Electronic Rates Markets &

Low Latency Interest Rate Swap Calculations

I Feel the Need, the Need for Speed

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Abstract

In electronic rates markets accuracy and low latency are threshold requirements that can be a barrier to market entry as they are essential to survive and compete. In this paper we outline how to achieve high performance by reducing swap pricing and risk calculations into trivial vector and matrix multiplication operations. This low latency approach has been successfully applied to Quant analytics in a live trading environment to create core trading competencies that can respond dynamically to market opportunities and challenges, as outlined in (Burgess, 2020) and (Burgess, 2022c).

We proceed as follows, firstly we discuss how to quote, price and book swaps for live execution purposes. Secondly, it is essential to understand the swap booking, schedule generation and pricing processes in detail in order to optimize swap calculations for low latency pricing, curve construction and risk. It follows that the majority of the calculation process can be reduced to static data that can be stored and reused rather than wastefully recomputed every calculation cycle. Thirdly, we discuss how to reduce calculations into their reduced primitive optimal state. We provide an Excel workbook to demonstrate the key concepts, see https://bit.ly/LowLatencySwaps. Fourthly, we touch upon advanced concepts including curve Jacobian and algorithmic adjoint differentiation (AAD). Finally to complete the picture we note that once calculations are in their primitive state we can further accelerate performance using parallelization, hardware and other technology solutions.

Keywords:

Electronic Markets, Swap Execution Facility, SEF, Request-for-Quote, RFQ, Bloomberg, Interest Rate Swaps, IRS, Swap Schedules, High Speed, Low Latency, Dynamic Data, Static Data, Curve Jacobian, Algorithmic Adjoint Differentiation, Speed, Performance Acceleration.

1. Introduction

Interest rate swaps are predominantly traded in electronic markets using a 'Swap Execution Facility' (SEF) or electronic trading venue that allows market participants to buy and sell swaps in a regulated and transparent manner, as required under the Dodd-Frank Act, 2010 (LCH, n.d.). Popular SEFs include those managed by BGC, Bloomberg, GFI, Tradeweb, TP ICAP, Tradition et al. To assist with market connectivity service providers such as ION group (ION, n.d.) provide the tools to automate the swaps workflow connectivity and facilitate market aggregation and trade management.

On venue interest rate swaps are standardized and executed at high speeds (of the order of milliseconds/microseconds) and have very tight bid-offer spreads, typically 1/10th of a basis point (bps) i.e. 0.001%. Non-standard swaps are also executed quickly, via RFQ (request for quote) with some voice trading. In order to achieve low latency, ultra-fast swap pricing and curve calibration we focus on methods to optimize swap calculations before proceeding to explore additional performance acceleration via hardware and technology solutions.

We proceed by examining swap booking, schedule and pricing conventions in detail. Armed with this information we demonstrate how to optimize swap calculations by storing the vast majority of the swap trade and schedule information for reuse. This allows us to price all standard swaps of any maturity for a given float index from a single stored swap schedule and bypass many repetitive, wasteful calculations. This facilitates low latency swap curve calibration, trade pricing and execution. To conclude we discuss how curve Jacobians can be used for ultra-fast curve rebuilds and touch upon the use of AAD for real-time swap DV01 risk calculations.

2. Swap Quotes

There are different pricing and quotation conventions for different swap types as illustrated in (table 1).

Swap Type	Order Execution	Quote Type
Standard Swap	On Venue	Par Rate / Spread over US Treasuries
IMM Swap	On Venue	NPV Cash Value
OTC Swap	RFQ / Voice	NPV Cash Value

Table 1: Swap and Quote Types

2.1 Standard Swaps (Par Rate)

Standard Swaps price to par and have a Net Present Value (NPV) of zero at inception. They are quoted on venue as a par-rate i.e. the fixed rate that gives a zero NPV. This mitigates any price disputes that could arise from yield curve[1] model differences et al.

Figure 1: USD Swap Price and Risk (Source: Bloomberg, SWPM <GO>)

Bloomberg dynamic trade specifications are captured on the 3) main tab and static data is captured in the 4) details tab

[1] Yield curve models produce the forward rates and discount factors required for pricing swaps. They are proprietary and counterparties can configure these models with different data sources, calibration instruments, interpolation conventions etc., which consequently leads to pricing disputes and perhaps arbitrage opportunities.

Standard swaps have the following features, see (Swap Ex, n.d.),

- Spot Starting
- Quoted as a Par Rate
- Tenors in whole years: 1Y-30Y
- Notional in integer multiples
- No floating spreads

In USD markets standard swaps are often priced as a spread over US Treasuries (UST) as per (equation 1). The spread itself is fairly stable with most of the price movement determined by US treasury yields. Bloomberg quotes for such swaps are shown in (figure 2), where swaps with a liquid underlying UST are highlighted in orange and those without a corresponding liquid UST underlying are highlighted in grey to indicate these swap quotes are linearly interpolated.

$$\text{Par Rate \%} = \text{US Treasury Yield \%} + \text{Spread} \qquad (1)$$

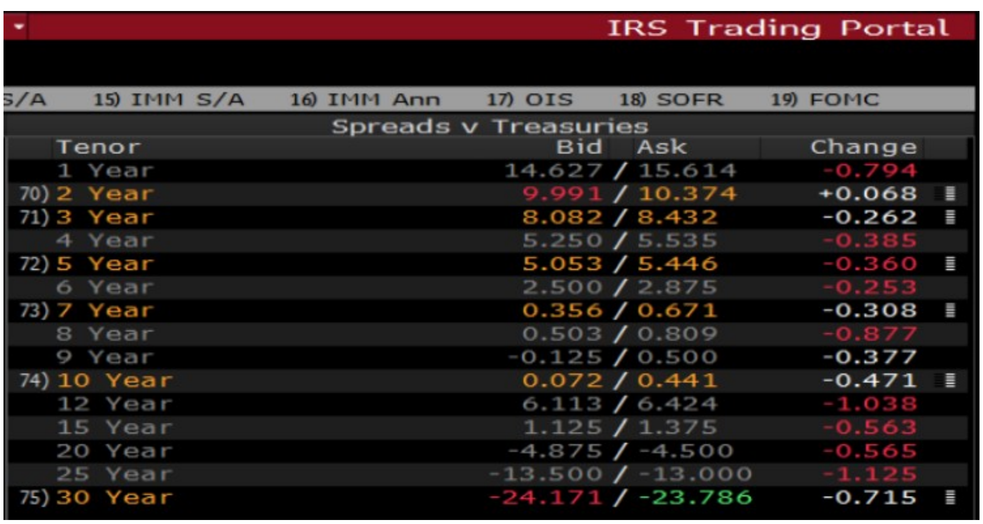

Figure 2: US IRS Trading Portal (Source: Bloomberg, March 2019)

The linear interpolation of quotes can present arbitrage opportunities, particularly when interpolating linearly over the large 10Y – 30Y range on the long end. Investors with good

UST and swap curve models could exploit arbitrages. To illustrate and for reference we decompose UST spread over quotes in (table 2).

	UST Yield (%)	Spread (bps)	Par Rate (%)	
1Y	0.0522%	14.00	0.1922%	
2Y	0.1647%	9.00	0.2547%	
3Y	0.2243%	8.00	0.3043%	
4Y	0.2941%	5.25	0.3466%	*
5Y	0.3341%	5.00	0.3841%	
6Y	0.3931%	2.50	0.4181%	*
7Y	0.4460%	0.35	0.4495%	
8Y	0.4736%	0.50	0.4786%	*
9Y	0.5070%	-0.10	0.5060%	*
10Y	0.5310%	0.08	0.5318%	
11Y	0.5254%	3.10	0.5564%	*
12Y	0.5186%	6.11	0.5798%	*
13Y	0.5660%	3.62	0.6022%	*
14Y	0.6125%	1.13	0.6237%	*
15Y	0.6632%	-1.88	0.6444%	*
20Y	0.7869%	-4.88	0.7382%	*
25Y	0.9547%	-13.50	0.8197%	*
30Y	1.1322%	-24.00	0.8922%	

* Interpolated

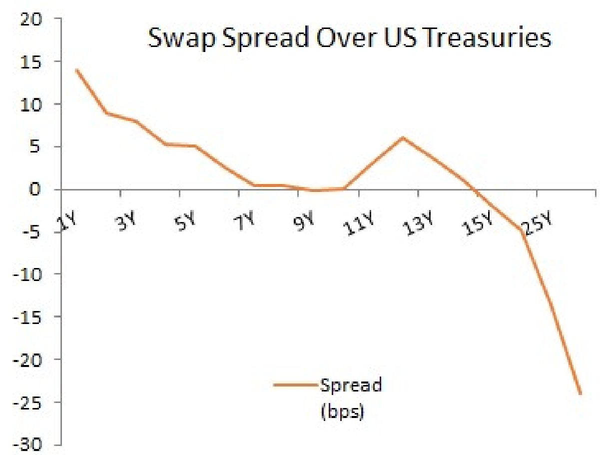

Table 2: US Swap Rates Quotes as a Spread to US Treasury Yields

2.2 IMM Swaps (NPV)

IMM Swaps have float leg resets and an expiration that follows 'International Money Market' (IMM) futures and options. IMM Swaps are similar to standard swaps having the following features, see (Swap Ex, n.d.),

- Quarterly Starting on 3rd Wednesday of March, June, September or December
- Quoted in cash NPV terms
- Tenors in whole years: 2Y, 3Y, 5Y, 7Y, 10Y and 30Y (varies by market)
- Notional in integer multiples
- No floating spreads

The most standardized IMM swaps are 'Market Agreed Coupon' or MAC swaps Clarus (n.d.), where the fixed coupon is standardized and usually the par-rate at the time rounded to the nearest 25 bps. This standardization has several advantages, namely, it increases liquidity and makes it easy for clearing houses and 'Central Clearing Parties' (CCPs) to compress and net down such swaps into a single trade. IMM swaps at inception often trade close to par, but are quoted in cash NPV terms.

2.3 OTC Swaps (NPV)

Over-the-counter swaps are bespoke or customized swaps that have no particular pattern, traded to meet individual corporate requirements. Such swaps rarely price to par and therefore are priced in cash value NPV terms.

3. Swap Pricing

Firstly standard swaps can be quoted directly as a fixed spread over US Treasury yields. Such swaps do not require a yield curve model; instead traders closely monitor the spread, which can be done manually or by using algorithms. Consequently quoting standard swaps is a trivial exercise that can be done at high speed using (equation 1) above. For IMM swaps traders monitor the TED spread, which captures the difference between treasury yields and IMM Eurodollar Futures. TED is an acronym formed from T-Bill and ED, with the latter being the ticker symbol for Eurodollar futures contracts.

3.1. Net Present Value (NPV)

A detailed overview of swap pricing and the formulae to use can be found in (Burgess, 2016). For fixed-float interest rate swaps (IRS) not trading at par (zero cash value) we quote and price swaps in cash NPV terms as follows,

$$Swap\ PV = \phi \left(\underbrace{\sum_{i=1}^{n} N\, r\, \tau_i\, P(t_E, t_i)}_{Fixed\ Leg} - \underbrace{\sum_{j=1}^{m} N\big(\, l_{j-1} + s\big)\, \tau_j\, P(t_E, t_j)}_{Float\ leg} \right) \tag{2}$$

where N denotes the swap notional, r the fixed rate, l_j the jth forward rate, τ_i the ith coupon period or year fraction and $P(t_E, t_i)$ the ith coupon discount factor and ϕ is an indicator function with $\phi = 1$ indicating we are receiving the fixed coupons and paying floating coupons and $\phi = -1$ when we are paying fixed and receiving floating coupons.

In order to evaluate (equation 2) we must calibrate swap curves to generate the required forward rates and discount factors, see (Burgess 2019) and (Burgess 2020) for more information.

3.2. Optimizing Net Present Value Calculations for Speed

For IRS with regular schedules with no initial or final stub periods[2] we can decompose and express (equation 2) in annuity format as below,

$$Swap\ PV = \phi \left(\underbrace{\mathrm{N\ r\ A}^{Fixed}}_{Fixed\ Leg} - \underbrace{\sum_{j=1}^{m} N\ l_{j-1}\ \tau_j\ P\big(t_E, t_j\big)}_{\substack{Float\ leg \\ excluding\ spread}} - \underbrace{N\ s\ A^{Float}}_{Float\ Spread} \right) \tag{3}$$

where annuity terms are denoted, $A^{Fixed} = \sum_{i=1}^{n} \tau_i P(t_E, t_i)$ and $A^{Float} = \sum_{j=1}^{m} \tau_j P(t_E, t_j)$. The annuity representation from (equation 3) makes it possible to optimize calculation performance and price swaps with different maturities using a single swap schedule.

3.3 Par Rates

Similarly, for a standard par swaps with zero NPV we quote such swaps as a par rate i.e. the fixed rate that makes the swap NPV zero. The par rate can be calculated by rearranging (equation 3) for the fixed rate r with the NPV set to zero and noting that notional N terms cancel as follows,

$$Par\ Rate, p = \frac{\sum_{j=1}^{m} l_{j-1}\ \tau_j\ P\big(t_E, t_j\big) - s\ \mathrm{A}^{Float}}{\mathrm{A}^{Fixed}} = \frac{PV(Float\ Leg)}{\mathrm{A}^{Fixed}} \tag{4}$$

[2] A stub is a reference to a swap coupon that is of irregular length. For example a swap paying regular three month coupons would be considered to have a stub as the first or last coupon was for two months say.

4. Swap Booking

Standard swaps trades comprise of dynamic and static data, the static data is predominantly required for swap schedule generation as shown in (figure 3) below. To facilitate low latency trade booking and execution, practitioners store swap static data in named templates, called generators or generator templates i.e. we store the blue static data displayed in (figure 3) and name that data "USD_SWAP_3M" for easy identification and reuse.

	Swap Generator Template	USD_SWAP_3M	
Dynamic Trade Info	LEG TYPE	LEG1:FIXED	LEG2:FLOAT
	PAY / RECEIVE	PAY	RECEIVE
	NOTIONAL	1,000,000	1,000,000
	FIXED RATE (%)	1.00%	-
	FLOAT SPREAD (BPS)	-	0.00
	EFFECTIVE DATE / LAG	2D	2D
	MATURITY DATE / TENOR	2Y	2Y
Static Data + Schedule Info	LEG CURRENCY	USD	USD
	NOTIONAL EXCHANGE	NONE	NONE
	LEVERAGE	1.00	1.00
	FRONT STUB INDEX	-	NATURAL
	BACK STUB INDEX	-	NATURAL
	VALUATION CURRENCY	USD	USD
	FORECAST INDEX	-	USD3M
	DISCOUNT INDEX	USDOIS	USDOIS
	INDEX COMPOUND METHOD	-	NONE
	SPREAD COMPOUND METHOD	-	NONE
	ROLL DAY	END	END
	STUB TYPE	SHORT START	SHORT START
	FIXING BUS DAY ADJUSTMENT	-	MODIFIED_FOLLOWING
	FIXING CALENDAR	-	NY+LDN
	FIXING LAG	-	2D
	FIXING IN-ADVANCE / IN-ARREARS	-	IN-ADVANCE
	ACCRUAL FREQUENCY	SEMI-ANNUAL	QUARTERLY
	ACCRUAL BUS DAY ADJUSTMENT	MODIFIED_FOLLOWING	MODIFIED_FOLLOWING
	ACCRUAL CALENDAR	NY	NY
	ACCRUAL DAYCOUNT	30/360	ACT/360
	PAYMENT FREQUENCY	SEMI-ANNUAL	QUARTERLY
	PAYMENT BUS DAY ADJUSTMENT	MODIFIED_FOLLOWING	MODIFIED_FOLLOWING
	PAYMENT CALENDAR	NY	NY
	PAYMENT LAG	2D	2D

Figure 3: **Standard Swap Booking Template**

(**black** fields are dynamic and variable and **blue** fields are static, constant trade parameters)

Using the swap generator template **"USD_SWAP_3M"** from (figure 3) we can fully represent all standard swaps quickly using shorthand as illustrated in (figure 4). Traders would typically specify the few dynamic trade components and a generator template to reference the fixed static data and schedule information. This is common practice in Bloomberg, ION, Murex and many other trading systems, although it is not always that obvious.

Swap Generator	Pay/Rec Fixed	Notional	Fixed (%)	Spread (bps)	Maturity	Trade Handle
USD_SWAP_3M	PAY	1,000,000	1.00%	0.00	1Y	USD_SB3_1Y:0
USD_SWAP_3M	PAY	1,000,000	1.00%	0.00	2Y	USD_SB3_2Y:0
USD_SWAP_3M	PAY	1,000,000	1.00%	0.00	3Y	USD_SB3_3Y:0
USD_SWAP_3M	PAY	1,000,000	1.00%	0.00	4Y	USD_SB3_4Y:0
USD_SWAP_3M	PAY	1,000,000	1.00%	0.00	5Y	USD_SB3_5Y:0
USD_SWAP_3M	PAY	1,000,000	1.00%	0.00	6Y	USD_SB3_6Y:0
USD_SWAP_3M	PAY	1,000,000	1.00%	0.00	7Y	USD_SB3_7Y:0
USD_SWAP_3M	PAY	1,000,000	1.00%	0.00	8Y	USD_SB3_8Y:0
USD_SWAP_3M	PAY	1,000,000	1.00%	0.00	9Y	USD_SB3_9Y:0
USD_SWAP_3M	PAY	1,000,000	1.00%	0.00	10Y	USD_SB3_10Y:0
USD_SWAP_3M	PAY	1,000,000	1.00%	0.00	11Y	USD_SB3_11Y:0
USD_SWAP_3M	PAY	1,000,000	1.00%	0.00	12Y	USD_SB3_12Y:0
USD_SWAP_3M	PAY	1,000,000	1.00%	0.00	13Y	USD_SB3_13Y:0
USD_SWAP_3M	PAY	1,000,000	1.00%	0.00	14Y	USD_SB3_14Y:0
USD_SWAP_3M	PAY	1,000,000	1.00%	0.00	15Y	USD_SB3_15Y:0
USD_SWAP_3M	PAY	1,000,000	1.00%	0.00	20Y	USD_SB3_20Y:0
USD_SWAP_3M	PAY	1,000,000	1.00%	0.00	25Y	USD_SB3_25Y:0
USD_SWAP_3M	PAY	1,000,000	1.00%	0.00	30Y	USD_SB3_30Y:0

Figure 4: Standard Swap Booking using Swap Generator Template Shorthand

As illustrated in (figure 4) trade information can be stored in a trade object called a trade handle. The trade handle is given a sensible name and the handle name is used to access trade information. The handle has a counter suffix, which ticks with the market, but is only updated if there is a change to the trade information.

5. Swap Schedules

In order to generate the swap cash flow schedule required to price a swap we must specify quite a lot of swap schedule parameters as shown in blue in (figure 3) above. In this section we firstly give an overview of key swap schedule parameters and specifications. Secondly we illustrate how to build the swap schedule and thirdly how to use the swap schedule to price swaps.

5.1 Swap Schedule Specifications

We outline some of the key swap schedule parameters below,

- **Effective Lag**

The effective lag specifies the start of the swap relative to the trade date (T). This is typically 2 business days (2D) in many markets and specified as T+2.

- **Business Day Adjustment**

Whenever calculating a date, the business day adjustment tells us how to manage weekends and holidays. For swaps we need a business adjustment parameter for forward rate fixing dates, accrual dates and discount factor payment dates, which can all have different business day adjustment conventions.

Business Day Adjustment	Method
FOLLOWING	Roll forwards to the next business day
MODIFIED FOLLOWING	Roll forwards to the next business day, however roll backwards if rolling forwards takes us into the next month
PRECEDING	Roll backwards to the previous business day
MODIFIED PRECEDING	Roll backwards to the previous business day, however roll forwards if the rolling backwards takes us into the previous month
UNADJUSTED	No change

Table 3: Business Day Adjustment Conventions

- **Day Count**

The day count is needed to compute the number of coupon days of interest to apply.

Day Count	Method
ACT/360	The actual number of coupon days divided by 360. Here we assume 360 days per year.
ACT/365	The actual number of coupon days divided by 365. Here we assume 365 days per year.
ACT/ACT	The actual number of coupon days divided by the actual number of days in the year with special treatment for coupons running over year end.
30/360	Assumes whole months have 30 days and that there are 360 days in a year

Table 4: Day Count Methods

We briefly outline some of the popular day count methods above in (table 4), see (Delta Quants, n.d.) and (OpenGamma, n.d.) for more information.

- **Roll Day**

The roll day is used to determine unadjusted coupon end and payment dates. When no roll is specified the market convention is to assume coupons roll on the same day as the unadjusted maturity date to be consistent with the market default short start stub convention. Once we know the roll day we apply accrual business day conventions to determine the accrual end date and payment business day conventions for the coupon payment date as illustrated in (figure 12) below.

We can specify the accrual end or coupon roll day to be a specific day of the month 1-31 or specify IMM, EOM, START or END. The market default for swaps is to imply coupon dates by rolling backwards from the unadjusted maturity date using Roll Day = END, as per (figure 5).

Roll Day	Method
Specified as day of the month 1 - 31	The coupon accrual end date or roll date falls on the day of the month specified
IMM	Coupons accrue to the IMM roll date i.e. the 3rd Wednesday of March, June, September or December
EOM	Coupons accrue to the end of the month
START	Coupons accrue to the same day as the unadjusted effective date
END (Market Default)	Coupon accrue to the same day as the unadjusted maturity date

Table 5: Roll Day Conventions

For example, consider a 1Y swap with quarterly coupons then using the market default we repeatedly roll backwards from the unadjusted maturity date by three months until we reach (or overshoot the effective date). All dates that we land on are the unadjusted coupon roll dates (or accrual dates) as illustrated in (figure 5). Once we have determined all the unadjusted accrual dates, we apply the accrual business day adjustment and calendar from (figure 3) to determine the actual accrual dates adjusted for holidays. Fixing and payment lags are applied to these adjusted accrual dates as indicated in (figure 12).

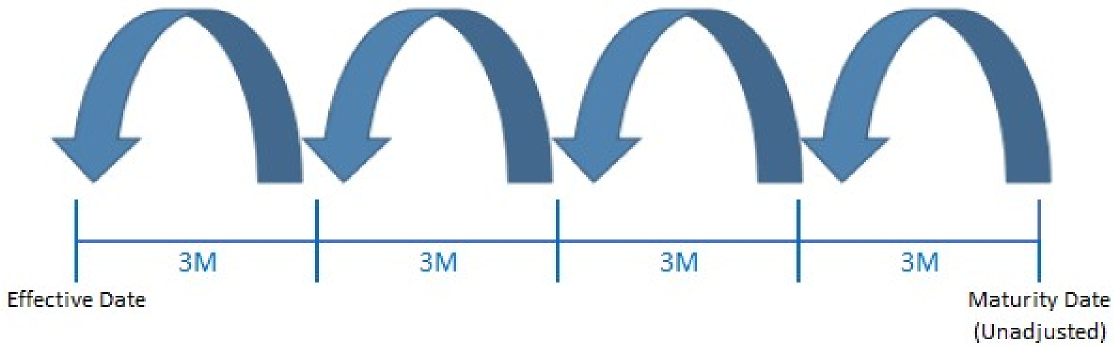

Figure 5: Roll Day Illustration **Rolling Backwards** from Maturity Date (Roll Day = End)

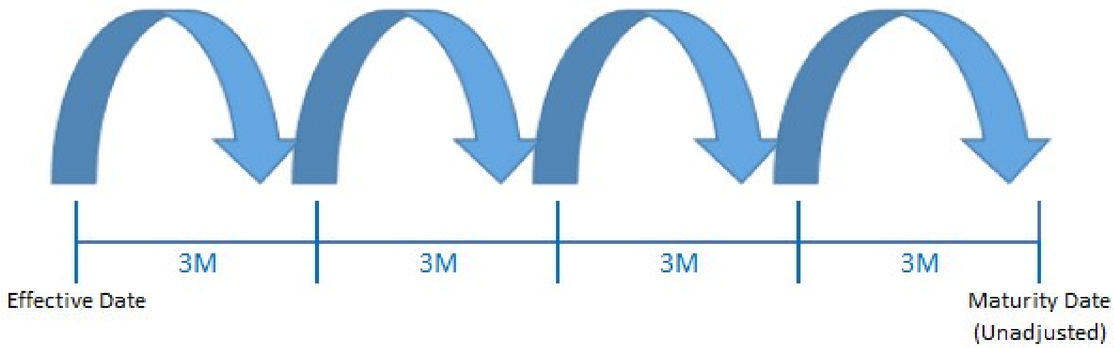

Figure 6: Roll Day Illustration **Rolling Forwards** from Effective Date (Roll Day = Start)

- **Stub Type**

The stub type tells us how to determine coupon accrual periods for swaps of irregular length

Stub Type	Method
SHORT START (Market Default)	Roll backwards from the unadjusted maturity date taking regular coupon steps-sizes e.g. quarterly until we reach the unadjusted effective date. This will create an irregular short coupon at the start of the swap.
LONG START	Similar to short start, but the short start stub is glued to the first regular coupon to become a long coupon.
SHORT END	Roll forwards from the unadjusted effective date taking regular coupon steps-sizes e.g. quarterly until we reach the unadjusted maturity date. This will create an irregular short coupon at the end of the swap.
LONG END	Similar to short end, but the short end stub is glued to the last regular coupon to become a long coupon.

Table 6: Stub Types and Methodology

Say we have an IRS with maturity in 1 year and 2 months that pays quarterly interest, what should we do with the left-over 2 months? The irregular or left-over period is called the stub period as illustrated below.

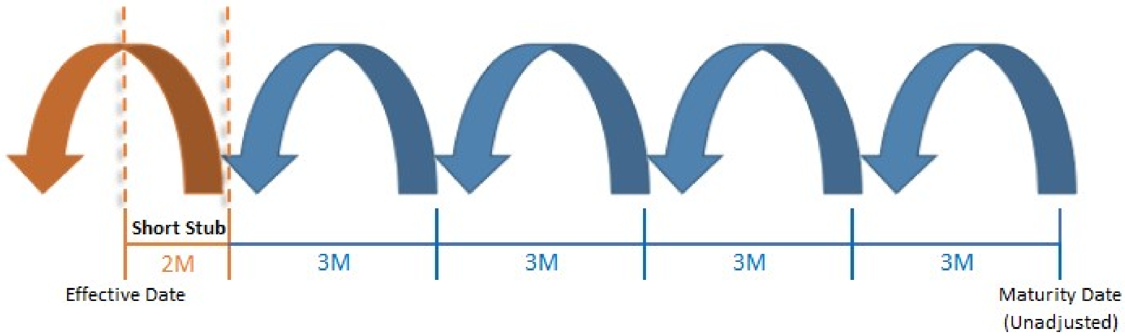

Figure 7: **Short Start** Stub Illustration **Rolling Backwards** from Maturity Date

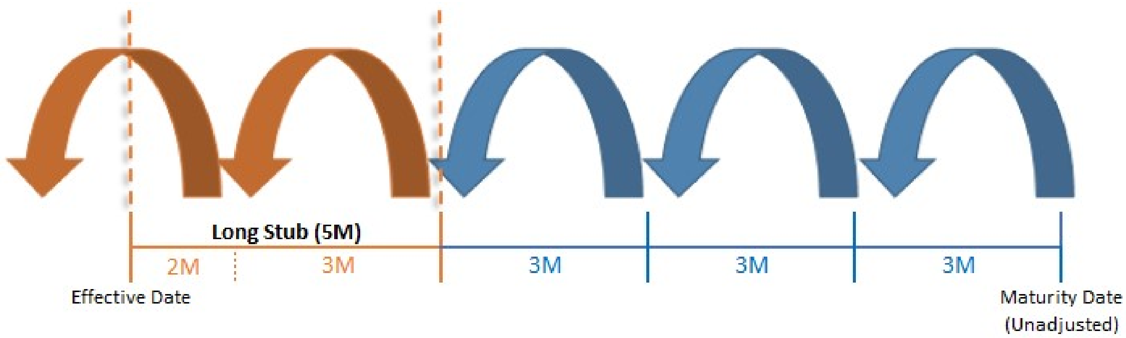

Figure 8: **Long Start Stub** Illustration **Rolling Backwards** from Maturity Date

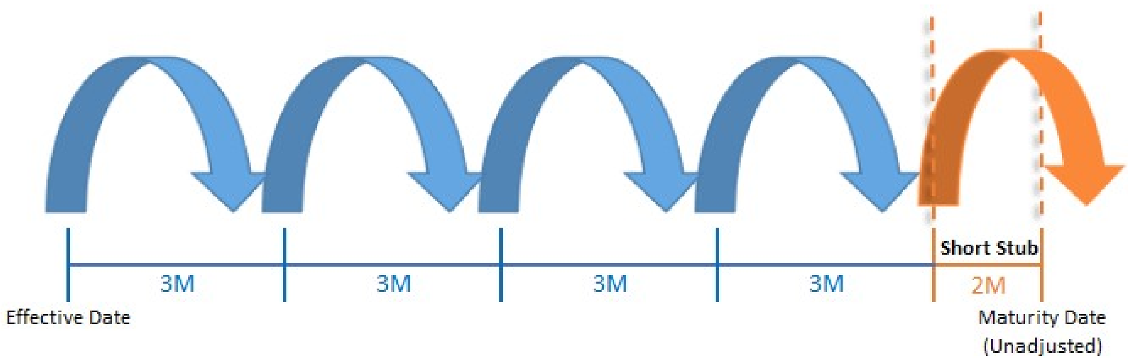

Figure 9: **Short End** Stub Illustration **Rolling Forwards** from Effective Date

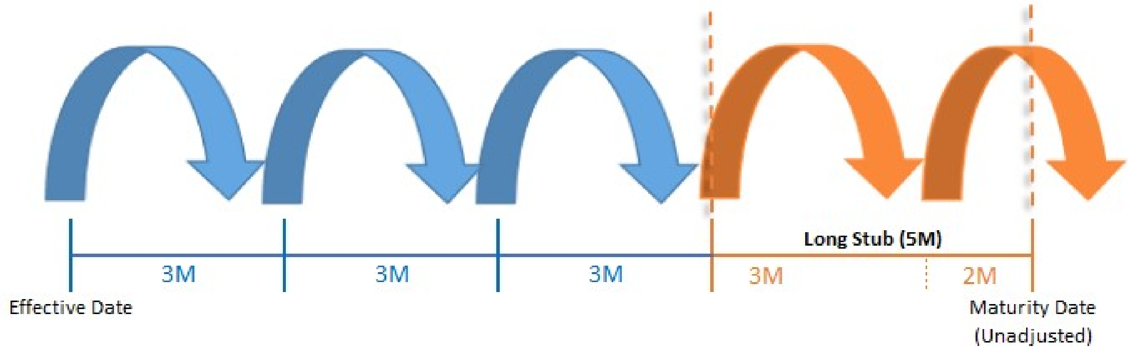

Figure 10: **Long End** Stub Illustration **Rolling Forwards** from Effective Date

Standard swaps do not have a stub. However, for swaps of irregular length the market default is to apply a short start stub. We note that irregularly sized floating coupons (floating stubs) require stub rates, which are linearly interpolated rates customized to the length of the stub period. For example, a 2M stub period would require a 2M stub rate implied by linearly interpolating the 1M and 3M market forward rates.

- **Swap Start and End Dates**

When booking swaps we should always book swaps with an unadjusted maturity date without applying any business day adjustment. The accrual and payment convention will manage any holidays relating to final coupon accruals and payments at maturity. This is to allow us to compute the correct coupon roll dates, which are typically implied from the unadjusted maturity date.

5.2 Swap Schedules & Coupon Dates

Swap start or effective dates are determined relative to the trade date as an 'Effective Lag'. The effective lag is typically 2D (T+2), indicating that swap coupons begin to accrue interest 2 business days from the trade date as illustrated in (figure 11). The first swap coupon starts to accrue interest from the effective date **(including)** to the first coupon accrual end date **(excluding)**.

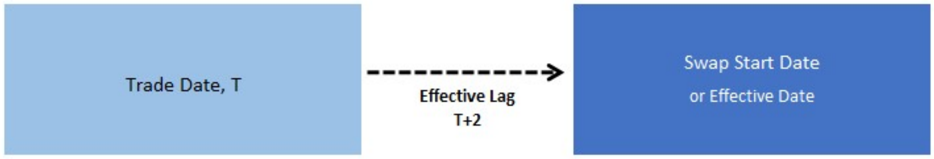

Figure 11: Effective Lag and Swap Start Date Illustration

Swap coupon dates are determined starting with the coupon accrual start date and use the swap trade conventions specified for the fixed leg in (figure 3) above. Knowing the coupon accrual start date we firstly determine the accrual end date, by rolling forward 6M as per the fixed leg accrual parameters. Secondly having determined the accrual end date we then imply the payment date using the fixed payment parameters and apply a 2D lag as per the fixed leg payment parameters. Finally, the next coupon accrual start date is this coupon's end date. We note that when computing coupon interest we **include** the accrual start date and **exclude** the accrual end date to ensure no double counting of interest days. We illustrate this process in (figure 12).

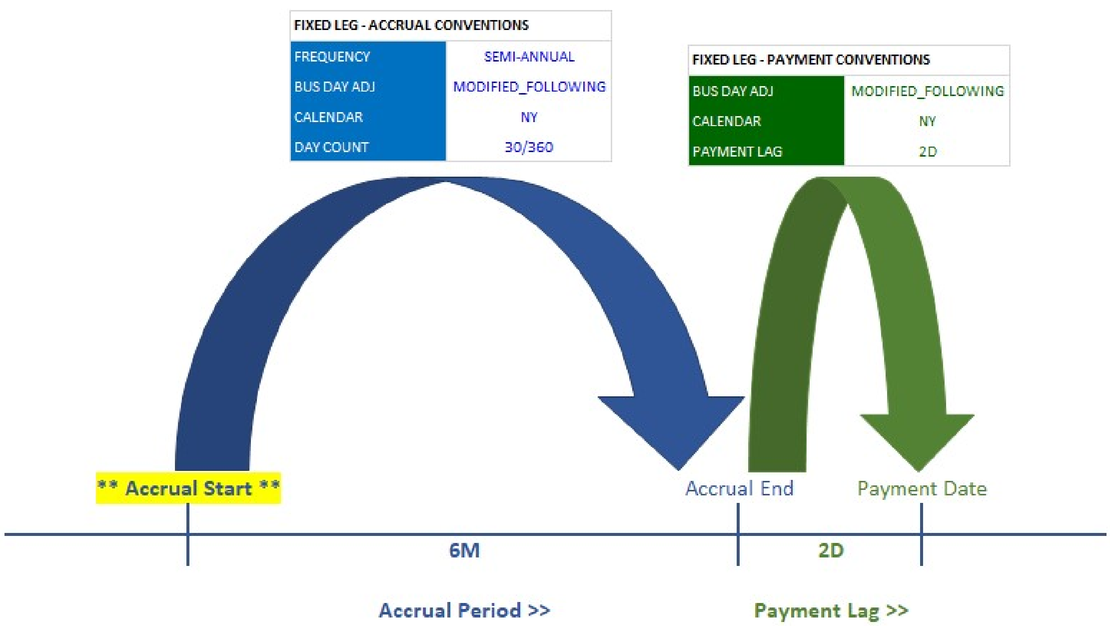

Figure 12: Illustration of Fixed Coupon Date Computation

For the swap floating leg we follow the same process as that for the fixed leg. However, conventions to be used are taken from the float leg specifications in (figure 3) and for the float leg we have the extra requirement to compute the forward rate fixing dates.

Starting with the accrual start date we roll forwards 3M to compute the accrual end date using float accrual settings. From the accrual end date we roll forwards 2D using float payment settings. Finally from the accrual start date we roll backwards using the 2D lag and the float leg fixing settings. We illustrate the float leg date calculation process in (figure 13).

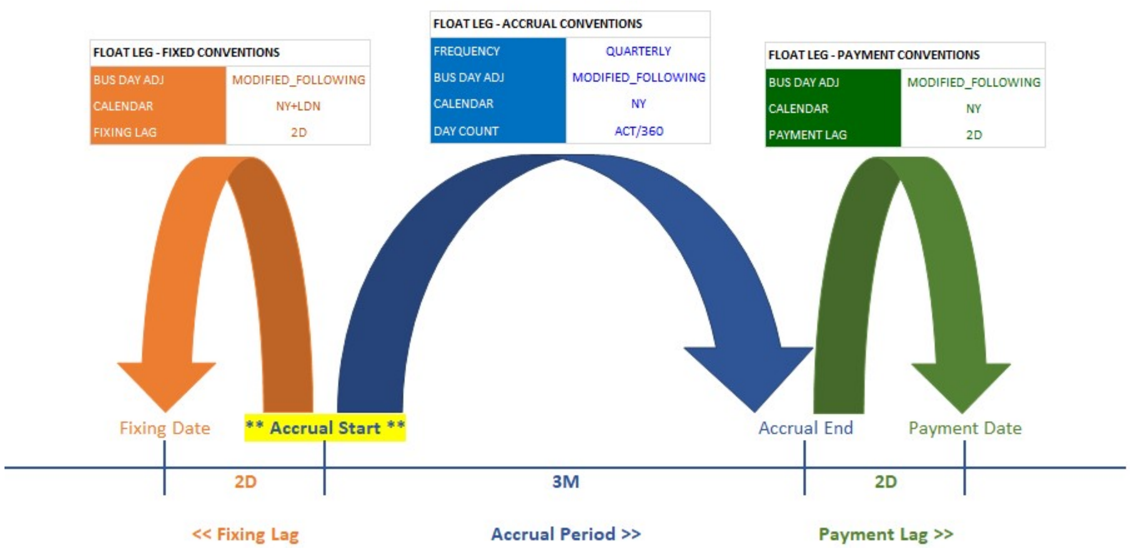

Figure 13: Illustration of LIBOR Float Coupon Date Computation (Fixing In-Advance)

For SOFR Swaps that use Risk-Free Rates (RFRs) instead of LIBOR forward rates the computation is similar however forward rates for RFR indices are computed daily and aggregated into a single rate known only in-arrears as shown in (figure 14).

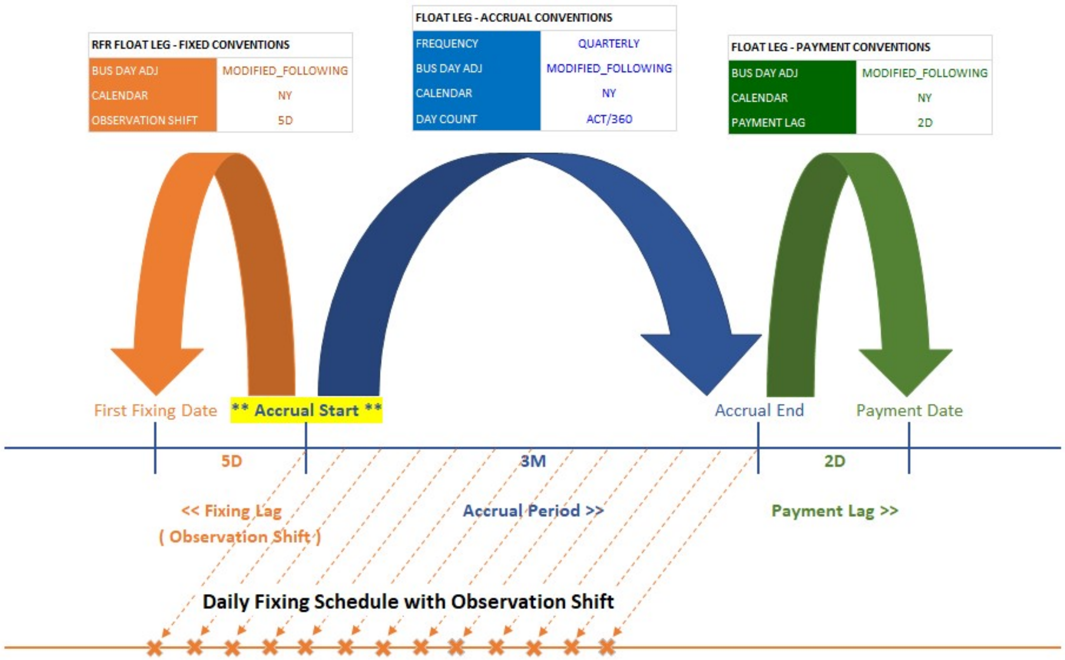

Figure 14: Illustration of RFR Float Coupon Date Computation

We note that RFR rates are set daily based on the previous days' market transactions with the total annualized rate only known in-arrears, see (Burgess, 2021d) for more details and the formulae to use.

5.3 Swap Schedules & Pricing

Putting everything together, the primary goal of the swap schedule determine the following three key parameters, which enable us to price a swap and quote the NPV or par rate accurately using (equations 3-4).

1. **Fixing Dates (Forward Rates)**

   This is the date we determine and set our floating interest rates

2. **Accrual Start and End Dates (Coupon Interest)**

   Required to determines the number of days' worth of interest to charge in conjunction with the day count basis e.g. 30/360

3. **<u>Payment Dates</u> (Discount Factors)**

The coupon payment date is also the date used for discount factor calculations

As illustrated in (figure 15) to price fixed coupons, accrual dates (blue) are needed to compute the coupon accrual year fractions, τ. We note that accrual year fractions **include** the accrual start date, however they **exclude** the accrual end date to ensure no double counting of interest days. Additionally payment dates (green) are required to compute the required discount factors, $P(t_E, t_i)$.

Year Fraction

Accrual Start	Accrual End
21-May-22	19-Nov-22
19-Nov-22	21-May-23
21-May-23	19-Nov-23
19-Nov-23	20-May-24

Discount Factor

Pay Date
19-Nov-22
21-May-23
19-Nov-23
20-May-24

τ

$P(t_E, t_i)$

Fixed Leg

N	r^{Fixed}	τ_i	$P(t_E, t_i)$	NPV^{Fixed}
1,000,000	1.0000%	0.50	0.997824	4,989
1,000,000	1.0000%	0.50	0.994549	4,973
1,000,000	1.0000%	0.50	0.991815	4,959
1,000,000	1.0000%	0.50	0.987801	4,939

$$PV^{Fixed\ Leg} = r \sum_{i=1}^{n} N_i \tau_i P(t_E, t_i)$$

Figure 15: Swap Schedule & Fixed Leg Pricing

To price the float leg of a swap, we calculate the float coupon accrual year fractions and discount factors as we did for the fixed leg. However, we have an additional requirement to implying forward rates from the forward fixing dates highlighted in (orange) in (figure 16).

Forward Rate	Year Fraction		Discount Factor
Fixing Date	Accrual Start	Accrual End	Pay Date
19-May-22	21-May-22	20-Aug-22	20-Aug-22
18-Aug-22	20-Aug-22	19-Nov-22	19-Nov-22
17-Nov-22	19-Nov-22	18-Feb-23	18-Feb-23
16-Feb-23	18-Feb-23	21-May-23	21-May-23
19-May-23	21-May-23	20-Aug-23	20-Aug-23
18-Aug-23	20-Aug-23	19-Nov-23	19-Nov-23
17-Nov-23	19-Nov-23	19-Feb-24	19-Feb-24
17-Feb-24	19-Feb-24	20-May-24	20-May-24

l_{j-1} τ $P(t_E, t_i)$

Float Leg

N	l_{j-1}	s	τ_j	$P(t_E, tj)$	NPV^{Float}
1,000,000	0.2800%	0.00	0.25	0.999302	700
1,000,000	0.2995%	0.00	0.25	0.997824	747
1,000,000	0.3385%	0.00	0.25	0.996240	843
1,000,000	0.3775%	0.00	0.25	0.994549	939
1,000,000	0.4165%	0.00	0.25	0.992752	1,034
1,000,000	0.4555%	0.00	0.25	0.991815	1,129
1,000,000	0.4750%	0.00	0.25	0.989860	1,175
1,000,000	0.5140%	0.00	0.25	0.987801	1,269

$$PV^{Float\ Leg} = \sum_{j=1}^{m} N_j (l_{j-1} + s) \tau_j P(t_E, t_j)$$

Figure 16: Swap Schedule & Float Leg Pricing

6. Low Latency Swap Calculations

Traditional systems and analytics recreate and price financial instruments many times for every computation and clock cycle throughout the day. This is highly wasteful and inefficient. Only a small part of the swap pricing computation is dynamic and requires recalculation, namely the calculation of forward rates and discount factors. Even then this can be done in an optimal manner that when combined with advanced techniques can allow swaps to be priced in real-time, typically in microseconds, that's one millionth of a second!

6.1 Low Latency Swap Pricing

When quoting swaps as a spread over treasuries as illustrated in (figure 2) and (table 2) we only need to monitor US Treasury yields and swap spreads. We do not require a yield curve model to quote standard swaps as a spread over treasuries. However, for NPV, par-rate and DV01 risk calculations a yield curve is required. Similarly, swaps trading with custom features and maturities also require a yield curve to imply forwards and discount factors necessary to compute the price and risk.

To help achieve low latency, swap generator templates provide an efficient short-hand to fully specify swaps, which would otherwise quite verbose to specify as shown (figure 3). Additionally, some market practitioners choose to store swap trades using a trade handle lookup object, whereby swap trade information is stored in memory for easy reference and only recomputed if there is a change or update to the underlying trade information.

As standard swap trade bookings are fixed and constant for any given trade date, this makes the use of trade object handles highly efficient as they do not require any updates and only need to be computed once at the start of the day before the market opens.

Furthermore, for a given float index such as USD 3M LIBOR we can price all USD Standard IRS using a single consolidated swap schedule as illustrated in (figure 17 and 18). This is because standard swap schedules are regular and can be fixed. Therefore for any given trade date swap schedules themselves can also be stored rather than recomputed. Moreover, as standard swaps are denominated in whole integer years as per (section 2.1 and 2.2) it is most efficient to build and store one swap schedule for the longest swap maturity, for example the 30Y maturity swap, and derive all shorter swaps schedules from that one single swap schedule.

To facilitate optimal pricing, the storage and reuse of a single swap schedule it is helpful to use the annuity pricing approach see (equations 3 and 4). This is so we can easily decouple the fixed par rate from each swap when evaluating par rates and NPVs for swaps of different maturities.

We illustrate how to price standard swaps for all maturities from a single swap schedule and how to compute the NPV, par rate and DV01 below in (figures 17 and 18).

Swap Generator	Pay/Rec Fixed	Notional	Fixed (%)	Spread (bps)	Maturity	Trade Handle	NPV	Par Rate (%)	DV01[3]
USD_SWAP_3M	RECEIVE	1,000,000	1.0000%	0.00	1Y	USD_SB3_1Y:0	8,018	0.1922%	99
USD_SWAP_3M	RECEIVE	1,000,000	1.0000%	0.00	2Y	USD_SB3_2Y:0	14,721	0.2547%	198
USD_SWAP_3M	RECEIVE	1,000,000	1.0000%	0.00	3Y	USD_SB3_3Y:0	20,511	0.3043%	295
USD_SWAP_3M	RECEIVE	1,000,000	1.0000%	0.00	4Y	USD_SB3_4Y:0	25,556	0.3466%	391
USD_SWAP_3M	RECEIVE	1,000,000	1.0000%	0.00	5Y	USD_SB3_5Y:0	29,962	0.3841%	486
USD_SWAP_3M	RECEIVE	1,000,000	1.0000%	0.00	6Y	USD_SB3_6Y:0	33,800	0.4181%	581
USD_SWAP_3M	RECEIVE	1,000,000	1.0000%	0.00	7Y	USD_SB3_7Y:0	37,127	0.4495%	674
USD_SWAP_3M	RECEIVE	1,000,000	1.0000%	0.00	8Y	USD_SB3_8Y:0	39,986	0.4786%	767
USD_SWAP_3M	RECEIVE	1,000,000	1.0000%	0.00	9Y	USD_SB3_9Y:0	42,414	0.5060%	859
USD_SWAP_3M	RECEIVE	1,000,000	1.0000%	0.00	10Y	USD_SB3_10Y:0	44,442	0.5318%	949
USD_SWAP_3M	RECEIVE	1,000,000	1.0000%	0.00	11Y	USD_SB3_11Y:0	46,096	0.5564%	1,039
USD_SWAP_3M	RECEIVE	1,000,000	1.0000%	0.00	12Y	USD_SB3_12Y:0	47,400	0.5798%	1,128
USD_SWAP_3M	RECEIVE	1,000,000	1.0000%	0.00	13Y	USD_SB3_13Y:0	48,374	0.6022%	1,216
USD_SWAP_3M	RECEIVE	1,000,000	1.0000%	0.00	14Y	USD_SB3_14Y:0	49,037	0.6237%	1,303
USD_SWAP_3M	RECEIVE	1,000,000	1.0000%	0.00	15Y	USD_SB3_15Y:0	49,407	0.6444%	1,389
USD_SWAP_3M	RECEIVE	1,000,000	1.0000%	0.00	20Y	USD_SB3_20Y:0	47,340	0.7382%	1,808
USD_SWAP_3M	RECEIVE	1,000,000	1.0000%	0.00	25Y	USD_SB3_25Y:0	39,793	0.8197%	2,206
USD_SWAP_3M	RECEIVE	1,000,000	1.0000%	0.00	30Y	USD_SB3_30Y:0	27,870	0.8922%	2,585

Figure 17: Low Latency Swap Pricing: NPV, Par-Rate and DV01

Fixed Leg

	τ_i	$P(t_E, t_i)$	A^{Fixed}
	0.50	0.995012	0.50
1Y	0.50	0.990050	0.99
	0.50	0.985112	1.49
2Y	0.50	0.980199	1.98
	0.50	0.975310	2.46
3Y	0.50	0.970446	2.95
	0.50	0.965605	3.43
4Y	0.50	0.960789	3.91
	0.50	0.955997	4.39
5Y	0.50	0.951229	4.86

Float Leg

	l_{j-1}	τ_j	$P(t_E, t_j)$	A^{Float}	Unit NPV^{Float}
	0.1250%	0.25	0.997503	0.25	0.00
	0.1768%	0.25	0.995012	0.50	0.00
	0.2165%	0.25	0.992528	0.75	0.00
1Y	0.2500%	0.25	0.990050	0.99	0.00
	0.2795%	0.25	0.987578	1.24	0.00
	0.3062%	0.25	0.985112	1.49	0.00
	0.3307%	0.25	0.982652	1.73	0.00
2Y	0.3536%	0.25	0.980199	1.98	0.01
	0.3750%	0.25	0.977751	2.22	0.01
	0.3953%	0.25	0.975310	2.47	0.01
	0.4146%	0.25	0.972875	2.71	0.01
3Y	0.4330%	0.25	0.970446	2.95	0.01
	0.4507%	0.25	0.968022	3.19	0.01
	0.4677%	0.25	0.965605	3.44	0.01
	0.4841%	0.25	0.963194	3.68	0.01
4Y	0.5000%	0.25	0.960789	3.92	0.01
	0.5154%	0.25	0.958390	4.16	0.01
	0.5303%	0.25	0.955997	4.39	0.02
	0.5449%	0.25	0.953610	4.63	0.02
5Y	0.5590%	0.25	0.951229	4.87	0.02

Figure 18: Single Consolidated Swap Schedule for all USD 3M Swaps

(For brevity we show the first 5Y of the 30Y schedule)

6.2 Low Latency Yield Curves

Being able to price blocks of standard instruments as shown in (figure 17) is not just useful for quoting on trading screens and venues, but is also useful for yield curve calibration, where we have to calibrate multiple calibration instruments simultaneously as shown in (figure 19). In interest rate markets yield curve models are required to imply forward rates and/or discount factors from liquid market instruments for pricing and risk purposes.

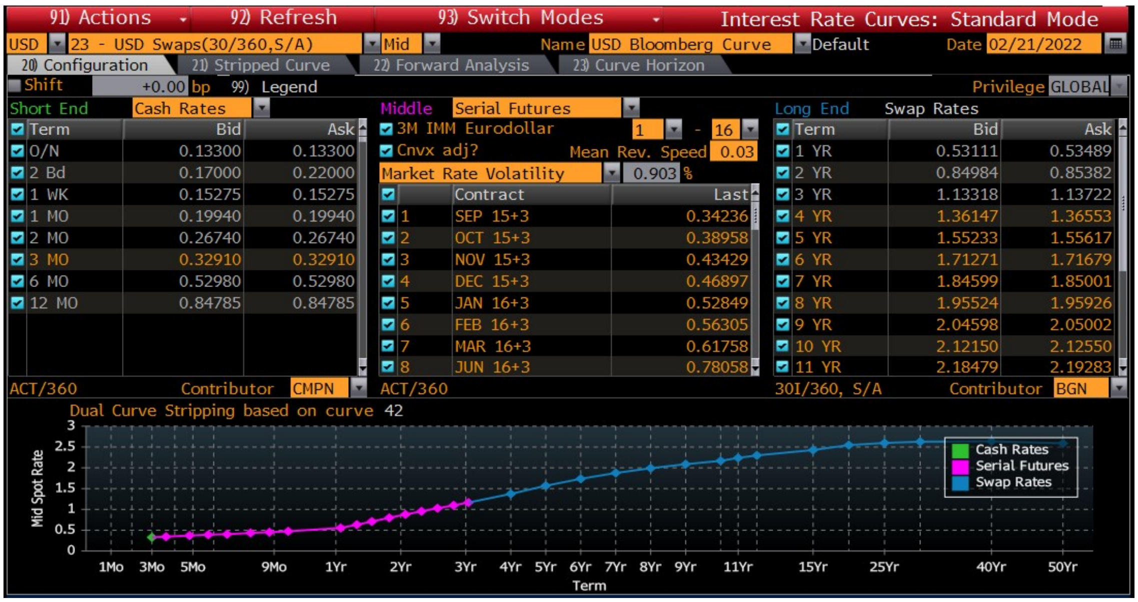

Figure 19: USD 3M Yield Curve (Source: Bloomberg ICVS USD <GO>)

Swaps are typically quoted to 0.001% i.e. 1/10th of a basis point (bps). Additionally, we are required to price such swaps not just accurately but quickly to compete for business on high frequency trading venues, with 10 milliseconds often considered modest performance and high performance in the order of microseconds (one millionth of a second) or more. For many practitioners calibrating yield curves to such high performance is hard to achieve, see (Burgess, 2019).

One strategy that high frequency firms and electronic rates practitioners employ to calibrate yield curves at high speed is to employ a hybrid methodology, where one builds both fast and slow curves. The slow curve runs constantly in the background and performs a full recalibration and the fast curve runs on top of that and holds true for small changes in the underlying calibration instruments. In my experience a slow curve might take 2-10 milliseconds to calibrate and the fast curve 1-2 microseconds.

Full Calibration (Slow)

When performing full calibration, ideally we should target 2-10 milliseconds, before hardware acceleration. To achieve low latency it is recommended to store useful results and intermediate steps for reuse as follows.

Firstly we compute the swap schedule for the longest swap instrument and derive all other swap schedules from it as outlined in (section 6.1) and (figure 17). We recommend that we compute this schedule once and store it for reuse. Secondly when performing a full curve calibration we often solve for forwards and discount factors using a gradient descent solving solver such as Newton-Raphson (Burgess, 2021a), where the curve Jacobian is computed as a by-product. We should store this important information. The Jacobian tells us how forwards and discount factors change for a small change in market data, see (Burgess, 2021b). Thirdly we should also store the results from the curve calibration solver routine[3], so that the next time the solver is run we can use these results as our new initial guess, see (Burgess 2021a). This often leads to a solver solution in just one Newton-Raphson iteration. Fourthly and finally we should store the forward rate and discount factor outputs which are needed for the fast incremental calibration.

Incremental Calibration (Fast)

Knowing the forward rates and discount factors from the full calibration process, we use the curve Jacobian to imply new forward rates and discount factors. The curve Jacobian tells us how forwards and discount factors change for a small unit change in market data[4]. We imply the new forward rates and discount factors by multiplying the curve Jacobian by the change in market data and add the incremental changes in forwards and discount factors to the base line results.

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[3] The calibration routine will use a pre-specified state variable, which could be zero rates, forward rates, discount factors or a function thereof, from which the curve calibration interpolator logic will solve for and imply forwards and discount factors for instrument pricing.

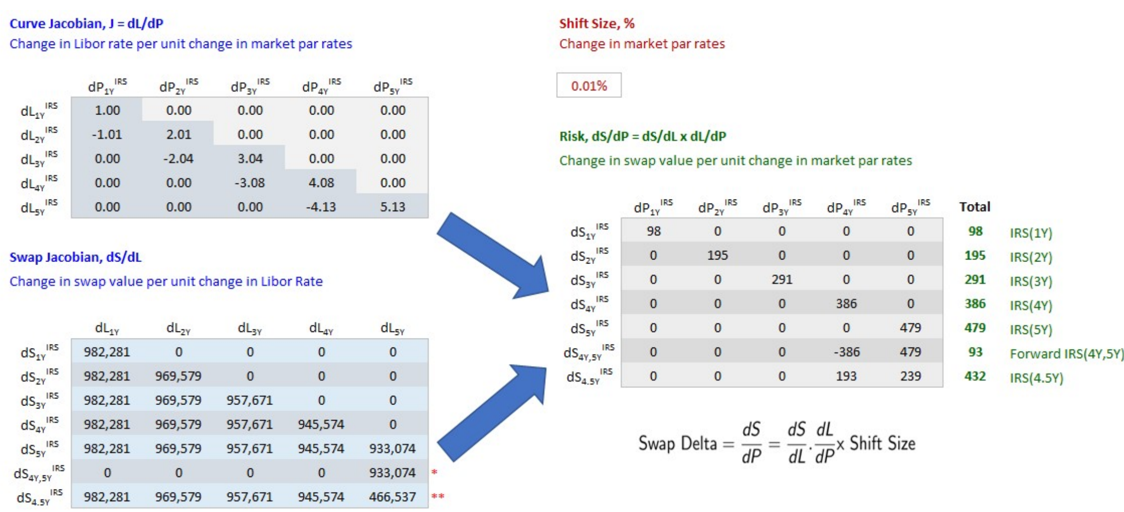
[4] The unit change or shift-size is defined by the model builder to be something small often around 1/100th of a basis point.

6.3 Low Latency DV01 Risk

Swap DV01 risk captures the sensitivity of interest rate swaps to a one basis point change in interest rates. It comprises of a PV01[5] (or annuity) component measuring how the swap price or NPV changes given a one bps change in forward rates and a DF01 (discount factor) component measuring how the swap NPV changes due to the impact of the forward rate change on discount factors.

As standard swaps trade at par i.e. have zero value there is no total discount factor risk as all cash flows net to zero. Consequently for standard swaps the DV01 and PV01 calculations are the same. To convince yourself, try pricing a par swap on the Bloomberg terminal using **SWPM USD <GO>** - you will see that par swaps have a DV01 that match their PV01.

Now the PV01 is a trivial calculation equivalent to the unit annuity scaled by one basis point and multiplied by the trade notional. This information can be quickly gleaned from the swap schedule information shown in (figure 18). For non-standard swaps some practitioners quickly compute the swap DV01 risk using algorithmic differentiation (AAD), see (Burgess, 2022).



Curve Jacobian, J = dL/dP
Change in Libor rate per unit change in market par rates

	dP_{1Y}^{IRS}	dP_{2Y}^{IRS}	dP_{3Y}^{IRS}	dP_{4Y}^{IRS}	dP_{5Y}^{IRS}
dL_{1Y}^{IRS}	1.00	0.00	0.00	0.00	0.00
dL_{2Y}^{IRS}	-1.01	2.01	0.00	0.00	0.00
dL_{3Y}^{IRS}	0.00	-2.04	3.04	0.00	0.00
dL_{4Y}^{IRS}	0.00	0.00	-3.08	4.08	0.00
dL_{5Y}^{IRS}	0.00	0.00	0.00	-4.13	5.13

Swap Jacobian, dS/dL
Change in swap value per unit change in Libor Rate

	dL_{1Y}	dL_{2Y}	dL_{3Y}	dL_{4Y}	dL_{5Y}	
dS_{1Y}^{IRS}	982,281	0	0	0	0	
dS_{2Y}^{IRS}	982,281	969,579	0	0	0	
dS_{3Y}^{IRS}	982,281	969,579	957,671	0	0	
dS_{4Y}^{IRS}	982,281	969,579	957,671	945,574	0	
dS_{5Y}^{IRS}	982,281	969,579	957,671	945,574	933,074	
$dS_{4Y,5Y}^{IRS}$	0	0	0	0	933,074	*
$dS_{4.5Y}^{IRS}$	982,281	969,579	957,671	945,574	466,537	**

Shift Size, %
Change in market par rates

0.01%

Risk, dS/dP = dS/dL x dL/dP
Change in swap value per unit change in market par rates

	dP_{1Y}^{IRS}	dP_{2Y}^{IRS}	dP_{3Y}^{IRS}	dP_{4Y}^{IRS}	dP_{5Y}^{IRS}	Total	
dS_{1Y}^{IRS}	98	0	0	0	0	98	IRS(1Y)
dS_{2Y}^{IRS}	0	195	0	0	0	195	IRS(2Y)
dS_{3Y}^{IRS}	0	0	291	0	0	291	IRS(3Y)
dS_{4Y}^{IRS}	0	0	0	386	0	386	IRS(4Y)
dS_{5Y}^{IRS}	0	0	0	0	479	479	IRS(5Y)
$dS_{4Y,5Y}^{IRS}$	0	0	0	-386	479	93	Forward IRS(4Y,5Y)
$dS_{4.5Y}^{IRS}$	0	0	0	193	239	432	IRS(4.5Y)

$$\text{Swap Delta} = \frac{dS}{dP} = \frac{dS}{dL}.\frac{dL}{dP} \text{x Shift Size}$$

Figure 20: Illustration of Bucketed Swap DV01 calculations using Curve Jacobian(s)

[5] The PV01 denotes the present value of one basis point (bps).

As illustrated in (figure 20) bucketed DV01 risk calculations can be efficiently computed using the curve Jacobian and a swap trade Jacobian, see (Burgess 2021b). The secret sauce here is in knowing how to form the swap trade Jacobian accurately in the correct shape to agree with the curve Jacobian[6], which may involve risk redistribution if the pricing and hedging yield curves comprise of different calibration instruments.

6.4 Hardware, Cloud and Other Considerations

Finally we note that all of the above methods mentioned relate to optimizing code routines prior to using hardware and cloud acceleration tools. Once fully optimized the swap pricing, curve calibration and risk calibration calculations should reduce to simple vector and/or matrix multiplication operations, which can be performed in-real time. To squeeze additional latency we can look at parallelizing vector and matrix multiplication logic, see (Savine, 2018), as curve Jacobian matrices can be large. The end result is that we can build all curves, price swaps and perform risk calculations in real-time; this includes bucketed risk and full delta ladders.

6 Conclusion

In electronic rates market accuracy and low latency are threshold requirements that can present a barrier to market entry and are essential to for any business to succeed and capture market share in this space. We discussed how to achieve low latency by reducing swap calculations to a primitive optimal state comprising of raw vector and matrix operations, which when combined with curve Jacobian and AAD techniques dramatically improves pricing performance, curve construction and risk calculations. We can accelerate further using parallelization techniques and dedicated hardware solutions. All of these techniques when combined can be used capture arbitrage opportunities in electronic rates markets and give a competitive advantage.

[6] This is required for matrix multiplication purposes.

References

Burgess, N. (2016), How to Price Swaps in Your Head - An Interest Rate Swap & Asset Swap Primer Available at: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2815495

Burgess, N. (2019), Libor Benchmark Reform: An Overview of Libor Changes and Its Impact on Yield Curves, Pricing and Risk. Available at https://ssrn.com/abstract=3479833

Burgess, N. (2020), Strategic Analysis of Japanese Megabanks – Core Competency Development for Competitive Advantage. Available at: https://ssrn.com/abstract=3645553

Burgess, N. (2021a), Quant Notes - How to Solve & Minimize Complex Equations using the Newton-Raphson Method. Available at: https://ssrn.com/abstract=3893253

Burgess, N. (2021b), NYU Yield Curve Seminar - An Overview of Yield Curve Calibration & LIBOR Reform. Available at: https://ssrn.com/abstract=3820145

Burgess, N. (2021c), Machine Earning – Algorithmic Trading Strategies for Superior Growth, Outperformance and Competitive Advantage. International Journal of Artificial Intelligence and Machine Learning, 2(1), 38-60. doi: 10.51483/IJAIML.2.1.2022.38-60.1, Available at: https://ssrn.com/abstract=3816567

Burgess, N. (2021d), Interest Rate Swap Compounding Formula.
Available at: https://ssrn.com/abstract=3882163

Burgess, N. (2022), Algorithmic Adjoint Differentiation (AAD) for Swap Pricing and DV01 Risk. Available at: https://ssrn.com/abstract=4101486

Clarus (n.d.), MAC Swap Trading.
Available at: https://www.clarusft.com/mac-swap-trading/

Delta Quants (n.d.), Day count Conventions Described.
Available at: http://www.deltaquants.com/day-count-conventions

ION (n.d.), Simplify your IRS trading business
Available at: https://iongroup.com/markets/products/swaps/

LCH (n.d.), Essential Swap Clearing
Available at: https://www.lch.com/services/swapclear/essentials

OpenGamma (n.d.), Strata Day Count Conventions. Available at:
https://strata.opengamma.io/apidocs/com/opengamma/strata/basics/date/DayCounts.html

(Savine A., 2018), Textbook: Modern Computational Finance: AAD and Parallel Simulations, ISBN 978-1119539452. Available at: https://www.amazon.co.uk/Modern-Computational-Finance-Parallel Simulations/dp/1119539455

SwapEx (n.d.), SwapEx Contract Specifications, USD LIBOR Interest Rate Swaps: Fixed-to-Floating. Available at:
http://www.swapex.com/swapex2/media/cms_page_media/67/SwapEx%20Contract%20Specifications%20(USD%20LIBOR%20IRS).pdf