Quantum optics laboratory

L. Zampieri

May 4, 2020

1 Number states and coherent states

In studying electromagnetic radiation, in quantum optics, there are two main bases that can be used to describe the Hilbert space. On one hand, the eigenvectors of the number operator $\hat{n}$ can be studied:

$$|n\rangle \quad \text{s.t.} \quad \hat{n}\,|n\rangle = n\,|n\rangle$$

On the other hand, the eigenvectors of the annihilation operator $\hat{a}$, named *coherent states*, can be considered:

$$|\alpha\rangle \quad \text{s.t.} \quad \hat{a}\,|\alpha\rangle = \alpha\,|\alpha\rangle$$

Observe that the base given by coherent states is overcomplete. For the sake of simplicity, we consider radiation in a monodimensional box, surrounded by perfectly-conductive walls and with defined wavevector k and spin.

1.1 Thermal states

Number states are interesting in the theoretical study of electromagnetic radiation: in fact, the Hamiltonian of the system can be written as:

$$\mathcal{H} = \hbar\omega(\hat{n} + {}^1\!/_2)$$

From this last expression it's clear that the number states are eigenvectors even for the Hamiltonian operator: this means that they're stationary states, namely they don't change in the temporal evolution. Moreover, they have also a defined energy:

$$E_n = \hbar\omega(n + {}^1\!/_2)$$

Thermal states are states given by a classical overlapping of different number states, whose probability distribution is described by the Boltzmann distribution:

$$P_n = \frac{e^{-\beta E_n}}{\sum_n e^{-\beta E_n}}$$

That states can be described using the density matrix:

$$\rho = \sum_n P_n\,|n\rangle\!\langle n|$$

It's easy to show that the average number of photons in the cavity $\bar{n}$ can be written as:

$$\bar{n} = \frac{1}{\exp\{\beta\hbar\omega\} - 1}$$

and the probability of finding n photons in the cavity:

$$P_n = \frac{\bar{n}^n}{(1+\bar{n})^{n+1}} \tag{1}$$

1.2 Coherent states

Coherent states, instead, are interesting due to the fact that they're very close to classical states. In fact, if one computes the expectation value of the electric field for a coherent state in a cavity, the result is a plane wave: of course, there is anyway a fluctuation due to the quantum vacuum oscillations.

The coherent states can be written as linear combination of number states as:

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum \frac{\alpha^n}{\sqrt{n!}} |k\rangle$$

Therefore, the average number of photons in the cavity is

$$\bar{n} = |\alpha|^2$$

and the probability of observing a number of photons n in the cavity is given by

$$P_n = e^{-\bar{n}} \frac{\bar{n}^n}{n!} \tag{2}$$

which is a Poisson distribution.

1.3 Experimental setup

To observe the two different type of states, an experimental setup as the one in fig. 1 is prepared. A laser beam, reflecting on a disk of sandpaper, hits a single photon detector (SPD). The latter is connected to a TimeTagger which assigns a time tag, with a resolution of 81ps, to each SPD click. The data are then read and saved from the PC.

The sandpaper disk is mounted on a motorized support: it can be put into rotation applying a proper voltage.

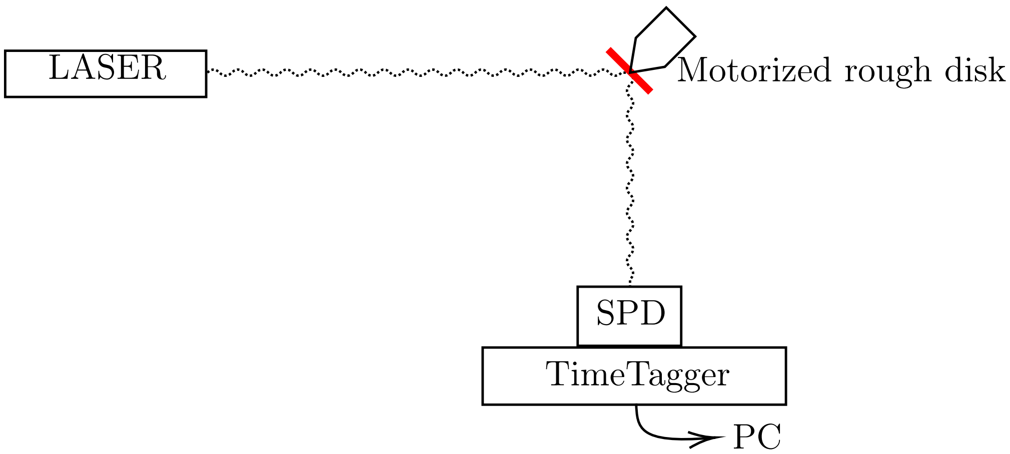


Figure 1: Experimental setup

1.4 Data analysis

Data are then collected and time tags are analyzed. Both with the disk in quiet and the disk spinning, dataset of around 20 seconds are collected. For each dataset, time tags are split in intervals, each one of $2 \cdot 10^5$ clock ticks (which means around 15µs). The number of registered event in each interval is counted and the histogram in fig. 2, in which the occurrences of each number of counts per time interval are plotted, is drawn.

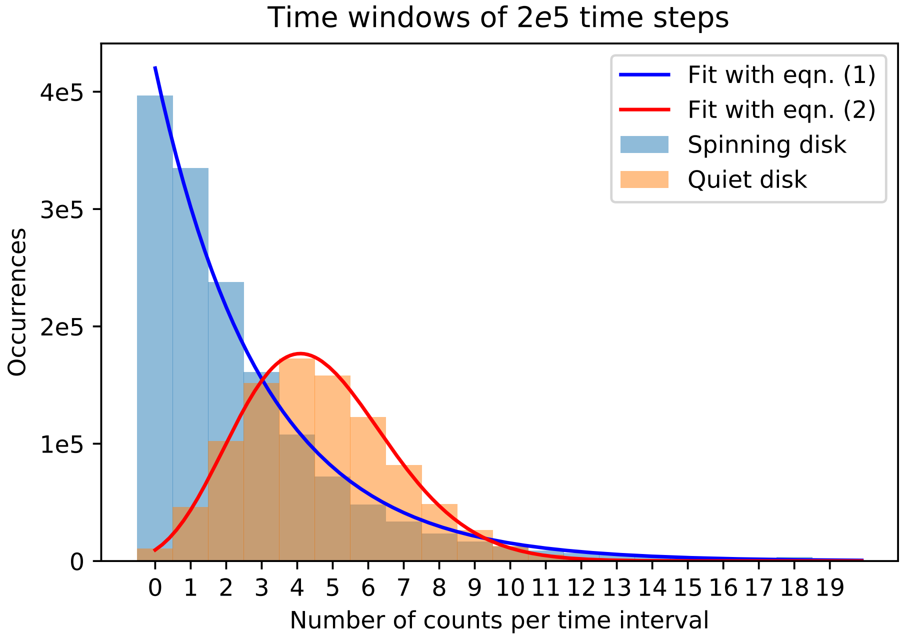


Figure 2: Occurrence of each number of counts per time interval

Larger time intervals, even if would be more statistically interesting due to the higher counts and therefore the higher-resolution plot, have not physical meaning, because the interval size would become comparable with the coherence length of the laser. Smaller time intervals, on the other hand, produce a too small statistic for the higher counts.

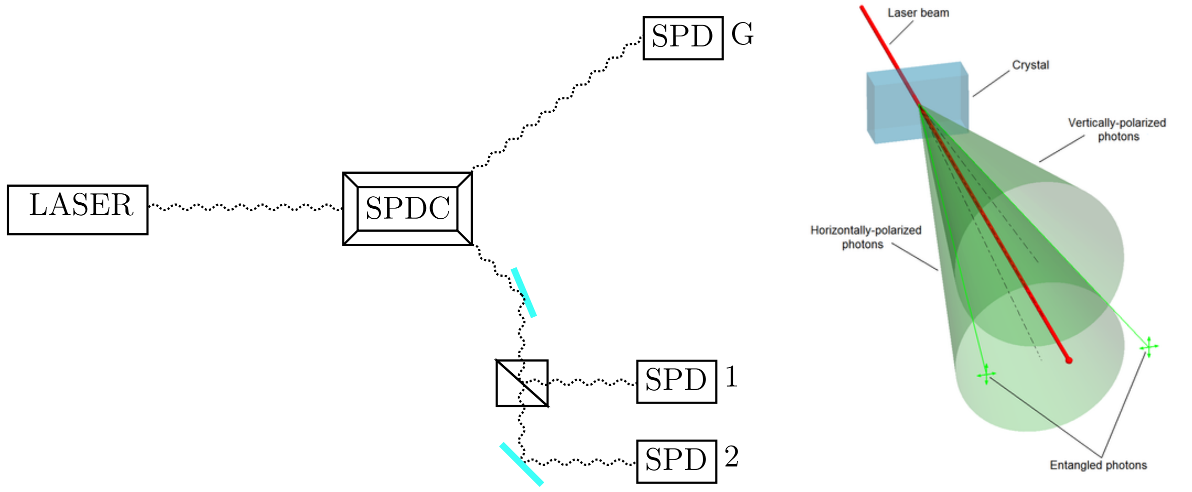
From the plot, it's clear that the two dataset follow different distributions. While the quiet-disk dataset is more Poisson-like, the spinning-disk one fit better with a exponential function. This is due to the fact that the beam produced by the laser is well described by a coherent state. When reflecting on the quiet disk, it keep its coherence and therefore the probability distributions of the photons follows distribution in eqn. (1). On the other hand, if the disk is spinning the beam is mixed and interferes with himself, due to the roughness of the disk and the continuos changing of the reflection point. Therefore, the beam assume a disordered nature, well described by a thermal state. The probability distribution of photons, therefore, follows eqn. (2).

This is more clear fitting the data with the proper distributions (properly normalized to the total number of considered windows). The fits are reported in fig. 2. From the fit, an estimation of the average frequency of photons hitting the target can be extracted.

$$\bar{n}_{Coherent} = (284 \pm 1)\text{kHz}$$
$$\bar{n}_{Thermal} = (157 \pm 5)\text{kHz}$$

Observe that no further information can be derived from this frequency: our previous described models regards stable light modes on closed cavity. No assertions can be done about for example temperature, α or ω in our open-space system.

2 Photon indivisibility



(a) Experimental setup

(b) Spontaneous parameter down conversion crystal (from English Wikipedia user J-Wiki / GFDL)

2.1 Classical point of view

To verify the indivisibility of the photon and the inadequacy of the classical undulatory theory, an experimental setup as the one in fig. 3(a) has been prepared. A laser beam hit a SPDC: it's a particular non-linear crystal which is able, from a classical undulatory point of view, of splitting a laser beam in two conical beams, each one with frequency half of the initial one and with different polarization, which intersects in two points (see fig. 3(b)). From one of the two intersection points, the beam is collected from a single photon detector, called *gate*. The beam coming from the other intersection point, instead, reach a 50/50 beam splitter. The reflected beam, then, reach the single photon detector 1, while the transmitted one reach the detector 2. With a timetagger, detectors *clicks* are collected and coincidences analyzed. In particular, the interesting quantity is

$$g^2 := \frac{N_{12G}N_G}{N_{1G}N_{2G}}$$

where N_G is the number of clicks of the gate, N_{1G} and N_{2G} the number of coincidences between gate and each other detector and N_{12G} the number of coincidences between the three detectors.

In a classical point of view, the detectors are independent. In a timetagger time-step, the probability of a detector to click is $p_i I_i$, when p_i is the detector sensitivity and I_i is the intensity hitting the detector. Therefore, considering T time steps,

$$\begin{aligned} N_G &= Tp_G I_G \\ N_{1G} &= Tp_G I_G p_1 I_1 \\ N_{2G} &= Tp_G I_G p_2 I_2 \\ N_{12G} &= Tp_G I_G p_1 I_1 p_2 I_2 \end{aligned}$$

which lead to

$$g^2 = \frac{Tp_G I_G p_1 I_1 p_2 I_2 \cdot Tp_G I_G}{Tp_G I_G p_1 I_1 \cdot Tp_G I_G p_2 I_2} = 1 \tag{3}$$

2.2 Quantum point of view

From a quantum point of view, the entire phenomena can be interpreted differently. The non-linear crystal, for each* hitting photon produces two photons, each one with different polarization and energy half of the initial. Taking photons from the intersections between the deflection cones, one obtain a couple of polarization-entangled photons. One photon hit the gate, while the other photon found the beam splitter. Therefore, it evolves in a superposition of transmitted or detected state. But, when one of the two detector reveal the photon, the wavefunction collapse and therefore is impossible that the other detector reveal it. Therefore, if the frequency of photons couple produced by the crystal is low with respect to the time resolution of the sistem (i.e., if the probability that in the same time two photons hit the gate is negligible), the number of triple coincidences is zero:

$$g^2 = \frac{Tp_{G12} \cdot Tp_G}{Tp_{G1} \cdot Tp_{G2}} = 0 \tag{4}$$

2.3 Real detectors

Real detectors, however, are not perfect: their efficiency is limited and background noise is present. To model them, let's consider a time interval of length $\mathrm{d}t$ such that the probability of having two photons hitting the crystal in the time step is negligible.

Let p be the probability of revealing a photon, and p_B the probability of a background click. Therefore, for each detector:

$$p_{\text{click}} = \delta_{\text{photon}} p + p_B$$

where δ_{photon} is 1 if there is a photon hitting the detector, 0 otherwise.

Let $2f$ be the probability of a photon being doubled by the crystal. Therefore, three cases can be distinguished: photon hitting both gate and detector 1, with probability f; photon hitting both gate and detector 2, with probability f; no photon, with probability $1-2f$. Observe that the losses on the light path are absorbed in p.

Therefore, considering the three cases:

$$\begin{aligned}
p_G &= f(p+p_B) + f(p+p_B) + (1-2f)(0+p_B) = 2fp + p_B \\
p_1 &= f(p+p_B) + f(0+p_B) + (1-2f)(0+p_B) = fp + p_B \\
p_2 &= f(0+p_B) + f(p+p_B) + (1-2f)(0+p_B) = fp + p_B \\
p_{G1} &= f(p+p_B)(p+p_B) + f(p+p_B)(0+p_B) + (1-2f)(0+p_B)(0+p_B) = fp^2 + 3fpp_B + p_B^2 \\
p_{G2} &= f(p+p_B)(0+p_B) + f(p+p_B)(p+p_B) + (1-2f)(0+p_B)(0+p_B) = fp^2 + 3fpp_B + p_B^2 \\
p_{G12} &= f(p+p_B)(p+p_B)(0+p_B) + f(p+p_B)(0+p_B)(p+p_B) + \\
&\quad + (1-2f)(0+p_B)(0+p_B)(0+p_B) = 2fp^2p_B + 4fpp_B^2 + p_B^3
\end{aligned}$$

and substituting in eq. (4):

$$g^2 = \frac{p_{G12}p_G}{p_1p_2} = \frac{(2fp^2p_B + 4fpp_B^2 + p_B^3)(2fp+p_B)}{(fp^2 + 3fpp_B + p_B^2)^2}$$

which is 0 in the ideal case when $p_B = 0$, but can be different from 0 otherwise.

2.4 Experimental data

An acquisition of about 45s has been performed, and data has been collected. To prevent timetagger bias, which result in a different length of odd and even time steps, all the timetags are rounded up to the nearest even number. Then, data should be rescaled to consider possibly different length of the path each photon must travel to reach the detectors. To make this, the differences between the clicks

*Not for each, only for a very small part, due to efficiency. But in our argument the non-deflected photons can be neglected.

of detectors 1 and 2 and the nearest previous and next clicks of gate detector has been computed. The results are shown in fig. 3.

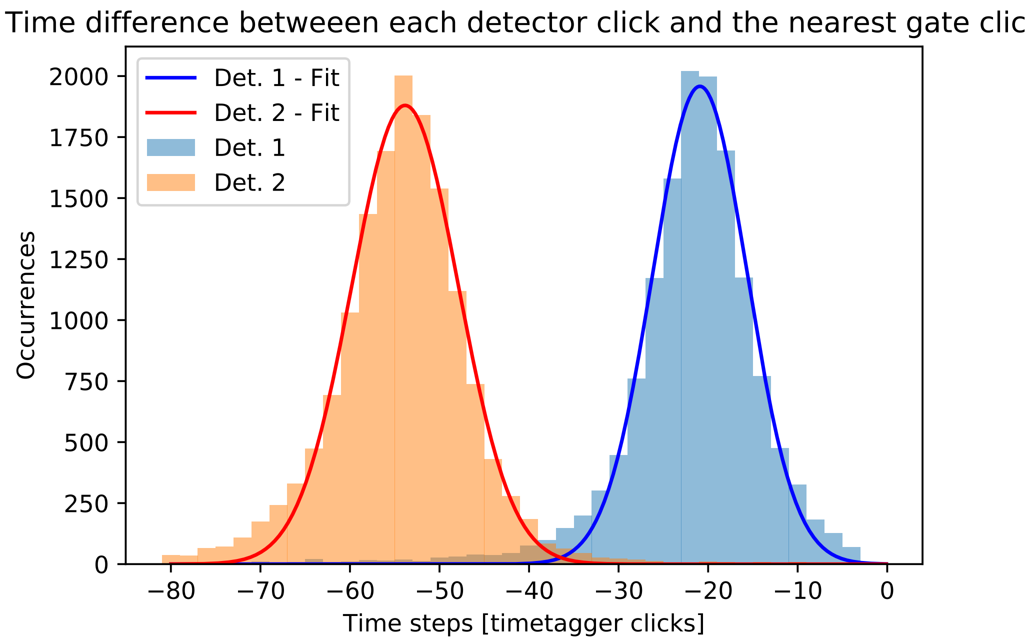


Figure 3: Time differences. 1 timetagger click equals to 80.955ps
.

Even if data are widely spread on a wide spectra, two peaks (containing no more than 2% of the total data points) are clearly visible. This two peaks are the ones concerning the coincidences. They're fitted through a gaussian, which allow us to determine the centroid, and data are shifted such that for both the detectors the centroid is in zero.

The sigmas of the fits give as a suggestion about the coincidences tolerance. In the following, two events are considered coincident if their time distance is less than five times the maximum sigma between the two fits:

$$w = 5\max(\sigma_1, \sigma_2) = 2.4\text{ns}$$

The following result are found

Acquisition time	~ 44s	
Gate counts	1554000	± 1000
Det. 1 counts	1057000	± 1000
Det. 2 counts	1000000	± 1000
Coincidences G-1	14000	± 100
Coincidences G-2	14800	± 100
Coincidences G-1-2	3	± 2

Table 1: Results. Poissonian errors has been considered.

which lead to, considering a poissionian error on the counts, to:

$$g^2 = 0.02 \pm 0.01$$

This result is not compatible (at more than 5σ) with the classical prediction: we can therefore reject the classical hypothesis.

From the data, considering a time interval of $2w$ and the upstream probability considerations, a roughly estimation of the system parameters can be done:

$$T = \frac{44\text{s}}{2 \cdot 2.4\text{ns}} \sim 9 \cdot 10^9$$
$$p_G \sim \frac{N_G}{T} \sim 0.00017$$
$$p_1 \sim p_2 \sim \frac{N_1 + N_2}{2T} \sim 0.00011$$
$$p_{G1} \sim p_{G2} \sim \frac{N_{G1} + N_{G2}}{2T} \sim 0.0000016$$

and solving the system:

$$f \sim 0.002 \qquad p \sim 0.03 \qquad p_B = 0.00005$$
$$g^2_{\text{expected}} \sim 0.01$$

The expected g^2, finally, is fully compatible with the found one!

In the end, we can affirm that the classical theory can be rejected with a 5σ confidence, and that the quantum approach is compatible with the found results.