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title: "Code TS2"  
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Sala  
date: "Second Assignment"  
output: html_document  
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```{r}  
library(depmixs4)
library(ggplot2)
library(dplyr)
library(patchwork)
library(ggthemes)

setwd("/Users/Alessandro/Desktop")

Load data
data <- read.csv("/Users/Alessandro/Desktop/data_assHMM.csv")
data$real_interest <- data$ITAbond_nom - data$inflation
data$Date <- as.Date(data$time, format = "%Y_%m_%d")

Homogeneous HMM
set.seed(123)
hmm_model <- depmix(real_interest ~ 1, data = data, nstates = 3,
family = gaussian(), ntimes = nrow(data))
hmm_fit <- fit(hmm_model, verbose = TRUE)
```

```
iteration 0 logLik: -379.9362
iteration 5 logLik: -337.6639
iteration 10 logLik: -267.6844
iteration 15 logLik: -263.5532
iteration 20 logLik: -223.3614
iteration 25 logLik: -220.234
iteration 30 logLik: -220.1418
iteration 35 logLik: -220.0943
iteration 40 logLik: -220.0627
iteration 45 logLik: -220.0376
iteration 50 logLik: -220.0146
iteration 55 logLik: -219.9907
iteration 60 logLik: -219.9632
iteration 65 logLik: -219.9291
iteration 70 logLik: -219.8861
iteration 75 logLik: -219.8367
iteration 80 logLik: -219.7919
iteration 85 logLik: -219.7625
iteration 90 logLik: -219.7483
iteration 95 logLik: -219.7427
iteration 100 logLik: -219.7407
iteration 105 logLik: -219.7401
iteration 110 logLik: -219.7399
iteration 115 logLik: -219.7398
iteration 120 logLik: -219.7398
converged at iteration 121 with logLik: -219.7398
```

```
hmm_post <- posterior(hmm_fit)

data$hmm_state <- factor(hmm_post$state, levels = 1:3,
labels = c("Recession", "Stable", "Boom"))
data$prob1 <- hmm_post$S1 # Boom
data$prob2 <- hmm_post$S2 # Stable
data$prob3 <- hmm_post$S3 # Recession

hmm_means <- sapply(hmm_fit@response, function(r) r[[1]]@parameters$coefficients)
data$hmm_implied <- sapply(hmm_post$state, function(s) hmm_means[s])

Set theme
theme_clean_base <- theme_minimal(base_size = 12) +
theme(
 panel.background = element_rect(fill = "white", color = NA),
 plot.background = element_rect(fill = "white", color = NA)
)

theme_custom <- theme_clean_base + theme(
 panel.grid = element_blank(),
 panel.grid.major.y = element_line(colour = "#e3e1e1", linetype = 2),
 plot.title.position = 'plot',
 legend.position = 'top',
 legend.title = element_blank()
)

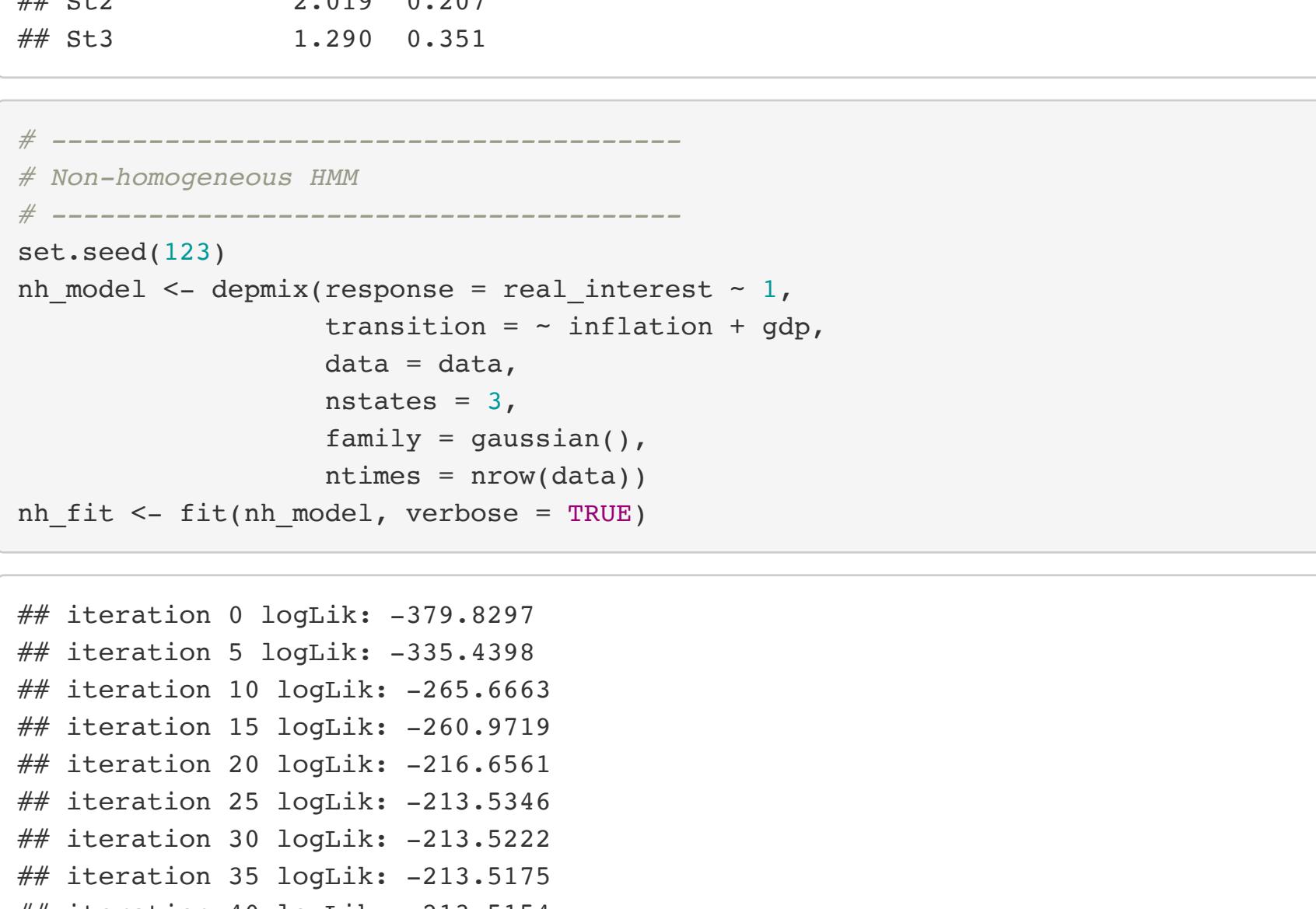
theme_set(theme_custom)

Plot 0: Homogeneous HMM States
p1 <- ggplot(data, aes(x = Date)) +
 geom_line(aes(y = real_interest), color = "darkblue", linewidth = 0.7) +
 geom_point(aes(y = real_interest, color = hmm_state), size = 1.3) +
 scale_color_manual(values = c("Boom" = "green", "Stable" = "blue", "Recession" = "red")) +
 labs(title = "Panel (a): Real Interest Rate and HMM States (Homogeneous)",
x = "", y = "Real Interest Rate", color = "State") +
 theme_custom

Plot 1: State probabilities
p2 <- ggplot(data, aes(x = Date)) +
 geom_line(aes(y = prob1), color = "red", linetype = "solid", linewidth = 0.5) +
 geom_line(aes(y = prob2), color = "blue", linetype = "solid", linewidth = 0.5) +
 geom_line(aes(y = prob3), color = "green", linetype = "solid", linewidth = 0.5) +
 labs(title = "Panel (b): HMM derived state probabilities", x = "Date", y = "Probability") +
 theme_custom

Combine vertically
combined_plot <- p1 / p2
plot(combined_plot)
```

Panel (a): Real Interest Rate and HMM States (Homogeneous)



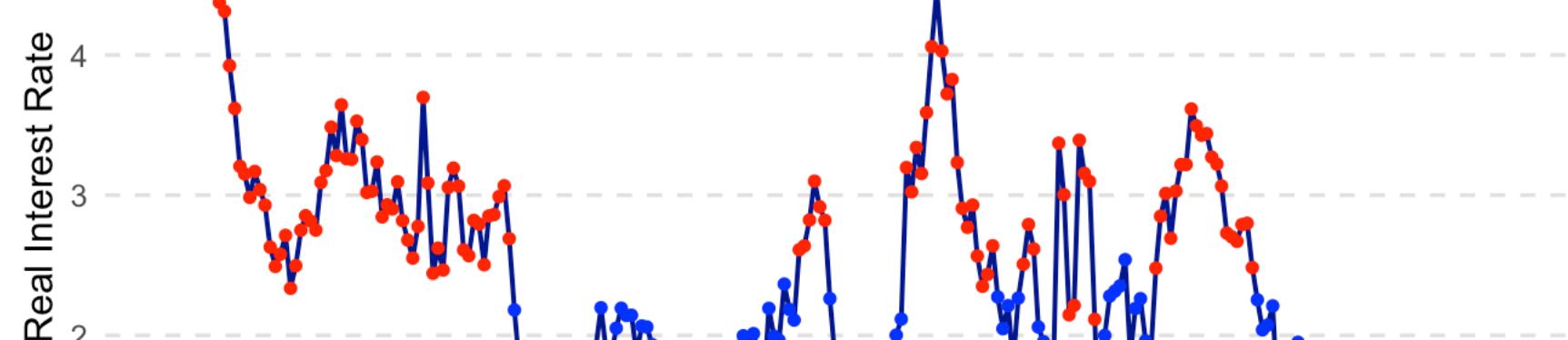
Panel (b): HMM derived state probabilities



```
ggsave("combined_plot.png", plot = combined_plot, width = 10, height = 8)
```

```
Plot 2: Real vs Implied (Homogeneous)
p3 <- ggplot(data, aes(x = Date)) +
 geom_line(aes(y = real_interest), color = "darkblue", linewidth = 0.7, alpha = 0.7) +
 geom_line(aes(y = hmm_implied), color = "red", linetype = "dashed", linewidth = 0.8) +
 labs(title = "Real vs Implied Interest Rate (Homogeneous HMM)",
x = "Date", y = "Interest Rate") +
 theme_custom
plot(p3)
```

Real vs Implied Interest Rate (Homogeneous HMM)



```
ggsave("hmm_implied_plot.png", plot = p3, width = 10, height = 5)
```

```
Provide MLE
summary(hmm_fit)
```

```
Initial state probabilities model
pr1 pr2 pr3
1 0 0

Transition matrix
toS1 toS2 toS3
fromS1 0.948 0.052 0.000
fromS2 0.085 0.811 0.104
fromS3 0.000 0.065 0.935

Response parameters
Resp 1 : gaussian
Rel.(Intercept) Rel.sd
St1 3.169 0.737
St2 2.019 0.207
St3 1.290 0.351
```

```

Non-homogeneous HMM

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```
set.seed(123)
nh_model <- depmix(response = real_interest ~ 1,
transition = ~ inflation + gdp,
data = data,
nstates = 3,
family = gaussian(),
ntimes = nrow(data))
nh_fit <- fit(nh_model, verbose = TRUE)
```

```
iteration 0 logLik: -379.8297
iteration 5 logLik: -335.4398
iteration 10 logLik: -265.6663
iteration 15 logLik: -260.9719
iteration 20 logLik: -216.6561
iteration 25 logLik: -213.5346
iteration 30 logLik: -213.5222
iteration 35 logLik: -213.5175
iteration 40 logLik: -213.5154
iteration 45 logLik: -213.5139
iteration 50 logLik: -213.5137
iteration 60 logLik: -213.5136
iteration 65 logLik: -213.5136
iteration 70 logLik: -213.5135
converged at iteration 73 with logLik: -213.5135
```

```
nh_post <- posterior(nh_fit)
```

```
data$nh_state <- factor(nh_post$state, levels = 1:3,
labels = c("Recession", "Stable", "Boom"))
```

```
nh_means <- sapply(nh_fit@response, function(r) r[[1]]@parameters$coefficients)
data$nh_implied <- sapply(nh_post$state, function(s) nh_means[s])
```

```
Plot 3: NH-HMM States
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```
p4 <- ggplot(data, aes(x = Date)) +
 geom_line(aes(y = real_interest), color = "darkblue", linewidth = 0.7) +
 geom_point(aes(y = real_interest, color = nh_state), size = 1.3) +
 scale_color_manual(values = c("Boom" = "green", "Stable" = "blue", "Recession" = "red")) +
 labs(title = "Real Interest Rate and NH-HMM States",
x = "Date", y = "Real Interest Rate", color = "State") +
 theme_custom
ggsave("nh_hmm_states_plot.png", plot = p4, width = 10, height = 5)
```

Real Interest Rate and NH-HMM States



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Plot 4: Real vs Implied (Non-Homogeneous)
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```
p5 <- ggplot(data, aes(x = Date)) +
 geom_line(aes(y = real_interest), color = "darkblue", linewidth = 0.7, alpha = 0.7) +
 geom_line(aes(y = nh_implied), color = "red", linetype = "dashed", linewidth = 0.8) +
 labs(title = "Real vs Implied Interest Rate (NH-HMM)",
x = "Date", y = "Interest Rate") +
 theme_custom
ggsave("nh_hmm_implied_plot.png", plot = p5, width = 10, height = 5)
```

Real vs Implied Interest Rate (NH-HMM)



```
plot(p5)
```