Pressure Drop Prediction of Coriolis Mass Flowmeters

- 3 major components of a CMF:
  - straight tube section
  - curved tube section
  - flanges / manifolds

$$\Delta P = \left[\underbrace{\frac{\rho \cdot V^2}{2}}_{\cancel{\text{Straight tube}}}\right] \underbrace{\left[\overbrace{f \cdot \frac{L}{D_H}}^{\substack{\text{Re, KE Model} \\ \text{straight tube}}} + \overbrace{n_{bends} \cdot \frac{angle_{bend}}{90.0} \cdot K_B}^{\substack{\text{KE Model} \\ \text{curved tube}}} + \overbrace{K_M}^{\substack{\text{Re, KE Test Data} \\ \text{Manifolds}}}\right]}_{\cancel{\text{Curved Tube}}} \quad \cancel{\text{Manifold}}$$

- ρ = fluid density
- V = mean fluid velocity

- <u>Straight Tube section</u>: $f \cdot \dfrac{L}{D_H}$

look at Fluid PDF docs

- f: friction factor
  - can be found using Moody chart
  - can also be found using Colebrook equation:

$$\frac{1}{\sqrt{f}} = -2.0 \cdot \log\left[\frac{\frac{\varepsilon}{D_H}}{3.7} + \frac{2.51}{Re\sqrt{f}}\right]$$

"relative roughness" of tube wall $\longrightarrow$ MMI uses $\varepsilon = 1.524\,\cancel{mm} \times 10^{-6}\,m$
$= 5 \times 10^{-6}\,ft$

- need to use iterative method to solve for f using Colebrook equation

<u>or</u> use Haaland equation (deviates <2% from Colebrook over entire range of turbulent Re)

$$f \approx \left[-1.8 \cdot \log\left(\frac{6.9}{Re} + \left(\frac{\frac{\varepsilon}{D_H}}{3.7}\right)^{1.11}\right)\right]^{-2}$$

hydraulic diameter $\rightarrow$ $D_H = \dfrac{4 \times A}{WP} = ID$

D_H reduces to ID for circular tubes

A = cross sectional area

WP = wetted perimeter

- Velocity: $V = \dfrac{\left(\frac{\dot{m}}{2}\right)}{(\rho A)}$

- Reynolds Number: $Re = \dfrac{\rho V D_H}{\mu}$

- <u>Curved Tube Section</u>: $n_{bends} \cdot \dfrac{angle_{bend}}{90.0} \cdot K_B$

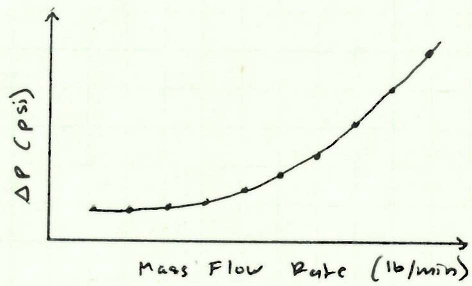
K_B: obtained from Rig 3, use $\dfrac{\varepsilon}{D_H} = 0$ (only for 1 90° bend, so eqn includes n_{bends} (# bends) and $\dfrac{angle_{bend}}{90}$ (angle of bend) to account for other bends)

- $\frac{\varepsilon}{D_H}$ line in Fig 3 has no equation, so a least squared polynomial curve fit was done to find equation for K_B if $\frac{\varepsilon}{D_H} = 0$:

$$K_B = 1\times10^{-4}\left[\frac{R}{D_H}\right]^4 - 3.5\times10^{-3}\left[\frac{R}{D_H}\right]^3 + 3.18\times10^{-2}\left[\frac{R}{D_H}\right]^2 - .1382\left[\frac{R}{D_H}\right] + .3288$$

- <u>Manifold section</u>: K_M
  - we used equations to find pressure drop in straight and curved sections
  - need to obtain an equation experimentally for K_M to find pressure drop in the manifold

$\boxed{\text{Step 1}}$: collect and plot pressure drop vs. MFR. Data should encompass entire flow range of meter
don't include MFR's that correspond to $\Delta P < 1$ psi $\rightarrow$ uncertainty in ΔP measurement



$\boxed{\text{Step 2}}$: Get Least squares curve fit equation for ΔP vs $\dot{m}$. Solving for B_1 and B_2 in eqn using Excel. $\rightarrow$ $\Delta P = B_1 \cdot \dot{m}^2 + B_2 \cdot \dot{m} + 0$

$\boxed{\text{Step 3}}$: Use the equation from step 2 w/ the original equation to get an equation for K_M

~~Origo~~ Original Eqn: $\Delta P = \frac{\rho V^2}{2}\left[f \cdot \frac{L}{D_H} + n_{bends} \cdot \frac{\alpha_{bend}}{90} \cdot K_B + K_M\right]$

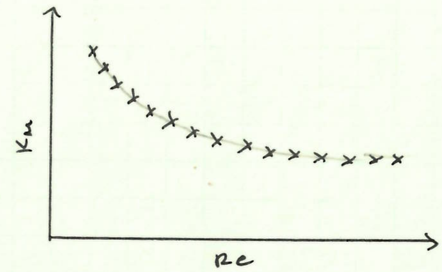
$\downarrow$

get ΔP from $\Delta P = B_1 \cdot \dot{m}^2 + B_2 \cdot \dot{m}$ $\rightarrow$

$$\frac{2\Delta P}{\rho V^2} - \left(f \cdot \frac{L}{D_H}\right) - \left(n_{bends} \cdot \frac{\alpha_{bend}}{90} \cdot K_B\right) = K_M$$

Variables	Description	Where to get value
ΔP	Pressure Drop	$\Delta P = B_1 \cdot \dot{m}^2 + B_2 \cdot \dot{m}$
ρ	density	fluid properties
V	Flow tube Velocity	$V = \left(\frac{\dot{m}}{2}\right)/(\rho A)$
f	friction Factor	$f \approx \left[-1.8\log\left(\frac{6.9}{Re} + \left(\frac{\varepsilon/D_H}{3.7}\right)^{1.11}\right)\right]^{-2}$ ← (Haaland Equation)
L	Total Flow Tube Length	sensor properties
D_H	Hydraulic Diameter	$D_H = \frac{4\times A}{WP}$ = ID for circular X-A
n_{bends}	# of bends	sensor properties
α_{bend}	bend angle	sensor properties
K_B	Loss coef for Tube Bends	$K_B = 1\times10^{-4}\left[\frac{R}{D_H}\right]^4 - 3.5\times10^{-3}\left[\frac{R}{D_H}\right]^3 + 3.18\times10^{-2}\left[\frac{R}{D_H}\right]^2 - .1382\left[\frac{R}{D_H}\right] + .3288$

$\boxed{\text{Step 4}}$ Plot K_M (values! not curve) vs. Re. Important to cover entire range of Re for the meter, especially low Re.



$\boxed{\text{Step 5}}$ Curve fit data from step 4 using the equation:

$$K_M = A_1 + A_2 e^{(-A_3 Re)} + A_4 e^{(-A_5 Re)}$$

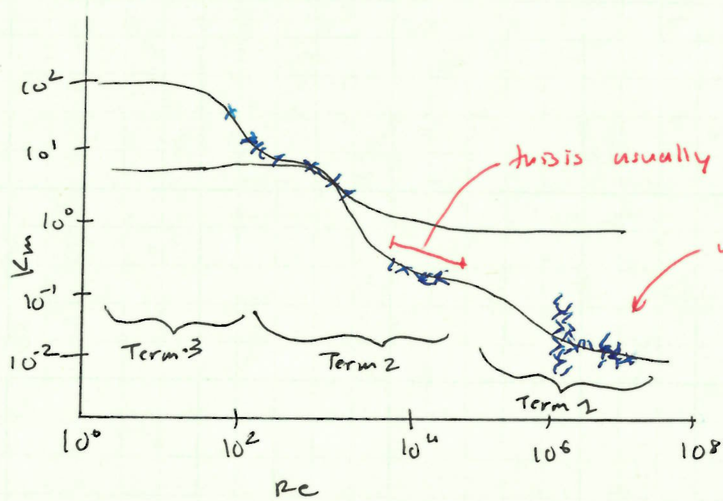
Will need to do one curve fit for high Re and one for low Re. Discontinuity around Re = 1000 (related to change in friction factor term at Re = 946). If you use 2 different fits, you get a smooth transition between the 2 equations.

Table Curve = curve fitter

$\boxed{\text{Step 6}}$ Calculate ΔP using original equation and equation for K_M from step 5.

- 12/12/2016 : Scott Meeting
  - we rarely have enough pressure drop data for Re, we have small subsection of data we need due to small Re range

$$K_m = \underbrace{A_1}_{\text{Term 1}} + \underbrace{A_2 e^{-A_3 Re}}_{\text{Term 2}} + \underbrace{A_4 e^{-A_5 Re}}_{\text{Term 3}} \rightarrow \text{eqn. fits experimentally obtained } K_m \text{ values}$$



- need to pick good guesses for $A_1 - A_5$ to get good fit in MATLAB
  - A_1 : driving term for high Re
  - A_2, A_3 : " " mid Re
- set A_1 to highest Re
- set A_2 to median value, term 2 goes to zero as Re $\rightarrow \infty$

· from experimental data: need flow rate, ΔP, ρ, μ
  - copy and paste into MATLAB data format ("matlab data.xls")
· Kmanifold.m : main script
  - has manifold at low Re (does not just shoot off to infinity)
  - the Re and Km it asks you for are to force some data points to use in the fit
  - use the Km at Re = 946 to use in ~~the~~ curve fit for high Re
· can look up Kmanifold in fluids textbook (should asymptote around 1 at high Re)
· Pressure_drop.m: plots different meters ~~over~~ at a given Re
· PresDropvsRe.m: plots different meters over Re range
· need to make sure meters ΔP don't cross, may need to retire fit so it doesn't cross another meter's ΔP vs. Re

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