Using AMPL/CPLEX to Model and Solve the Electric Vehicle Routing Problem (EVRP) with Heterogeneous Mixed Fleet

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Abstract: Vehicle Routing Problem(VRP) is one of the most important and classical issues in the logistics distribution field. However, the excessive consumption of oil resources makes a dramatic increase in emissions of carbon dioxide in the atmosphere which causes a deterioration of the environment around us for the past few years. The electric vehicle(EV) is a better alternative which operates with batteries instead of using gasoline.In this paper, we present the Electric Vehicle Problem(EVRP) and describe it with a mathematical programming model. Then, we verity the model via the mathematic program software called AMPL/CPLEX and the mathematic instances are extracted from the Solomon's instances. We propose a programming algorithm as an exact solution approach for the EVRP. Problem examples and numerical calculation have been provided to evaluate the solution approach and the optimality.

Key Words: Vehicle Routing Problem, Electric Vehicle, AMPL/CPLEX

1 INTRODUCTION

In 1959, Dantzig and Ramser put forward the VRP firstly which has been the front of Operational Research and Combinatorial Optimization over the past twenty years [1]. The main task of the VRP problem is scheduling vehicles and planning transportation routes in the logistics distribution activities at the most optimized cost such as the total driving distance, the total driving time, the total fuel cost, the quantity of the vehicle or the combination of above situations. Decisions in routing problem affect directly the distribution activities' performance and efficiency, which are significant for a logistics firms to make more profit.

In general, vehicle consumes gasoline as fuel whose cost is extremely high and releases poisonous gases to atmosphere which is tremendously harmful to human health. Recently, the idea of energy saving and emission reduction has become a global hot topic, therefore, we replace fuel vehicle of VRP with electric vehicle, which draws forth a new problem called EVRP. Thus, there must be some charging stations in the EVRP. Although, in terms of environment and operation cost, electric vehicle is a better option compared with the traditional fuel vehicle and the driving distance of the electric vehicle with fully charged battery is usually short and there are a few charge stations in the way, which called "Range anxiety". There must exist plenty of difficulties before electric vehicle replaces fuel vehicle. So in this case, the task allocation between vehicles, route selection, scheduling, speed selection, the access to charging infrastructure and calculation of the charging time and etc. constitutes a very complex optimization problem. The promotion and application of the modern energy vehicle leads to challenge to the traditional logistics mode, which also is an opportunity to logistic field in improving the logistic cost and delivery quality.

In this paper, we address the factor of serving a set of customers and considering that the vehicle need to stop at the charging stations and charge battery ensure the logistic activity during the trip. There are plenty of literatures researchers are studied about that is denoted as Electric-VRP(EVRP) due to the use of electric vehicle. The formal representation and the algorithm of classical VRPs model can review literatures such as Laporte et al. (2000) [2], Toth and Vigo (2002) [3], Golden et al. (2008) [4], and Eksioglu et al. (2009) [5]. Recently, the green concept of VRP problems (i.e., Green VRP) attracts the academia's research' interests and creates a better surrounding for the development of the logistic distribution (Lin et al., 2014; Demir et al., 2014; Xiao & Konak, 2015a; Xiao & Konak, 2015b) [6][7][8][9]. These problems emphasis more on the protection of the atmospheric environment, require logistics vehicles to make a balance between reducing fuel consumption as well as CO2 emissions and vehicle operating costs, which no longer regard the traditional economic cost as the only decision goal and deal with VRP problems at a systematic and green vision. Erdogan and Miller-Hooks (2012) studied using "alternative energy" vehicle of VRP problems, considering the "gas stations" of the vehicles are in smaller quantities that are in a large area, which is aimed at planning the routes [10]. This is the earliest research of new energy vehicle after the "refueling" problem of literature which considers the driving distance restriction of the "alternative energy vehicles". Mirchandani et al. established the EVRP model in which electric vehicles exchange the batteries during the trip, setting that vehicles have to visit the battery swap station

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for a certain distance [11]. This mathematic model is similar to the VRP model of consuming fuel with the same spanking recharge pattern. Yet, it is required that the battery specification of all electric vehicles must be same for workers to carry the battery tray easily, which increases the logistic cost in total for a long time. Yang and Sun present an electric vehicles battery swap stations location routing problem (BSS–EV– LRP), which aims to determine the location strategy of battery swap stations(BSSs) and the routing plan of a fleet of electric vehicles(EVs) simultaneously under battery driving range limitation [12]. However, a limited number of researches study on the EVRP and their mathematical model is simplified that consider rarely about charge station during the trip. Moreover, China has the largest extensity and inputs most efforts in promoting and utilizing of the electric vehicle which occupies a leading place all over the world in practice. Oppositely, the research on the relational theory and method is still at the initial stage. It is significant that we need to launch the research about electric vehicle in logistic field, only in this way can we keep pace with the practical development of the EVRP.

2 PROBLEM FORMULATION

In this section, we firstly introduce the classical VRP problem mathematic model. The VRP problem studied in the literatures is described as follows.

A number of m vehicles which is noted as V and indexed as h, set out from the depot in the two-dimensional coordinate plane, are going to visit and serve a set of n customers/nodes, noted as N and indexed as i. It is required that each customer is accessible at least and only once. The distance between site i and site j is known as d_{ij}. At the same time, the quantity of the customers of each vehicle that is assigned is demanded to be same as much as possible (or the difference is less than 1). Generally speaking, the total cost is the plus of the distance that all vehicles has traveled in the whole driving process.

The objective function is to minimize the sum of driving distance over the entire trip, denoted by total distance(Total_dis).

$$\min.\ Total\_dis = \sum_{h\in V}\sum_{i,j\in N, i\neq j} x_{ijh}\cdot D_{ij}$$

The notations used in the model are listed as bellows:

Parameters:

N set of nodes, n=card(N)

I index of customer that $i\in N$

V set of vehicle

D_{ij} the distance between node i and node j

M A large number

Variable:

x_{ijh} Binary variable indicating if node i and j are connected

u_i Non-negative integer variable indicating the visiting sequence

However, this model assumes that the demand of the node is zero and the capacity of the vehicle is unlimited. While in fact, VRP problem widely exists in a variety of fields such as milk delivery, garbage collection and customer pick up-send service, which are always related to the specific capacity. In other words, it seems that the logistic distribution activities always refer to the load which plays a significant role in the aspect of the efficiency of the enterprise.

Then we put forward the following problems in this paper: in the process of the logistic distribution activities of some enterprises, we consider the demand of the customer node and the capacity of the vehicle in this paper. Meanwhile, it is considered that the types of the logistics vehicles in the logistics system are varied with the complexity of the logistics activities. Therefore, this paper defines a new set of vehicle type and also demand that each vehicle is limited by its capacity.

Therefore, there are several new factors we premeditate:

Definition 1 p_{hi}: Non-negative variable indicating the remaining power of the vehicle h which is at the node i.

Definition 2 R_i: Binary parameter means if node i is a charge station, 1 stands that i is a charge station, or conversely.

Definition 3 a_i: Parameter denotes the demand of the customer node i.

Definition 4 C_h: Parameter denotes the capacity of the vehicle h.

Definition 5 L_h: Parameter shows the longest distance of completely charged vehicle h.

Based on these definitions and the classical VRP mathematic model, we can build an Electric Vehicle Routing Problem(EVRP) model.

Objective:

$$\min.\ Total\_dis = \sum_{h\in V}\sum_{i,j\in N, i\neq j} x_{ijh}\cdot D_{ij}$$

Constraints:

$$\begin{aligned}
&\sum_{h\in V}\sum_{j\in N, i\neq j} x_{jih} = 1 && \forall i\in N, i>0; R_i = 0\\
&\sum_{h\in V}\sum_{j\in N, j\neq i} x_{ijh} = 1 && \forall i\in N, i>0; R_i = 0\\
&\sum_{i\in N; i>0} x_{0ih} \leq 1 && \forall h\in V\\
&\sum_{j\in N, j\neq i} x_{jih} = \sum_{j\in N, j\neq i} x_{ijh} && \forall i\in N, h\in V\\
&c_h \geq \sum_{i,j\in N, j\neq i, j>0} x_{ijh}\cdot a_j && \forall h\in V; R_j = 0 \qquad (1)\\
&p_{hi} \geq L_h && \forall i\in N; h\in V; R_i = 1\\
&p_{hi} \leq L_h && \forall i\in N; h\in V\\
&u_j - u_i \geq 1 - M\cdot(1-x_{ijh})\\
&\qquad \forall i,j\in N, i\neq j, i>0, j>0; h\in V\\
&\sum_{i,j\in N; j\neq i; j>0} x_{ijh_1} \leq= 1 + \sum_{i,j\in N; j\neq i; j>0} x_{ijh_2}\\
&\qquad \forall h_1, h_2\in V; h_1\neq h_2\\
&p_{hi} - p_{hj}\cdot\left(1-R_j\right) \geq D_{ij} - M\cdot(1-x_{ijh})
\end{aligned}$$

$$\forall i, j \in N, i \neq j; h \in V$$

In Eq. (1), the first and second constraints show the visit principle of every node, which mean that if i isn't a charge station, vehicles enter node i and leave node i once. The third constraint indicates that all vehicles which take part in the logistic activity set out from the depot. This constraint guarantees the routing of each vehicle constitutes a circle. And the fourth constraint expresses the move in and move out balance of each node. The fifth constraint denotes the capacity limitation constraint. We can obverse the plus of the node demand of the same vehicle is no more than than the vehicle capacity. The sixth constraint indicates that the vehicle is fully charged when leave a charge station. The seventh constraint indicates the remaining power are limited to the battery capacity. The eighth constraint signifies the requirement of the visit sequence. If the x_{ijh} is 1, that is vehicle h leave node i and enter the node j, the visit sequence of j is must more than the visit sequence of i. The ninth constraint denotes that any two vehicles' visit sequence of the difference is no more than 1. The tenth constraint expresses that the remaining power of the vehicle is enough to drive to the next charge station.

It is clear that the EVRP model has taken capacity limitation into consideration and replaced the fuel vehicle with electric vehicle, which reflects the actual logistic distribution activities better.

3 COMPUTATIONAL TEST

For the purpose of verifying the feasibility of the EVRP model, we adopt the original internationally recognized problem Solomon's instances and add the coordinates of the charging stations, in which there are 21 nodes including 1 depot, 8 charging stations and 12 customer nodes with its demand. Besides, we set 3 vehicles start from the depot with its own capacity. Table 1 lists the data of the nodes of the EVRP problem. Table 2 lists the data of the vehicles of the EVRP problem [13].

Table1. Data of nodes

Node	X_i	Y_i	R_i	a_i
0	0.5	0.5	1	0
1	0.590166	0.609209	0	68
2	0.203093	0.189873	1	0
3	0.253639	0.921892	0	27
4	0.532339	0.957156	0	47
5	0.166748	0.105726	0	63
6	0.027142	0.714106	1	0
7	0.332709	0.551532	0	49
8	0.805947	0.263135	1	0
9	0.640516	0.349604	1	0
10	0.754375	0.407247	0	68
11	0.486731	0.665212	0	23
12	0.039934	0.575807	1	0
13	0.65443	0.942022	0	62
14	0.478125	0.363525	1	0
15	0.312899	0.003089	0	57
16	0.60368	0.755598	1	0
17	0.390284	0.450103	0	17
18	0.957052	0.170122	0	76
19	0.592204	0.787748	0	27
20	0.760299	0.837808	1	0

Table2. Data of vehicles

Vehicle	C_h	L_h
1	200	0.6
2	200	0.7
3	200	0.8

The EVRP computational tests were carried out on a Mac book with a 2.9G Intel Core i7 CPU. The bi-objective model for EVRP was coded by a mathematic program language called AMPL and solved by MIP solver CPLEX. Based on the data, we applied MIP-based algorithm to verify the EVRP problem. The optimal solution ran by AMPL/CPLEX is arranged in Table 3. It is observed that the final load of each vehicle is less than its capacity limitation 200 and the accumulated load of the vehicle is the summation of the customer nodes demand which is planned in the same routing circle. And each vehicle is fully charged when it leaves the charge station in which the driving distance of the remaining power is equal to the longest distance of fully charged vehicle. What's more, all vehicles which take part in the logistics distribution activity set out from the depot. On the foundation of the data and the operation result, we accomplish the nodes distribution figure and the final vehicle routing plan with the drawing software which is called Tecplot. The final results are shown in figure 1 and figure 2.

Table3. Output of example

Vehicle	C_h	X_i	Y_i	a_i	$\sum a_i$	P_{hi}
1	200	0.500000	0.500000	0	0	0.60
		0.332709	0.551532	49	49	0.42
		0.640516	0.349604	0	49	0.60
		0.957052	0.170122	76	125	0.23
		0.805947	0.263135	0	125	0.60
		0.754375	0.407247	68	193	0.27
		0.500000	0.500000	0	193	0.60
2	200	0.500000	0.500000	0	0	0.70
		0.390284	0.450103	17	17	0.12
		0.478125	0.363525	0	17	0.70
		0.312899	0.003089	57	74	0.30
		0.166748	0.105726	63	137	0.12
		0.203093	0.189873	0	137	0.70
		0.027142	0.714106	0	137	0.70
		0.253639	0.921892	27	164	0.38
		0.603680	0.755598	0	164	0.70

		0.592204	0.787748	27	191	0.30
		0.500000	0.500000	0	191	0.70
3	200	0.500000	0.500000	0	0	0.80
		0.486731	0.665212	23	23	0.63
		0.532339	0.957156	47	70	0.31
		0.654430	0.942022	62	132	0.19
		0.603680	0.755598	0	132	0.80
		0.590166	0.609209	68	200	0.65
		0.500000	0.500000	0	200	0.80

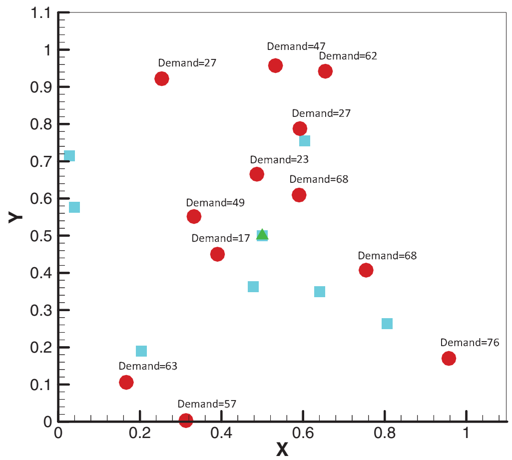

Fig.1 Nodes Distribution

In the figure above, the red circular stands for the customer node, the blue square represents the charge station and the green triangle stands for the depot. Besides, there is the definition that the depot in the EVRP example is also a charge station, which leading to the result that all vehicles set out from the depot with the fully charged batteries.

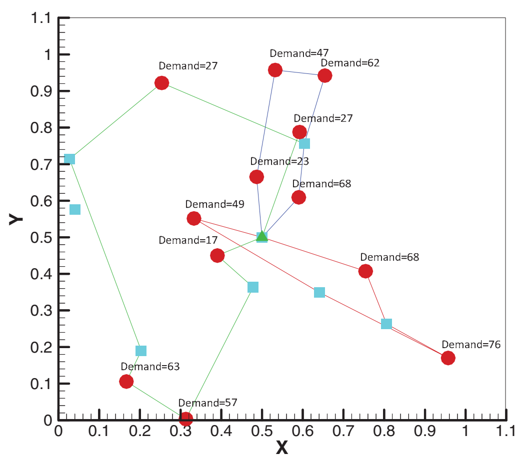

Fig.2 Vehicle Routing Plan
Total_Distance=5.0692

In the figure 2, the three color lines mean that there are three vehicles take part in the logistic distribution activity. It can be observed that all the vehicles start from the depot and return back to the depot again whose coordinate is (0.500000, 0.500000) painting with the green triangle. All vehicles visit several customer nodes and the charge stations. And the lines in the same color constitute three closed loop. In the figure 2, we can read that the green routing and the blue routing both pass through the charge station at the (0.603680,0.755598), which verities the charge station is free for all vehicles and the correctness of the mathematical model in turn. Beyond that, the AMPL operation result is that the total_Distance is 5.0692.

4 CONCLUSIONS AND FUTURE RESEARCHES

In this paper, we firstly analysis the background of the VRP problem and introduce the necessity of researches on the EVRP problem. We discuss the research status of the EVRP in the domestic and foreign literatures. Then, we describe the EVRP problem in mathematic bi-objective MIP model. Finally, computational test on AMPL/CPLEX is operated and the result give the verification of the feasibility of the EVRP model. The future research plans are on developing heuristic algorithm for large-sized problems and consider the influence of time.

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