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Model fitting and latent component extraction

We developed 4 models within the dynamic linear model framework: the Random Walk Plus Noise (RWPN) model, the Locally Linear Trend model(LLT), the Classical Seasonal and the Fourier Seasonal models. The last two were estimated directly on the raw series, while RWPN and LLT on a deseasonalized time series. Parameters were found through maximum likelihood while state estimation was performed via Kalman filtering.

As first step, we pre-process the original monthly temperature anomalies to remove seasonality before fitting trend-only models. We apply STL decomposition to decompose the time series into trend, seasonal, and remainder components. We extracted the estimated seasonal component and subtract it from the original series to obtain a deseasonalized time series that retains only trend and irregular fluctuations. By removing the seasonal cycle in advance, we ensure that RWPN and LLT focus just on modeling the underlying trend and stochastic variation, without being confounded by periodic seasonal effects.

After having modified the series, the unknown variance parameters of RWPN and LLT models were estimated through the Maximum Likelihood Estimation (MLE). For the RWPN, we estimated two parameters: observation variance and state variance while for the LLT, we calculated three parameters: observation variance, level variance, and slope variance. In MLE, the parameter estimates are obtain through an iterative optimization procedure and the initial values are set to 100, Moreover, positivity constraints are imposed with lower bounds to preserve randomness property. The Hessian matrix is computed to obtain asymptotic standard errors by inverting the observed Hessian. The final outputs are the MLE parameter estimates and their corresponding standard errors for each model.

The Kalman smoother computes retrospective estimates of the unobserved state sequence, conditioning each state Thetat on the entire dataset rather than just on past data.The algorithm runs backward from the final filtered state, recursively updating the state estimates.

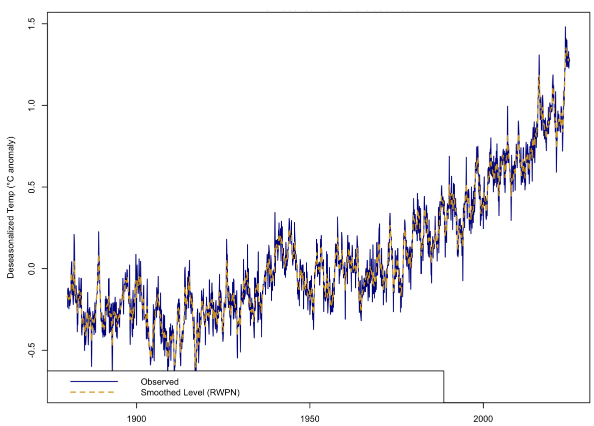

Figure 1: RWPN: Smoothed Level vs. Observed

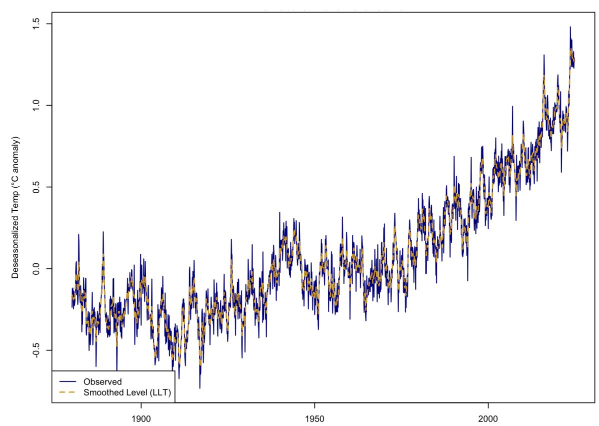

Figure 2: LLT: Smoothed Level vs. Observed Time

It produces smoother, less variable estimates than filtering, as it conditions on more information. This procedure is essential for reconstructing hidden trends (e.g., level, slope) once all data are observed. This visualization compares the observed deseasonalized temperature anomalies to the smoothed level estimates produced by RWPN and LLT. In both plots, the smoothed levels closely follow the observed data, indicating a good fit with accurate reconstruction of the underlying trend. These smoothed estimates provide a precise latent trend estimate, useful for interpretation and for forecasting beyond the observed period. The inclusion of future prediction (forecasting step) relies on the last smoothed state and the model dynamics to project the trend forward, offering continuity between in-sample smoothing and out-of-sample prediction.

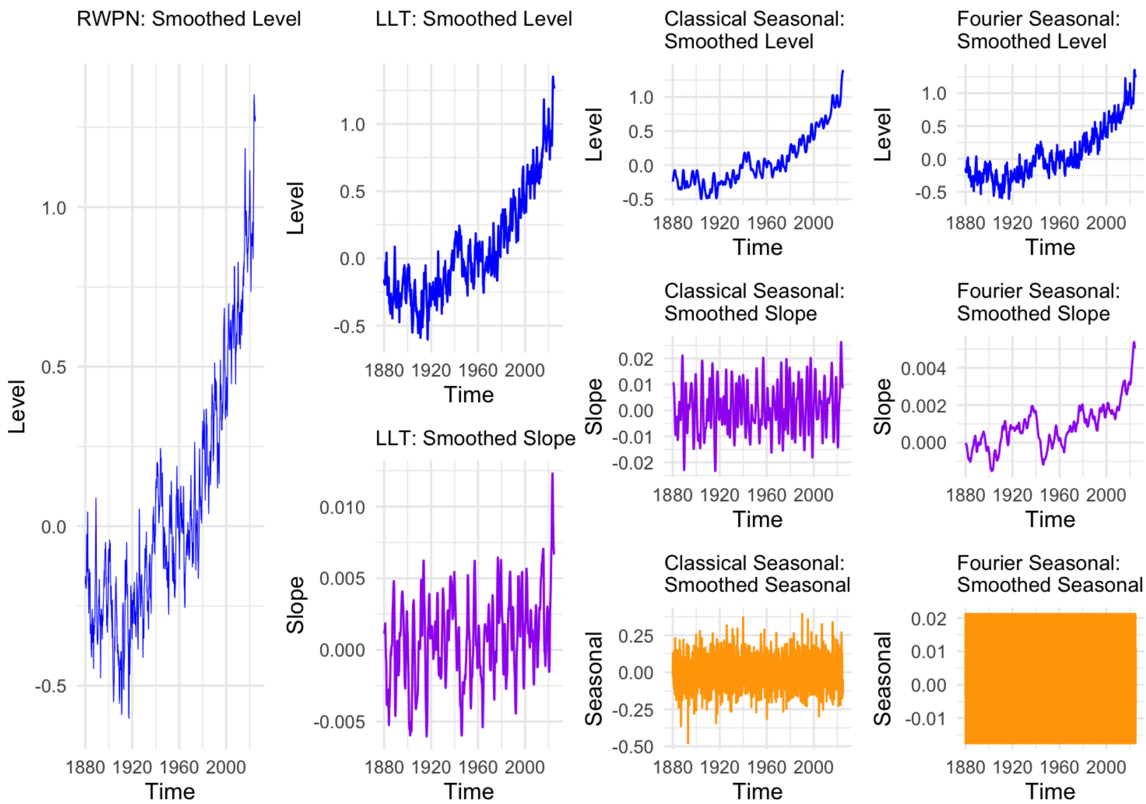

Figure 3: Summarizing Plots

Model Comparison Interpretation

RWPN model achieves the best AIC (-6373) and BIC (-6362), meaning it provides the strongest likelihood-based fit while penalizing for simplicity. LLT model follows with slightly worse AIC/BIC, reflecting additional complexity (slope term) with only marginal improvement in fit. The Fourier seasonal model under-performs RWPN and LLT on AIC/BIC, suggesting that its harmonic seasonal structure does not capture meaningful seasonal variation in the data. The Classical Seasonal model achieves the lowest RMSE (0.00997), meaning it reconstructs the historical data most precisely in-sample, but its extremely low log-likelihood (-541) and positive AIC (+1091) indicate poor probabilistic fit. These differences likely arise because the temperature anomaly series contains a weak seasonal signal and is predominantly driven by trend (level and slope dynamics). The Classical Seasonal model forces strong seasonal factors into the model regardless of their true contribution, leading to excellent mechanical reconstruction but poor generalization and likelihood fit. Conversely, RWPN and LLT focus on stochastic trend evolution, which better aligns with the data's structure. The low explanatory variance of seasonality (6 per cent) supports why models emphasizing trend outperform those emphasizing seasonal components in likelihood metrics. In short, the better statistical fit of RWPN arises because the data is trend-dominated with minimal systematic seasonality.

These results show that the seasonal component explains approximately 6.02 per cent of the total variance in the observed series, while the combined level and slope components explain about 93.77 per cent of the variance. This indicates that the underlying variability in the time series is predominantly driven by long-term trend and slope dynamics, with a relatively small contribution from seasonal fluctuations. The seasonal pattern is present but not a dominant source of variation in the data.

DLM vs HMM, which is the more suitable?

Both Dynamic Linear Models and Hidden Markov Models are examples of latent-state TS models formulated under the state-space framework. They share a common goal: to estimate unobserved states (denoted as θ_t or S_t) that drive the observed process. To achieve this, both models employ recursive estimation algorithms, specifically the Kalman filter for DLMs and the forward-backward algorithm for HMMs.

Both DLMs and HMMs can be viewed as special cases of a general state-space model, where the observed data Y_t follows a conditional distribution $f(Y_t \mid \text{state}_t)$, and the latent state evolves according to a stochastic process defined by $g(\text{state}_{t-1})$.

Focusing on DLMs, these models assume that the latent states are continuous and evolve under Gaussian linear dynamics. This structure allows DLMs to directly model components such as stochastic trend, slope, and seasonality as continuous quantities, thereby enabling a smooth reconstruction of underlying gradual processes. The smoothing property of DLMs helps filter out short-term fluctuations, producing robust and interpretable trend estimates. Each of the latent components, level, slope, and seasonal effects, are not only interpretable but also evolve continuously over time, which supports nuanced temporal analysis.

However, DLMs come with inherent assumptions and limitations. Standard DLMs are built on the premise of linearity and Gaussianity, meaning both the state transition and observation equations are modeled as linear Gaussian systems. While this structure ensures tractable and closed-form inference, it also restricts the model's flexibility in capturing nonlinear dynamics, non-Gaussian shocks, or heavy-tailed noise. To overcome these limitations, the DLM framework can be extended in several directions. One approach involves nonlinear state-space models, such as those estimated using the Extended or Unscented Kalman Filters. Another path is through Generalized Linear State-Space Models, which accommodate non-Gaussian observation distributions. Alternatively, Bayesian hierarchical state-space models can be employed, estimated via simulation-based methods like particle filters or Markov Chain Monte Carlo.

While these extensions enhance the model's flexibility, they also significantly increase computational complexity and make estimation more challenging. Moreover, a key drawback of DLMs is their inability to detect abrupt regime shifts or discrete state transitions, as their smoothing nature tends to gloss over such changes. In scenarios where structural breaks or latent regimes are present, DLMs may under-represent these discrete changes. In such cases, HMMs, which incorporate discrete latent states and a state transition matrix, are better suited for capturing sudden regime switches.

Despite this, for the task of modeling global temperature anomalies, DLMs are generally more appropriate than HMMs. This is because the time series in question exhibits a continuous and gradually evolving trend, rather than abrupt structural breaks. Furthermore, the gradual nature of climate variability aligns closely with DLMs' assumption of continuous latent states, making them a natural choice for this type of application.

Although HMMs are powerful tools for identifying hidden regime switches, DLMs offer a better theoretical and empirical fit for trend-dominated, smoothly evolving time series such as global temperature anomalies.