

# Stochastic Simulation

## Discrete simulation / event-by-event

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# Discrete event simulation



- Continuous but asynchronous time
- Systems with discrete state-variables
  - ◇ Inventory systems
  - ◇ Communication systems
  - ◇ Traffic systems - (simple models)
- even-by-event principle

# Elements of a discrete simulation language/program



- Real time clock
- State variables
- Event list(s)
- Statistics

# The event-by-event principle



- Advance clock to next event to occur
- Invoke relevant event handling routine
  - ◇ collect statistics
  - ◇ Update system variables
- Generate and schedule future events - insert in event list(s)
- return to top

# Analysing steady-state behaviour

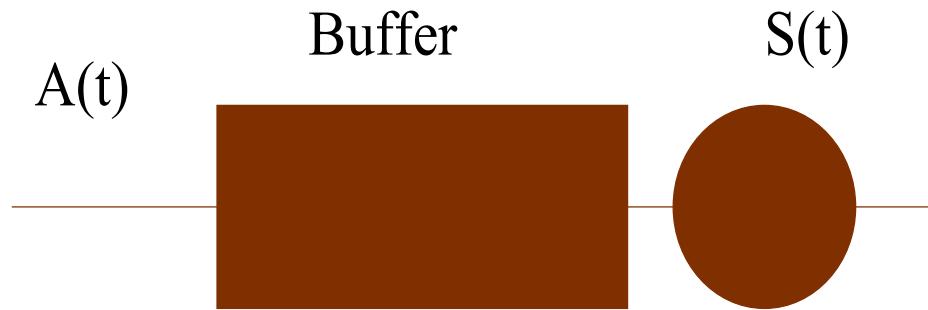


- Burn-in/initialisation period
  - ◇ Typically this has to be determined experimentally
- Confidence intervals/variance estimated from sub-samples

# Queueing systems



- Arrival process
- Service time distribution(s)
- Service unit(s)
- Priorities
- Queueing discipline



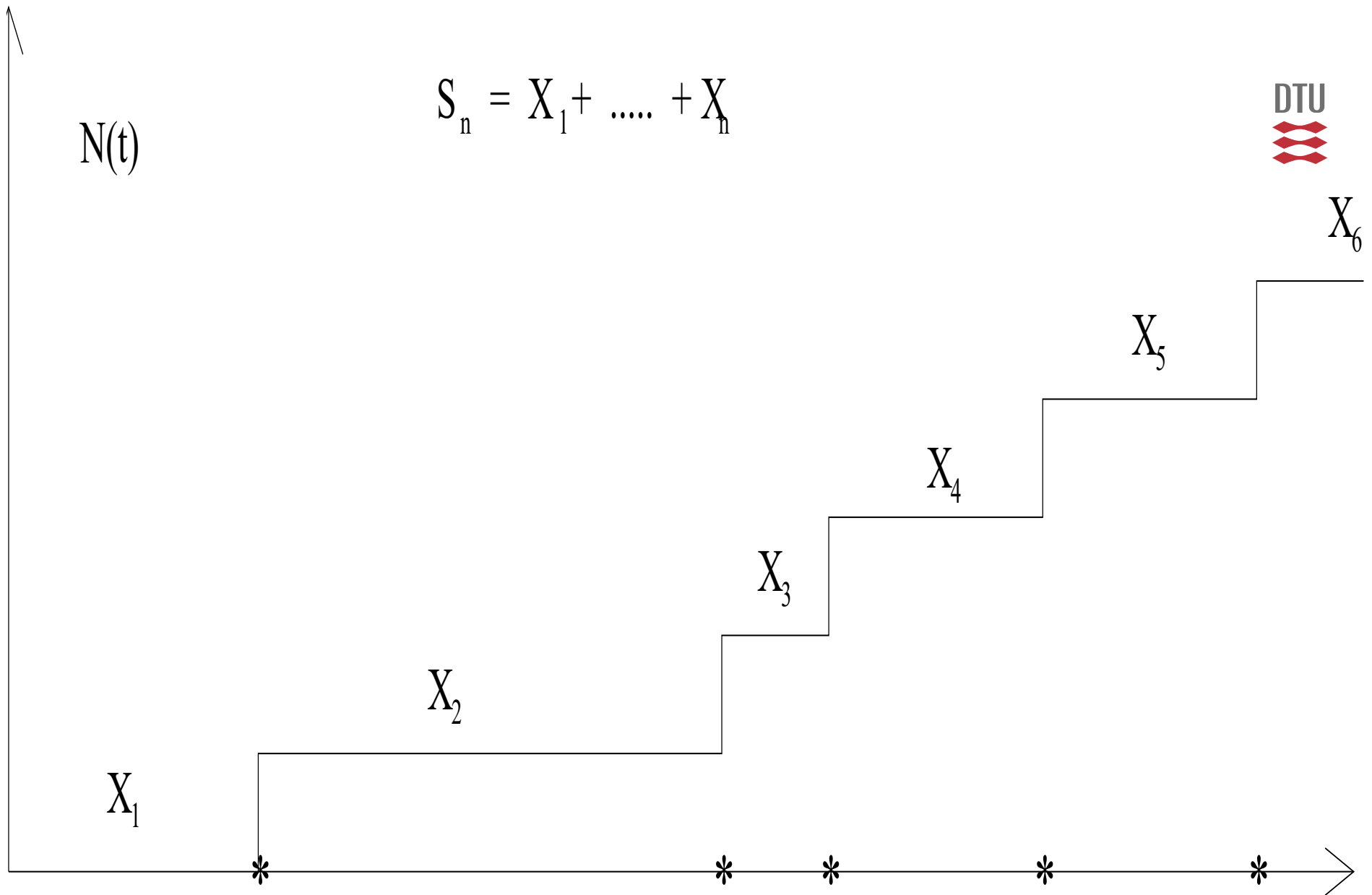
- $A(t)$  - Arrival process
- $S(t)$  - Service process (service time distribution)
- Finite or infinite waiting room
- One or many servers
- Kendall notation:  $A(t)/S(t)/N/K$ 
  - ◇  $N$  - number of servers
  - ◇  $K$  - room in system (sometime  $K$  only relates to waiting room)

# Performance measures



- Waiting time distribution
  - ◇ Mean
  - ◇ Variance
  - ◇ Quantiles
- Blocking probabilities
- Utilisation of equipment (servers)
- Queue length distribution





# Poisson process



- Independently exponentially distributed intervals

$$P(X_i \leq t) = 1 - e^{-\lambda t}$$

- Poisson distributed number of events in an interval. Number of events in non-overlapping intervals independent

$$N(t) \sim P(\lambda t) \Leftrightarrow P(N(t) = n) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}$$

- If the intervals  $X_i$  are independently but generally distributed we call the process a renewal process

# Sub-samples - precision of estimate



- We need sub-samples in order to investigate the precision of the estimate.
- The sub-samples should be independent if possible
- For independent subsamples the standard deviation of our estimate will be proportional to  $\sqrt{n}^{-1}$

# Confidence limits based on sub-samples



- We want to estimate some quantity  $\theta$
- We obtain  $n$  different (independent) estimates  $\hat{\theta}_i$ .
- The central limit theorem motivates us to construct the following confidence interval:

$$\bar{\theta} = \frac{\sum_{i=1}^n \hat{\theta}_i}{n} \quad S_{\theta}^2 = \frac{1}{n-1} \left( \sum_{i=1}^n \hat{\theta}_i^2 - n\bar{\theta}^2 \right)$$

$$\left[ \bar{\theta} + \frac{S_{\theta}}{\sqrt{n}} t_{\frac{\alpha}{2}}(n-1); \bar{\theta} + \frac{S_{\theta}}{\sqrt{n}} t_{1-\frac{\alpha}{2}}(n-1) \right]$$

# Confidence limits based on sub-samples - based on normal distribution



$$\left[ \bar{\theta} + \frac{S_{\theta}}{\sqrt{n}} u_{\frac{\alpha}{2}}; \bar{\theta} + \frac{S_{\theta}}{\sqrt{n}} u_{1-\frac{\alpha}{2}} \right]$$

# General statistical analysis



- More generally we can apply any statistical technique
- In the planning phase - experimental design
- In the analysis phase
  - ◇ Analysis of variance
  - ◇ Time-series analysis
  - ◇ ⋮