Monetary Theory and Policy: Takehome Assignment 3

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Instructions

Please read the following carefully.

- Your solution to this take-home assignment must be sent to email to the TA by the deadline, *i.e.* 23.59 (11.59 p.m.) of the last available day. Note: the e-mail of the TA is `piero.dedominicis@unibocconi.it`;
- Please name your file *Surname_Name_IDnumber_Takehome2* before sending it;
- You are strongly encouraged to use simple text editors, such as Word, or LaTeX. However, you are **required** to submit your solution in PDF format (max 5 MB);
- The deadline is strict: should you not meet it, the assignment will not be graded.

Warning You may find it helpful to collaborate with other students, particularly at empirical applications. However, you are required to submit your own copy: identical solutions will receive a zero grade.

1 The New Keynesian Supply Side

In this exercise you will work out the supply side of the New Keynesian model. The economy is populated by a large number N of firms producing a differentiated good. Each firm i produces a variety of the good, label it i as well, according to a linear production function:

$$Y(i) = AN(i) \tag{1}$$

where $Y(i)$ is the output of variety i, $N(i)$ is the labor it uses and A is a common productivity term. These producers are *monopolistically competitive*, hence they take demand for their variety as given and choose the optimal price level $P(i)$.
Varieties i can be interpreted as imperfectly substitutable inputs in the production of a representative firm producing a final good using the varieties as intermediate inputs. Alternatively, they can be imperfect substitutes in the preferences of the consumer. In this question, we explore this second interpretation.

Question 1

Before delving into the firm's problem, we are going to derive the optimal demand for every different variety of good i that the firm takes as given when optimizing. For simplicity, we omit time indexes. The representative consumer wants to maximize the level of consumption of the aggregated final good C, by combining consumption of every different variety of good i, $C(i)$:

$$C = \left(\sum_{i=1}^{N} C(i)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2}$$

This consumption aggregator is also known as Dixit-Stiglitz consumption aggregator, and $\varepsilon > 1$ is the elasticity of substitution across goods of different variety.
In this question you will derive the demand for each differentiated good i. To do so, solve the problem of the representative agent seeking to pick $C(i)$ for $i = 1, \ldots, N$ to maximize his net consumption $PC - \sum_{i=1}^{N} P(i)C(i)$ where C is given by (2), where P is the general price level and $P(i)$ is the price of variety i, and $\sum_{i=1}^{N} P(i)C(i)$ is the level of expenditure. Show that at the optimum

$$C(i) = \left(\frac{P(i)}{P} \right)^{-\varepsilon} C \tag{3}$$

which is the downward-sloped demand for each variety with price elasticity equal to $-\varepsilon$.

Hint Substitute (2) into the expression for net consumption and take the first order condition with respect to $C(i)$. You do not need to show that (2) is concave.

Question 2

Solve the problem of the producer of variety i, which wants to set the price $P(i)$ to maximize profits $P(i)Y(i) - WN(i)$ subject to (1) and (3), and find the optimal pricing condition:

$$P(i) = \frac{\varepsilon}{\varepsilon - 1} \frac{W}{A} \tag{4}$$

where you can define $\mu \equiv \varepsilon/(\varepsilon - 1)$. Explain why the optimal pricing condition also represents a labor demand condition.

Remark From the goods market clearing you have that $C(i) = Y(i)$ and so $Y = C$, hence you can use $Y(i) = \left(\frac{P(i)}{P} \right)^{-\varepsilon} Y$ as a constraint for the above problem. The intuition is that the monopolist knows the optimal demand and take is as given.

Question 3

First, compare the pricing condition (4) with the outcome you would have obtained in perfect competition. Second, think of what is the real compensation of labor in the economy with perfect competition. How does the real wage relate to the marginal product of labor? Third, consider the economy with monopolistic competition (and for simplicity flexible prices). How does your answer change?

Question 4

Now we reintroduce time indexes. In class, you showed that the optimal labor supply schedule that solves the problem of the household reads out as follows:

$$\frac{W_t}{P_t} = C_t N_t^{a-1} \tag{5}$$

with $a > 1$. Use aggregate technology and market clearing ($Y_t = A_t N_t = C_t$) and (5) to show that the real wage is given by

$$\frac{W_t}{P_t} = \frac{Y_t^a}{A_t^{a-1}} \tag{6}$$

What is the real marginal cost? What is the elasticity of the real marginal cost to output Y_t? Briefly give the economic intuition.

Question 5

Combine the optimal conditions for labor demand and labor supply to write the firm's optimal price as a function of the mark-up μ, aggregate output Y_t, labor productivity A_t, and the aggregate price level P_t. Then, define the natural level of output Y_t^n and derive its expression. Briefly comment on the determinants of the natural level of output. Finally, use the expression of Y_t^n to obtain the firm's optimal price as a function of Y_t, Y_t^n, and P_t.

Question 6

Assume that a fraction $s \in (0,1)$ of firms has sticky prices, whereas the remaining $(1-s)$ fraction sets its price flexibly. Write the last pricing condition you derived in the previous question in loglinear terms. Then, characterize the loglinear price set by flexible and sticky price firms. For sticky price firms assume for simplicity $y_t^{n,e} = y_t^e$ Finally, aggregate those prices to derive the loglinear expression for the aggregate price level.

Question 7

Derive the Phillips curve. Then, explain how a deviation of inflation from expected inflation causes a deviation of output from its natural level, and provide an intuition for this result.

2 The Phillips Curve

The focus of the original Phillips curve was the correlation between wage inflation and unemployment (wage Phillips curve). In the previous exercise, you derived the New Keynesian Phillips curve, which links deviations of output from its natural level to unexpected movements in the inflation rate (price Phillips curve). Economists have looked at both versions of the Phillips curve, as the two types of correlation may reflect different behavior in the structural factors driving the economy.
In the first part of this empirical exercise, you are going to explore the historical behavior of the "classical" Phillips curve, that is the price Phillips curve (although we will use unemployment rate as a measure of slackness, rather than the output gap). In the second part of the assignment, you will study alternative definition of the Phillips curve, using different measures of labor market slackness as predictor for both price inflation rate and wage inflation rate. For the first part, you have to download the following quarterly series from FRED:

1. *Unemployment Rate*, Seasonally Adjusted

   (Unit: **Percent**. Aggregation Method: Average)

2. *Consumer Price Index for All Urban Consumers: All Items in U.S. City Average*, Index 1982-1984=100, Seasonally Adjusted

   (Unit: **Percent Change from Year Ago**. Aggregation Method: Average)

These two series should be downloaded from $1951Q1$ up to the latest available observation.

Question 8

Construct a series for expected price inflation measured as

$$\tilde{\mathbb{E}}_{t-1}\left[\pi_t\right] = \frac{1}{K}\sum_{k=1}^{K}\pi_{t-k} \tag{7}$$

where π_t is the CPI inflation rate, setting $K = 4$. What is equation (21) capturing? Briefly comment. Then, provide a scatter plot in which you compare the excess CPI inflation $\pi_t - \tilde{\mathbb{E}}_{t-1}\left[\pi_t\right]$ and the unemployment rate (u_t), i.e. the former will be on the y-axis, whereas the latter will be on the x-axis. Data for this plot will span the period 1960 – onward.
Overlay a linear fit and the associated coefficient of determination (R2). Your plot will show an empirical version of the Phillips curve. Comment on the figure you obtain. In particular, relate your answer to the expression you derived in question 7.

Question 9

Reproduce the same graph of the previous question, by decade. Specifically, for each decade plot the excess CPI inflation rate $\pi_t - \tilde{\mathbb{E}}_{t-1}\left[\pi_t\right]$ and the unemployment rate (u_t). Let the 2000s decade end in 2007Q3, and include the remaining observations in one 2007Q4-2020 "decade". In each panel, include a linear fit and the associated correlation coefficient. Comment on the evolution of the New Keynesian Phillips curve over time. In particular, try to provide reasons for the sudden flattening experienced in the 2010s.

Question 10

Now we are going to assess how different labor market slackness measures predict price and wage inflation over the last two decades. We will consider two different measures for inflation: the *CPI inflation rate* π_t (the same that you have considered in the previous questions) and an inflation measure for wages and salaries, that is the Employment Cost Index for wage and salaries of private industry workers. We will refer to this alternative measure as the *wage growth rate* (or *wage inflation rate*) and denote it with the symbol π_t^w. As measures for labor market slackness, we will use the same that you have already used for the first takehome assignment, namely the unemployment rate, the unemployed-job openings ratio and the quits rate. Download the following time series for the labor market in the US, at quarterly frequency, from the FRED website from 2002Q1 until 2023Q2 (the Employment Cost Index on the FRED website is available since only 2002Q1):

1. *Unemployment Level, Thousands of Persons*, Seasonally Adjusted

   (Unit: **Thousands of Persons**. Aggregation Method: Average)

2. *Job Openings: Total Nonfarm*, Seasonally Adjusted

   (Unit: **Level in Thousands**. Aggregation Method: Average)

3. *Unemployment Rate*, Seasonally Adjusted

   (Unit: **Percent**. Aggregation Method: Average)

4. *Quits: Total Nonfarm*, Seasonally Adjusted

   (Unit: **Rate**. Aggregation Method: Average)

5. *Employment Cost Index: Wages and Salaries: Private Industry Workers*, Index Dec 2005=100, Seasonally Adjusted

   (Unit: **Percent Change from Year Ago**. Aggregation Method: Average)

6. *Consumer Price Index for All Urban Consumers: All Items in U.S. City Average*, Index 1982-1984=100, Seasonally Adjusted

   (Unit: **Percent Change from Year Ago**. Aggregation Method: Average)

Standardize both inflation series (the CPI inflation rate and the wage growth rate) using only data points before 2018 (excluded) and plot them in the same graph. Do the two series always move in the same direction? Why could it be important to consider both changes in *CPI inflation rate* and *wage growth rate*?

Hint Think about different structural factors that can drive the two types of inflation.

Question 11

Construct two series for expected inflation measured using the two different measures for inflation:

$$\tilde{\mathbb{E}}_{t-1}\left[\pi_t\right] = \frac{1}{K}\sum_{k=1}^{K}\pi_{t-k} \tag{8}$$

$$\tilde{\mathbb{E}}_{t-1}\left[\pi_t^w\right] = \frac{1}{K}\sum_{k=1}^{K}\pi_{t-k}^w \tag{9}$$

where π_t is the *CPI inflation rate* and π_t^w the *wage growth rate* (Employment Cost Index), setting $K = 4$ (equation (22) is the same as equation (21)).
As before, plot the comparison between different inflation measures and different measures for labor market slackness. In total, you have to produce six plots: three in which you compare the *excess CPI inflation* $\pi_t - \tilde{\mathbb{E}}_{t-1}\left[\pi_t\right]$ with each measure of labor market slackness and three in which you compare the *excess wage growth* $\pi_t^w - \tilde{\mathbb{E}}_{t-1}\left[\pi_t^w\right]$ with each measure of labor market slackness. The three measures of labor market slackness are *unemployment rate*, *unemployed per job opening* and *quits rate* (remember to use $1 -$ *quits rate* as measure of slackness). Remember to plot the inflation rates on the y-axis and the slackness measures on the x-axis.
Moreover, for each one of the six plots, separate by colors the following periods:

- in red, the period 2002-2009;
- in blue, the period 2010-2018;
- in green, the period 2019-2023.

For each of these periods, overlay a linear fit and the associated coefficient of determination (R2) on each plot.
Finally, answer the following questions: do different measures of labor market slackness predict different excess CPI inflation before the pandemic? Does your answer change if we consider excess wage growth? Now focus on the last period (2019-2023), do different measures of labor market slackness predict different excess CPI inflation? Does your answer change if we consider excess wage growth? Always in the period 2019-2023, what predictor explains the largest share of variance for excess CPI inflation and excess wage growth? Why do you think this could be the case?

Hint When answering the questions above comment on the slope and the R-squared of the relations you estimate

References

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