Università degli Studi di Padova

Dipartimento di Fisica e Astronomia Galileo Galilei

REALIZATION OF A QUANTUM INTEGER-SPIN CHAIN WITH CONTROLLABLE INTERACTIONS

C. Senko, P. Richerme, J. Smith, A. Lee, I. Cohen, A. Retzker, and C. Monroe

Phys. Rev. X 5, 021026 (2015)

Introduction to Quantum Hardware

Prof. Caterina Braggio
Prof. Carmelo Mordini

Alessandro Miotto

Introduction


PHYSICAL REVIEW X **5,** 021026 (2015)

Realization of a Quantum Integer-Spin Chain with Controllable Interactions

C. Senko,[1] P. Richerme,[1] J. Smith,[1] A. Lee,[1] I. Cohen,[2] A. Retzker,[2] and C. Monroe[1]
[1]*Joint Quantum Institute, University of Maryland Department of Physics and National Institute of Standards and Technology, College Park, Maryland 20742, USA*
[2]*Racah Institute of Physics, The Hebrew University of Jerusalem, Jerusalem 91904, Givat Ram, Israel*
(Received 6 October 2014; revised manuscript received 16 March 2015; published 17 June 2015)

The physics of interacting integer-spin chains has been a topic of intense theoretical interest, particularly in the context of symmetry-protected topological phases. However, there has not been a controllable model system to study this physics experimentally. We demonstrate how spin-dependent forces on trapped ions can be used to engineer an effective system of interacting spin-1 particles. Our system evolves coherently under an applied spin-1 *XY* Hamiltonian with tunable, long-range couplings, and all three quantum levels at each site participate in the dynamics. We observe the time evolution of the system and verify its coherence by entangling a pair of effective three-level particles ("qutrits") with 86% fidelity. By adiabatically ramping a global field, we produce ground states of the *XY* model, and we demonstrate an instance where the ground state cannot be created without breaking the same symmetries that protect the topological Haldane phase. This experimental platform enables future studies of symmetry-protected order in spin-1 systems and their use in quantum applications.

DOI: 10.1103/PhysRevX.5.021026

Subject Areas: Atomic and Molecular Physics, Condensed Matter Physics, Quantum Physics

I. INTRODUCTION

A major area of current research is devoted to developing experimentally controllable systems that can be used for quantum computation, quantum communication, and quantum simulation of many-body physics. To date, most experiments have focused on the use of two-level systems ("qubits") for computation and communication [1,2] and for the study of spin-$1/2$ (or spinless) many-body phenomena [3,4]. However, there are a variety of motivations for performing experiments in higher-dimensional Hilbert spaces. Contrary to the intuition that enlarging the spin degree simplifies calculations by making them semiclassical [5], spin $> 1/2$ systems inherently have more complexity and cost exponentially more resources to classically simulate. For instance, it is computationally easy to find the ground-state energy of a spin-$1/2$ chain with nearest-neighbor-only interactions in one dimension; for systems with spin $7/2$ or higher, the problem is known to belong to the QMA-complete complexity class, which is a quantum analog of the classical NP-complete class [6,7]. The difficulty of this problem for intermediate spin values, such as spin 1, is still an open question. From a more practical point of view, controllable three-level systems ("qutrits") are useful for quantum logic, since they can substantially simplify certain operations within quantum algorithms [8,9] and can enhance the efficiency of quantum communication protocols [10].

When individual three-level systems are coupled together, they can be used to encode the physics of interacting spin-1 particles. Such systems have attracted a great deal of theoretical interest following Haldane's conjecture that antiferromagnetic Heisenberg spin-1 chains, as opposed to spin-$1/2$ systems, have a finite energy gap that corresponds to exponentially decaying correlation functions [11,12]. This so-called Haldane phase possesses a doubly degenerate entanglement spectrum [13] and a nonlocal string order [14,15], which is related to the order appearing in spin liquids [16] and in the fractional quantum Hall effect. These characteristics suggest that the Haldane phase is one of the simplest known examples of a symmetry-protected topological phase of matter [14].

In addition to their interesting many-body properties, topological phases may be exploited in a more applied setting. The Haldane phase is useful for quantum operations (for instance, as a perfect quantum wire) [17,18] and can be destroyed only by crossing a phase transition. The finite energy gap in topological spin-1 systems makes them a potential candidate for long-lived, robust quantum memories [19], and schemes using symmetry-protected spin-1 phases for measurement-based quantum computation have also been proposed [20,21].

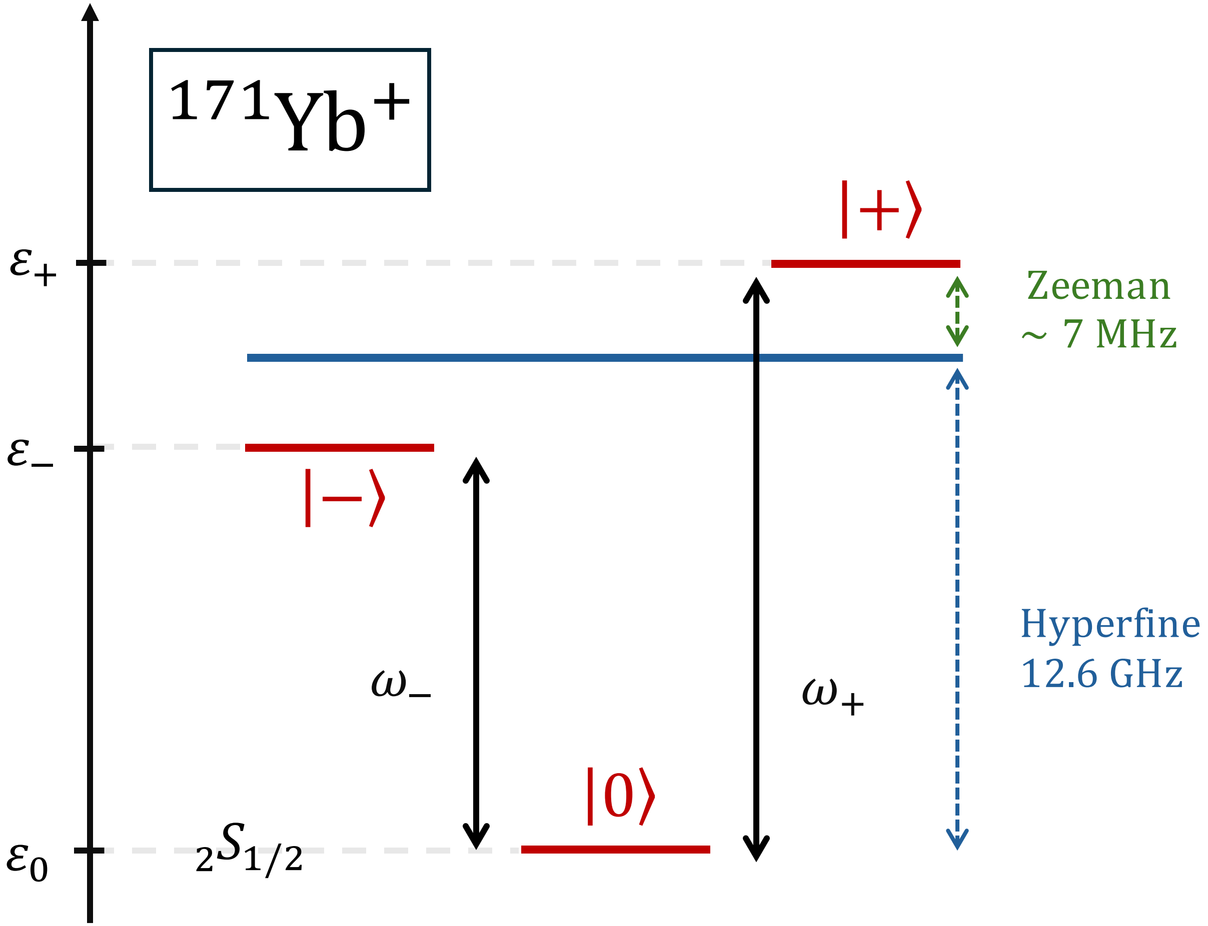
Several groups have developed controllable three-level quantum systems by using pairs of photons [22], superconducting circuits [23], or dressed states in trapped ions

This research centers on the **engineering and study** of a one-dimensional **spin-1 "*qutrit*"** chain using trapped $^{171}\mathrm{Yb}^{+}$ ions

The primary objective was to successfully realize and control a **spin-1 XY Hamiltonian** and its evolution

This work establishes a robust experimental platform for future investigations into spin-1 chains, particularly their topological properties

Three-level system: $^{171}\mathrm{Yb}^+$



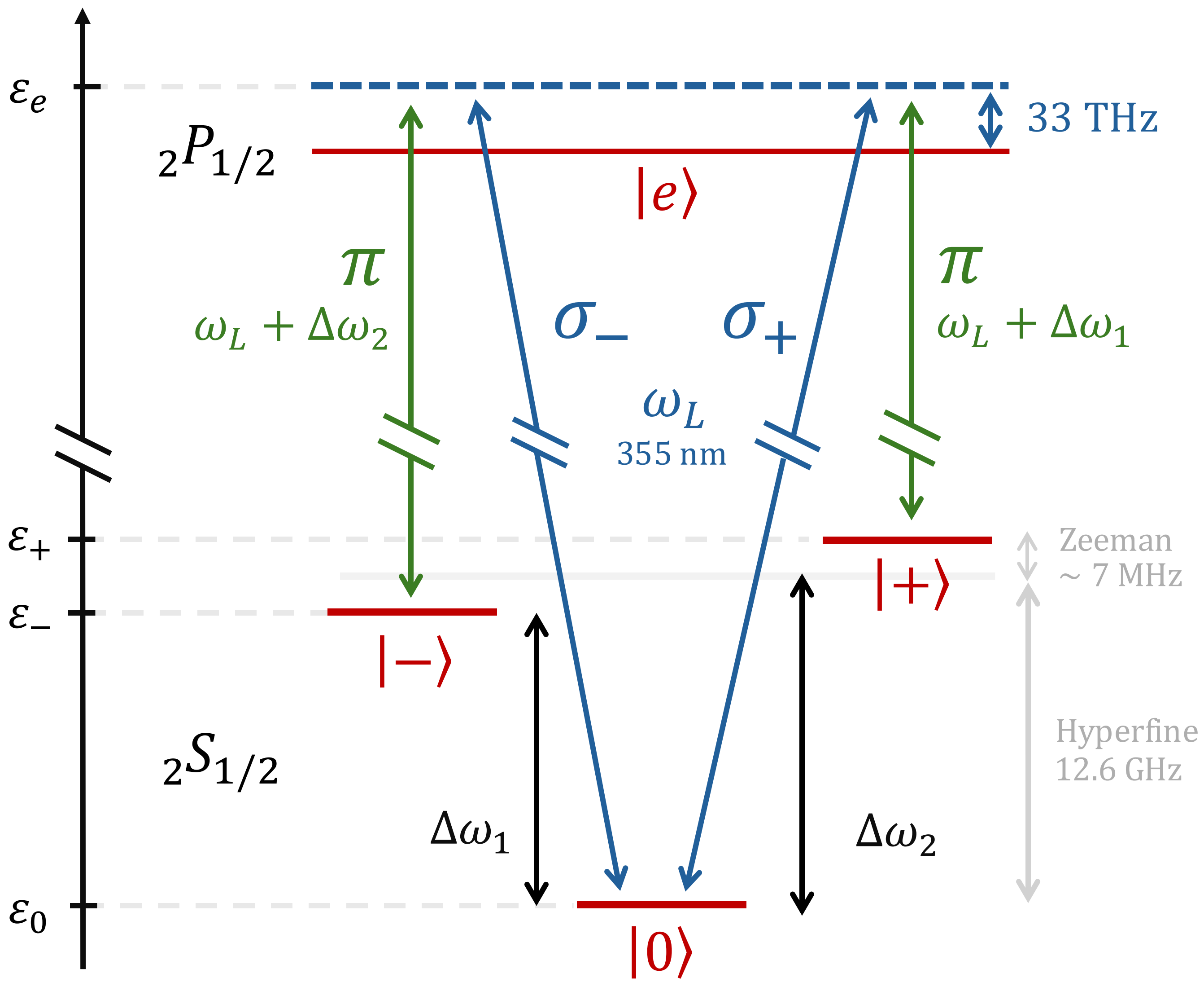
Hyperfine splitting separates the ground state of $^{171}\mathrm{Yb}^+$ (nuclear spin $I = 1/2$) into two energy levels: $F = 0{,}1$

A magnetic field of $\sim 5\ G$ lift the degeneracy of $F = 1$, causing Zeeman splitting between the $m_F = \pm 1$ sublevels

$$S_z|F, m_F\rangle = \hbar m_f |F, m_F\rangle$$

$$|+\rangle = |F = 1, m_F = 1\rangle$$
$$|-\rangle = |F = 1, m_F = -1\rangle$$
$$|0\rangle = |F = 0, m_F = 0\rangle$$

Three-level system: Raman transitions



Raman induced transition using two linearly polarized lasers:

- **σ-laser: polarized orthogonal to the magnetic field. Provide equal superposition of σ_+ and σ_-**

$$\boldsymbol{E}_\sigma = \mathcal{E}_\sigma \boldsymbol{\epsilon}_\sigma \cos(\boldsymbol{k}_\sigma \cdot \boldsymbol{r} - \omega_L t + \varphi_\sigma)$$

- **π-laser: polarized along to the magnetic field. Beat notes can be applied to tune the frequency**

$$\boldsymbol{E}_\pi = \mathcal{E}_\pi \boldsymbol{\epsilon}_\pi \cos(\boldsymbol{k}_\pi \cdot \boldsymbol{r} - \omega_\pi t + \varphi_\pi)$$

$$\omega_\pi = \begin{cases} \omega_L - \Delta\omega_1 & \text{target } |e\rangle \leftrightarrow |-\rangle \\ \omega_L - \Delta\omega_2 & \text{target } |e\rangle \leftrightarrow |+\rangle \end{cases}$$

Three-level system: Raman transitions

$$H_0 = \begin{pmatrix} \varepsilon_+ & 0 & 0 & 0 \\ 0 & \varepsilon_0 & 0 & 0 \\ 0 & 0 & \varepsilon_- & 0 \\ 0 & 0 & 0 & \varepsilon_e \end{pmatrix} \qquad H_{\text{int}} = \begin{pmatrix} 0 & 0 & 0 & g_+^* \\ 0 & 0 & 0 & g_0^* \\ 0 & 0 & 0 & g_-^* \\ g_+ & g_0 & g_- & 0 \end{pmatrix}$$

Photon coupling terms for $i = \{0, -, +\}$

$$g_i = \Omega_i e^{i(\boldsymbol{k}_i \cdot \boldsymbol{r} - \omega_i t)}$$

with $\Omega_i \propto \mathcal{E}_i \boldsymbol{d}_{ie} \cdot \boldsymbol{\epsilon}_i$ the (complex) Rabi frequency. H_{int} couple the ground states $|i\rangle$ with the excited state $|e\rangle$

By choosing $K = \mathrm{diag}(\omega_L - \Delta\omega_1, \omega_L, \omega_L - \Delta\omega_2, 0)$ for the unitary transformation $U(t) = \exp(iKt)$ we get the following Hamiltonian. Under the RWA, the fast-rotating component can be ignored

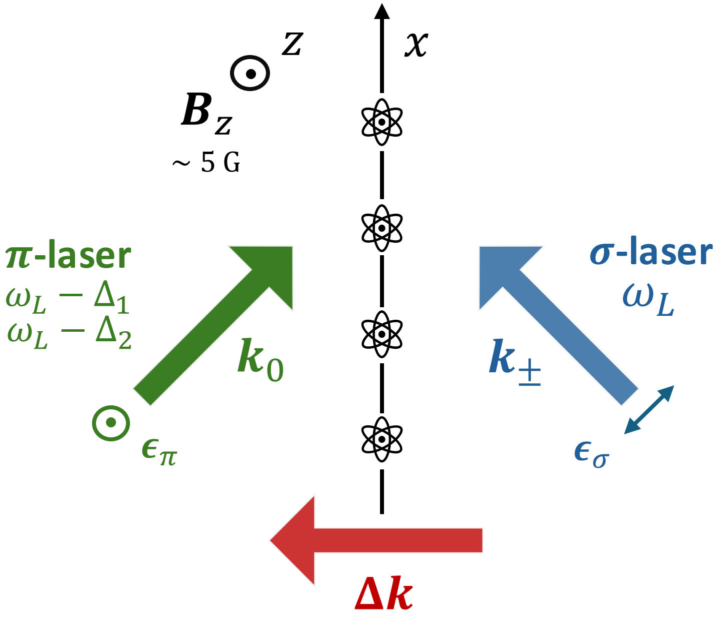
$$\widetilde{H} = UHU^\dagger + iU\dot{U} = \begin{pmatrix} \Delta_+ & 0 & 0 & \Omega_+^* e^{-i\boldsymbol{k}_+ \cdot \boldsymbol{r}} \\ 0 & \Delta_0 & 0 & \Omega_0^* e^{-i\boldsymbol{k}_0 \cdot \boldsymbol{r}} \\ 0 & 0 & \Delta_- & \Omega_-^* e^{-i\boldsymbol{k}_- \cdot \boldsymbol{r}} \\ \Omega_+ e^{i\boldsymbol{k}_0 \cdot \boldsymbol{r}} & \Omega_0 e^{i\boldsymbol{k}_0 \cdot \boldsymbol{r}} & \Omega_- e^{i\boldsymbol{k}_- \cdot \boldsymbol{r}} & \varepsilon_e \end{pmatrix} = \begin{pmatrix} \widehat{\boldsymbol{\omega}} & \widehat{\boldsymbol{\Omega}}^\dagger \\ \widehat{\boldsymbol{\Omega}} & \varepsilon_e \end{pmatrix}$$

Three-level system: Raman transitions

$$\tilde{H} = \begin{pmatrix} \hat{\boldsymbol{\omega}} & \hat{\boldsymbol{\Omega}}^* \\ \hat{\boldsymbol{\Omega}} & \varepsilon_e \end{pmatrix}$$

We decompose the Hilbert space in a slow and fast subspaces and perform the adiabatic elimination, which returns the effective 3-level hamiltonian

$$H_{\text{eff}} = \hat{\boldsymbol{\omega}} - \frac{1}{\varepsilon_e}\hat{\boldsymbol{\Omega}}^\dagger \hat{\boldsymbol{\Omega}} = \begin{pmatrix} \Delta\omega_1 & g_{-0}^* & g_{-+}^* \\ g_{-0} & 0 & g_{0+}^* \\ g_{-+} & g_{0+} & \Delta\omega_2 \end{pmatrix} \qquad g_{ij} = -\frac{\Omega_i^*\Omega_j}{\varepsilon_e} e^{i(\boldsymbol{k}_j - \boldsymbol{k}_i)\cdot\boldsymbol{r}}$$



Based on the experimatal setup, it is possible to simplify the Hamiltonian

$$\Omega \coloneqq \Omega_+ = \Omega_- = \Omega_0$$

$$\Delta\boldsymbol{k} \coloneqq \boldsymbol{k}_0 - \boldsymbol{k}_- \approx \boldsymbol{k}_0 - \boldsymbol{k}_+$$

$$\boldsymbol{k}_+ - \boldsymbol{k}_- \approx 0$$

$$H_{\text{eff}} = \begin{pmatrix} \Delta\omega_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta\omega_2 \end{pmatrix} - \frac{|\Omega|^2}{\varepsilon_e}\begin{pmatrix} 0 & e^{-i\Delta\boldsymbol{k}\cdot\boldsymbol{r}} & 1 \\ e^{+i\Delta\boldsymbol{k}\cdot\boldsymbol{r}} & 0 & e^{-i\Delta\boldsymbol{k}\cdot\boldsymbol{r}} \\ 1 & e^{+i\Delta\boldsymbol{k}\cdot\boldsymbol{r}} & 0 \end{pmatrix}$$

Three-level system: Raman transitions

In the Lamb-Dicke regime $|\Delta\mathbf{k}\cdot\langle\boldsymbol{r}\rangle| \ll 1$ it is possible to expand $e^{\pm i\Delta\boldsymbol{k}\cdot\boldsymbol{r}} \approx 1 \pm i\eta(a+a^\dagger)$

$$H_{\text{int}} \propto e^{-i\Delta\boldsymbol{k}\cdot\boldsymbol{r}}S_+ + e^{+i\Delta\boldsymbol{k}\cdot\boldsymbol{r}}S_- \approx -i\eta\left(a+a^\dagger\right)S_+ + i\eta\left(a+a^\dagger\right)S_-$$

Using $H_0 = \omega a^\dagger a + \mu S_z$, in the RWA only the terms S_+a, $S_-a^\dagger$ can be considered.
This terms corresponds to the excitation/de-excitation of the atom and the annihilation/creation of a phonon

$$\begin{aligned} a^\dagger &\mapsto e^{i\omega t} \\ a &\mapsto e^{-i\omega t} \end{aligned} \qquad \begin{aligned} S_+ &\mapsto e^{i\mu t} \\ S_- &\mapsto e^{-i\mu t} \end{aligned} \qquad H_{\text{eff}} \approx -S_+ a e^{i(\mu-\omega)t} + S_- a^\dagger e^{-i(\mu-\omega)t}$$

Considering a multi-mode harmonic oscillator $\sum_m \omega_m a_m^\dagger a_m$ and a chain of ions i, the interaction Hamiltonian become:

$$\boxed{H = \sum_{i,m} \frac{i\eta_{i,m}\Omega_i}{2\sqrt{2}} \left(-S_+^i a_m e^{i(\mu-\omega_m)t} + S_-^i a_m^\dagger e^{-i(\mu-\omega_m)t}\right)}$$

XY model

$$H(t) = \sum_{i,m} \frac{i\eta_{i,m}\Omega_i}{2\sqrt{2}} \left(-S_+^i a_m e^{i(\mu-\omega_m)t} + S_-^i a_m^\dagger e^{-i(\mu-\omega_m)t}\right)$$

In the **dispersive regime** **(the optical beat note is far from each normal modes $|\mu - \omega_m| \gg \eta_{i,m}\Omega_\mathrm{i}$), the phonons are only virtually excited. This can be seen by performing the Magnus expansion:**

$$U(t) = \exp\left\{-i\int_0^t dt_1 H(t_1) - \frac{1}{2}\int_0^t dt_1 \int_0^{t_1} dt_2 [H(t_1), H(t_2)]\right\}$$

XY model Hamiltonian

$$H_\mathrm{eff} = \sum_{i<j} \frac{J_{ij}}{4}\left(S_+^i S_-^j + S_-^i S_+^j\right) + \sum_{i,m} V_{im}\left[\left(2a_m a_m^\dagger + 1\right)S_z^i - \left(S_z^i\right)^2\right]$$

$$J_{ij} = \Omega_i\Omega_j \sum_m \frac{\eta_{im}\eta_{jm}}{2(\mu - \omega_m)}$$

$$V_{im} = \frac{(\eta_{im}\Omega_i)^2}{8(\mu - \omega_m)}$$

XY model

TWO-BODY INTERACTION
XY-MODEL HAMILTONIAN

For large , the Ising matrix J_{ij} can be approximated by a long-range antiferromagnetic coupling

$$J_{ij} \simeq \frac{J_0}{|i-j|^\alpha}$$

Nearest-neighbor Ising coupling: $J_0 \simeq \mathcal{O}(1\,\mathrm{kHz})$
Range of interaction (tunable): $0 < \alpha < 3$

$$H_{\mathrm{eff}} = \overbrace{\sum_{i<j} \frac{J_{ij}}{4}\left(S_+^i S_-^j + S_-^i S_+^j\right)} + \underbrace{\sum_{i,m} V_{im}\left[\left(2a_m a_m^\dagger + 1\right)S_z^i - \left(S_z^i\right)^2\right]}$$

ONE-BODY INTERACTION

For short chain, or for long range interaction ($\alpha \lesssim 0.5$) V_{ij} can be factored out

$$\approx B\sum_{i=1}^{N} S_z^i + D\sum_{i=1}^{N}\left(S_z^i\right)^2$$

Preparation

MOTIONAL-STATE PREPARATION The system is cooled near the motional ground state $\langle \bar{n} \rangle \approx 0.05$

SPIN-STATE PREPARATION Optical pumping to the $|00 \ldots 0\rangle$ state. The total spin z-component $\mathcal{S}_z = \sum_i S_z^{(i)} = 0$. Because the hamiltonian commute with $\mathcal{S}_z$, then the dynamics is restricted to the subspace $\mathcal{S}_z = 0$

- → **PRO:** Protection against fluctuation of the magnetic field $\Delta \boldsymbol{B}(t)$. This results in coherence times which could, in principle, reach seconds
- → **CON:** Hilbert space for the chain of N ions partially reduced to $\sim 3^N/2\sqrt{N}$. Stil exponentially greater than the $\sim 2^N$ for a spin $1/2$ system

Measurement

It is not possible to discriminate between the state $|+\rangle$ and $|-\rangle$ because both appears bright under laser-induced fluorescence

A π rotation can be used to move from $|0\rangle$ state to the $|\pm\rangle$ state

Example for two ion chain:

DARK STATE $|0\rangle$

First ion is bright: $|+\rangle$ or $|-\rangle$

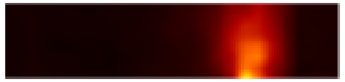
Second ion is dark: $|0\rangle$

$|?,0\rangle$

$\xrightarrow{\pi}$

$|0\rangle \leftrightarrow |+\rangle$

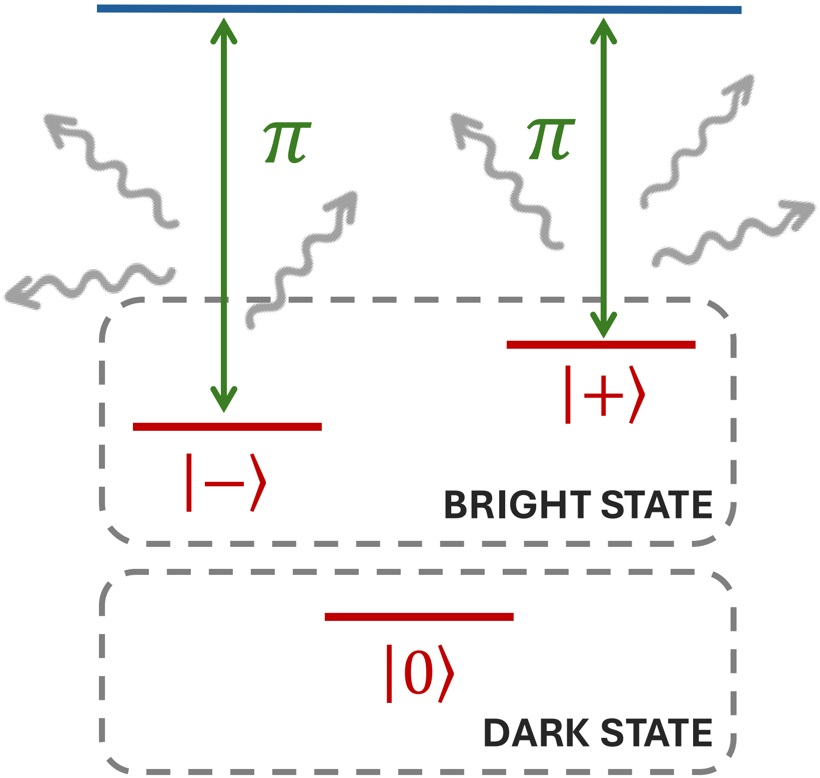
DARK STATE $|+\rangle$



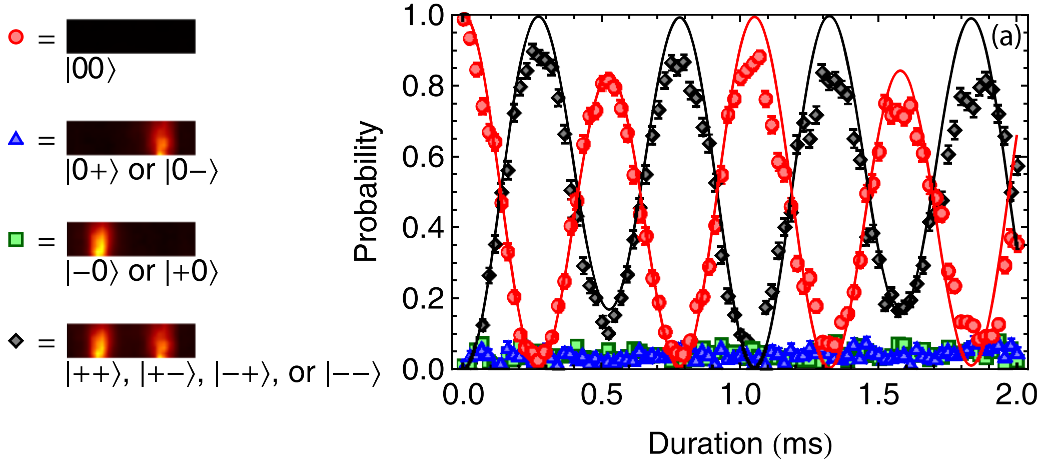
First ion is dark: $|+\rangle$

Second ion is bright: $|0\rangle$ or $|-\rangle$

$|+,0\rangle$

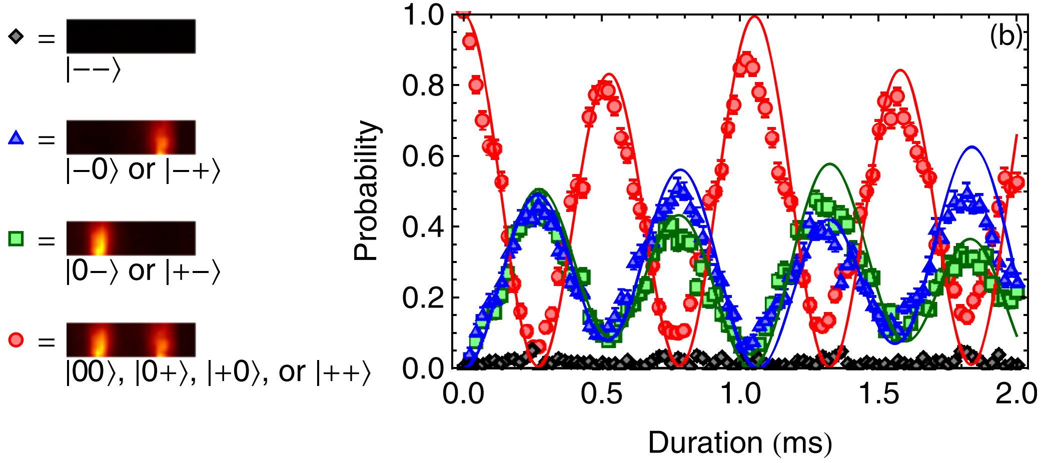


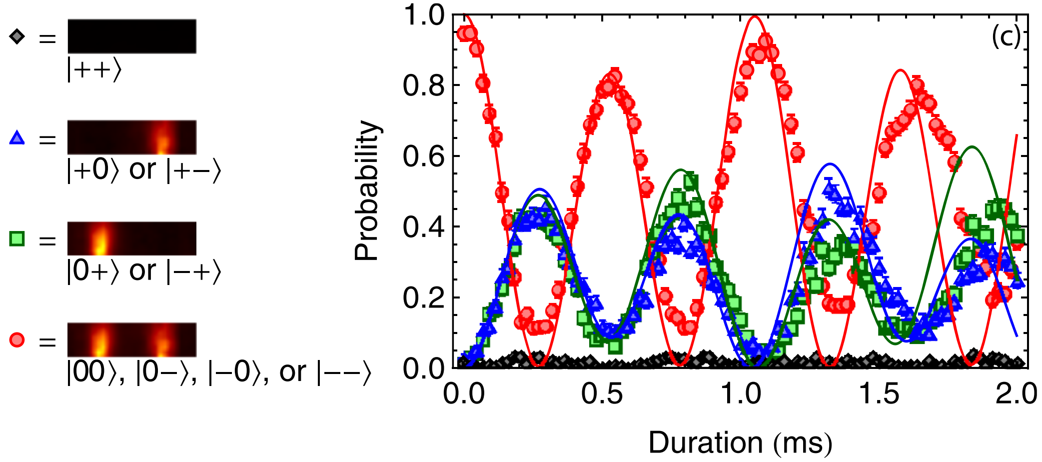
Coherent dynamics of two spins



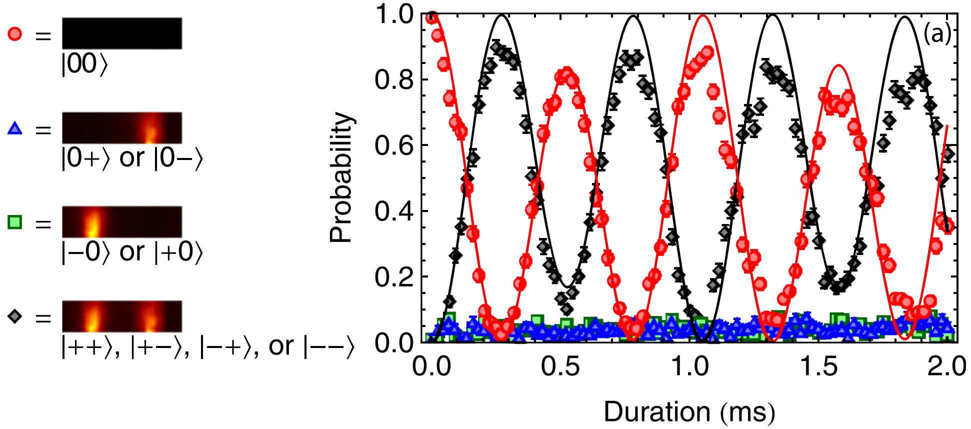
- **Rabi flopping between $|00\rangle$ and the entangled state $\frac{1}{\sqrt{2}}(|+-\rangle + |-+\rangle)$, following the evolution of the XY Hamiltonian**
- **The slight divergence can be attributed to a position-dependent S_z^i and $(S_z^i)^2$ shifts. A numerical evolution of the system, when fitted to experimental data, produces the following result:**

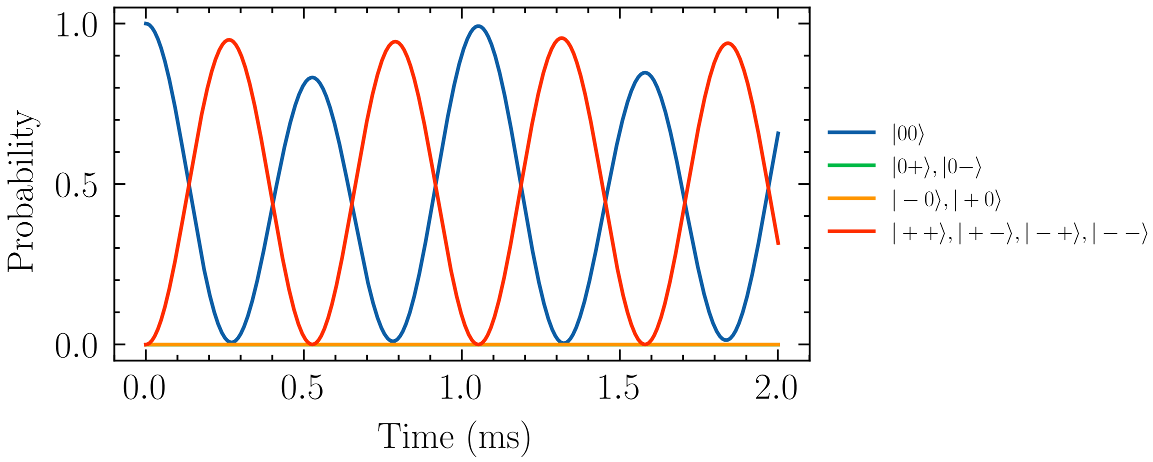
$$(200\ \mathrm{Hz})S_z^2 + (150\ \mathrm{Hz})(S_z^2)^2$$

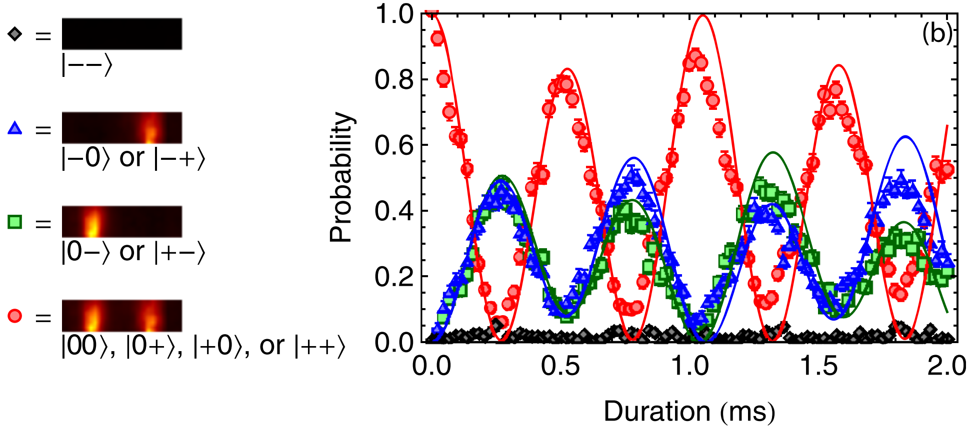


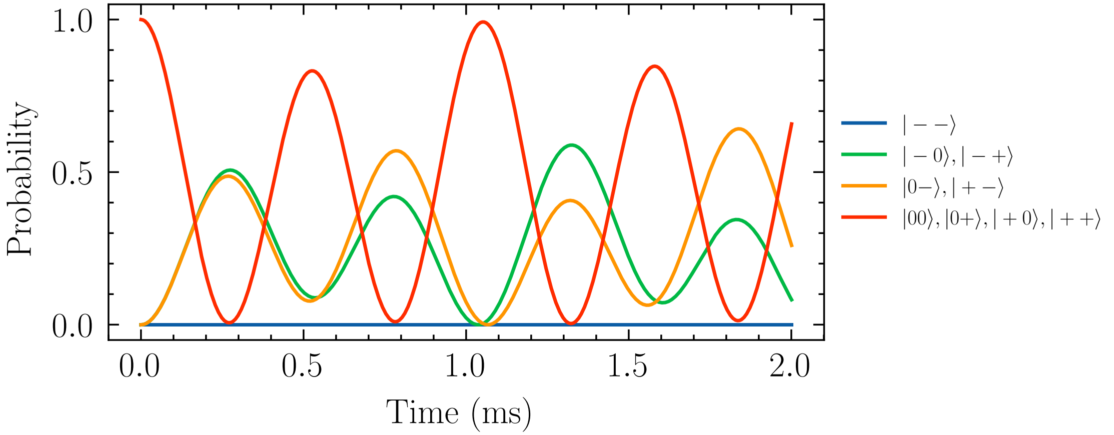


Coherent dynamics of two spins









$$H = J_{12}(S_+^1 S_-^2 + S_-^1 S_+^2) + \alpha S_z^2 + \beta (S_z^2)^2$$

$J_{12} \approx 0.5/\sqrt{2}t_0 = 1.31\ \mathrm{kHz}$

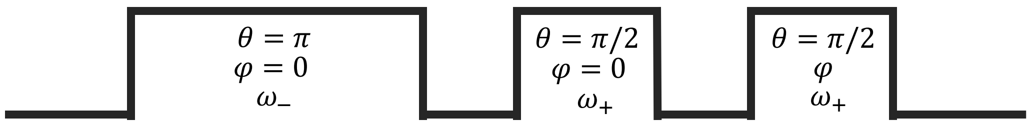
$\alpha \approx 200\ \mathrm{Hz}$ $\quad \beta \approx 150\ \mathrm{Hz}$

At time $t_n = (2n+1)/(2\sqrt{2}J_{12})$ time system is in the entangled state $\frac{1}{\sqrt{2}}(|+-\rangle + |-+\rangle)$

Coherent dynamics of two spins: entanglement verification

The effectiveness of the entanglement procedure can be studied through a sequence of global rotations

$$R_{0\pm}(\theta,\varphi) = \exp\left\{\frac{i\theta}{2}\sum_k \left[e^{\pm i\varphi}|\pm\rangle\langle 0|_k + e^{\mp i\varphi}|0\rangle\langle \pm|_k\right]\right\}$$



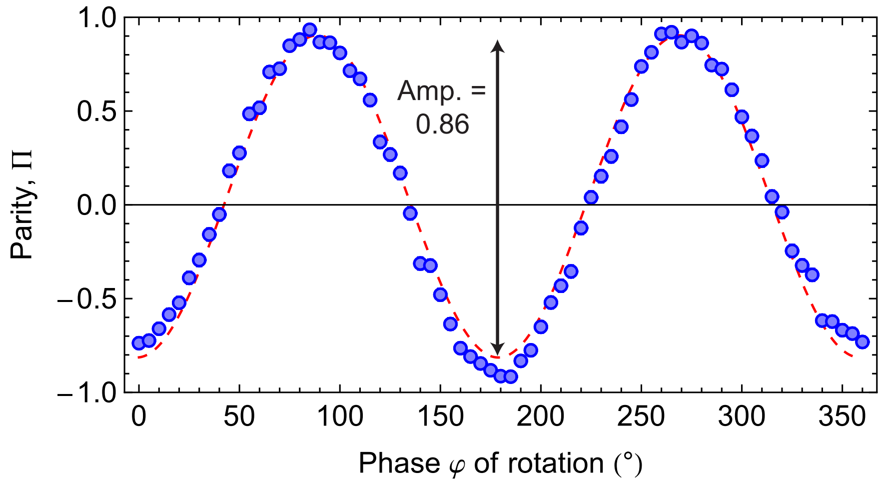
$R_{0-}(\pi, 0)$ $R_{0+}(\pi/2, 0)$ $R_{0+}(\pi/2, \varphi)$

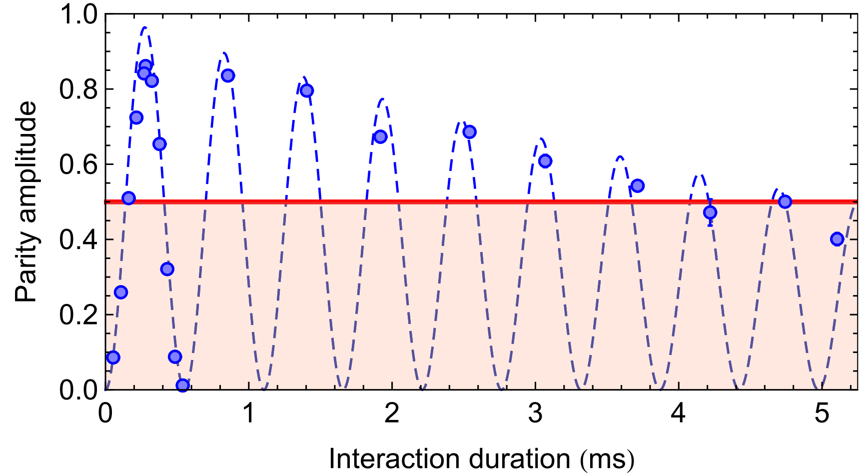
$$\frac{|+-\rangle + |-+\rangle}{2} \rightarrow \frac{|+0\rangle + |0+\rangle}{2} \rightarrow \frac{|00\rangle + |++\rangle}{2} \rightarrow \frac{ie^{i\varphi}}{2}\left(\sin\varphi\,(|00\rangle + |++\rangle) + \cos\varphi\,(|0+\rangle + |+0\rangle)\right)$$

A third φ-rotation is applied to probe the final state. This is done by using the state parity given by:

$$\Pi(\varphi) = \sum_{j=0}^{2}(-1)^j P_j(\varphi) \approx C - A\cos(2\varphi)$$

An amplitude $A > 1/2$ certifies an entangled state between the two ions





Laser intensity fluctuations, along with point instability and **inhomogeneities of V_{ij}** across the chain, are the two main sources of **decoherence**, which leads to dephasing over time.

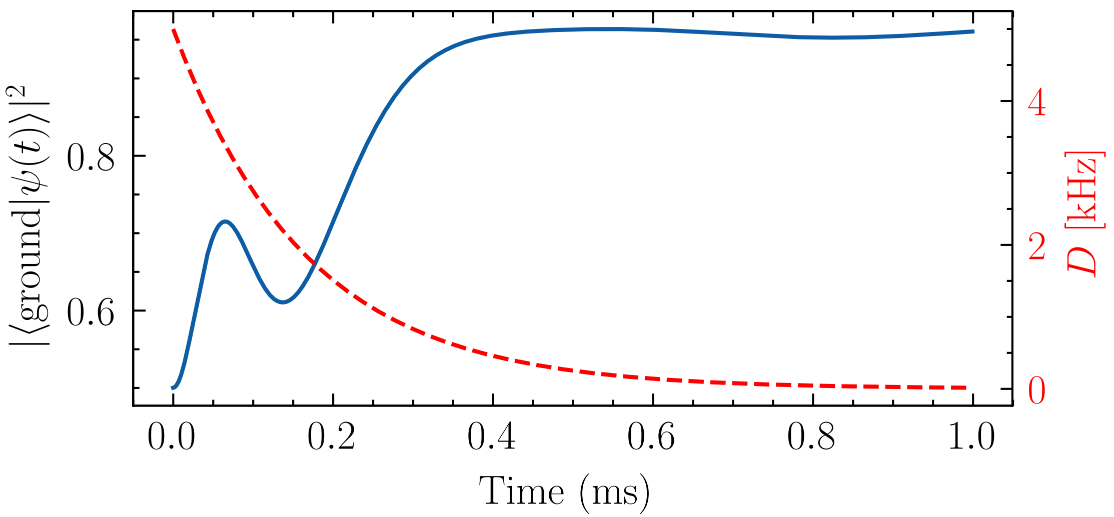
Ground state production

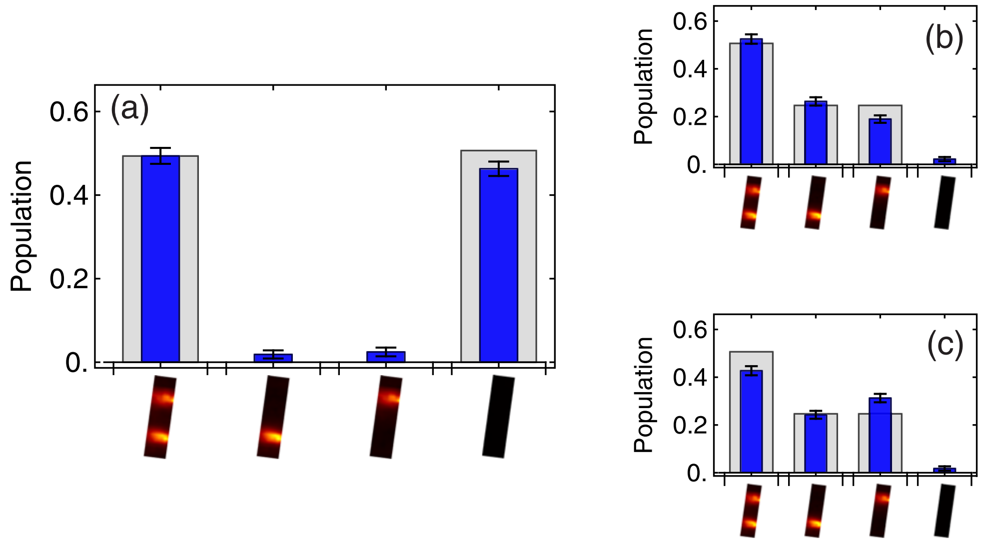
By shifting the beat frequency of the Raman laser to $\omega_{\pm} \mp \mu - D$, it is possible to introduce a tunable $\left(S_z^i\right)^2$ term.

This term is useful to lead the system from $|00\rangle$ state to the XY hamiltonian ground state $\frac{1}{\sqrt{2}}|00\rangle - \frac{1}{2}(|+-\rangle + |-+\rangle)$

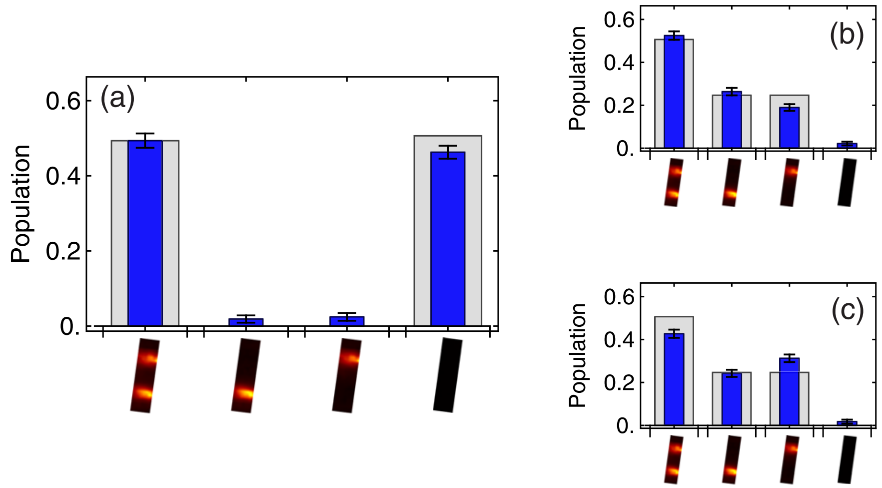
$$H = \sum_{i<j} \frac{J_{ij}}{4}\left(S_+^i S_-^j + S_-^i S_+^j\right) + B \sum_{i=1}^{N} S_z^i - D(t) \sum_{i=1}^{N} \left(S_z^i\right)^2$$

$$D(t) = D_0 e^{-t/\tau} \qquad D_0 = 5\ \text{kHz},\quad \tau = 0.167\ \text{ms}$$





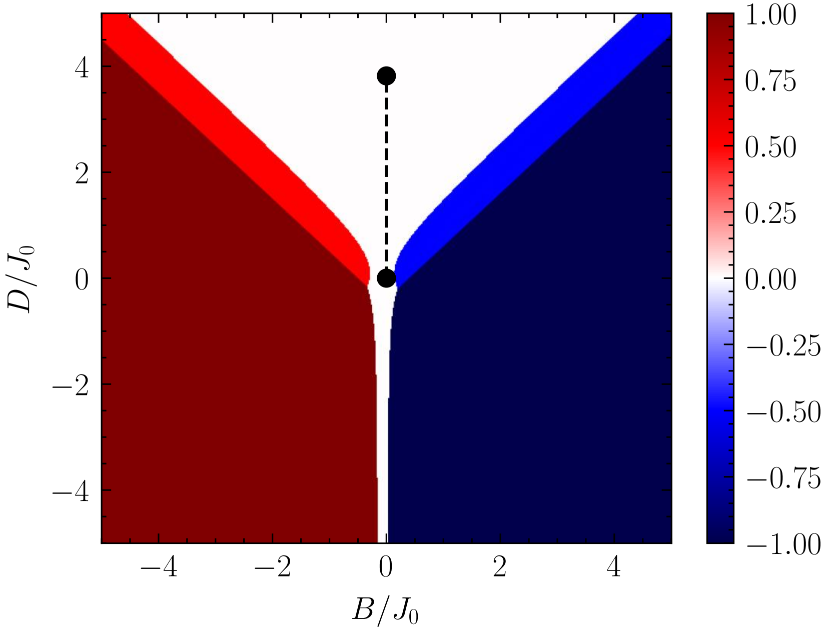
Ground state production: two ions

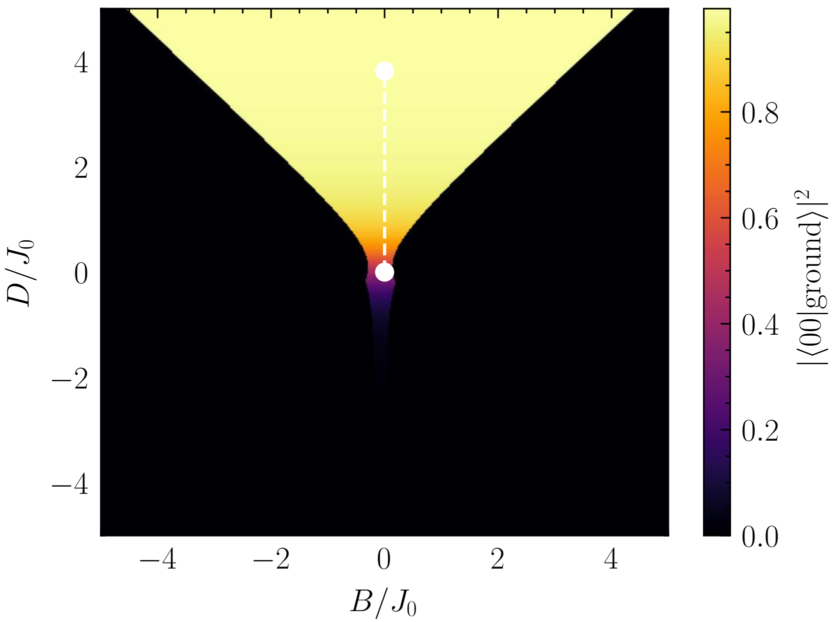


Ground state production via adiabatic ramp

$$H = \sum_{i<j} \frac{J_{ij}}{4}\left(S_+^i S_-^j + S_-^i S_+^j\right) + B \sum_{i=1}^{N} S_z^i - D(t) \sum_{i=1}^{N} \left(S_z^i\right)^2$$

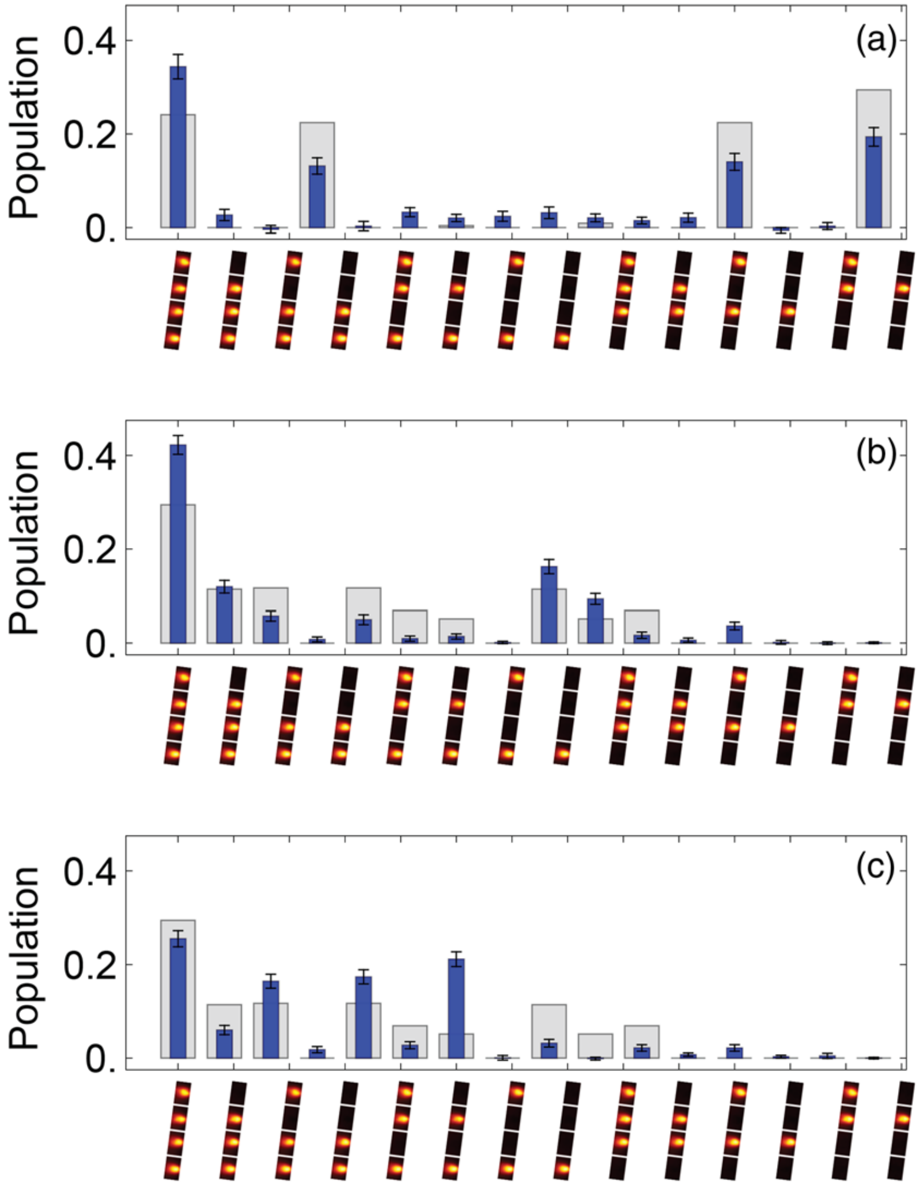
$$D(t) = D_0 e^{-t/\tau} \qquad D_0 = 5\ \mathrm{kHz}, \quad \tau = 0.167\ \mathrm{ms}$$

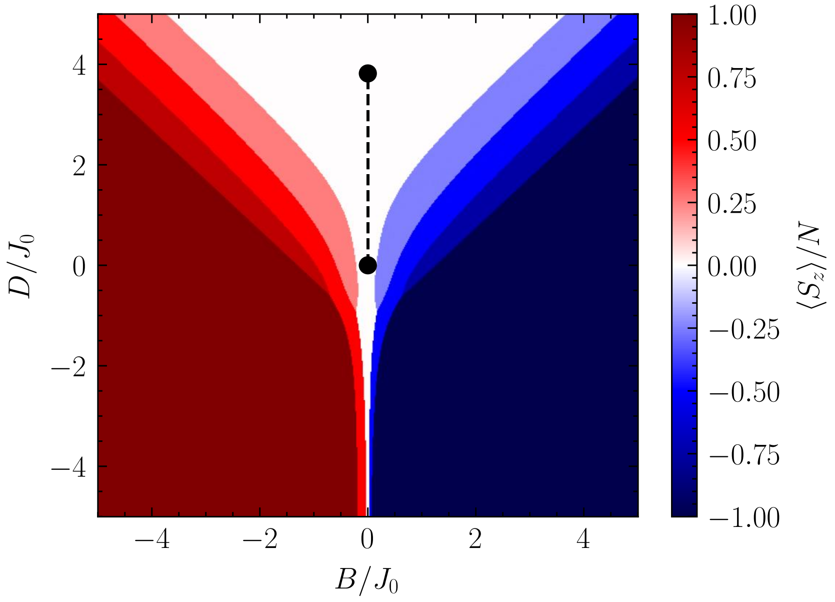


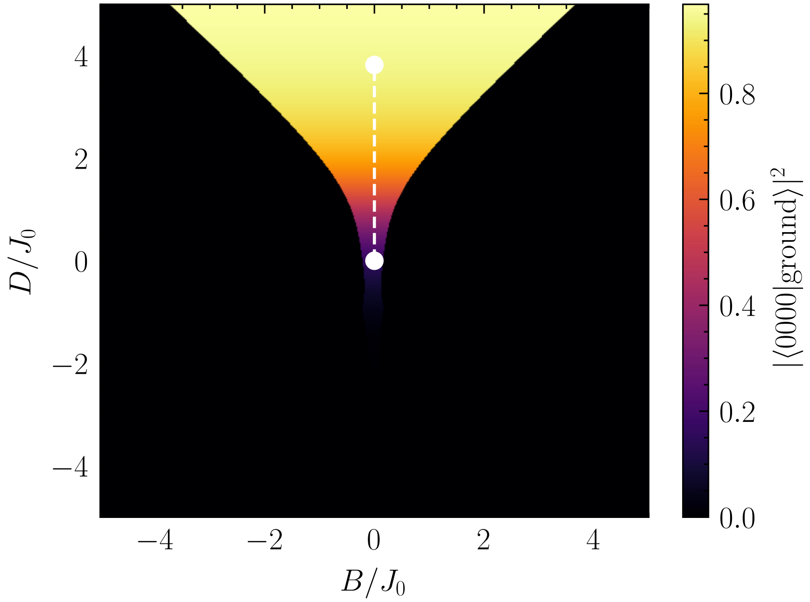


QuTip simulation of a two spin-1 ion chain. On the left the ground state magnetization for different values of D and B. On the right the overlap with the $|00\rangle$ state. The dashed line rapresents the adiabatic ramp

Ground state production: four ions

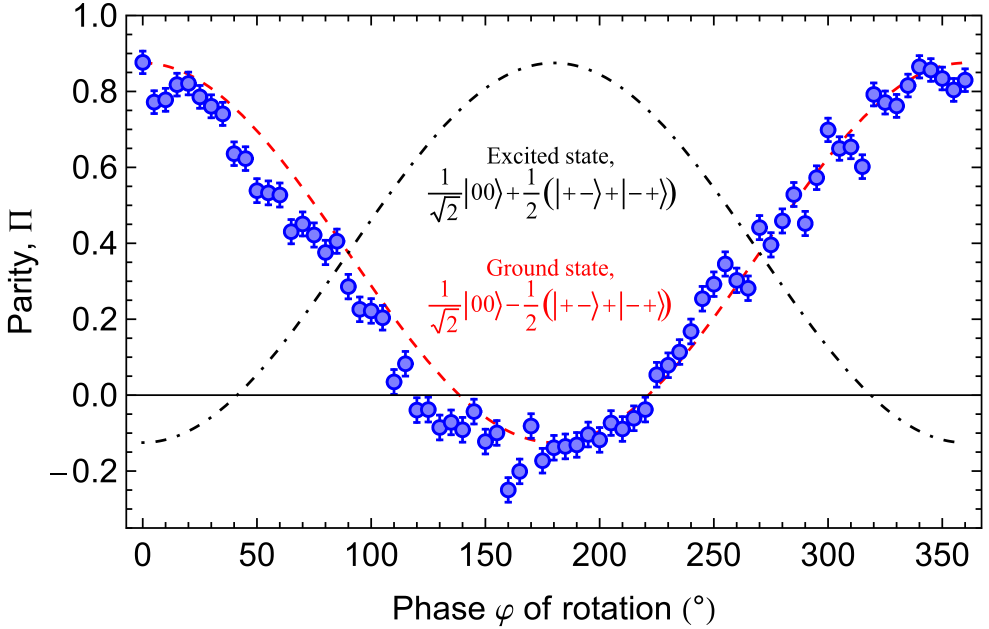






QuTip simulation of a four spin-1 ion chain. On the left the ground state magnetization for different values of D and B. On the right the overlap with the $|0000\rangle$ state. The dashed line rapresents the adiabatic ramp

Ground state verification



Because measurement in the S_z basis discards phase information, it becomes critical to detect the true ground state, as it differs from the highest excited state by only a relative phase

GROUND STATE	$\frac{1}{\sqrt{2}}\|00\rangle - \frac{1}{2}(\|+-\rangle + \|-+\rangle)$
HIGHEST EXCITED STATE	$\frac{1}{\sqrt{2}}\|00\rangle + \frac{1}{2}(\|+-\rangle + \|-+\rangle)$

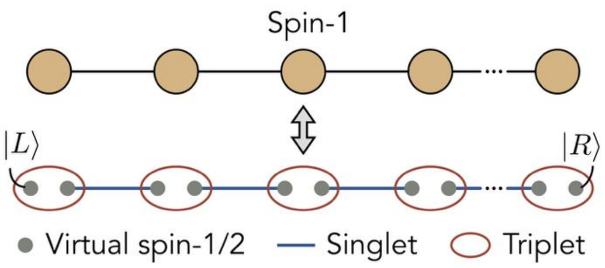
After a global rotation given by $R_{0-}(\pi/2,\varphi)R_{0+}(\pi/2,0)$, the parity Π become dependent on φ

$$\Pi(\varphi) = \begin{cases} \frac{3}{8} + \frac{1}{2}\cos\varphi & \text{For the ground state} \\ \frac{3}{8} - \frac{1}{2}\cos\varphi & \text{For the highest excited state} \end{cases}$$

Toward Haldane phase

The Haldane phase is a symmetry-protected topological phase that exists in one-dimensional antiferromagnetic spin-1 chains. Its topological properties and symmetries enable applications beyond theoretical studies, such as application on fault-tolerant quantum systems

Smith et al. (2023)



A solvable model in the Haldane phase is the Affleck–Kennedy–Lieb–Tasaki (AKLT) system

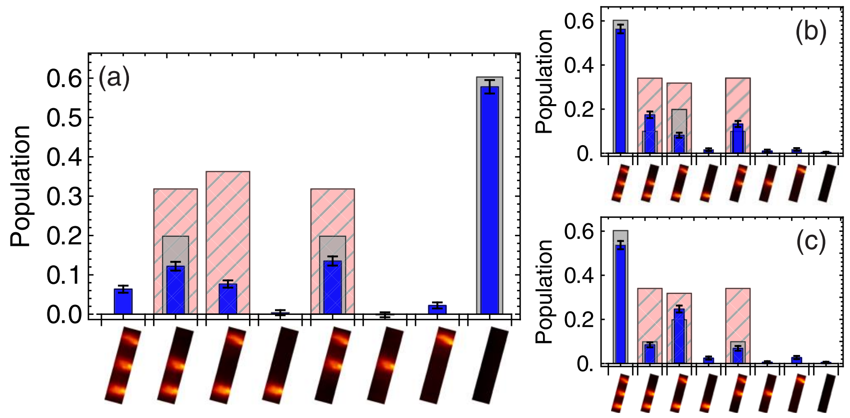
$$H = \sum_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + \frac{1}{3}(\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1})^2 \qquad \boldsymbol{S}_i = \begin{pmatrix} S_x^i \\ S_y^i \\ S_z^i \end{pmatrix}$$

Using a chain of three ions and a coupling $\alpha = 0.36$ it was possible to achieve an overlap of 99.9% with the AKLT ground state:

$$|\psi\rangle_{\text{gs}} = \sqrt{0.16}(|0-+\rangle - |0+-\rangle + |-+0\rangle - |+-0\rangle) + \\ +\sqrt{0.18}(|+0-\rangle - |-0+\rangle)$$

Haldane-like phase: antisymmetric under left-right spatial inversion of the chain and global rotation $|+\rangle \leftrightarrow |-\rangle$

However, the adiabatic ramp is not able to find the true antisymmetric state because of the initial symmetric state preparation.



First-excited state preparation for three ions chain ater the adiabatic ramp