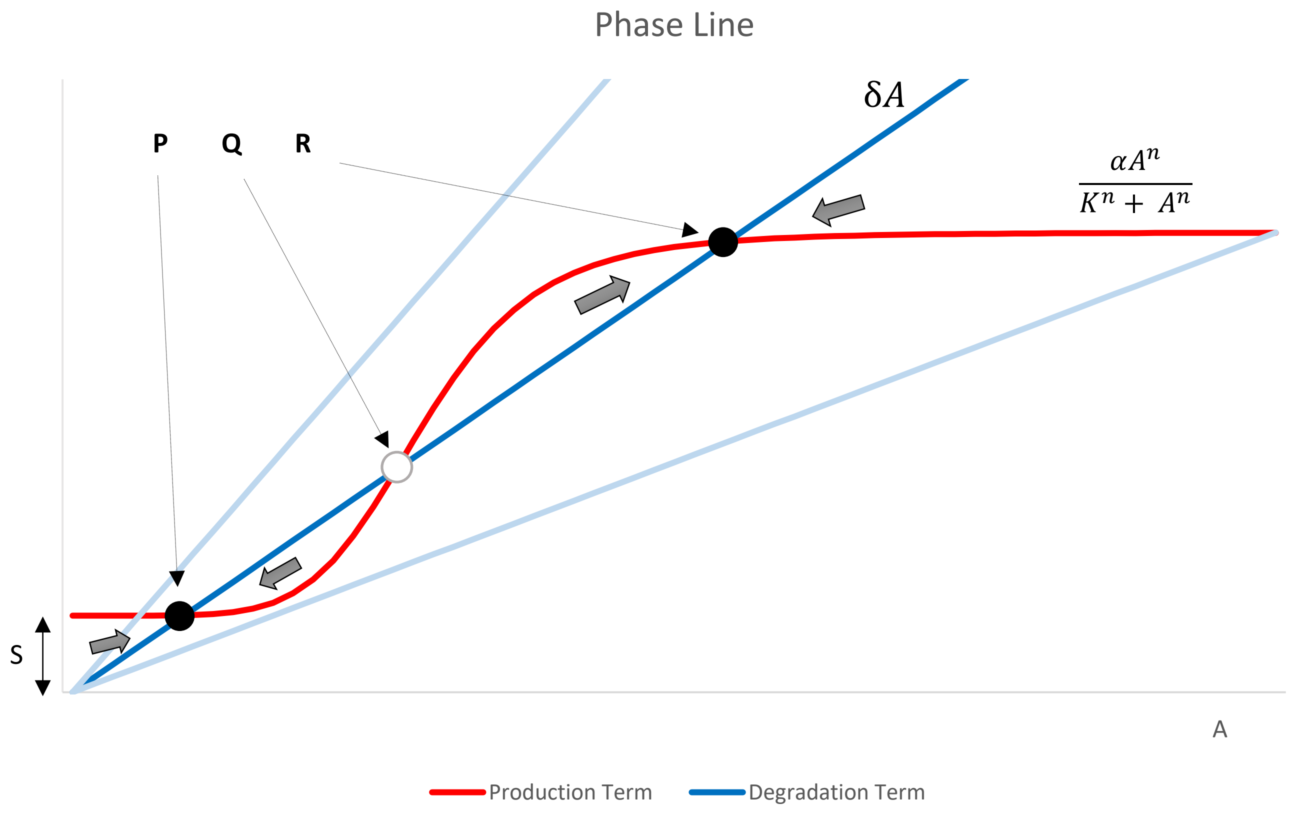
Assignment 2 – Bifurcation Diagrams

Plot the bifurcation diagram of the positive feedback circuit with cooperativity and an additional signal S as a function of the decay rate delta.

In the phase line graph for a **positive feedback circuit** the production term and the degradation term of the differential equation $\frac{dA}{dt} = S + \frac{\alpha A^n}{K^n + A^n} - \delta A$ are plotted separately, and their intersection points P and R represent the stable equilibria of the system while intersection point Q represents an unstable equilibrium.



The values of these points are obviously depending on the parameters of the equation (*S, α, n, K, and δ*).
In our case, we want to plot the bifurcation diagram of the circuit as a function of the **decay rate δ**, so we basically want to understand what happens to the graph when δ is changing.

As shown by the blue lines, when δ is smaller, the only intersection point between the production term and the degradation term will be the stable equilibria which we called R, thus our **gene will be highly expressed**.

So, from this first observation we can start plotting the bifurcation diagram: starting from δ -> 0, at the beginning we will only have one stable equilibrium, basically stuck at the same value of A*.

Now, at some point in the growth of δ, a **saddle node bifurcation** will appear: the degradation term will intersect with the production term in another two points, P and Q.

P is another stable equilibrium at which our gene is in its ON state, like R; but in this case, it will be expressed at a much lower level, basically just thanks to the input signal S.

With increasing δ, P will now be stuck always at the same value of A*, while Q will start increasing, pulling away from P and getting closer and closer to R, until another blue-sky bifurcation appears and the two equilibria stop existing.