
Pressure Drop Prediction of Coriolis Mass Flowmeters

Tony Pankratz and Gary Pawlas

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Abstract:

Variables:

Variables	Description	Units
Δp	Pressure drop	Pa
ρ	Fluid Density	Kg/m ³
V	Tube fluid velocity	m/s
f	Friction factor	
L	Total Flow tube length	m
D_H	Hydraulic diameter	m
ID	Internal Tube diameter	m
D_T	Flow tube internal diameter	m
nbends	# of flow tube bends	
bendangle	Bend angle for each flow tube bend	degrees
K_B	Loss coefficient for tube bends	
K_M	Loss coefficient for flanges and manifold	
e	Relative roughness	m
Re	Reynolds # = $\frac{\rho V D_H}{\mu}$	
A	Fluid cross sectional area = $\pi \cdot \left(\frac{ID}{2}\right)^2$	m ²
WP	Wetted Perimeter = $\pi \cdot ID$	m
R_C^2	Curve fit parameter	
A_1 - A_5	Coefficients for the K_M equation	
\dot{m}	Total mass flow rate	kg/s
R	Tube bend radius	m
μ	Viscosity	cp

Introduction:

The need for an accurate pressure drop prediction is very important to Micro Motion. The sales force uses the meter sizing program every day to predict pressure drop across a Coriolis

Mass Flowmeter (CMF). An accurate pressure drop prediction is also needed when making viscosity measurements. The previous pressure drop equations in the sizing program predicts pressure drop fairly well, if the fluid has a viscosity similar to water. However, the previous pressure drop equations over-predicts, sometimes as much as 50%, the pressure drop for gaseous process fluids.

The work contained in this document was driven by a need to make a more accurate pressure drop prediction for gaseous process fluids and to add a pressure drop prediction for Basis (our new product line). Time was spent reviewing the previous pressure drop procedure with little progress made in understanding the procedure. Therefore, a new procedure (the current procedure) was developed and implemented. The idea and basis for this procedure came from previous work completed by Gary Pawlas and Tony Pankratz in an engineering report titled "Pressure Drop and Computational Fluid Dynamics" (#10130), Reference (3).

A CMF (from a fluid mechanics point of view) is made up of three major components, straight tube sections, curved tube sections, and flanges/manifolds. Equation (1) is now being used to predict the pressure drop of a CMF that incorporates the pressure drop associated with the three components. In Equation (1) the three terms in the brackets account for the pressure drop associated with the straight tubes, the curved tube sections, and the manifolds, respectively. Notice that Equation (1) is only dependent on meter geometry, fluid properties, and tube roughness.

$$\Delta P = \frac{\rho \cdot V^2}{2} \left[\underbrace{f \cdot \frac{L}{D_H}}_{\text{Straight Tube}} + \underbrace{(nbends) \left(\frac{\text{bendangle}}{90.0} \right) \cdot K_B}_{\text{Curved Tube}} + \underbrace{K_M}_{\text{Manifold}} \right] \quad (1)$$

Handwritten annotations in red:

- Fluid density (pointing to ρ)
- mean fluid velocity (pointing to V)
- friction factor (pointing to f)
- total flow tube length (pointing to L)
- hydraulic diameter (pointing to D_H)
- curved tube coef. (pointing to K_B)
- manifold coef. (pointing to K_M)

Background:

In most fluid mechanics texts there is a section that explains viscous flow in pipes, Reference [1], Reference [2]. As a subset of viscous flow a section is contributed to pressure drop calculations in piping systems. All the equations described in this paper except Equation (1) can be found in most fluid mechanics texts. Equation (1) shows the relation describing the pressure drop in a CMF, where ρ is the fluid density, V is the mean fluid velocity, f is the friction factor, L is the total flow tube length, D_H is the hydraulic diameter, K_B is a curved tube coefficient, and K_M is the manifold coefficient.

The friction factor, f , in Equation (1) can be obtained using the Moody chart (Figure (1)) or the Colebrook equation (Equation (2)). In Equation (2), $\frac{\epsilon}{D_H}$ is called the relative roughness of

the tube wall which is determined by Figure (2). The value of ϵ that is used for all Micro Motion drawn tubing is 0.000001524 m (0.000005 ft), which was determined by experimentation, see Ref (3). In order to solve the Colebrook equation an iterative solution scheme must be used. However, Haaland (Reference [2], pp. 314) published an equation, Equation (3), that deviates less than 2% from the Colebrook equation over the entire range of turbulent Rey-

holds numbers. Using Equation (3) one can solve directly for the friction factor instead of using an iterative method. Therefore, the Haaland equation is used to calculate the friction factor in this work.

colebrook equation \rightarrow solve for f using iterative method

$$\frac{1}{\sqrt{f}} = -2.0 \cdot \log \left(\frac{\frac{\epsilon}{D_H}}{3.7} + \frac{2.51}{Re\sqrt{f}} \right) \quad (2)$$

Haaland eqn: can solve for f directly, deviates less than 2% from colebrook

$$f \approx \left[-1.8 \cdot \log \left(\frac{6.9}{Re} + \left(\frac{\frac{\epsilon}{D_H}}{3.7} \right)^{1.11} \right) \right]^{-2} \quad (3)$$

The flow tubes hydraulic diameter D_H in Equation (3) is calculated using Equation (4), where A is the cross-sectional area and WP is the wetted perimeter. Note for circular tubes D_H reduces to the internal tube diameter.

$$D_H = \frac{4 \times A}{WP} = ID \quad (4)$$

\nwarrow cross-sectional area
 \swarrow for circular tubes
 \nwarrow wetted perimeter

The velocity of the fluid in the flow tubes, appearing in Equation (1) can be calculated using Equation (5), where \dot{m} is the total mass flow rate, and A is the flow cross sectional area. Note the velocity is the fluid flow tube velocity, not the process line fluid velocity, and is based on half the total mass flow rate. The Reynolds number, Re , can be calculated using Equation (6).

$$V = \frac{\left(\frac{\dot{m}}{2} \right)}{(\rho A)} \quad (5)$$

\nwarrow total mass flow rate
 \swarrow flow cross-sectional area

$$Re = \frac{\rho V D_H}{\mu} \quad (6)$$

\nwarrow fluid flow tube velocity (not process line fluid velocity)

The curved tube coefficient, K_B , in Equation (1) can be obtained from Figure (3). The value of $\frac{\epsilon}{D_H}$ used for calculating K_B is 0. K_B in Figure (3) is only valid for one 90° bend, so the

multiplier $\left[(\text{nbends}) \left(\frac{\text{bendangle}}{90.0} \right) \right]$ was added in Equation (1) to account for CMFs that have more than one bend and bends that are different than 90° . No equation was given for the “ $\frac{\varepsilon}{D_H} = 0$ ” curve shown in Figure (3), so the line was digitized, plotted, and the data was used to obtain a least squared polynomial curve fit equation (Equation (7)).

$$K_B = 1 \times 10^{-4} \left[\frac{R}{D_H} \right]^4 - 3.5 \times 10^{-3} \left[\frac{R}{D_H} \right]^3 + 3.18 \times 10^{-2} \left[\frac{R}{D_H} \right]^2 - 0.1382 \left[\frac{R}{D_H} \right] + 0.3288 \quad (7)$$

The term K_M (M stands for manifold) in Equation (1) has to be determined by experimental analysis, which is described below in the **Procedure** section.

In review, a pressure drop calculation can be made for any style CMF using Equation (1) with Equations (3)- (7), and with an experimentally obtained equation for K_M . The flow information needed to make a pressure drop prediction is the mass flow rate, \dot{m} , density, ρ , viscosity, μ , and the dimensions of the CMF.

Appendix A contains a discussion of the pressure drop associated with the transition region from laminar to turbulent flow. The reason for the inclusion of Appendix A in this report is to contradict claims by other Mass flow companies that pressure drop can be very low at Re less than 4000.

Procedure:

The procedure for calculating pressure drop is shown in Figure (4). Figure (4) provides a concise one page summary of the procedure used to calculate pressure drop across a CMF. All the work contained in this paper was done in a spreadsheet, Excel (Reference [4]), so the description of the procedure will follow the format of the Excel documents. Please note that this procedure described below must be done for every **Micro Motion meter**. There is no universal pressure drop equation for all CMF.

for every meter? or for every meter type?

The first step in the procedure, as shown in Figure (4), is to collect and plot pressure drop versus mass flow rate data. The data should include mass flow rates that encompass the entire flow range of the meter. Mass flow rates that correspond to pressure drop values of less than 1 psi should not be included in the data set, due to uncertainties in the pressure drop measurement. For a discussion on the pressure drop uncertainties see Appendix B.

The second step in the procedure is to obtain a least squares curve fit equation for the pressure drop versus mass flow rate data collected in Step 1. The equation used to curve fit the data is a second order polynomial with a y intercept of zero [$\Delta P = B_1 \cdot \dot{m}^2 + B_2 \cdot \dot{m} + 0$]. The coefficients B_1 and B_2 are obtained from the curve fit procedure, (Excel was used to curve fit the data). Figure (5) shows an example set of data for the CMF100 along with the curve fit equation. Appendix A discusses the assumption of using a second order polynomial to curve fit the water pressure drop data. Also, Appendix A discusses the transition region between laminar and turbulent flow.

The third step shown in Figure (4) is to calculate values for K_M using Equation (8). The equation obtained in Step 2 is used to calculate the pressure drop values, Equation (3) is used to calculate f , and Equation (7) is used to calculate values for K_B .

$$K_M = \frac{2\Delta P}{\rho \cdot V^2} - f \frac{L}{D_H} - ((nbends) \left(\frac{bendangle}{90.0} \right) \cdot K_B) \quad (8)$$

Figure (6) shows an example of the K_M worksheet from Excel, which shows how the K_M values are calculated. The spreadsheet in Figure (6) was created by first entering the mass flow rate and pressure drop data. Next the flow tube velocity was calculated using Equation (5). Now the Re was calculated using Equation (6). Knowing the Re the friction factor, f , was calculated using Equation (3). Equation (7) was then used to calculate K_B . Finally K_M was calculated using Equation (8).

The fourth step is to plot the values of K_M versus Re . If high viscosity fluid pressure drop data is available the two K_M data sets can be combined and used in step 5.

The Fifth step in the procedure is to curve fit the data plotted in step 4. The equation used to curve fit the K_M data is given in Equation (9), where A_1 - A_5 are the constants obtained from curve fitting the data. Figure (7) shows an example of water K_M data and the curve fit equation.

$$K_M = A_1 + A_2 \cdot e^{(-A_3 Re)} + A_4 \cdot e^{(-A_5 Re)} \quad (9)$$

Equation (9) was used to fit the data for all the Basis and Elite meters to a R_C^2 value of at least 0.9995. The better the fit (an R_C^2 value of 1.0 is a perfect fit) the better the pressure drop prediction will be. Table Curve 2.0 was the software used to curve fit the K_M data, Reference [5].

When low Re K_M data is combined with water K_M data there is a discontinuity in the data around a Re of 1000, see Figure (8). The discontinuity is related to the change in the friction factor term at a Re of 946, see Appendix A. In order to curve fit the K_M data with a high accu-

7 racy two equations are used. One equation is for the low Re (below 946) and the second equation is for high Re (above 946). Figure (8) shows that there is a smooth transition between the two equations.

The sixth step is to calculate the pressure drop using Equation (1), combined with Equations (3)- (7), and (9).

Appendix C contains the document that was given to MIS describing the exact equations and program flow that was added to the sizing program.

Results

The results presented in this section are for a CMF 100, using only water for the curve fit of the K_M data unless other wise stated. However, similar results were obtained for the rest of the Elite and Basis lines. All the graphs presented in this section are plotted versus mass flow rate and Reynolds number. The Reynolds number graphs provide more insight into the effects of viscosity, and density on pressure drop.

Figure (9) shows the pressure drop prediction (water: $\rho=1000 \text{ Kg/m}^3$, $\mu=1.0 \text{ Cp}$) for the previous method, the current method, and the experimental data. Figure (10) shows the percent error for the previous and current methods. Notice the current method's percent error is well within $\pm 1\%$ when the mass flow rate is above 100 lb/min. The large errors associated with low flow rates can be explained by the pressure drop associated with mass flow rates below 100 lb/min are below 1 psi, which is the region were the pressure drop is uncertain. Figure (10) also shows the previous method on average over predicts the pressure drop by 20%. The new method shows a substantial improvement in the pressure drop prediction for water.

Figure (11) shows experimental pressure drop data for gaseous air with the current pressure drop prediction. The current method fits the experimental data very well. In order to clarify the pressure drop prediction shown in Figure (11) the 50 and 1400 psi data was plotted and analyzed. Figure (12) shows a comparison between experimental data, the current method, and the previous method for gaseous air at 50 and 1400 psi. The current pressure drop method is more accurate in predicting the actual pressure drop than the previous method, as can be seen in Figures (13) and (14). The current prediction method reduces the average % error from 20-50% (previous method) for the 50 psi gaseous air to about 0-14% (current method), and from 20-50% (previous method) to $\pm 3\%$ (current method) for the 1400 psi gaseous air.

The results presented so far are for water and gaseous process fluids. Figure (15) shows experimental data along with the predicted values for a water corn syrup mixture. The density and viscosity of the fluid was 77.15 lb/ft^3 (1235.9 Kg/m^3) and 13.3 Cp , respectively. Figure (16) shows the %error for the current and previous pressure drop prediction methods. In Figure (16) the previous method predicts pressure drop better than the current method when the mass flow rate is under 125 lb/min, which corresponds to pressure drops under 1 psi. However, the current method predicts pressure drop more accurately than the previous method when the mass flow rate is grater than 125 lb/min, as seen in Figure (16). The current method

7 provides some improvement in the pressure drop prediction over the previous method, but not as good as the improvement for water and gaseous process fluids.

| The reason for the poor pressure drop prediction can be seen in Figure (17), which shows water and corn syrup K_M values. Only water K_M values were used in the curve fit procedure in Step 5 of Figure (4). The values for K_M only go down to a Re of approximately 38,000, which does not cover the lower Re region where high viscosity fluids reside.

Improved pressure drop predictions can be obtained when K_M values that cover the entire Re range are included, as can be seen in Figure (18). Figure (18) shows experimental pressure drop data along with predicted values for three different viscosity fluids. The 200 cp fluid was the pure Sweetose corn syrup, the 20 cp fluid was a diluted mixture of Sweetose corn syrup and water, and the third fluid was water. Notice in Figure (18), that the variance between the five data points at each flow rate increases as the viscosity decreases. This variance is most likely due to the increase in turbulence as the Re increases. In short, the more turbulent the fluid flow (i.e. the higher the Re), the more variance in the pressure measurement. Another thing to notice in Figure (18) is that as the pressure drop falls below 1 psi the variance in the measurement is significant, which makes the data points un-usable. Figure (19) shows the % error for the predicted values for the three fluids. One interesting thing about Figure (19) is that over a very large Re range the % error is only $\pm 3\%$.

| One last important thing about the pressure drop prediction. The pressure drop prediction does not take into account for differences in customer installations, differences in flanges, and pressure tap placement and size. Mounting conditions and pressure tap placement both can effect the pressure drop measurement which in turn affects the % error of the predicted values. The meters that were tested were "normal meters", i.e. meters with standard flanges. Figure (20) shows a typical meter installation as the pressure drop data was taken. The distance from the flanges to the pressure taps varies from meter size to meter size, but most distances are around 5 inches on the inlet side and 10 inches on the outlet side.

Knowing that the pressure drop is very dependent on mounting and pressure tap location the accuracy of the pressure drop prediction can only be $\pm 10\%$. However, if the customer is very care full in mounting the meter (i.e. matching the flanges and pipe diameters) the actual pressure drop value can be lower than predicted. The pressure drop prediction is a good average of what the customers will actually experience.

Summary:

The results show an improvement in the pressure drop prediction for water and gaseous process fluids over the previous pressure drop prediction. The goals of this work are to improve the pressure drop prediction for gaseous process fluids and to add the Basis product line to the sizing program. The improvement in the pressure drop prediction using the new method meets

the two stated goals and was implemented. Pressure drop data was collected using higher viscosity fluids which improve the pressure drop prediction for viscous fluids to be within $\pm 10\%$. Currently the new pressure drop method has been implemented for the Basis and the Elite lines.

Bibliography

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2. White, Frank M., **Fluid Mechanics**, New York, New York, McGraw-Hill Publishing Company, 1986, pp. 287-371.
3. Gary Pawlas, Tony Pankratz, "Pressure Drop and Computational Fluid Dynamics", an internal MMI report #10130, 1994.
4. Microsoft Corporation, Excel, 1985-1994.
5. Jandel Scientific, TableCurve 2d Automated Curve Fitting Software, 1989-1994.

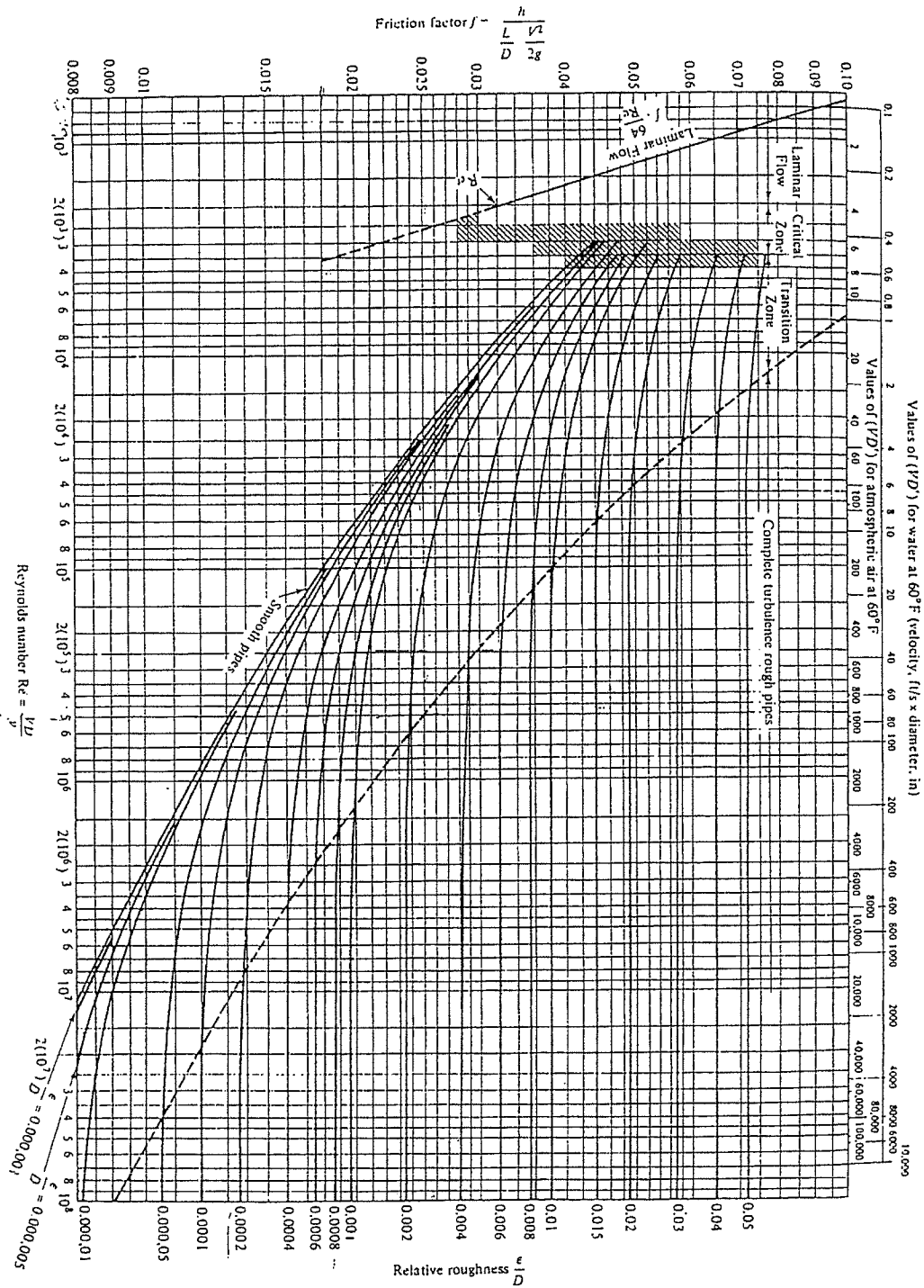


FIGURE 1. The Moody Chart for pipe friction with smooth and rough walls (From Reference [1])

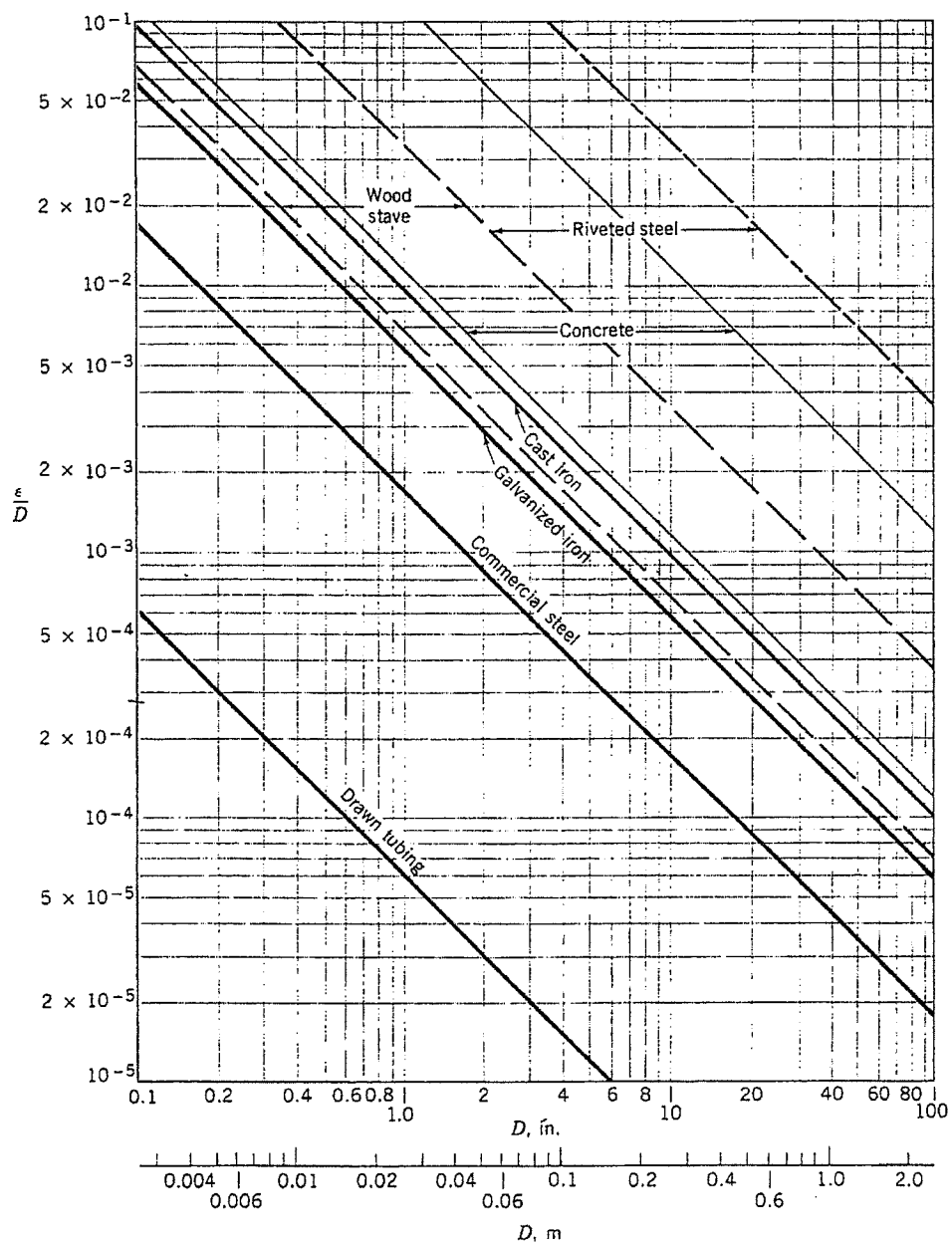


FIGURE 2. Relative Roughness of new pipes (From Reference [1])

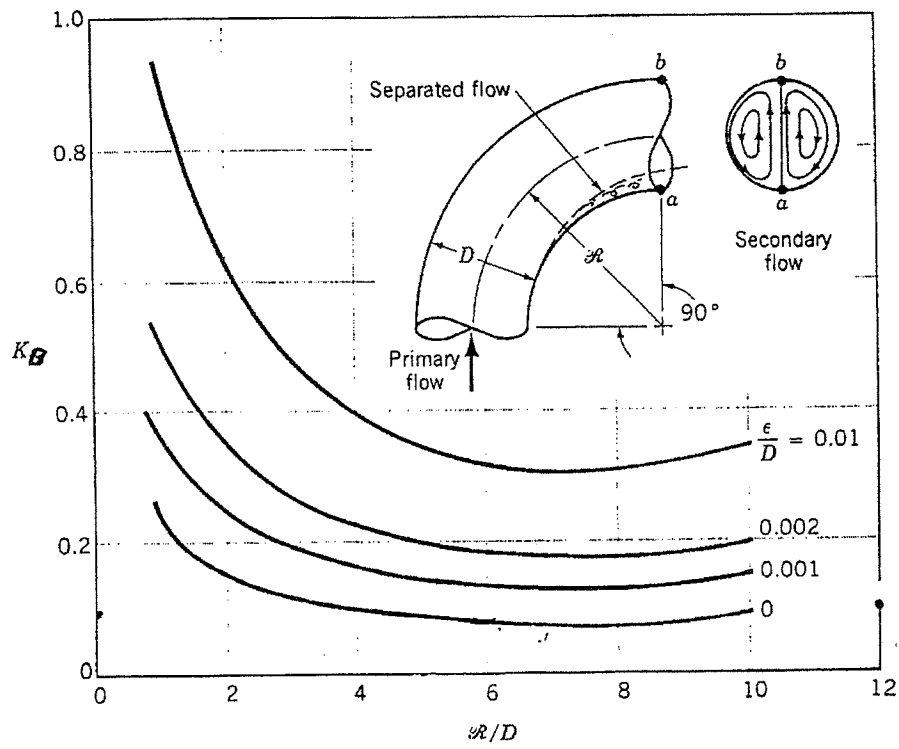
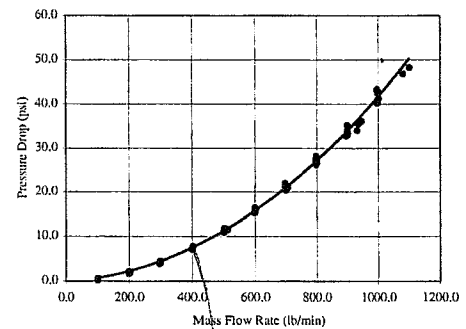


FIGURE 3. K_B minor loss coefficient (From Reference [1])

Step 1

Collect and plot Pressure drop vs. Mass Flow rate (usually water is the fluid). Try to obtain data for the entire flow range of the meter



Step 2

Obtain curve fit equation for pressure drop vs. mass flow rate data.

$$\Delta P = B_1 \cdot \dot{m}^2 + B_2 \cdot \dot{m} + 0$$

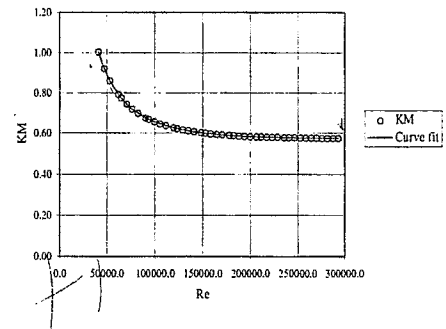
Step 3

Use Equation (6) with the equation obtained in Step 2 to calculate values of K_M .

$$K_M = \frac{2\Delta P}{\rho \cdot v^2} - f \frac{L}{D_H} - \left((nbends) \left(\frac{bendangle}{90.0} \right) \cdot K_B \right)$$

Step 4

Plot the K_M values obtained in Step 3 vs. Re. K_M values from higher viscosity fluids also can be used. It is important to cover the entire Re range of the meter, especially the low Re's.



Step 5

Use this equation to obtain a curve fit equation for the K_M data. Two curve fit equations are needed one for high Re and one for low Re.

$$K_M = A_1 + A_2 \cdot e^{(-A_3 Re)} + A_4 \cdot e^{(-A_5 Re)}$$

Step 6

Calculate pressure drop using Equation (1) with Equations (3)-(7), and the equations obtained in Step 5.

$$\Delta P = \frac{\rho \cdot v^2}{2} \left[f \cdot \frac{L}{D_H} + (nbends) \left(\frac{bendangle}{90.0} \right) \cdot K_B + K_M \right]$$

FIGURE 4. Graphical representation of the Procedure used to obtain an equation for K_M

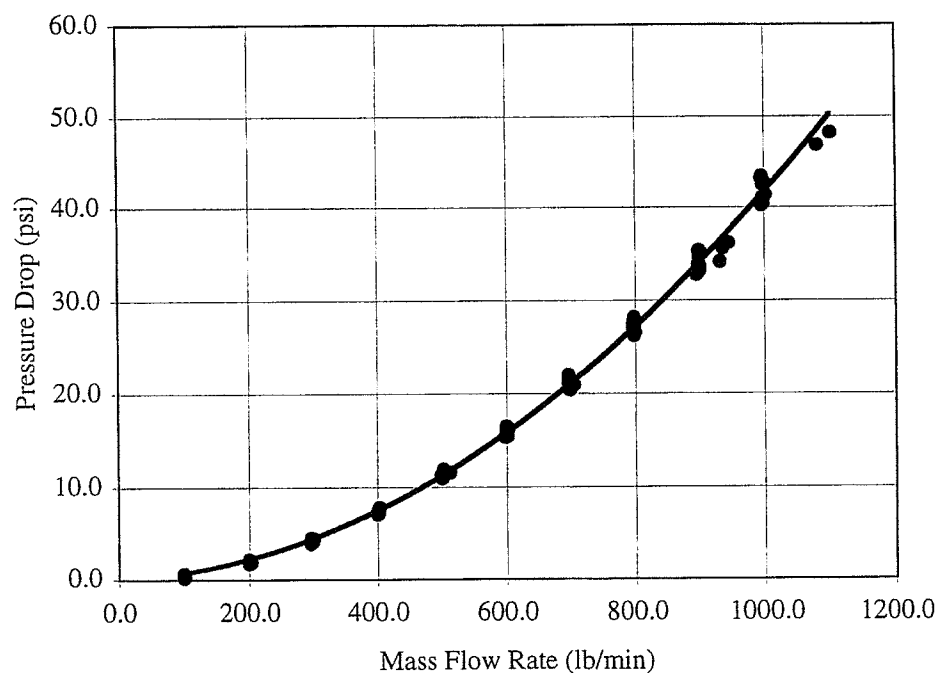


FIGURE 5. Pressure drop data for a CMF 100 (process fluid is water)

Mdot (lb/min)	Δp (psi)	V Tubes (ft/s)	Re	f Haaland	KB	KM
13.00	0.05	0.7617	3815.1	0.04109	0.1112	9.2081
40.00	0.19	2.3436	11738.9	0.02967	0.1112	2.9340
60.00	0.32	3.5153	17608.4	0.02673	0.1112	1.9928
80.00	0.49	4.6871	23477.8	0.02490	0.1112	1.5426
111.00	0.82	6.5034	32575.5	0.02306	0.1112	1.1840
120.00	0.93	7.0307	35216.8	0.02266	0.1112	1.1174
140.00	1.19	8.2024	41086.2	0.02189	0.1112	1.0033
160.00	1.48	9.3742	46955.7	0.02126	0.1112	0.9210
180.00	1.81	10.5460	52825.1	0.02073	0.1112	0.8592
210.00	2.35	12.3037	61629.3	0.02007	0.1112	0.7919
220.00	2.55	12.8896	64564.1	0.01988	0.1112	0.7742
240.00	2.96	14.0613	70433.5	0.01953	0.1112	0.7441
260.00	3.41	15.2331	76303.0	0.01922	0.1112	0.7195
280.00	3.89	16.4049	82172.4	0.01894	0.1112	0.6992
309.00	4.64	18.1040	90683.2	0.01858	0.1112	0.6756
320.00	4.94	18.7484	93911.4	0.01845	0.1112	0.6680
340.00	5.51	19.9202	99780.8	0.01824	0.1112	0.6559
360.00	6.11	21.0920	105650.3	0.01804	0.1112	0.6456
380.00	6.75	22.2638	111519.8	0.01786	0.1112	0.6367

FIGURE 6. K_M spreadsheet example for a CMF 100 (process fluid is water)

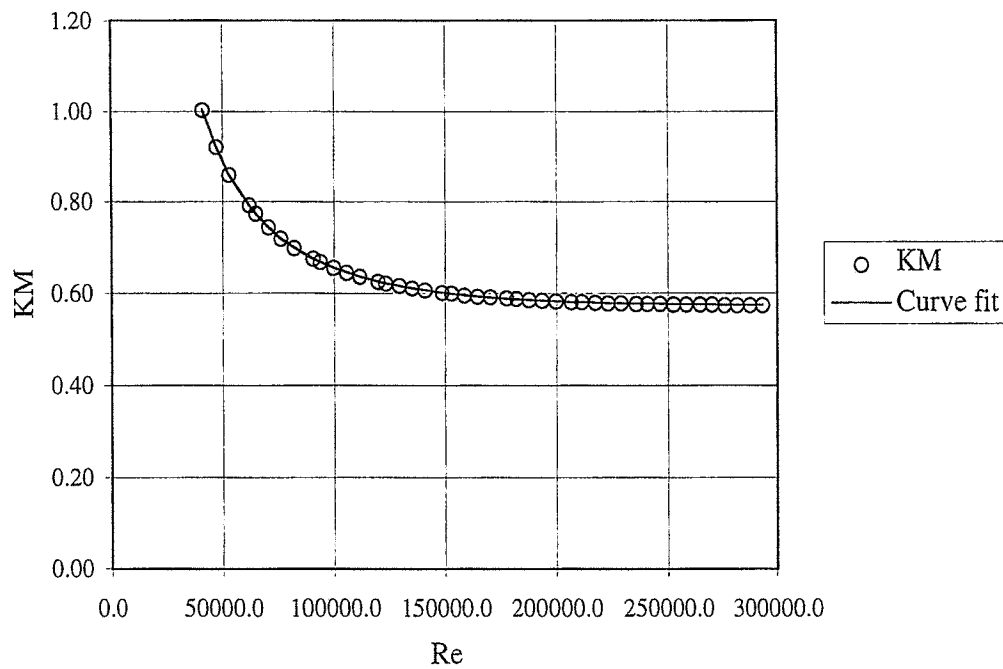


FIGURE 7. K_M chart example for a CMF 100 (process fluid is water)

CMF 100

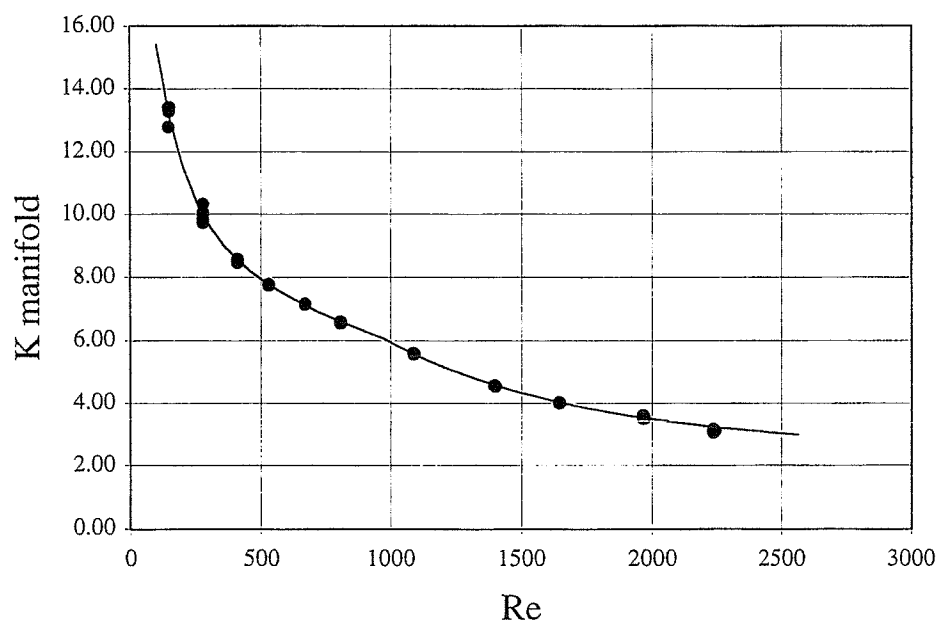
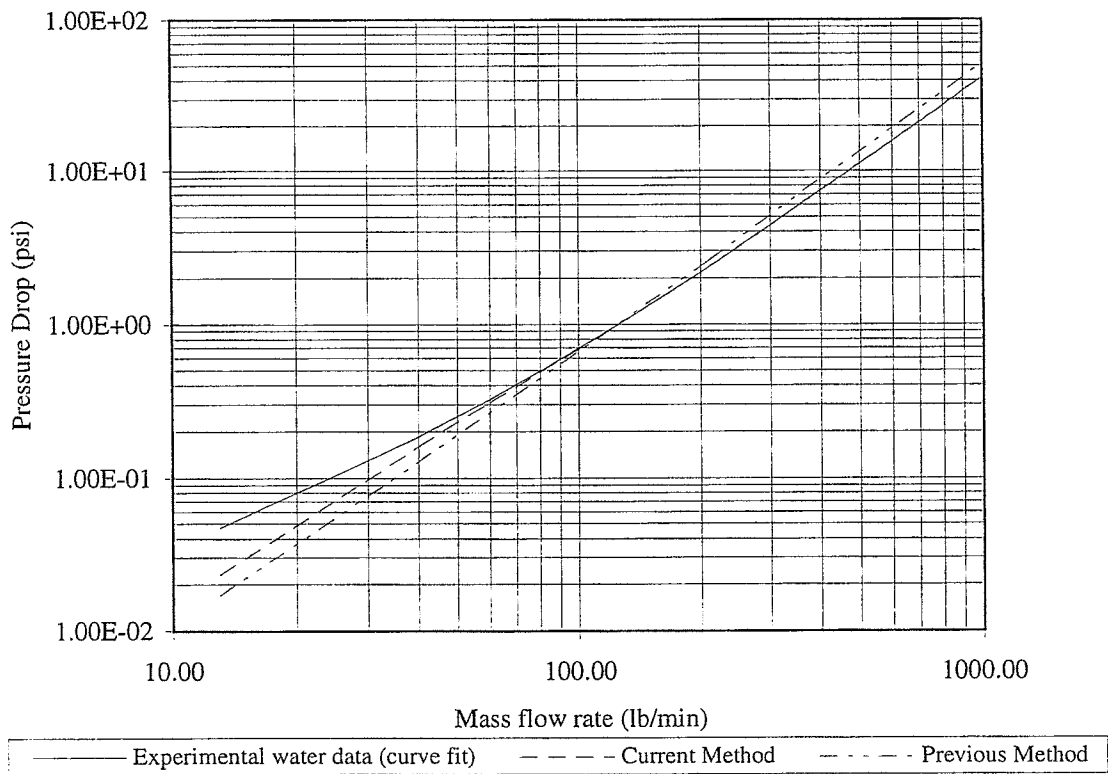
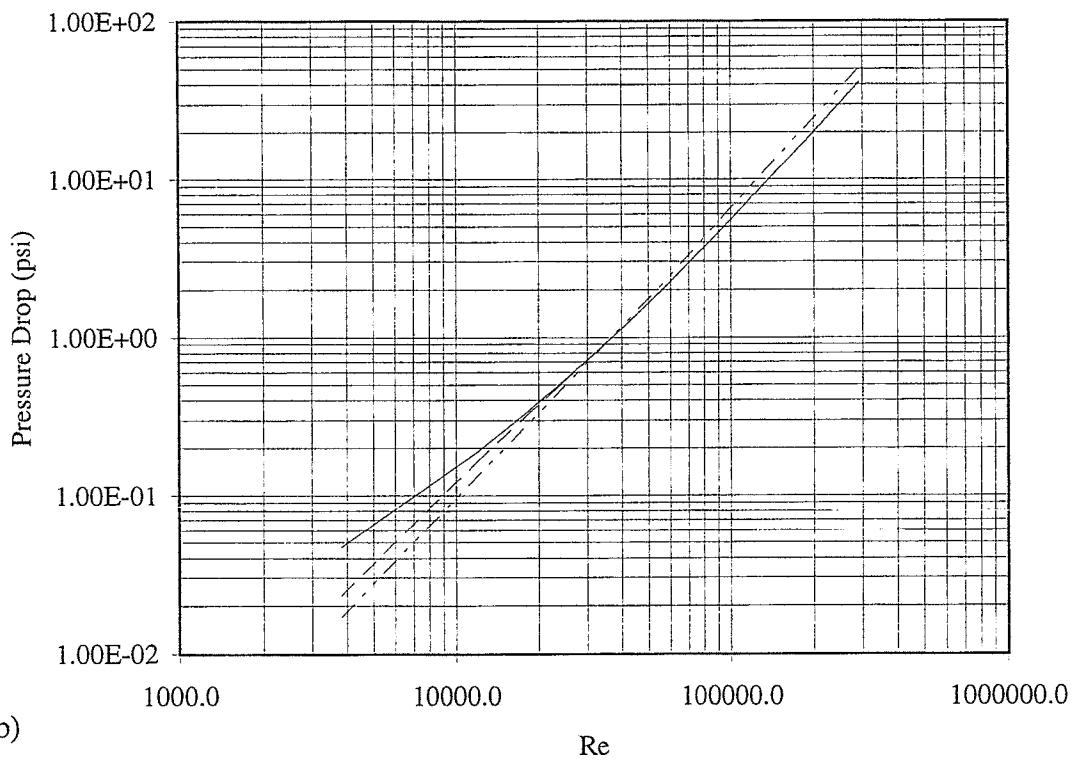


FIGURE 8. K_M chart example for a CMF 100 Combining water and corn syrup data



a)



b)

FIGURE 9. CMF 100 Pressure Drop Prediction and % error for water

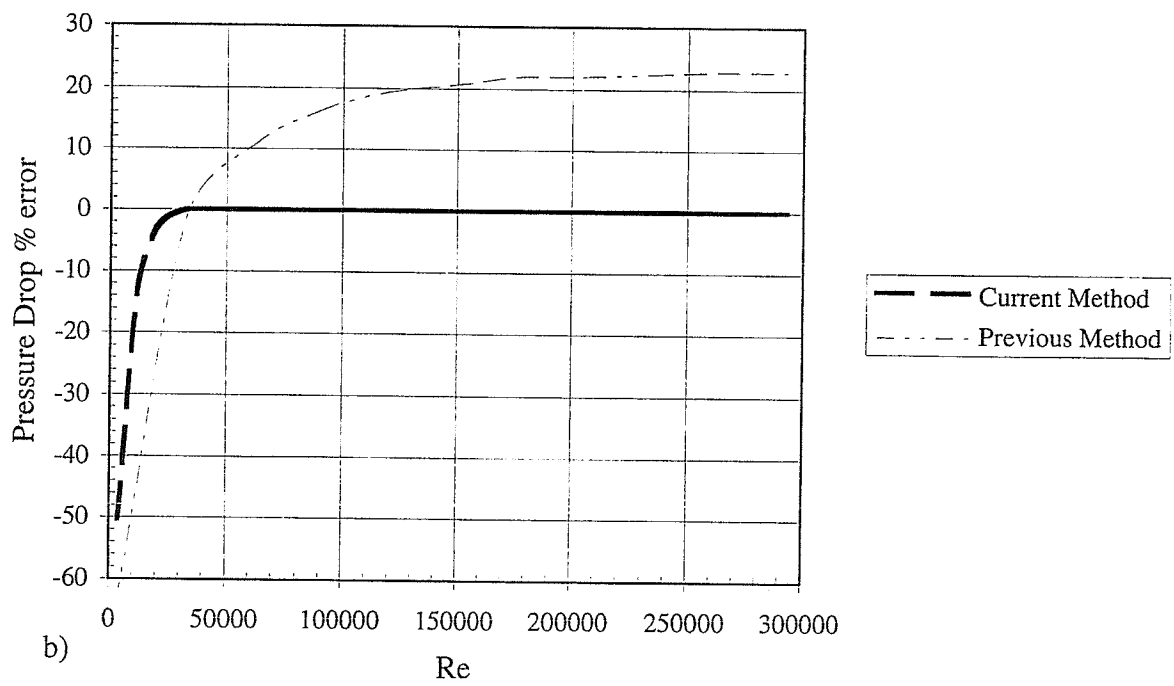
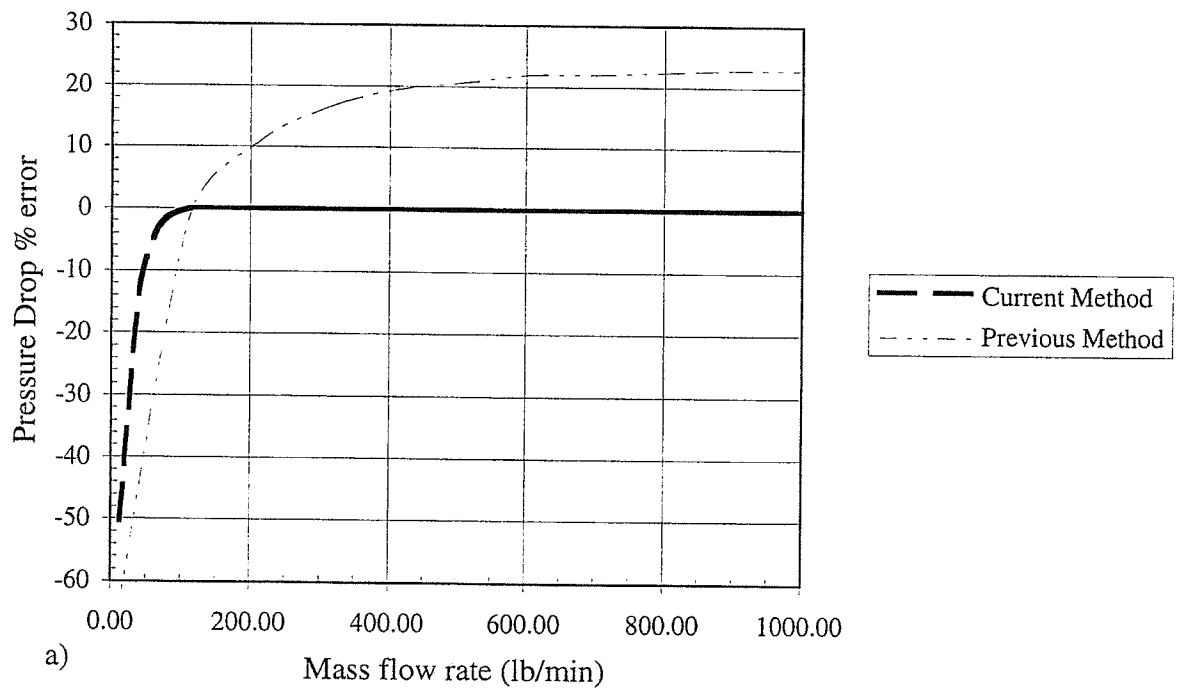


FIGURE 10. CMF 100 Pressure Drop Prediction and % error for water (Re)

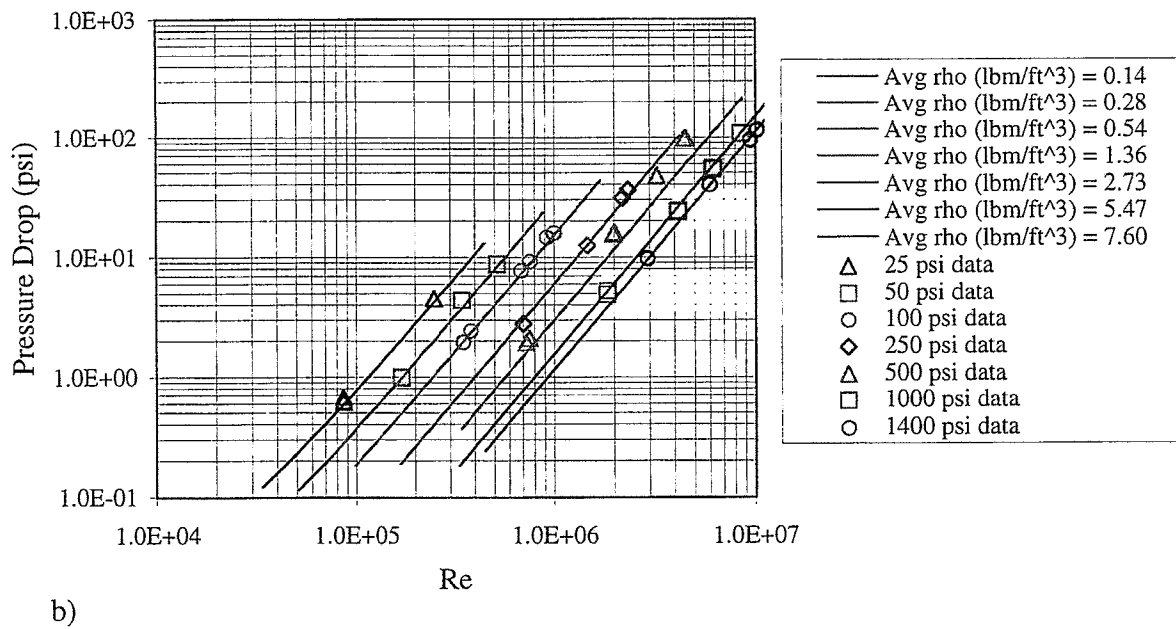
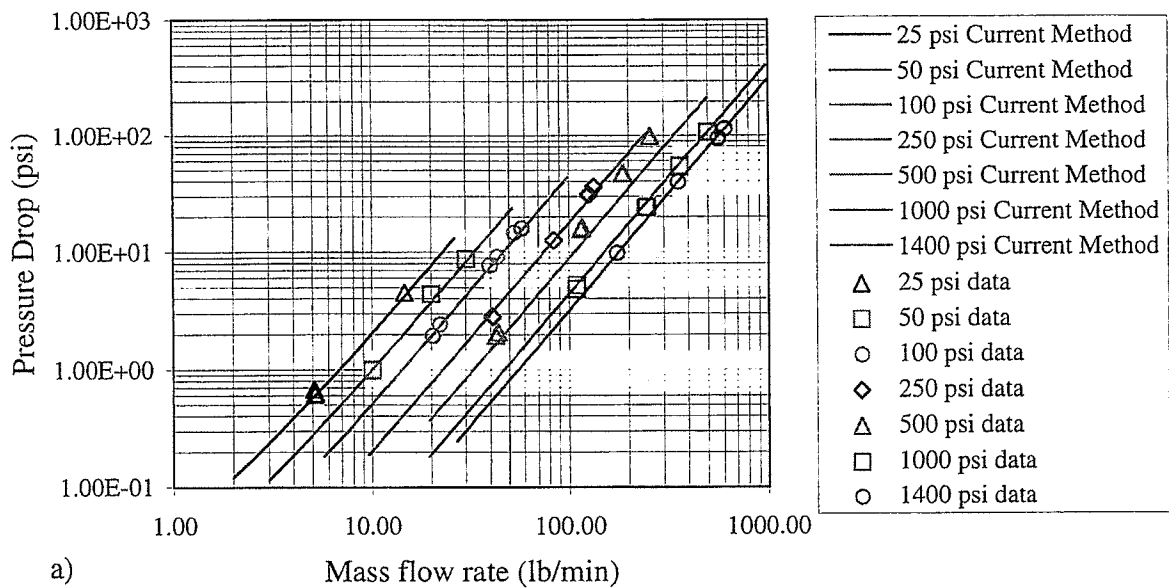
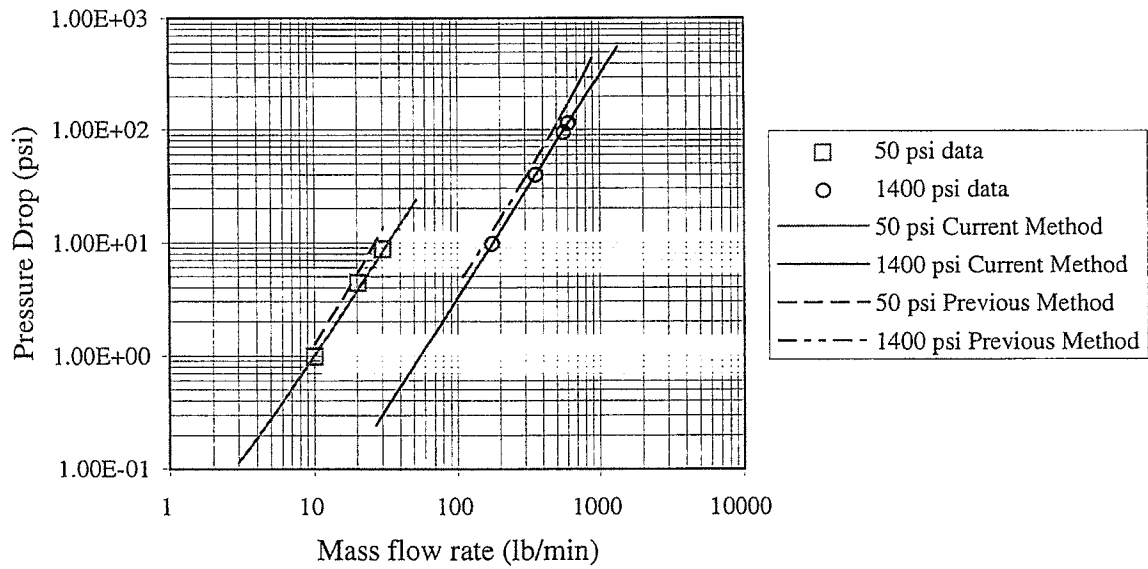
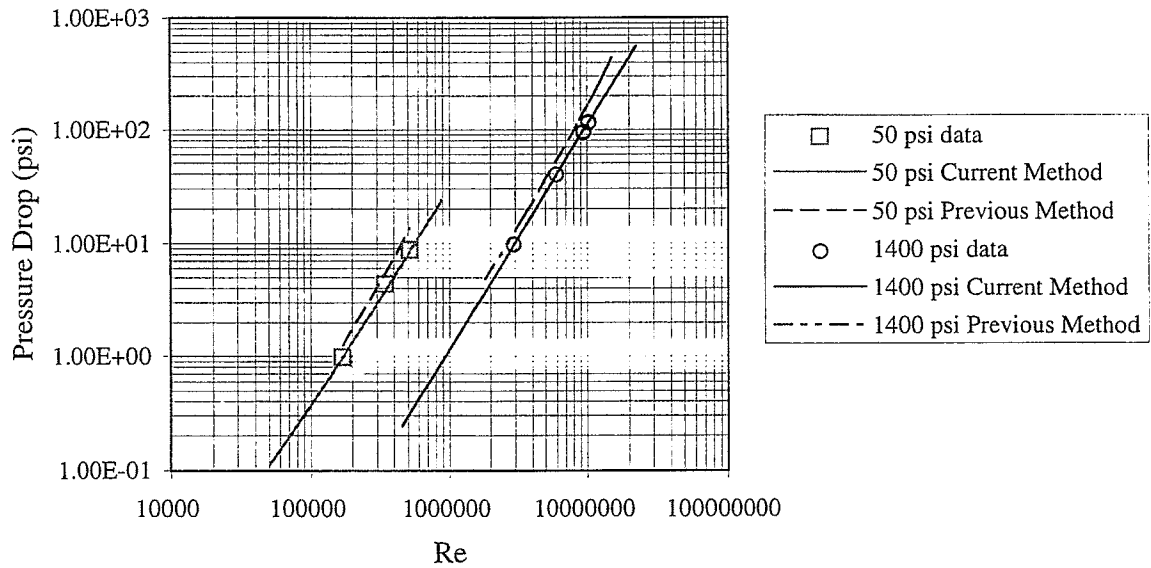


FIGURE 11. CMF 100 gaseous air pressure drop prediction (new method)

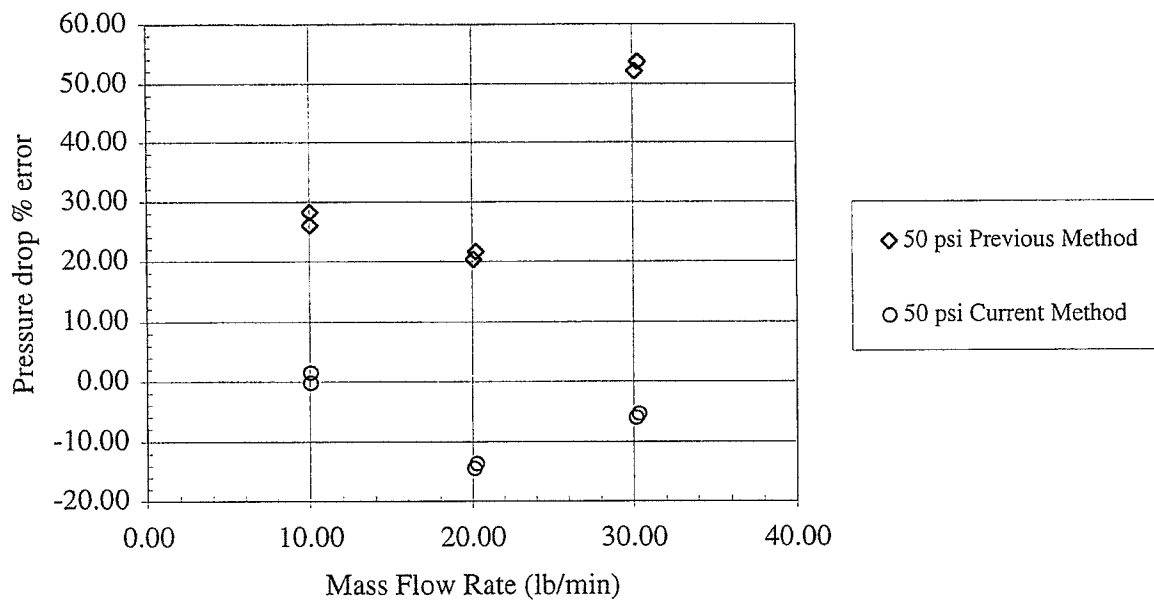


a)

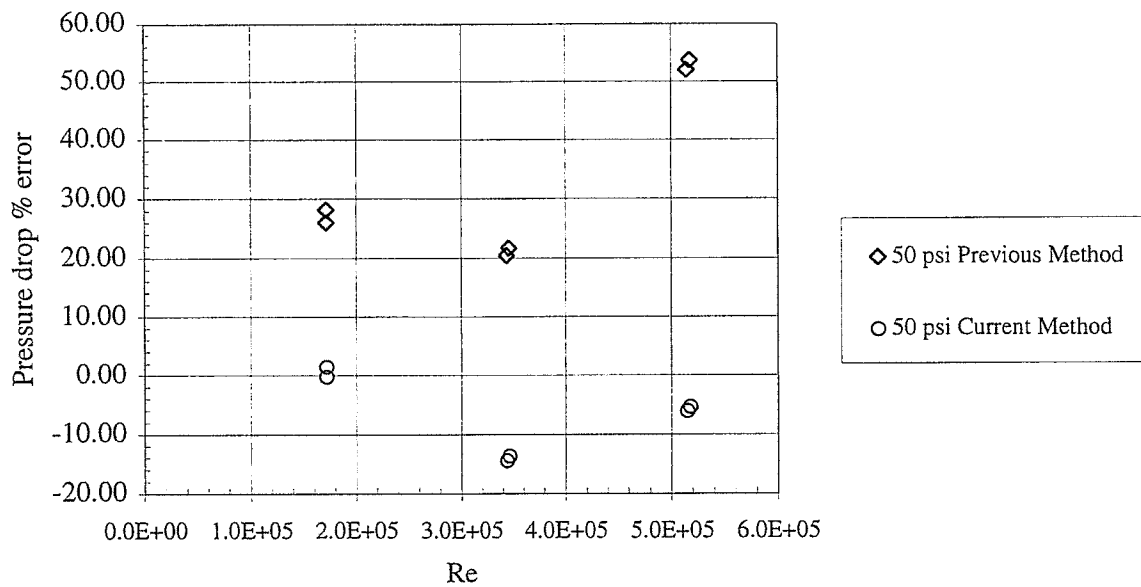


b)

FIGURE 12. CMF 100 gaseous pressure drop: Experimental data, Sizing program, and new prediction method.

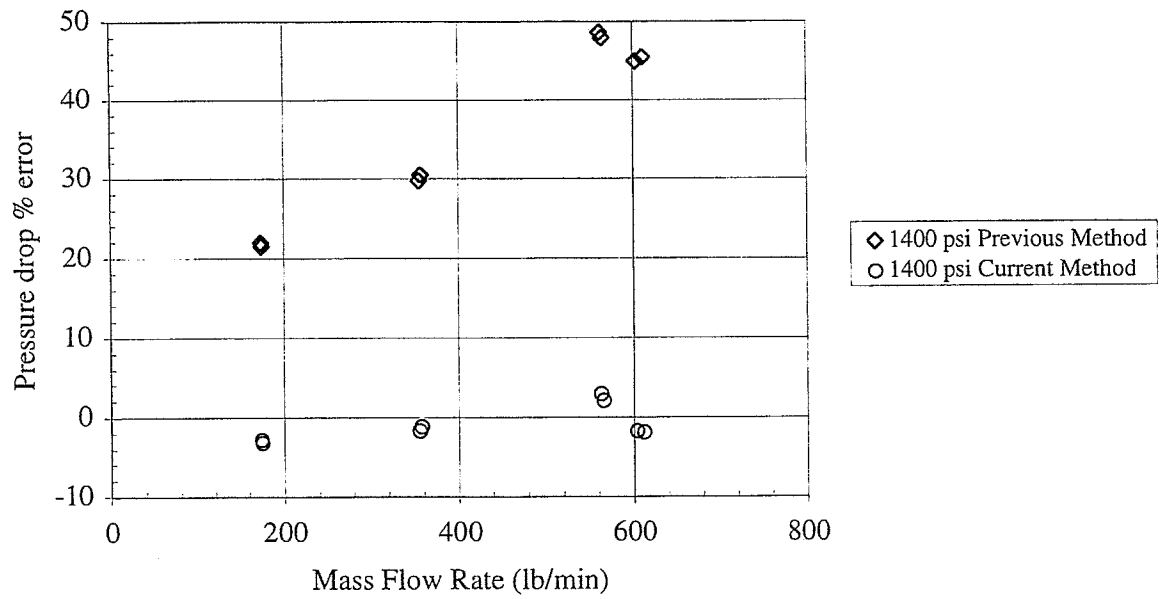


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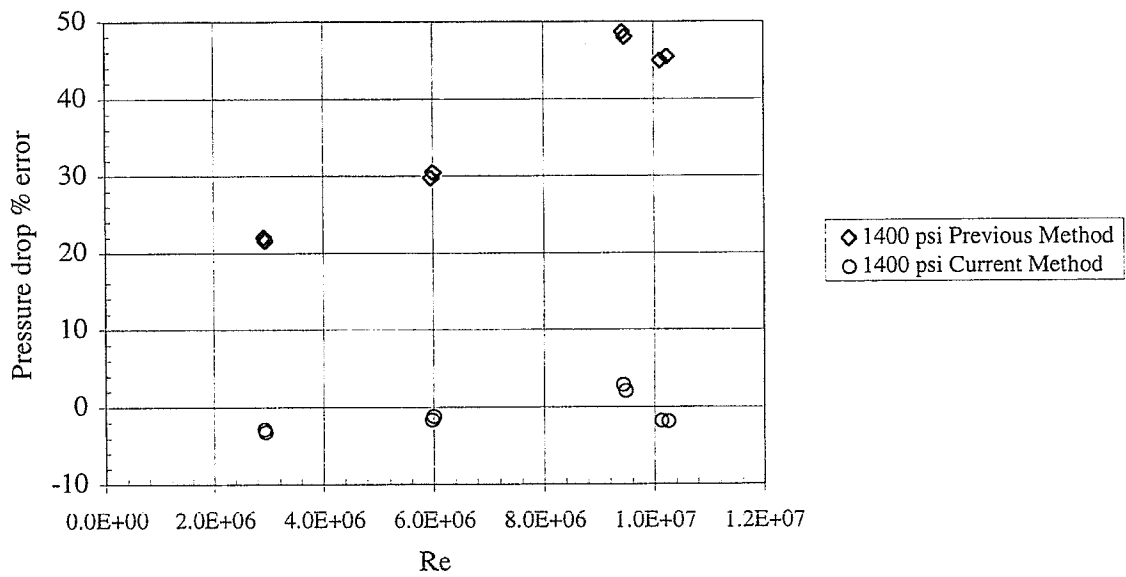


b)

FIGURE 13. CMF 100 50 psi gaseous air pressure drop prediction %error

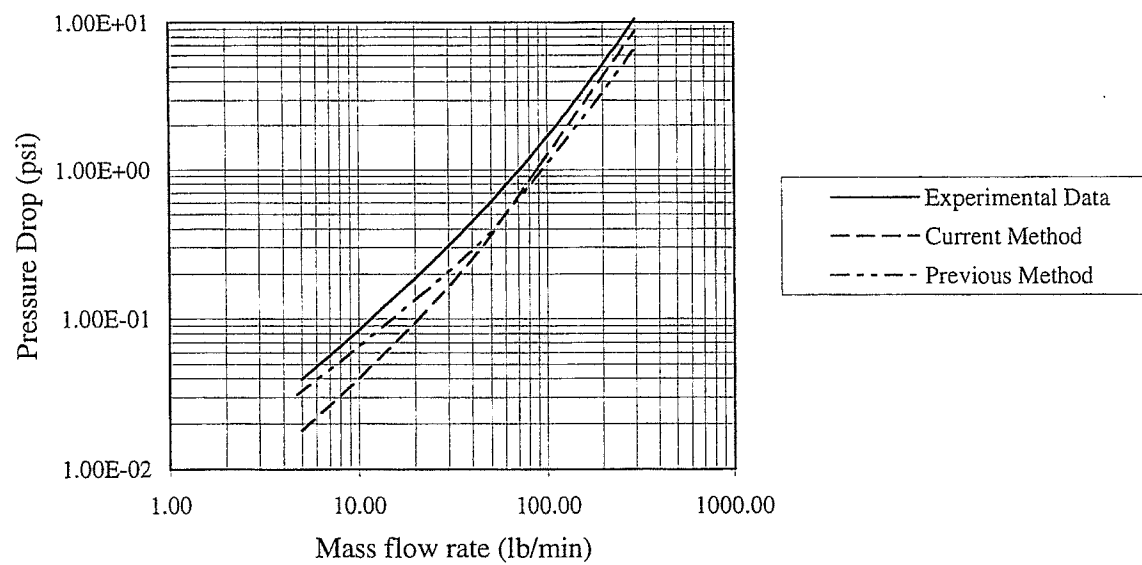


a)

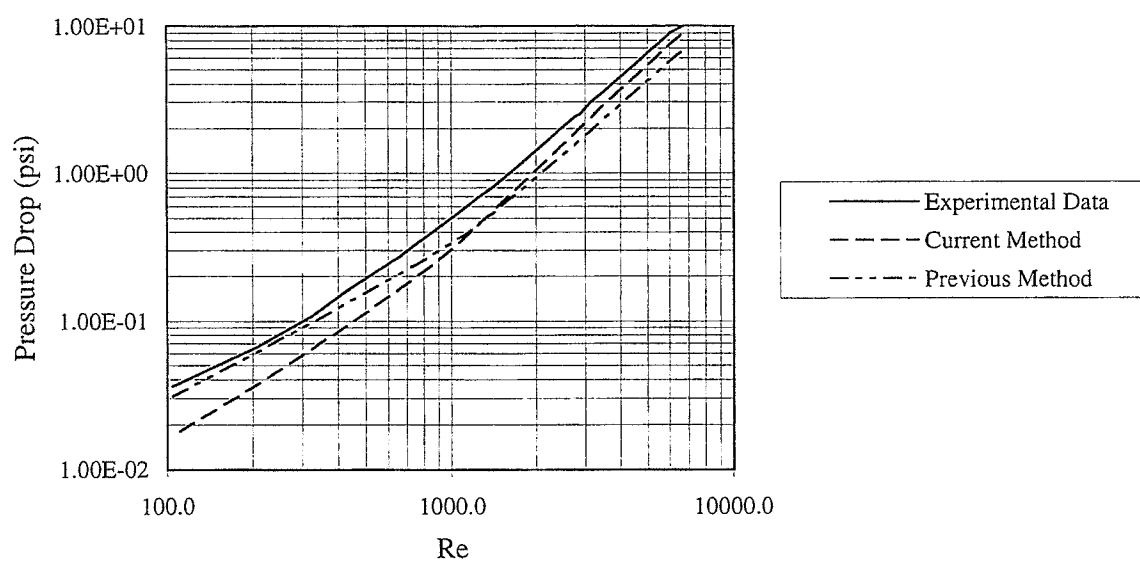


b)

FIGURE 14. CMF 100 1400 psi gaseous air pressure drop prediction %error

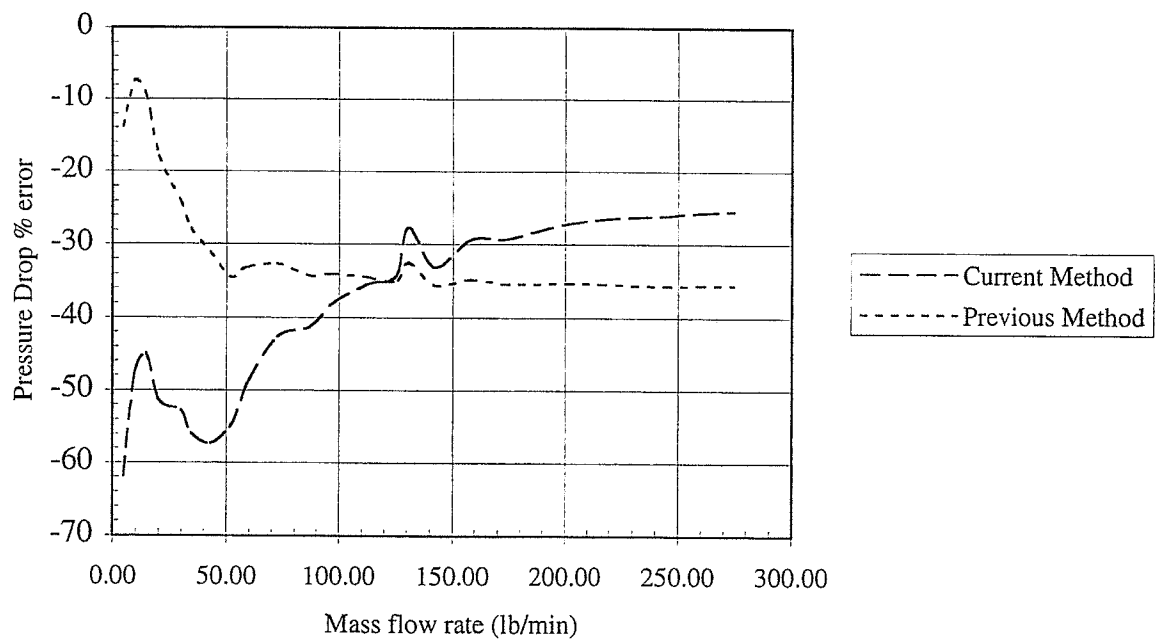


a)

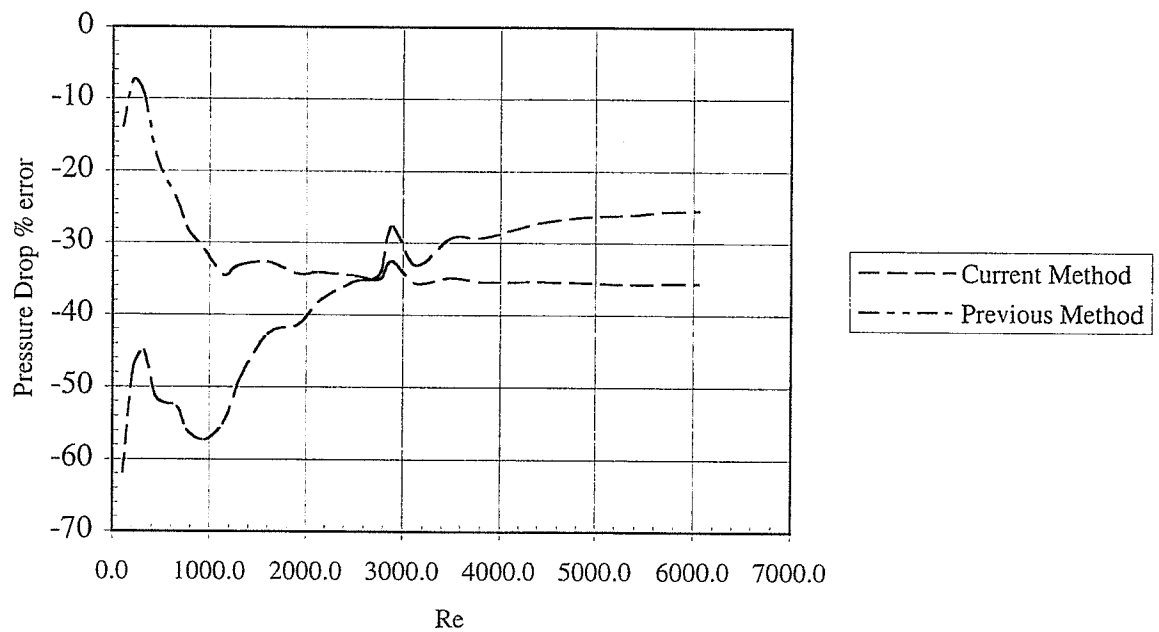


b)

FIGURE 15. CMF 100 Corn Syrup pressure drop prediction.



a)



b)

FIGURE 16. CMF 100 Pressure Drop For Corn Syrup %error

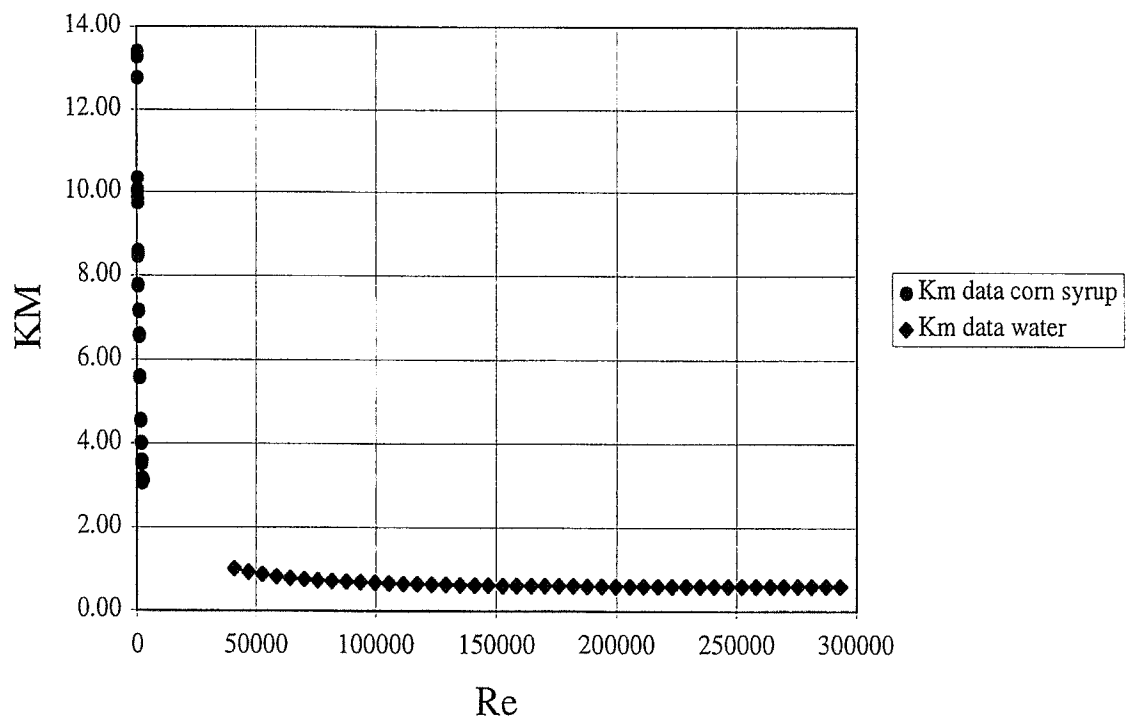


FIGURE 17. K_M data for water and Corn Syrup

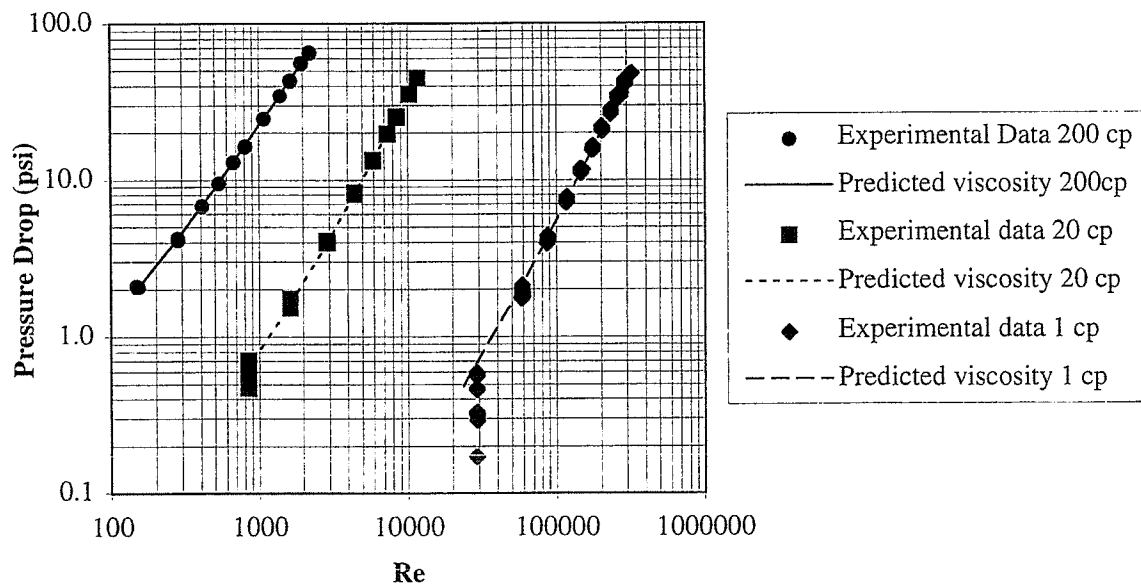
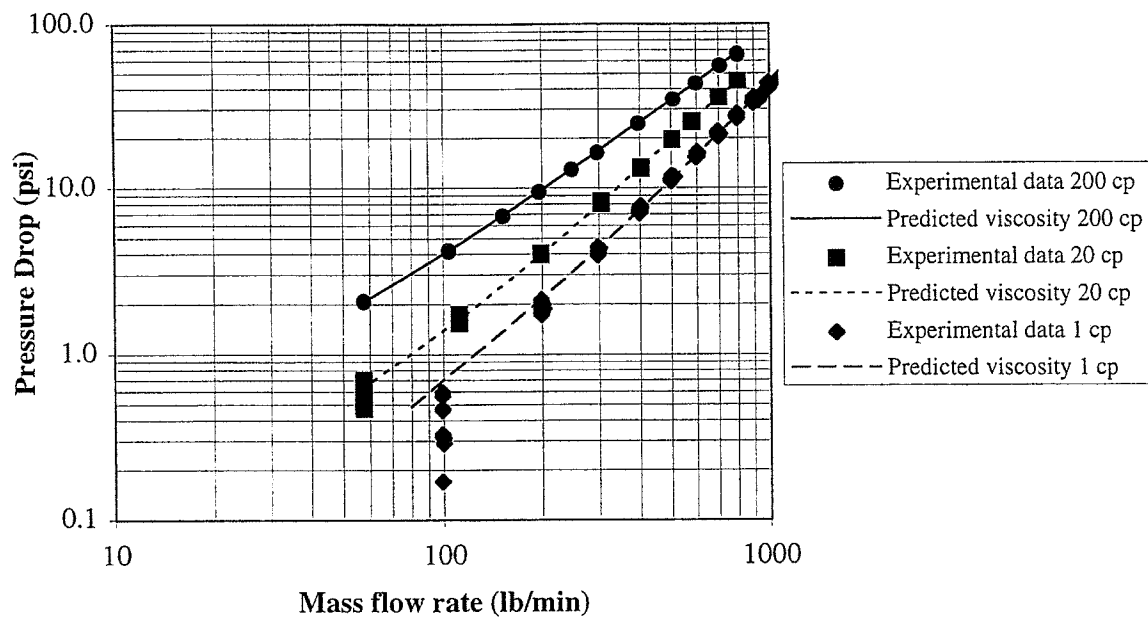


FIGURE 18. Improved mass flow rate predictions using K_M values that cover the entire Re range.

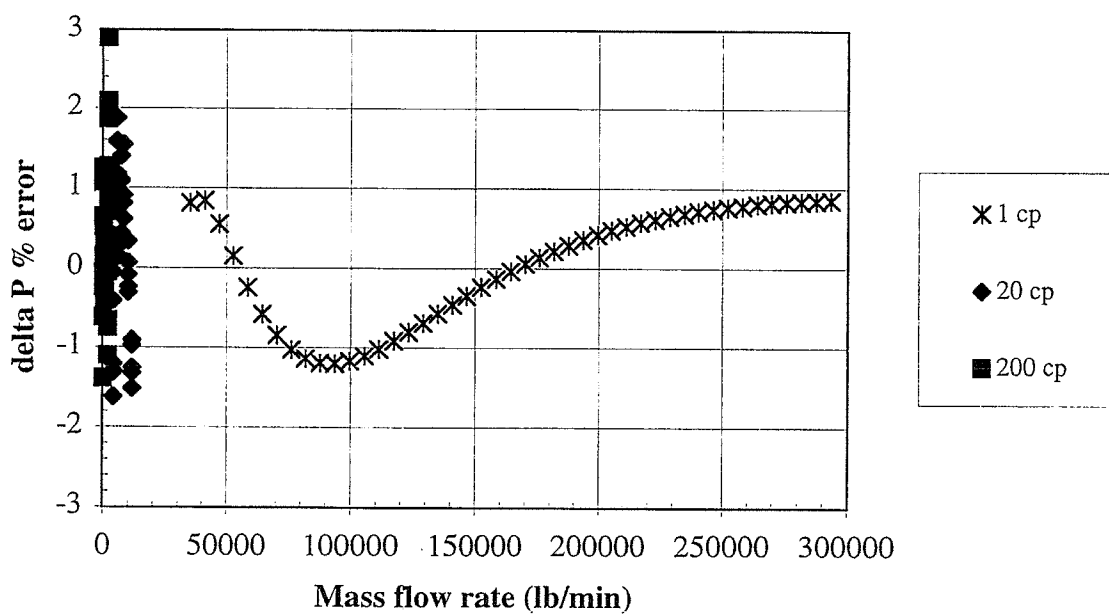
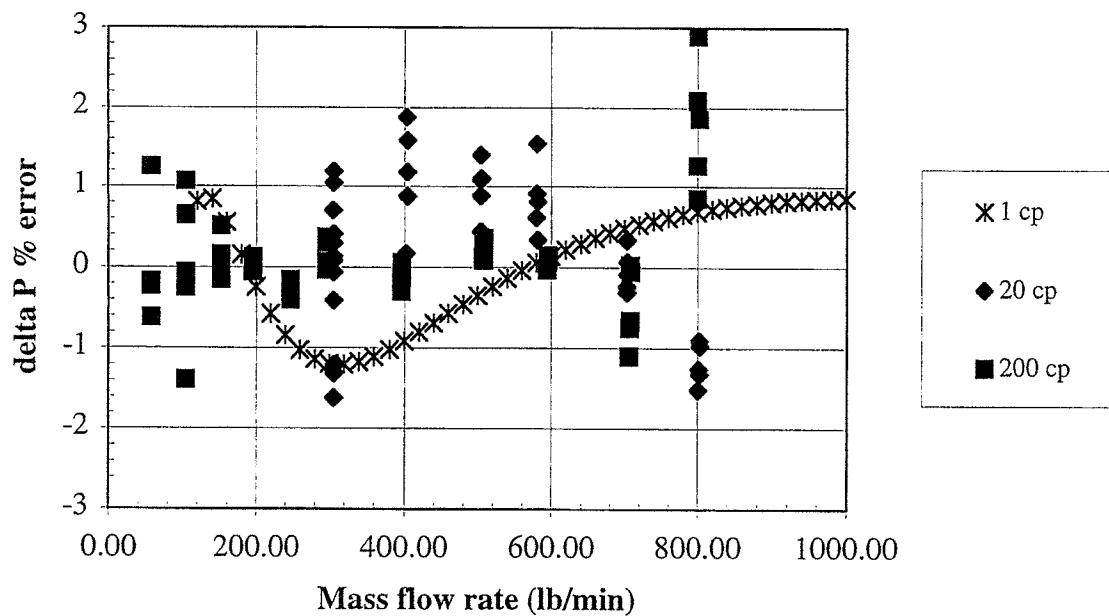


FIGURE 19. Improved pressure drop prediction % errors

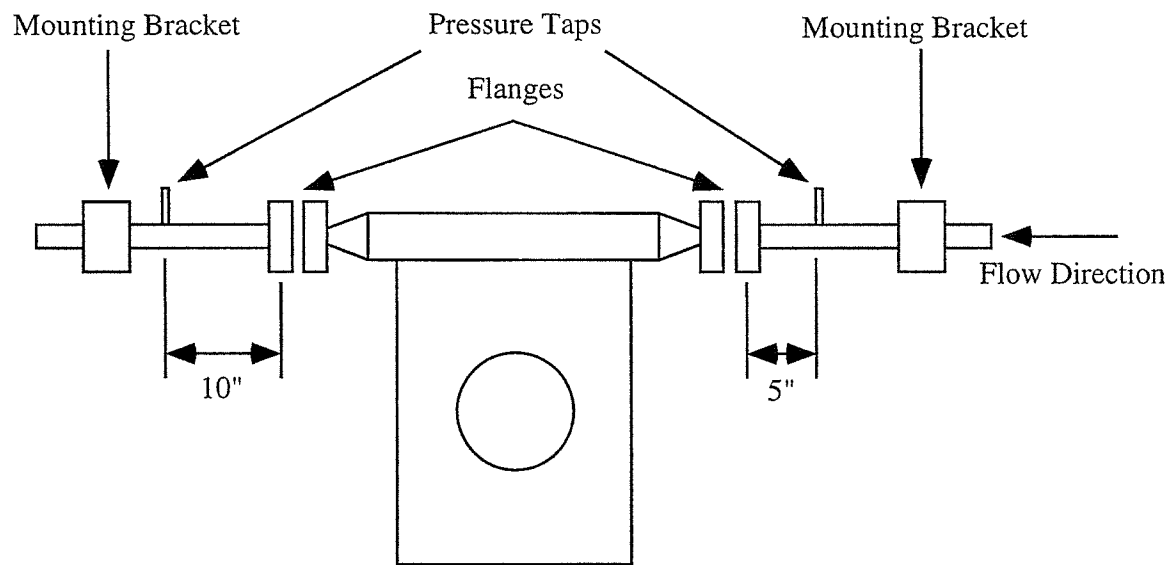


FIGURE 20. Diagram of a meter in the flow test stand. (all meters had the same set up with different spool pieces to match the meter inlet diameter)

Appendix A

Appendix A covers two main issues, first the transition region between laminar and turbulent flow, and second the assumption made about curve fitting the experimental pressure drop data with a second order polynomial. Just to review, the transition region from laminar to turbulent flow is an area that can not be accurately defined, as can be seen in Figure (21). The transition region usually spans from a Re of 2000 to about 4000. However, this transition region can vary depending on the roughness of the tubes and the pipe geometry. While in this region the friction factor, f , can have a step change at the laminar to turbulent transition point. The friction factor will follow the relation $f=64/Re$ in the laminar region until the turbulent transition point and then the friction factor will follow the Haaland equation, Equation (3).

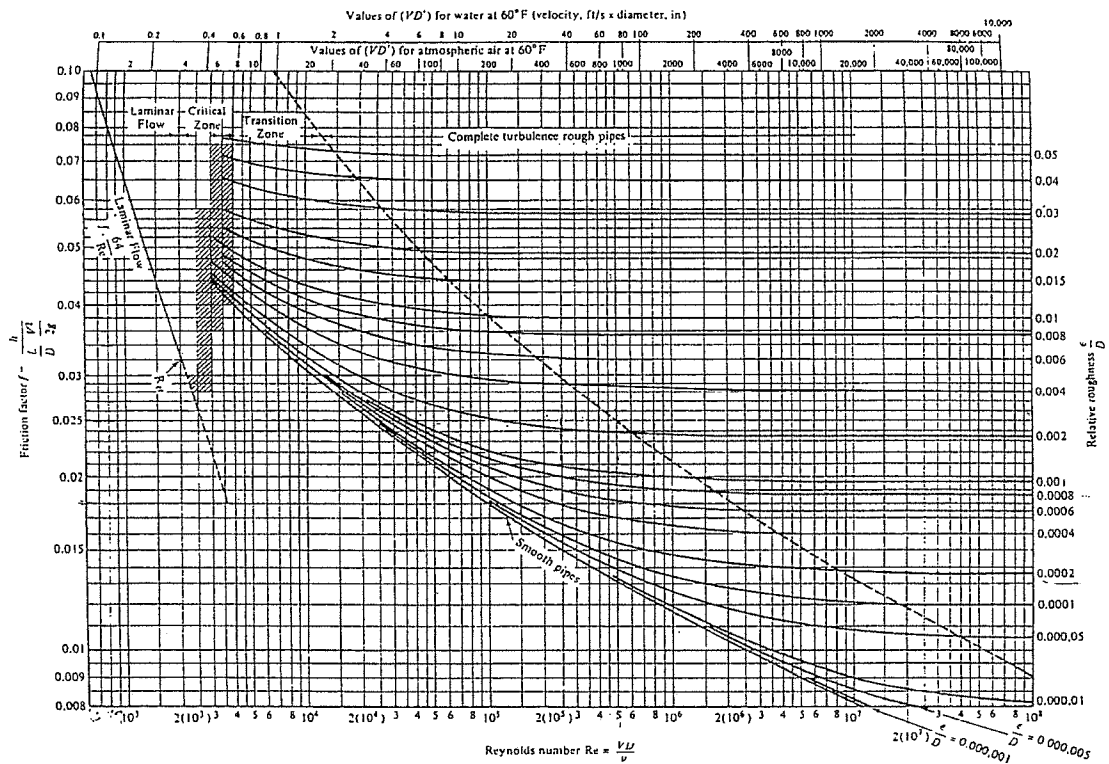


Fig. 6.13 The Moody chart for pipe friction with smooth and rough walls. This chart is identical to Eq. (6.64) for turbulent flow. (From Ref. 8, by permission of the ASME.)

FIGURE 21. The Moody Chart for pipe friction with smooth and rough walls (From Reference [1])

Recalling Equation (10) one can see that if the friction factor, f , has a step change the pressure drop will also have a step change. Remember that K_B is constant for all Re and K_M is a smooth curve.

$$\Delta P = \frac{\rho \cdot V^2}{2} \left[f \cdot \frac{L}{D_H} + (\text{nbends}) \left(\frac{\text{bendangle}}{90.0} \right) \cdot K_B + K_M \right] \quad (10)$$

Figure (22) shows an example plot of pressure drop versus Re for theoretical data with the assumed transition point at a Re of 3000 (A Re of 3000 was picked arbitrary). The transition point in real data will be dependent on tube roughness and tube geometry. The step change that occurs from the friction factor is very evident in the pressure drop data shown in Figure (22). The step change should also be evident in real world data if the pressure drop across CMF behaves according to Moody's chart.

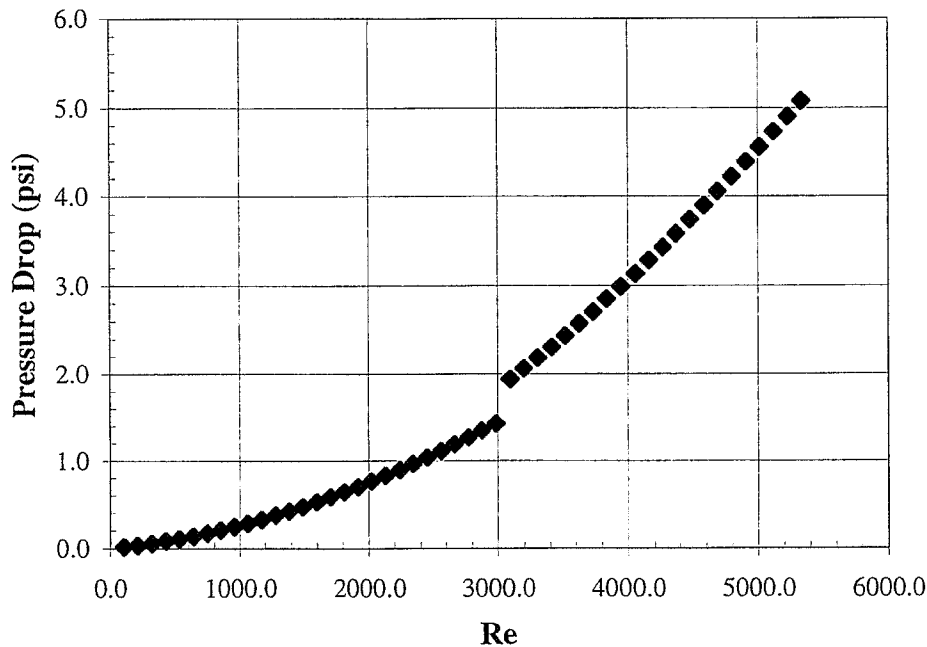


FIGURE 22. Example pressure drop versus Re data showing a step change

Figure (23) shows experimental pressure drop data for a CMF100 in the Automated Flow Test Facility 1200 (AFTF1200) stand. At each flow rate 30 data points were taken and plotted. It is quite evident that there is no step change in the pressure drop data shown in Figure (23). This suggests that there is a smooth transition of the friction factor from laminar to turbulent flow, no step change at a transition point. Figure (24) shows the same data as Figure (23) only plotted on a log-log graph. The curve fit lines in the graphs of Figures (23) and (24) are a 2nd order polynomial curve fit with an R^2 value of 0.9998. One could argue that the data below a Re of 1000 is linear, which is a valid argument, however the 2nd order polynomial fits the data exceptionally well.

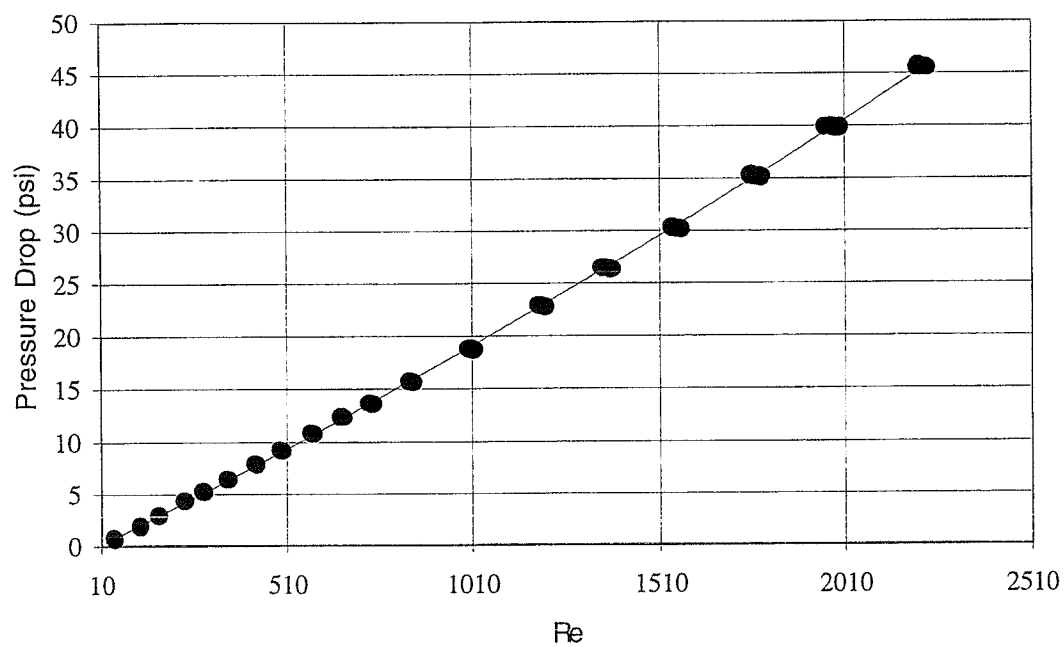
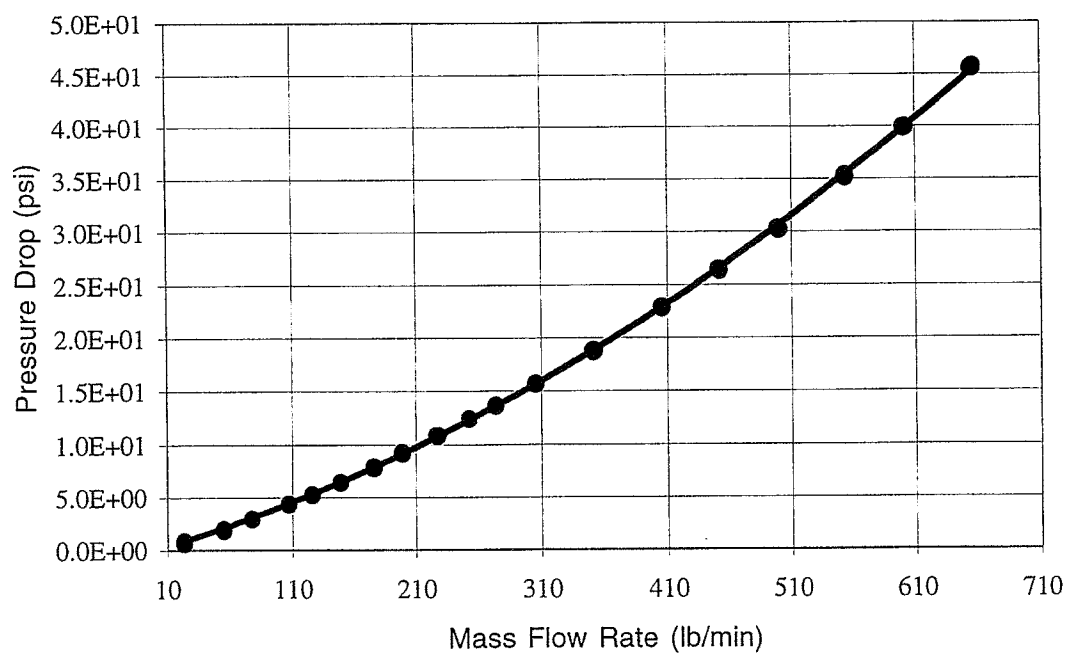


FIGURE 23. Experimental Pressure drop data

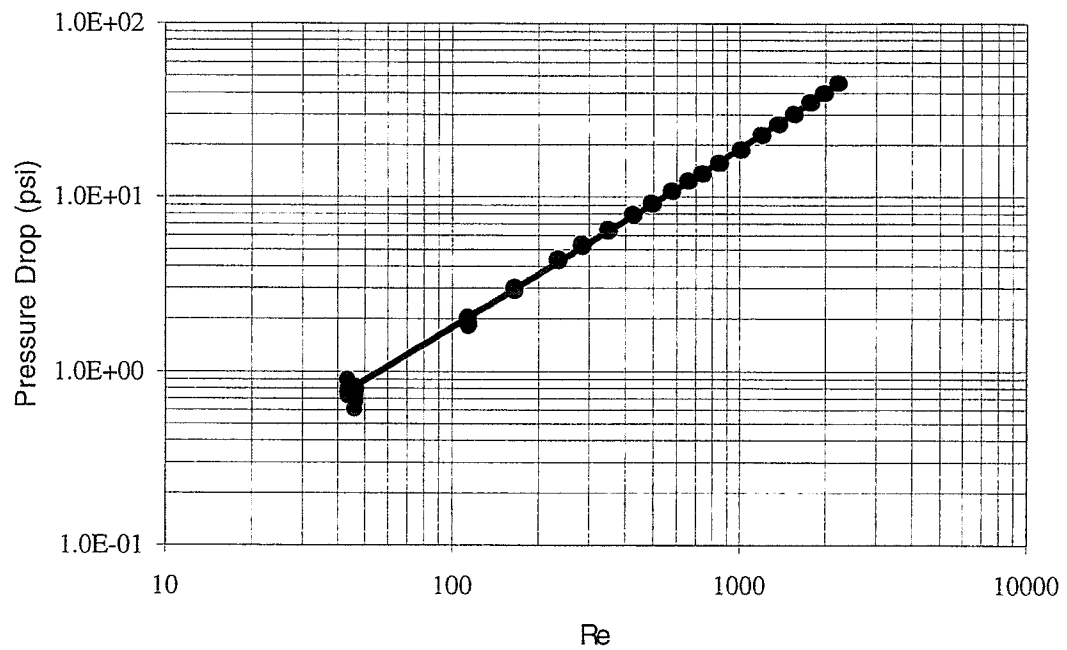
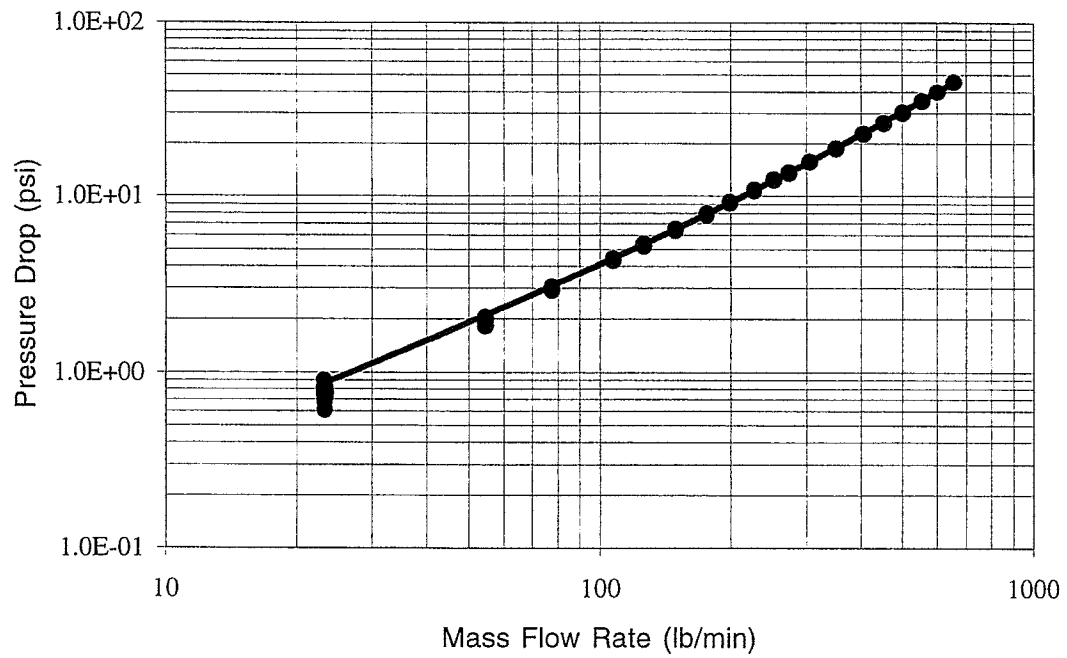


FIGURE 24. Experimental Pressure drop data

In order to incorporate a smooth transition from the laminar to the turbulent flow region for the Haaland equation, Equation (3), and $f = 64/Re$ the intersection must be found, see Figure (25). The intersection point of the two curves is at a Re of 946. For more information on the exact implementation of the friction factor equation see Appendix C.

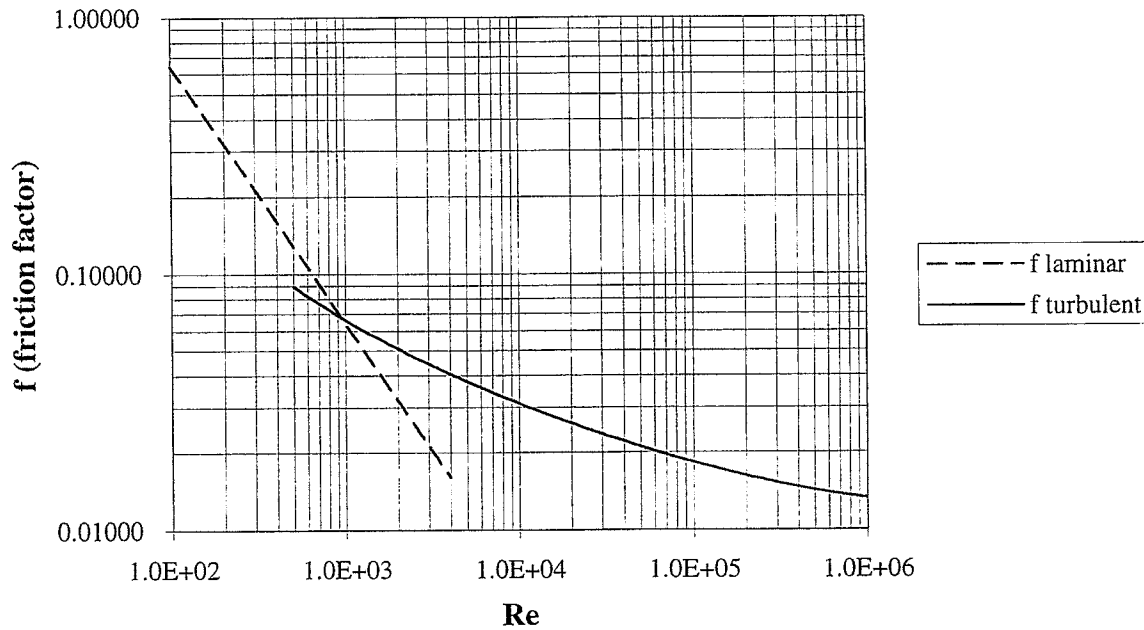


FIGURE 25. Friction factors laminar and turbulent regions

In summary, the experimental pressure drop data shows a smooth transition from the laminar to turbulent flow regions. A second order polynomial curve fit equation was used to represent the experimental data. Micro Motion's competitors are making an inaccurate pressure drop prediction in the transition region, due to the mistake of assuming that $f = 64/Re$ holds for CMF up to Re 's of 4000. When in actuality $f = 64/Re$ holds only up to a Re of 946.

Appendix B

Pressure drop uncertainties

Appendix B deals with the uncertainties in the pressure drop measurements. The Rosemont pressure transmitter that our flow stands use is the 1151 scaled from 0 to 100 psi. The pressure transmitters have a stated accuracy of $\pm 0.2\%$, which corresponds to ± 0.2 psi. This accuracy is reasonable when the pressure drop is above 1 psi. However it is unacceptable when the pressure drop is below 1 psi. For example Figure (26) shows experimental pressure drop data taken on water with a CMF100. Notice that the variation below 1 psi is much more pronounced than above 1 psi. Looking at the data below 1 psi the average is about 0.4 psi with a min of 0.18 and a max of 0.6 psi, which is within the stated accuracy of the pressure transducer.

The effect of the accuracy of the pressure drop measurement on the K_M value is amplified at low Re, which is proportional to flow tube velocity. The K_M values have a high variance due the pressure drop data being multiplied by $1/V^2$. This is significant when the velocity is much less than 1, which is in the lower pressure drop region. An example of the effect of the variance on the K_M value is shown in Figure (27).

$$K_M = \frac{2\Delta P}{\rho \cdot V^2} - f \frac{L}{D_H} - ((nbends) \left(\frac{bendangle}{90.0} \right) \cdot K_B) \quad (11)$$

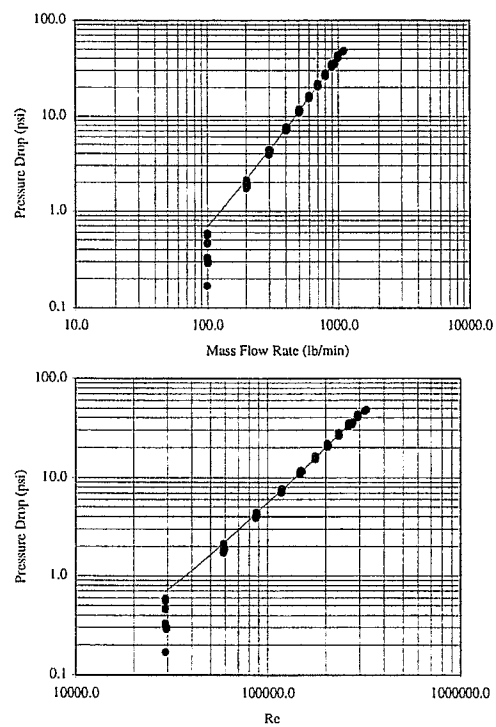


FIGURE 26. Experimental data water

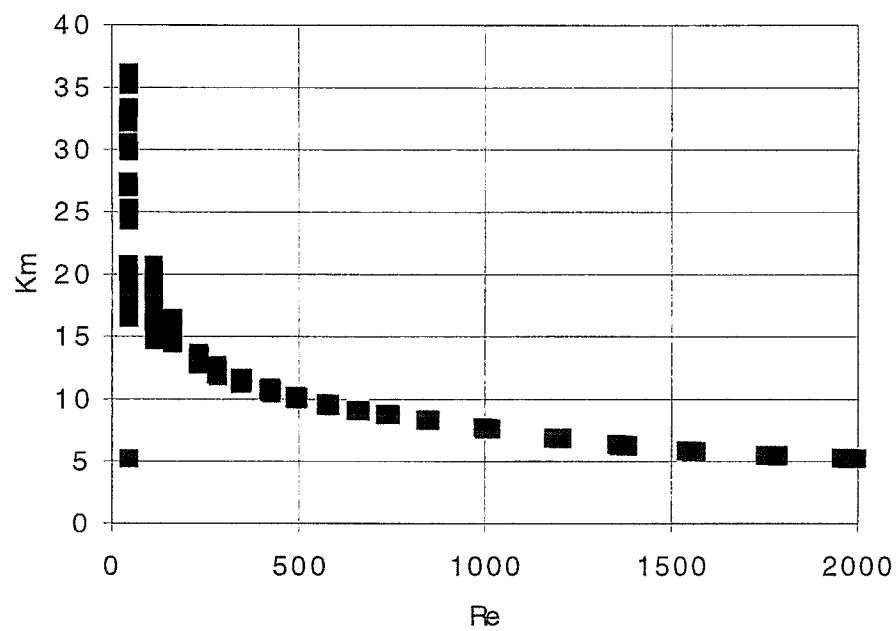


FIGURE 27. K_M values for corn syrup

Appendix C

This document describes the implementation of the new procedure for calculating pressure drop in the sizing program. All the equations described in this document are correct for SI units only.

User input

Table 1 shows the user input parameters needed to make a pressure drop calculation. The Meter code (mc), specifies the particular meter for the pressure drop calculation.

TABLE 1. User input parameters 2

variable name	Description	Units
rho	Density	$\frac{\text{kg}}{\text{m}^3}$
mu	Viscosity	cp
mdot	Mass flow rate	$\frac{\text{kg}}{\text{s}}$
mc	Meter code	

Data file input

Table 2 contains meter data used in the pressure drop calculation. This data is dependent on the selected meter. The data listed in Table 2 should be entered in a data file (most likely the current meter data file).

TABLE 2. Data file parameters 2

Variable name	Description	Units
id	tube Internal diameter	m
e	tube roughness	m
lt	flow tube length	m
nbends	Number of flow tube bends in the meter	
bend_radius	radius of the flow tube bends	m
b_a	flow tube bend angle	deg
a0	Type of curve fit for Km	
a1-a9	Km equation coefficients	

Subroutine output

The pressure drop (PD) subroutine described below outputs pressure drop in Pascals. However, in the PD subroutine tube velocity and Reynolds number are calculated which can be very useful to other subroutines (An example is the viscosity seizing program). Table 3, shows the output variables.

TABLE 3. Output from subroutine

Variable name	Description	Units
delta_p	Pressure drop	Pascals
v	Tube velocity	$\frac{m}{s}$
Re	Reynolds Number	

Program Outline

pi = 3.14159265358979324 : define pi

A = (pi * id^2) / 4.0 : Area in m^2

e_d = e/id : non-dimensional number used in the f eq

mu = mu * 1.0e-3 : change viscosity from cp to Kg/m/s

r_d = bend_radius / id : non-dimensional number used in the Kl eq

kl = b1 * r_d^4+b2 * r_d^3+b3 * r_d^2+b4 * r_d+b5 : non-dimensional number

v = (m_dot) / (2.0 * rho * A) : flow Tube velocity in m/s (flow tube velocity not pipe line velocity)

Re = (rho * v * id) / (mu) : Reynolds number non-dimensional (Re for the flow tubes)

: This if statement selects and calculates the friction factor (f). If the Re is below 946 then the flow is in the laminar range and f is calculated using the equation shown. However if the Re is

above 946 then the flow is most likely turbulent and the second equation is used.

```
if (Re<946.0)
    {
        f = 64.0 / Re                : non-dimensional number
    }
else
    {
        f = (-1.8 * log10(6.9 / Re + (e_d / 3.7)^1.11))^2.0      : non-dimensional
    }
number
```

: The next couple of if statements selects the proper curve fit equation for k_man.

```
if (a0 = 1)
    {
        k_man = a1 + a2 * exp(-a3 * Re) + a4 * exp(-a5 * Re)
    }

if (a0 = 2)
    {
        K_man = a1 + a2 / (1 + a2 * a3 * Re) + a4 / (1 + a4 * a5 * Re)
    }
```

: Now all the information has been calculated for use in the pressure drop equation.

$\Delta p = (v^2 * \rho / 2) * (f * L_t / id + nbends * (b_a / 90.0) * k_l + k_{man})$