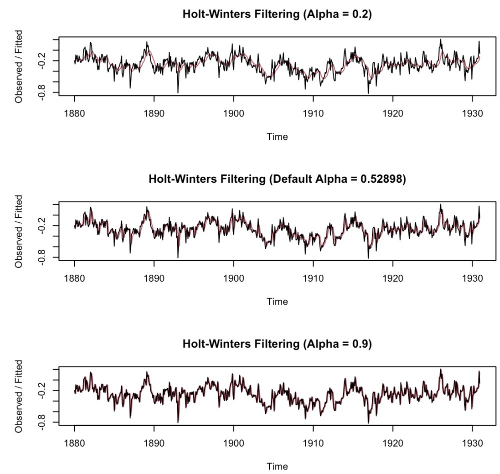

Figure 6: Comparison of different values of α in exponential smoothing.

2. One-Step-Ahead Predictions

The one-step-ahead predictions closely follow the observed values, indicating that the model accurately captures the short term variations during the early period.

3. Forecast Function Expression

For simple exponential smoothing, the one-step-ahead forecast is given by $\hat{y}_{t+1} = l_t$, and for any $h > 1$, the forecast remains constant: $\hat{y}_{t+1} = l_t$, and $\hat{y}_{t+h} = l_t$.

4. Extended Forecast Evaluation

Holt-Winters forecasting, fitted to data until December 1930, preserves trend and seasonality. While short-term predictions are accurate, long-term errors accumulate due to compounding uncertainty and unmodeled structural shifts. Exponential smoothing performs well in the short term but fails over longer horizons, as seen in Figure 7, where forecasts remain near the historical mean and systematically underpredict the trend.

The forecast errors, summarized in Table 1, confirm this pattern. Notably, errors grow substantially over time, highlighting the method's limitations for long-term forecasting in non-stationary data. The assumption that future temperature anomalies follow past behavior is unrealistic given the observed warming trend.

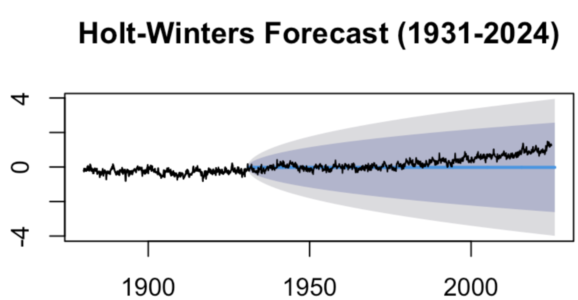

Figure 7: Holt-Winters Forecast from 1931 to 2024

Time Horizon	Forecast Error
1 Day	-0.0887
1 Year	-0.0754
5 Years	-0.1521
10 Years	-0.0730
50 Years	0.0048
90 Years	0.2454

Table 1: Forecast errors at different time horizons.