Structural Models

Alessandro Caggia

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Abstract

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1. Causality in Structural Models

1.1. General Setup

- Say that after solving a model comprising a system of equaitons (eg supply and Demand funcitons) you end up with an equation for Y, say $Y = F(x_1, x_2, \dots, x_n)$
- Now we are interested in the **Causal effect**, the response of Y to a *ceteris paribus* change in one variable. If we can vary the "cause" x_j independently, then:

$$\frac{\partial Y}{\partial x_j} = F_j'(x_1, x_2, \dots, x_n)$$

- This is the marginal causal effect of x_j on Y.
- If we assume linear equation: $Y = \beta X + \varepsilon$, we can use OLS to recover the parameter of interest.
- Under proper identification, $\beta_j = \frac{\partial \mathbb{E}[Y]}{\partial x_j}$ can be interpreted causally and β_j is called a structural parameter as it is derived from a structural model.

1.2. Interrelated Causes and Endogeneity

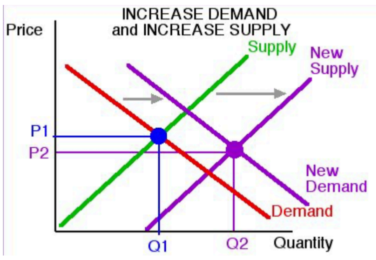
- When causes are interrelated, causal effect still exists but is harder to identify.
- Example: Demand and Supply

$$Y^d = Y^d(P^d, Z^d, U^d)$$
$$Y^s = Y^s(P^s, Z^s, U^s)$$

- If P^d and P^s could be varied exogenously:

$$\frac{\partial Y^d}{\partial P^d}, \quad \frac{\partial Y^s}{\partial P^s}$$

would be causal effects.
- But in equilibrium: $P^d = P^s = P$, $Y^d = Y^s = Y$. So you do not specifically observe the two equaitons above!
- You are now observing an equilirbium point / a series of equilibriu points (say (Y, P)) that is JOINTLY determined by S and D,
- The relaiton between y adn p that you are observing is totally in-informative as you have 2 unobservables (D and S) determining jus tone observable: price may be determined by a symultaneous shift in D amd supply.



- So causal interpretation of $\frac{\partial Y}{\partial P}$ is invalid due to endogeneity (simultaneity bias).
- You want to isolate shifts in D or S

1.3. Using Exogenous Variation: Reduced Form

- If we can vary Z^d and/or Z^s exogenously, we can recover causal effects (IV style).
- Reduced form (endogenous as a function of exogenous variables):

$$Y = Y(Z^d, U^d, Z^s, U^s)$$
$$P = P(Z^d, U^d, Z^s, U^s)$$

- If Z^s affects P and can be exogenously varied (no effect on Z^d, U^d, U^s), we can identify causal effect of Z^s on P and Y.
- idea is shiftng the suppyll curve while keeping the demand curve fixed
- Reduced Form Causal Effects of Z^s on P and Y (ceteris paribus):

$$\frac{\partial Y}{\partial Z^s}, \quad \frac{\partial P}{\partial Z^s}$$

2. Structural Model: Concrete Example

2.1. Model Setup

- Structural equations:

$$Y^d = \beta_0 + \beta_1 P^d + U$$
$$Y^s = P^s - Z$$

- In equilibrium: $P^d = P^s = P$, $Y^d = Y^s = Y$
- Structural form:

$$Y = \beta_0 + \beta_1 P + U$$
$$P = Y + Z$$

we cannot identify the structural coefficient β_1 representing the ceteris paribus marginal effect of P^d on Y^d

2.2. Solving the System

- Assumption: U and Z are independent!
- Solve the system, reduced form of the model:

$$Y = \frac{\beta_0}{1-\beta_1} + \frac{\beta_1}{1-\beta_1} Z + \frac{U}{1-\beta_1}$$
$$P = \frac{\beta_0}{1-\beta_1} + \frac{1}{1-\beta_1} Z + \frac{U}{1-\beta_1}$$

- Let $\gamma_0 = \frac{\beta_0}{1-\beta_1}, \gamma_1 = \frac{\beta_1}{1-\beta_1}, \gamma_2 = \frac{1}{1-\beta_1}$
- Reduced form causal effects:

$$\frac{\partial Y}{\partial Z^s} = \gamma_1, \quad \frac{\partial P}{\partial Z^s} = \gamma_2$$

- Recover structural parameter of interest from the reduced parameters:

$$\frac{\partial Y^d}{\partial P^d} = \beta_1 = \frac{\partial Y / \partial Z^s}{\partial P / \partial Z^s w}$$

The Exogenous variation in Supply allowed you to find the slope of Demand

Interpretation

- $\gamma_1 = \frac{\beta_1}{1-\beta_1}$ is causal effect of Z^s on Y.

- Reduced form coefficient is the composition of two effects:

$$\frac{\partial Y}{\partial Z} = \frac{\partial Y^s}{\partial P^s} \cdot \frac{\partial P}{\partial Z} = \beta_1 \cdot \frac{1}{1-\beta_1}.$$

Z is the exogenous variable generating the change in Y, while P is the intermediate cause.

3. Structural Models: Other Examples

3.1. Identifying the Supply Slope

- New model:

$$Y^d = \beta_0 + \beta_1 P^d + Z + U$$
$$Y^s = \beta_2 P^s$$

- Goal: identify β_2
- You can identify the slope of supply BECAUSE you have exogenous shocks in Demand!

4. Structural Approach and Potential Outcomes

- Assumptions needed to identify β in a structural equation are equivalent to potential outcome assumptions.
- Structural equation $Y_i = \beta D_i + \varepsilon_i$
  - Assumes homogeneous β (guarantees no treatemtnegfect heteroegenity)
  - Selection bias arises if D_i is correlated with ε_i (some unobserved characteristics are correlated with the treatement assignment)