Tenor Basis Spread Formulae

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Abstract

A Tenor Basis Swap, also known as a floating-floating interest rate swap, is a financial instrument whereby floating cashflows from two different interest rates are exchanged, typically floating interest rates determined from benchmark Libor indices of the same currency are exchanged e.g. 3M Libor vs 6M Libor cashflows.

Investors trade Tenor Basis Swaps for several reasons including to hedge or switch floating cashflows to a more preferred fixing frequency, for duration or risk management or to manage liquidity risk . The instrument is of course also used for speculative purposes.

Tenor Basis Swaps are typically quoted in financial markets by the Tenor Basis Spread. In this paper we outline the present value calculation for Tenor Basis Swap pricing and demonstrate how to calculate the Tenor Basis Spread.

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Notation

In this paper we use the following notation.

n	number of fixed coupons
m	number of floating coupons
p	par rate
l_i	ith LIBOR forecast rate
s	floating spread over libor
N	swap notional
$P(t_0, t_i)$	discount factor for the ith coupon
ϕ	Tenor Basis Indicator Function +1 when receiving the long tenor Libor Index and -1 when paying
τ_i	ith accrual period or year fraction
t_0	time to the valuation date in years, usually zero
t_i	time to the ith coupon payment date in years

1 Tenor Basis Swaps

A Tenor Basis Swap, also known as a floating-floating interest rate swap, is a financial instrument whereby floating cashflows from two different interest rates are exchanged, typically floating Libor[1] indices of the same currency are exchanged e.g. 3M Libor vs 6M Libor cashflows.

This instrument provides investors with a mechanism to switch borrowing / lending payments to a different frequency of choice. The produce has several uses, the more prominent being

- As a liquidity tool
- As a cashflow replication mechanism
- As a duration or risk management mechanism
- For speculative purposes

2 Tenor Basis Swap Pricing

A tenor basis swap can be priced using the following formula, for a 3x6 tenor basis swap[2].

[1] London Interbank Offer Rate

[2] 3x6 denotes that we are exchanging 3M Libor interest payments for 6M Libor payments

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$$PV(\text{Tenor Basis 3x6}) = \phi \left(\sum_{i=1}^{n} l_i^{6m} \tau_i^{6m} P(t_0, t_i) - \sum_{j=1}^{m} (l_j^{3m} + s) \tau_j^{3m} P(t_0, t_j) \right) \tag{1}$$

in the general case we write

$$PV(\text{Tenor Basis}) = \phi \left(\sum_{i=1}^{n} l_i \tau_i^{LongIndex} P(t_0, t_i) - \sum_{j=1}^{m} (l_j^{ShortIndex} + s) \tau_j^{ShortIndex} P(t_0, t_j) \right) \tag{2}$$

where long index denotes the Libor index with the longer tenor and short index the Libor index with the shorter tenor.

3 Tenor Basis Swap Spreads

Tenor Basis Swaps usually quote at par, meaning that the price of the swap is zero and that the PV or present value of each of the trade legs is the same. So in the case of a 3x6 Tenor basis swap the present value of the 3M Libor interest payments equals the present value of the 6ML payments. This is made possible by adding a basis spread s to the Libor Index with the lower frequency tenor in this case the 3M Libor leg.

Tenor Basis Swaps are typically quoted in the market place as Basis Spread s for a fixed maturity using the below formula, which is a rearrangement of (1) with the PV set to zero. The basis spread informs investors what adjustment or spread to add to the lower tenor Libor index so that the interest payments on both legs of the basis swap are equivalent. Rearranging equation (1) with $PV = 0$ leads to the following expression for the basis spread.

$$\begin{aligned} s &= \left(\frac{\sum_{i=1}^{n} l_i^{6m} \tau_i^{6m} P(t_0, t_i) - \sum_{j=1}^{m} l_j^{3m} \tau_j^{3m} P(t_0, t_j)}{\sum_{j=1}^{m} \tau_j^{3m} P(t_0, t_j)} \right) \\ &= \left(\frac{\sum_{i=1}^{n} l_i^{6m} \tau_i^{6m} P(t_0, t_i) - \sum_{j=1}^{m} l_j^{3m} \tau_j^{3m} P(t_0, t_j)}{Annuity^{3m}} \right) \end{aligned} \tag{3}$$

likewise for the general case we have

$$\begin{aligned} s &= \left(\frac{\sum_{i=1}^{n} l_i^{LongIndex} \tau_i^{LongIndex} P(t_0, t_i) - \sum_{j=1}^{m} l_j^{ShortIndex} \tau_j^{ShortIndex} P(t_0, t_j)}{\sum_{j=1}^{m} \tau_j^{ShortIndex} P(t_0, t_j)} \right) \\ &= \left(\frac{\sum_{i=1}^{n} l_i^{LongIndex} \tau_i^{LongIndex} P(t_0, t_i) - \sum_{j=1}^{m} l_j^{ShortIndex} \tau_j^{ShortIndex} P(t_0, t_j)}{Annuity^{ShortIndex}} \right) \end{aligned} \tag{4}$$

4 Conclusion

In summary we reviewed the present value calculation formula for a Tenor Basis Swap and how to calculate the Tenor Basis Swap spread, the later being how Tenor Basis Swaps are quoted in the financial marketplace.

References

[1] **Hull**, J (2011) Textbook: Options, Futures and Other Derivatives 8ed, Pearson Education Limited