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Four parameter data fit for Langmuir probes with nonsaturation of ion current

D. Desideri and G. Serianni
Consorzio RFX, Corso Stati Uniti, 4-35127 Padova, Italy

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The four parameter data fit for Langmuir probes is here generalized within the classical single Langmuir probe theory. The data fit accounts for the variation of the Debye sheath thickness of the probe with the applied voltage which results in a nonsaturation of the ion current. The new fitting formula has been applied to the RFX experimental data. With respect to the use of the simple exponential fit, in general a correction of the order of 20% on electron temperature and ion saturation current has been obtained. © *1998 American Institute of Physics.*
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I. INTRODUCTION

In thermonuclear fusion experiments single Langmuir probes are widely used. Their characteristic dimension perpendicular to the magnetic field direction is often quite small in order to minimize the incident energy flux density. A solution consists in adopting probes flush to the local limiter with very small angles of incidence (γ) with respect to the magnetic field direction.[1–3] A different solution consists in using cylindrical probes with an angle of incidence not too small (i.e., greater than 10°) but with a small cross-sectional diameter, that becomes comparable to the ion Larmor radius in experiments with relatively low magnetic field at the edge, as in reversed field pinch configuration.[4]

In the case of flush-mounted probes with a very small angle of incidence γ a nonsaturation of ion current has been observed, due to the variation of the probe sheath thickness with the applied voltage giving a noticeable change of the collecting area, and a four parameter data fit has been proposed, assuming a linearization of the Child–Langmuir relation.[1]

In this work the four parameter data fit is extended in the general case of nonsaturation of the ion current, regardless to the angle of incidence of the probe surface with respect to the magnetic field direction, and this new fit is obtained within the framework of the classical Langmuir probe theory. In Sec. II the theoretical work is reported and the new analytical fitting formula is presented. The resulting formula has been applied to the experimental data obtained on the reversed field pinch experiment RFX and a few examples are reported in Sec. III.

II. THE ANALYTICAL FITTING FORMULA

It is assumed that when a particle enters the probe sheath, it is collected by the probe: no presheath effects are considered. Within this hypothesis, the ion and electron collecting areas, by using a first order approximation, can be expressed as

$$A_i = A_{if}[1+\alpha(x_p - x_{pf})], \tag{1}$$

$$A_e = A_{ef}[1+\beta(x_p - x_{pf})], \tag{2}$$

where A_{if} and A_{ef} are respectively the ion and electron collecting areas when the probe is at the floating potential; x_p is the probe sheath thickness, and x_{pf} is the probe sheath thickness when the probe voltage is equal to the floating potential (V_f); α and β are two parameters that do not depend on the probe voltage.

The Child–Langmuir relation gives a $V^{3/4}$ law for the sheath. As a fair approximation for $-100 < V - V_p \leqslant 0$ (usual range for experimental data), a linear dependence has been assumed:[1]

$$(V_p - V)^{3/4} \approx \tfrac{1}{3}(V_p - V), \tag{3}$$

where the factor 1/3 has been obtained, for the voltage range proposed, by the least-squares method; V_p is the plasma potential, whereas V is the probe voltage.

By substituting Eq. (3) in the usual Child–Langmuir relation for the estimation of the voltage dependence of the probe sheath thickness x_p, the following expression is obtained:

$$x_p = \sqrt{\frac{8}{9}\,\epsilon_0\,\frac{1}{n}\,\sqrt{\frac{2}{ek(T_i+T_e)}}}\;\tfrac{1}{3}(V_p - V) = Z(V_p - V), \tag{4}$$

where $Z = (8.7\times 10^{-5})/\sqrt{J_s}$; $J_s = nec_s \exp(-\tfrac{1}{2})$ is the ion current density; $c_s = \sqrt{[k(T_i+T_e)]/m_i}$ is the ion sound velocity; n is the density, and e is the unit charge.

For $V \leqslant V_p$, from the Langmuir probe theory, the probe current is given by

$$I = -A_i \exp\left(-\frac{1}{2}\right) nec_s + A_e ne\sqrt{\frac{kT_e}{2\pi m_e}} \exp\left(e\,\frac{V}{kT_e}\right). \tag{5}$$

From Eq. (5), the expression of the floating potential is obtained:

$$V_f = \frac{kT_e}{2e}\left\{\ln\left[\left(\frac{A_{if}}{A_{ef}}\right)^2\left(1+\frac{T_i}{T_e}\right)\frac{2\pi m_e}{m_i}\right] - 1\right\}. \tag{6}$$

Combining Eqs. (5) and (6), and then Eqs. (1), (2), and (4), yields

$$I=-A_i\exp(-\tfrac{1}{2})nec_s\left[1-\frac{A_e}{A_i}\frac{A_{if}}{A_{ef}}\exp\left(e\frac{V-V_f}{kT_e}\right)\right]$$
$$=I_s[1+\alpha Z(V_f-V)]\left[1-\frac{1+\beta Z(V_f-V)}{1+\alpha Z(V_f-V)}\times\exp\left(e\frac{V-V_f}{kT_e}\right)\right],\qquad(7)$$

where $I_s=-A_{if}\,nec_s\exp(-\tfrac{1}{2})$.

Equation (7) can be rewritten in the form:

$$I=I_s[1+R(V_f-V)]\left[1-\frac{1+S(V_f-V)}{1+R(V_f-V)}\exp\left(e\frac{V-V_f}{kT_e}\right)\right]\qquad(8)$$

after introducing the R and S parameters: $R=\alpha Z$, $S=\beta Z$.

Equation (8) is characterized by two terms which are not contained in the standard current–voltage characteristic: the first one is relative to the ion saturation current; the second is the factor multiplying the exponential. Both corrections are negligible for $V\approx V_f$, and are meaningful for $V\ll V_f$ and $V\gg V_f$, respectively. The portion of the characteristic for $V\gg V_f$ is, however, avoided in the data analysis of single Langmuir probes, while the portion for $V\ll V_f$ is the ion saturation where the exponential is negligible. Therefore, for the usual voltage range of data fit, $V\leqslant V_f$, Eq. (8) can be simplified by keeping the correction on the ion saturation and neglecting the correction due to the electrons, resulting with:

$$I=I_s[1+R(V_f-V)]\left[1-\exp\left(e\frac{V-V_f}{kT_e}\right)\right].\qquad(9)$$

V_f and T_e are two of the four fitting parameters; the plasma density is obtained from I_s, when a good estimation of A_{if} is known. Moreover, once an estimation of the increment of the collecting area with the sheath (the α parameter) is known, with I_s and A_{if}, a cross check on the obtained value of the R parameter can be performed.

When the angle of incidence γ with the magnetic field direction becomes very small, a good estimation of A_{if} becomes difficult to be given: in this case, the method of calculating the plasma density proposed in Ref. 1 still holds. Equations (6) and (8) give:

$$I=I_s+I_sR(V_f-V)+I_e\exp\left(e\frac{V}{kT_e}\right),\qquad(10)$$

where $I_e=A_ene\sqrt{(kT_e)/(2\pi m_e)}$, while, using the expressions for I_s and R and Eqs. (1) and (4):

$$I_sR(V_f-V)=-nec_s\exp(-\tfrac{1}{2})(A_i-A_{if}).\qquad(11)$$

By estimating, as in Ref. 1, the change in the effective area for a cylindrical probe of radius r_p as

$$A_i-A_{if}=2r_p(x_p-x_{pf})\qquad(12)$$

with Eqs. (4) and (10) and the expression for Z, it results

$$I=A+(\Delta I)V+I_e\exp\left(e\frac{V}{kT_e}\right),\qquad(13)$$

where A is a constant, and $\Delta I=5.4\times10^{-14}r_p\sqrt{nc_s}$. Therefore, for hydrogen ions, on assuming the ion temperature to be equal to the electron temperature, the plasma density is

$$n=(\Delta I)^2r_p^{-2}\left(\frac{kT_e}{e}\right)^{-0.5}2.4\times10^{22},\qquad(14)$$

that is equal to the result of Ref. 1.

III. APPLICATION ON RFX

On RFX an automatic routine has been implemented in order to evaluate the single Langmuir probe current–voltage characteristic.[5] From a moving average procedure, a first estimation of the floating potential V_f^* is obtained. Then, by using a least-squares routine, the characteristic is cyclically fitted by varying at every step the maximum fit voltage (V_g) from the experimental available value down to V_f^*. A minimum temperature approach is used, with some criteria in order to reject too noisy data.[5]

The procedure has been used with two different fitting routines. One is the standard simple exponential fit, i.e., Eq.

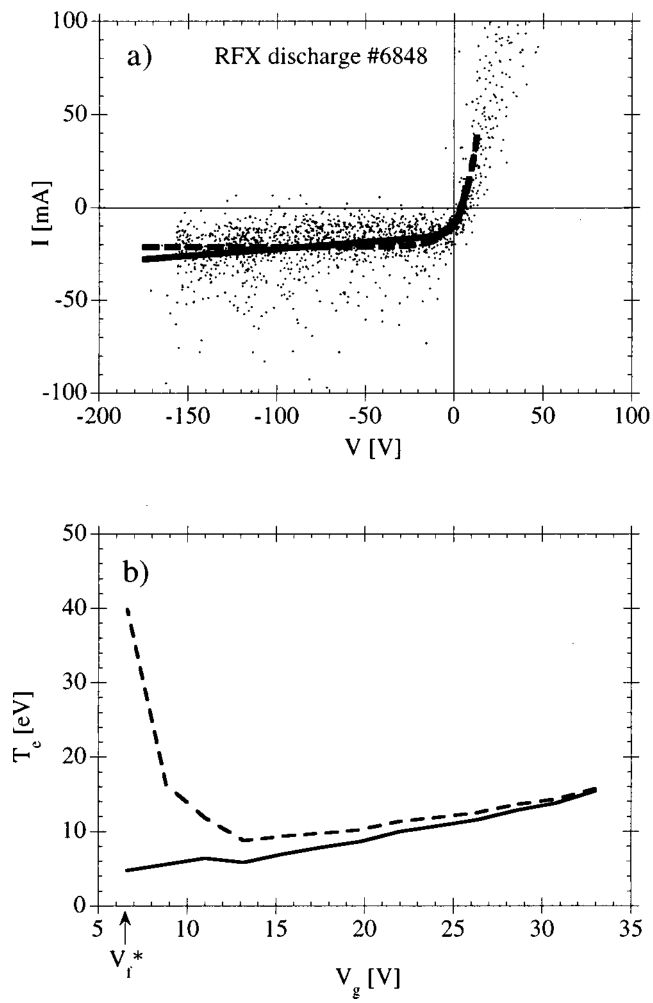

FIG. 1. (a) Current–voltage characteristic curve: experimental data (dots) and best fit using a minimum temperature approach with the simple exponential fit ($R=0$, dashed line) and four parameter data fit (solid line). (b) Comparison of electron temperature vs maximum fit voltage (V_g) using the simple exponential fit ($R=0$, dashed line) and the four parameter data fit (solid line). The probe diameter is 3 mm, with $\gamma=19°$.

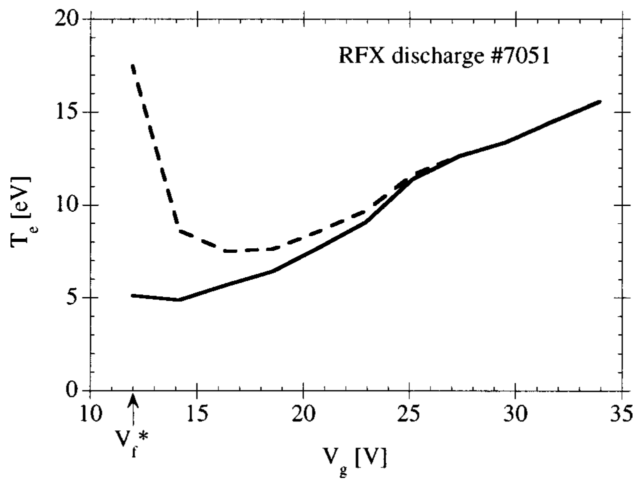

FIG. 2. Example of electron temperature vs the maximum fit voltage (V_g): the dotted line is relative to the simple exponential fit ($R=0$); the solid line is relative to the four parameter data fit. The probe diameter is 2 mm, with $\gamma=37°$.

(9) with $R=0$; the second is the formula reported in Eq. (9), i.e., with the four fitting parameters: R, I_s, V_f, and T_e.

In Fig. 1 an example is reported, relative to a cylindrical probe (3 mm diameter), inserted in a cylindrical hole (5 mm diameter), flush to a mushroom shaped limiter, with an angle (γ) of 19° with the magnetic field direction.[4] In Fig. 1(a) the current–voltage characteristic is presented, while in Fig. 1(b) the electron temperature T_e vs V_g (down to V_f^*) is reported in the two cases of simple exponential fit (dashed line) and of four parameter data fit (solid line). It can be noticed that when V_g is near to the floating potential, the simple exponential fit is no more acceptable, as the effect of nonsaturation of the ion current is important. The four parameter fit, instead, shows a regular behavior, as usually obtained in cases of ion current saturation.[6]

A similar example, relative however to a different geometry, is reported in Fig. 2. It is the case of a probe (2 mm diameter), inserted in a cylindrical hole (3 mm diameter), flush to a local plane limiter, mounted on a rotating arm, with $\gamma=37°$: an equipment that is similar to the system reported in Ref. 7. Again, applying the four parameter data fit causes the behavior of T_e vs V_g to become quite regular when V_g is near to V_f^*.

A correction on the electron temperature and ion saturation current of $\approx$20% has been obtained in general with the new four parameter data fit, with respect to the values obtained with the simple exponential fit.

In both examples the ion Larmor radius is $\approx$2 mm: therefore the probe characteristic dimension perpendicular to the magnetic field direction is comparable to the ion Larmor radius and, in this condition, the estimation of the probe collecting area is not straightforward. For the geometry of a square or cylindrical probe, flush to a local limiter, two models (one analytical, for γ between 12° and 42°, and one numerical) have been developed, in the hypothesis of noncollisionality, for the estimation of the ion collecting area and its dependence on the Debye sheath thickness.[8] The R parameter obtained from the data fitting is in good agreement with the values obtained from these models.[8]

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