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QUANTUM NON-DEMOLITION DETECTION OF SINGLE MICROWAVE PHOTONS IN A CIRCUIT

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Nature Phys 6, 663–667 (2010)

Introduction to Quantum Hardware

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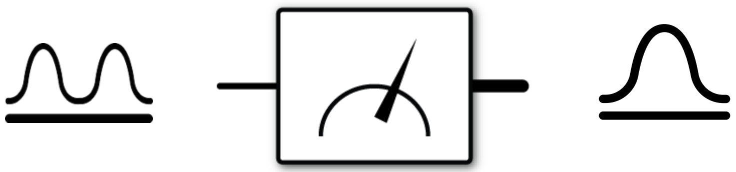
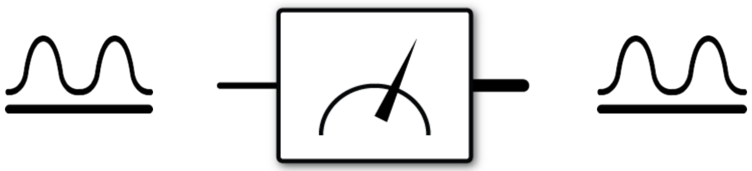
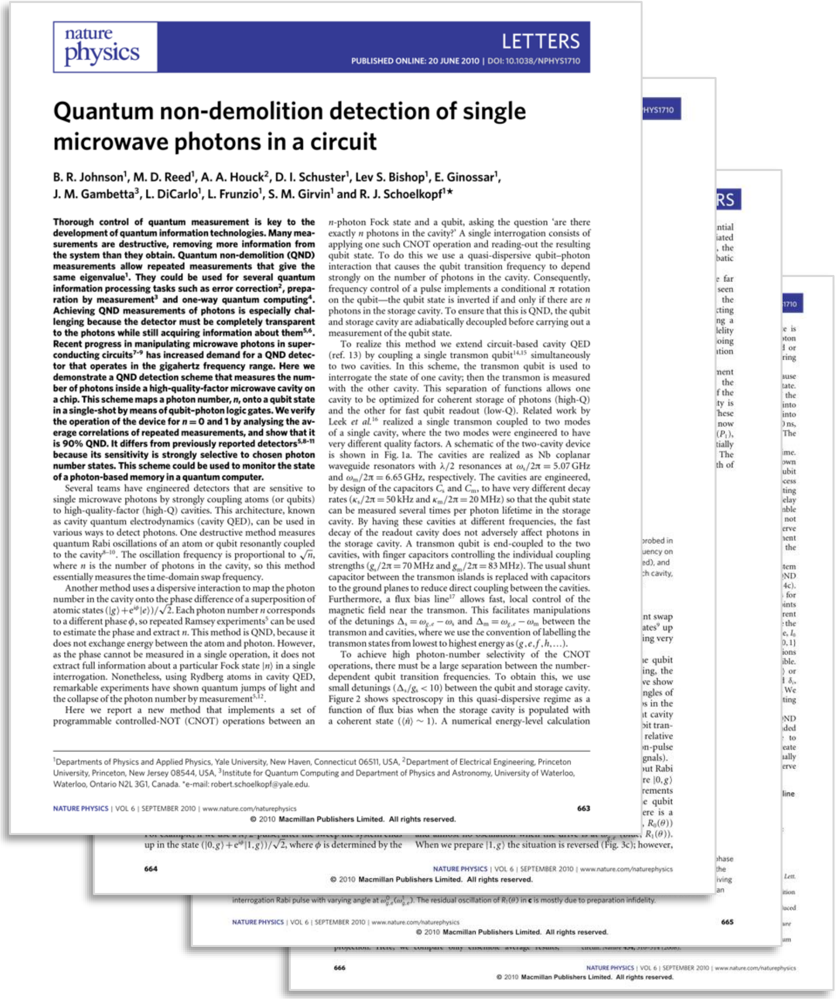
- **Quantum measurements** are often **destructive**, extracting information by **altering the system**'s state or destroying the measured particle
  (e.g., photon is detected by absorption)



- **Quantum Non-Demolition** (QND) measurement allows to repeatedly measure a quantum property of a system **without changing its state** or subsequent evolution



Introduction



nature physics

LETTERS

PUBLISHED ONLINE: 20 JUNE 2010 | DOI: 10.1038/NPHYS1710

Quantum non-demolition detection of single microwave photons in a circuit

B. R. Johnson[1], M. D. Reed[1], A. A. Houck[2], D. I. Schuster[1], Lev S. Bishop[1], E. Ginossar[1], J. M. Gambetta[3], L. DiCarlo[1], L. Frunzio[1], S. M. Girvin[1] and R. J. Schoelkopf[1]*

NATURE PHYSICS | VOL 6 | SEPTEMBER 2010 | www.nature.com/naturephysics

663

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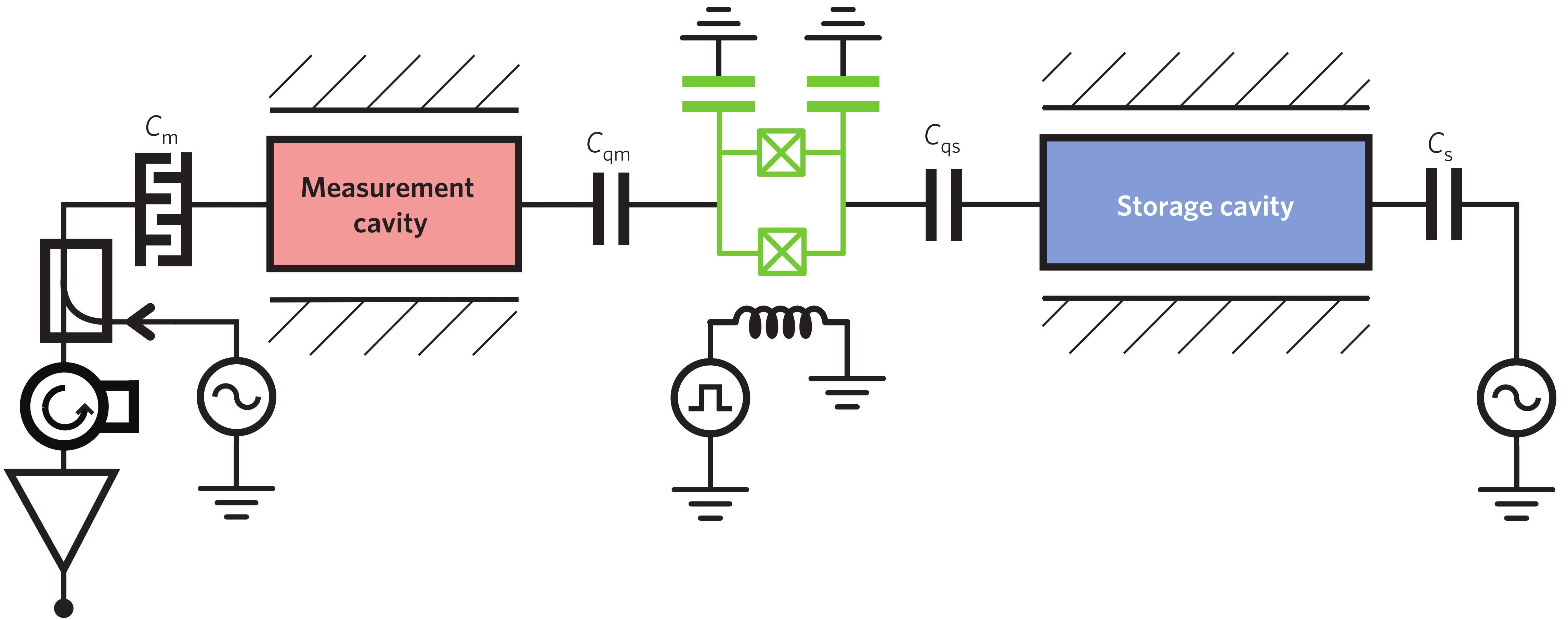
This research presents a QND scheme for **detecting single microwave photons** within a high-quality-factor cavity

The experimental setup consists of a superconducting transmon qubit coupled to a microwave **cavity**, which serves as a **storage element for the photons**

The core of the method involves **mapping the photon number onto the qubit state**, allowing for the state to be **measured without absorbing** or destroying the photons themselves

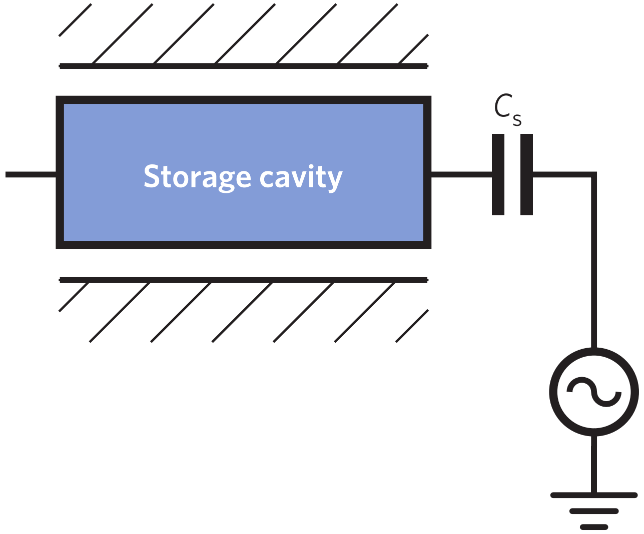
Experimental setup

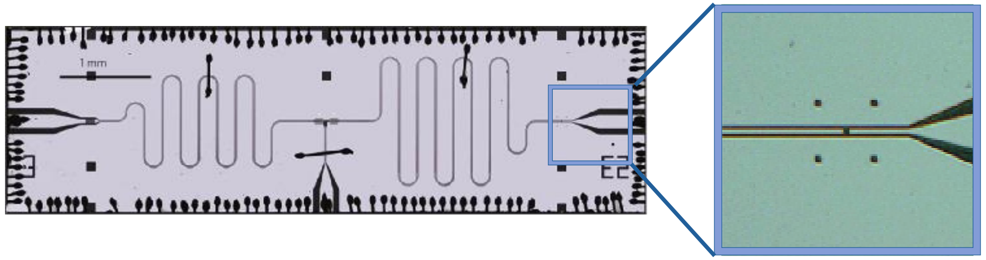


STORAGE CAVITY

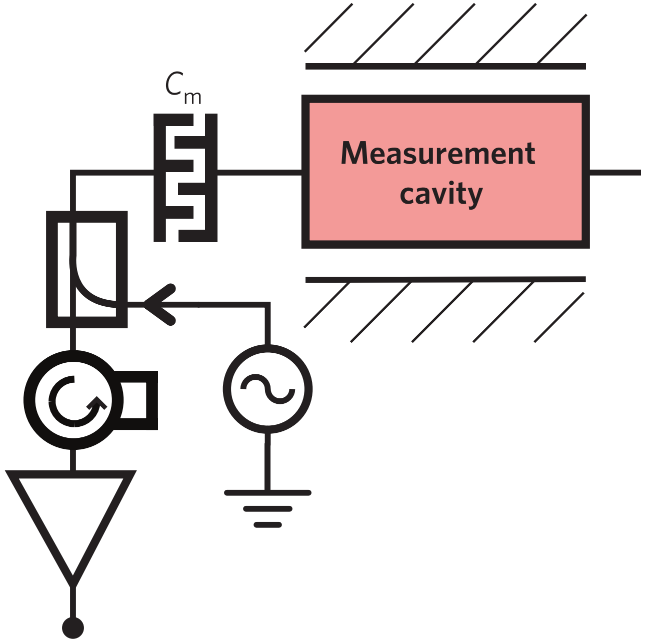
Niobium-based coplanar waveguide resonator with $\lambda/2$ resonance at $\omega_s/2\pi = 5.07\ \mathrm{GHz}$

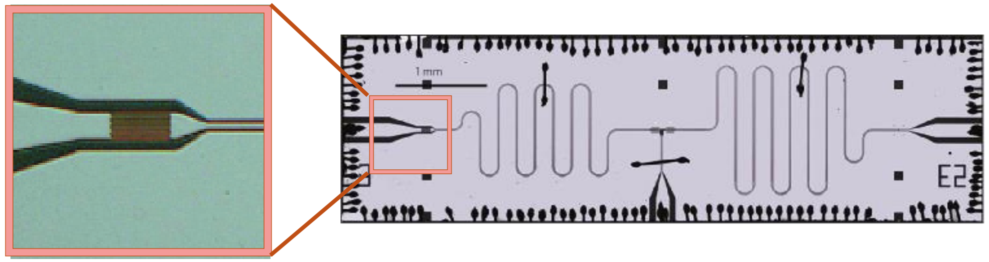
Is weakly coupled to an external microwave line by C_s capacitor: high-Q cavity with $\kappa_s/2\pi = 50\ \mathrm{kHz}$





Experimental setup





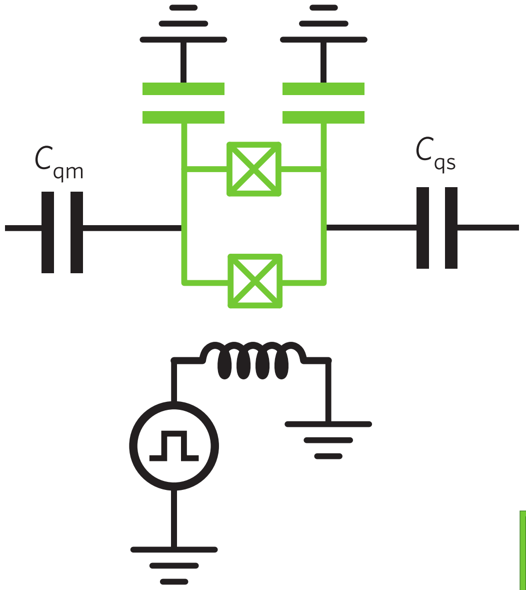
MEASUREMENT CAVITY

Niobium-based coplanar waveguide resonator with $\lambda/2$ resonance at $\omega_s/2\pi = 6.65\ \mathrm{GHz}$

Is strongly coupled to an external microwave line and a detection chain by C_m capacitor: low-Q cavity with $\kappa_m/2\pi = 20\ \mathrm{MHz}$

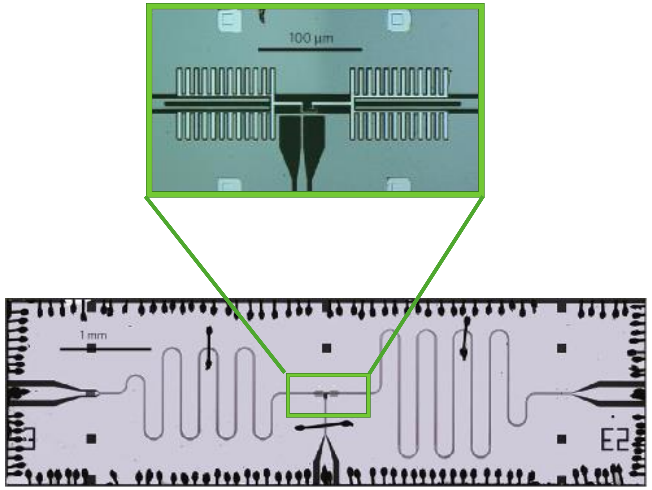
TRANSMON QUBIT

capacitively coupled to the measurement cavity (C_{qm}) and storage cavity (C_{qs})



A flux bias Φ_{ext} line allow to tune the transmon frequency ω_q

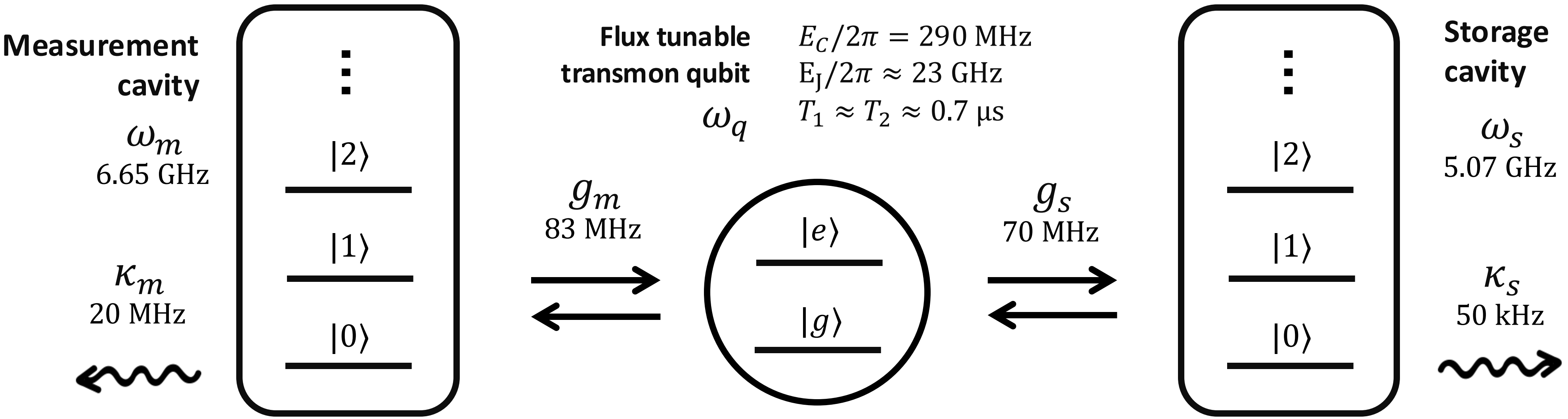
$$H_q = 4E_C(n-n_g)^2 - E_J(\Phi_{ext})\cos\varphi$$

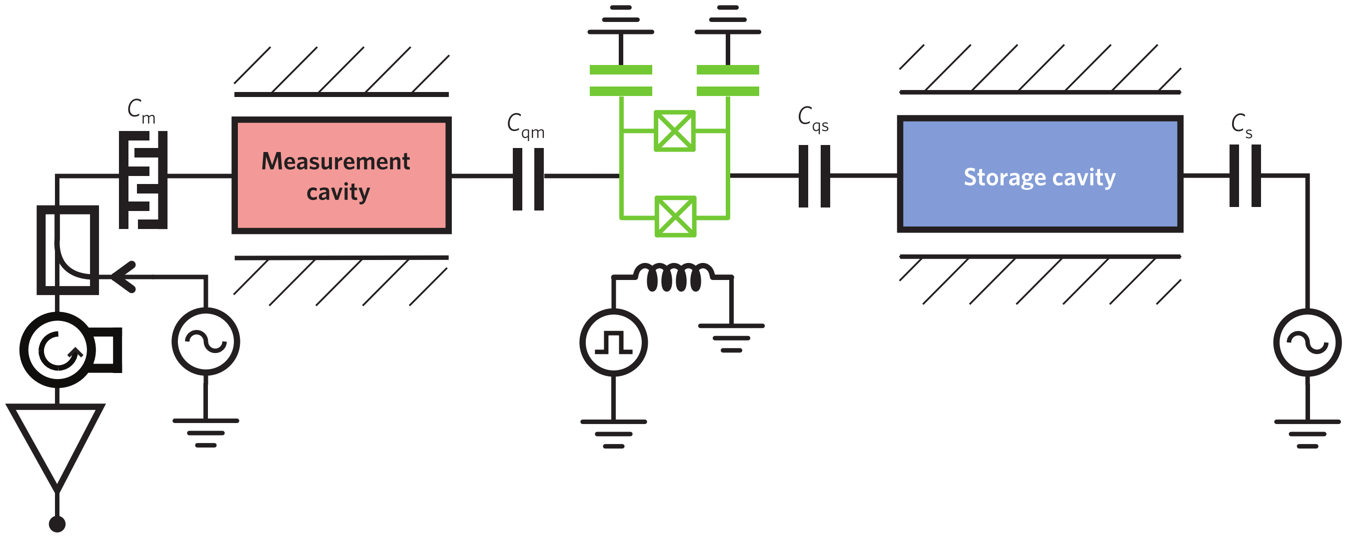


Coupling strengths:
$g_s/2\pi = 70$ MHz
$g_m/2\pi = 83$ MHz

Charging energy $E_C/2\pi = 290$ MHz
Josephson energy $E_J/2\pi \approx 23$ GHz
Coherence time $T_1 \approx T_2 \approx 0.7$ μm

Experimental setup





Qubit-cavity coupling

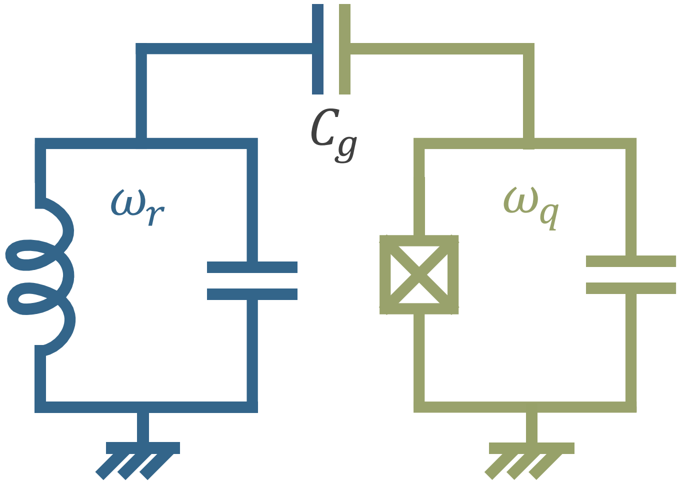
The cavity become a source of quantized gate voltage n_g, producing a charge bias in the junction

$$H_q = 4E_C(n - n_g)^2 - E_J \cos\varphi$$
$$n_g \mapsto n_r = (C_g/C_r)Q_r/2e$$

Cavity (LC resonator)

Quantum harmonic oscillator with frequency $\omega_r = 1/\sqrt{L_r C_r}$. The Hilbert space is $\mathcal{H} = \mathrm{span}\{|n = 0,1,2,\dots\rangle\}$ and a, $a^\dagger$ are the annihilation and creation operators

$$H_r = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right)$$



Transmon qubit

Superconducting qubit characterized by the Josephson energy $E_J = \Phi_0 I_c/2\pi$ and charging energy $E_C = e^2/2C_\Sigma$.

In the transmon regime ($E_J \gg E_C$) The system can be approximate as an anharmonic oscillator with anharmonicity $\alpha = -E_C$.

$$H_q \approx \hbar\omega_q b^\dagger b + \frac{\alpha}{2} b^\dagger b^\dagger b b \quad \begin{cases} b \to \sigma_- \\ b^\dagger \to \sigma_+ \end{cases}$$

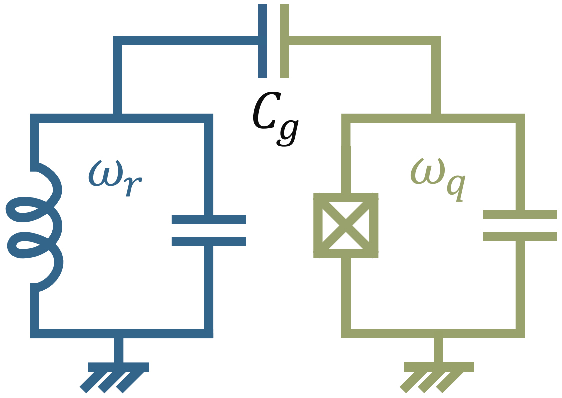
Restriction of the Hilbert space to a two-level system

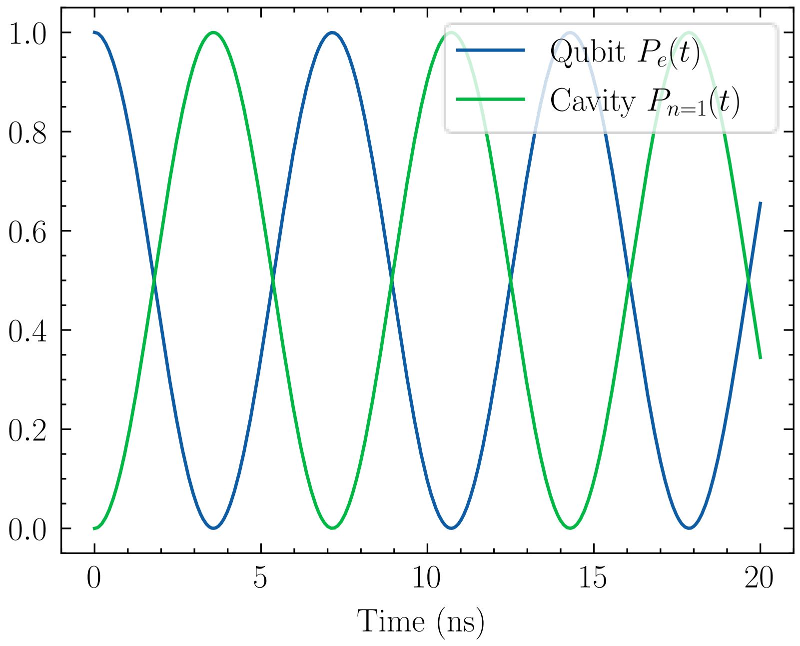
Jaynes-Cummings Hamiltonian

$$\boxed{H_{JC} = \frac{1}{2}\hbar\omega_q\sigma_z + \hbar\omega_r a^\dagger a + \hbar g(a\sigma_+ + a^\dagger\sigma_-)}$$

Blais, Alexandre, et al. "Circuit quantum electrodynamics." *Reviews of Modern Physics* 93.2 (2021): 025005.

Jaynes-Cummings Hamiltonian





$$H = \frac{1}{2}\hbar\omega_q\sigma_z + \hbar\omega_r a^\dagger a + \hbar g(a\sigma_+ + a^\dagger\sigma_-)$$

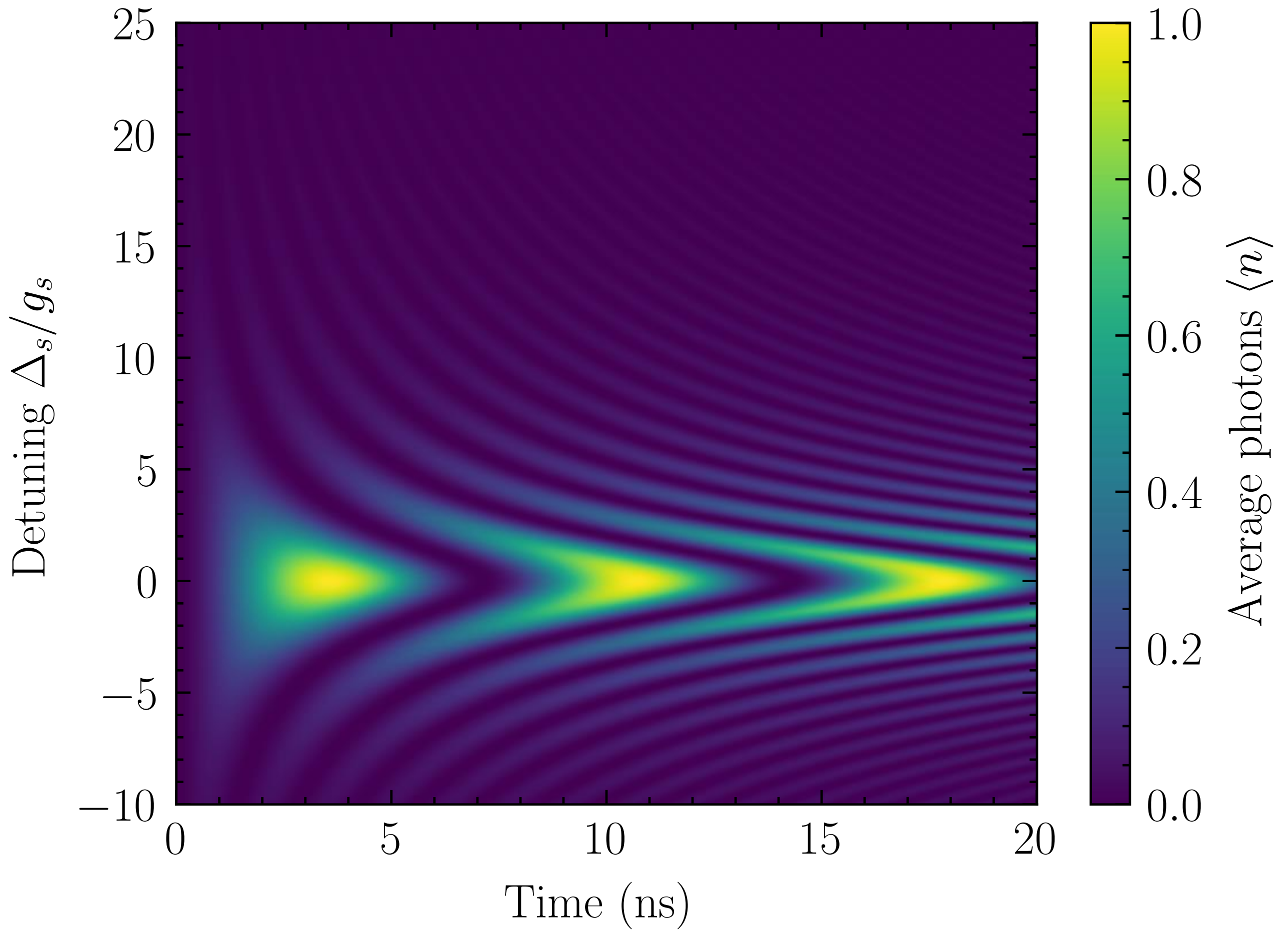
The total excitation number $N = a^\dagger a + \sigma_+\sigma_-$ commute with the JC Hamiltonian, therefore N is a constant of motion: energy is exchanged between the qubit and the cavity

$$E_{n,\pm} = \hbar\omega_r\left(n - \frac{1}{2}\right) \pm \frac{\hbar}{2}\sqrt{\Delta^2 + 4g^2 n}$$

detuning: $\Delta := \omega_q - \omega_r$

The system exhibit vacuum Rabi oscillations with frequency $\Omega_n^\Delta = \sqrt{\Delta^2 + 4g^2(n+1)}$ and amplitude $\left(2g\sqrt{n+1}/\Omega_n^\Delta\right)^2$

Dispersive and resonant regime

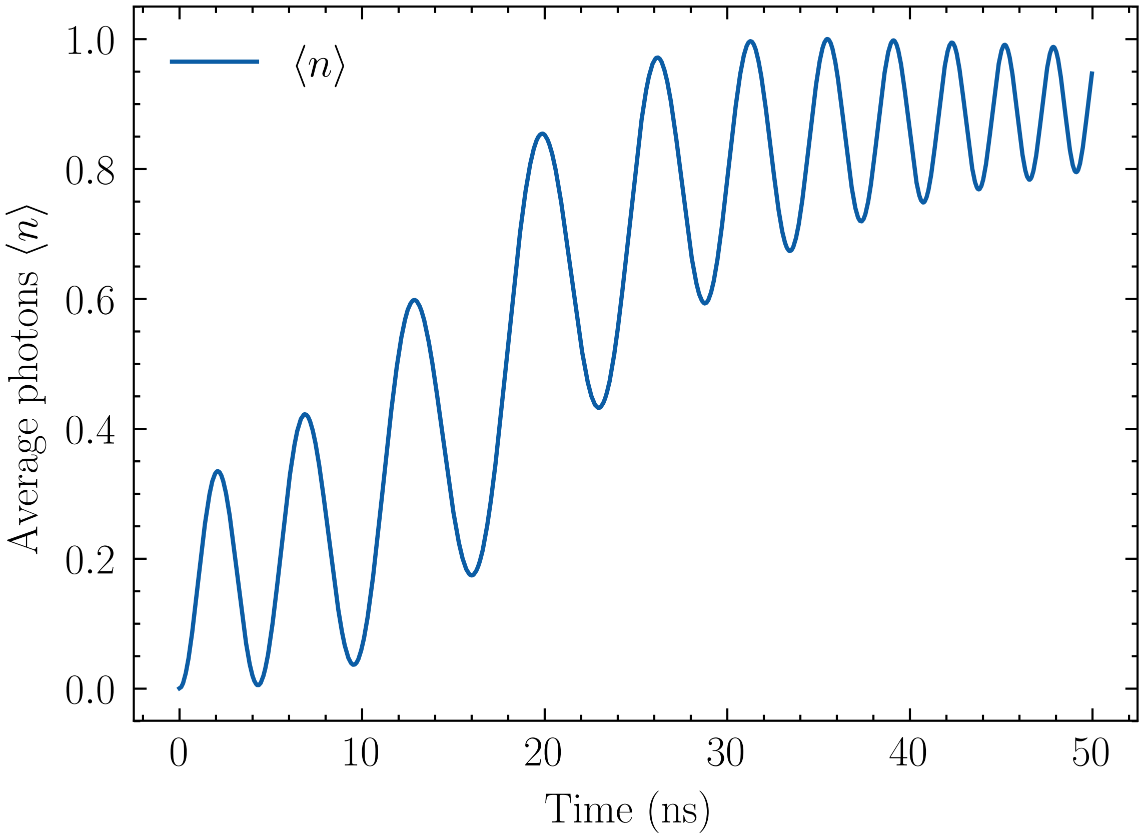


QuTip simulation of cavity-transmon full hamiltonian using the storage cavity parameters of the Johnson et al. (2011) experiment.

In the qubit-cavity interaction, two different regimes can be employed:

- → **Resonant $\Delta/g \approx 0$: strong qubit-cavity interaction. Useful for state preparation and manipulation**
- → **Dispersive $\Delta/g \gg 1$: weak qubit-cavity interaction. Useful for retain the cavity state or for QND detection (in semi-dispersive regime)**

Preparation protocol: resonant regime



Preparation protocol

- The transmon qubit is prepared to the excited state $|0, e\rangle$ and the detuning is set to be $\Delta_s/g_s \approx -3$
- The frequency of the qubit ω_q is changed by 600 MHz in 50 ns
- The qubit reverts to the ground state, exciting a photon in the cavity. The resulting state is $|1, g\rangle$

CNOT$_n$ operation: dispersive case

In the dispersive regime ($\Delta/g \gg 1$) a Schrieffer-Wolff transformation can be performed:

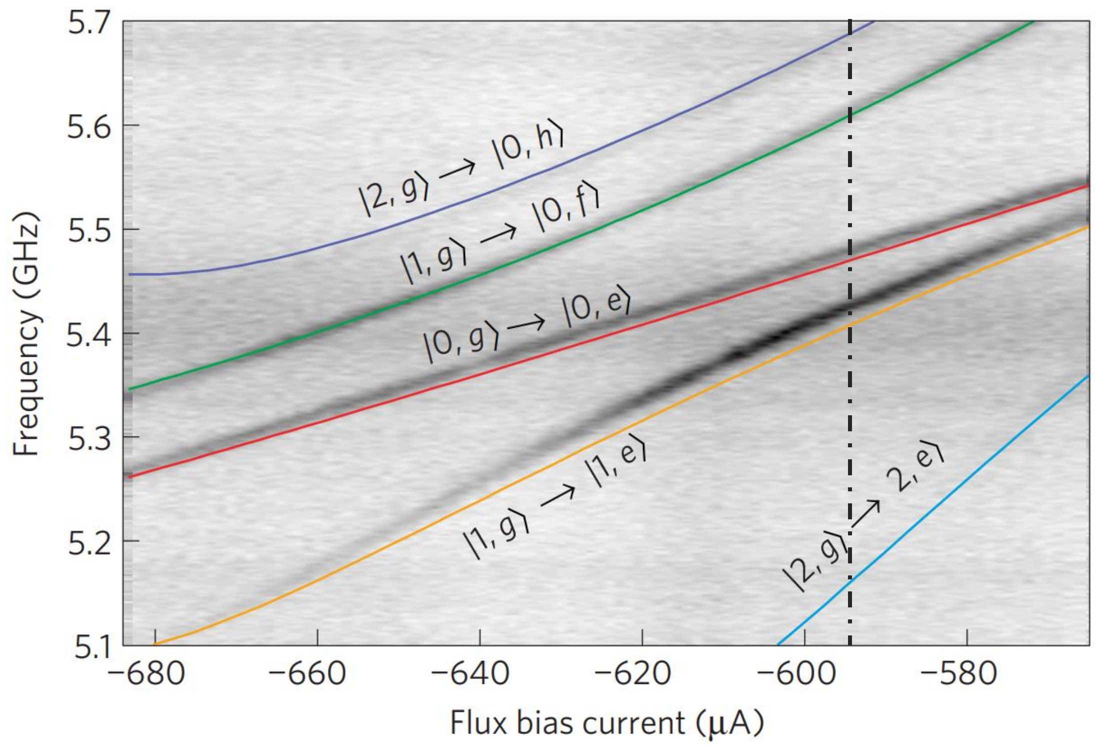
$$\left.\begin{aligned} H_0 &= \frac{1}{2}\hbar\omega_q\sigma_z + \hbar\omega_r a^\dagger a \\ V &= \hbar g(a\sigma_+ + a^\dagger\sigma_-) \end{aligned}\right\} \quad H \approx \frac{1}{2}\hbar\omega_q'\sigma_z + \hbar\omega_r' a^\dagger a + \hbar\chi a^\dagger a\sigma_z$$

Dispersive shift	$\chi = -\dfrac{g^2 E_C/\hbar}{\Delta(\Delta - E_C/\hbar)}$
Dressed frequencies	$\omega_r' = \omega_r - \dfrac{g^2}{\Delta - E_C/\hbar}$ $\omega_q' = \omega_q - \dfrac{g^2}{\Delta}$

$$\boxed{H \approx \frac{\hbar}{2}\left(\omega_q' + 2\chi a^\dagger a\right)\sigma_z + \hbar\omega_r' a^\dagger a}$$

The qubit state become dependent on the number of photons in the storage cavity

$$\omega_{ge}^{n=0} - \omega_{ge}^{n=1} = 2\chi$$

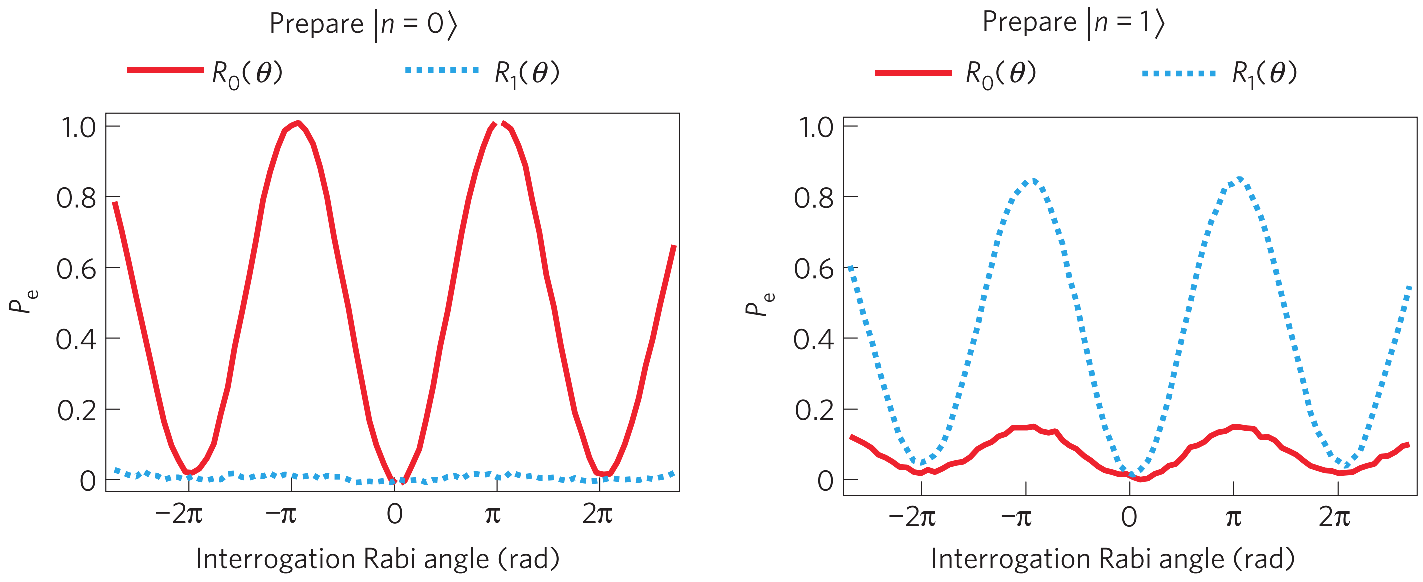


CNOT$_n$ operation: dispersive case

A drive at frequency $\omega_d^n = \omega_q' + 2\chi n$ is applied to the qubit. This drive becomes resonant and a π-pulse flips the qubit state only for a specific photon number n. This selective operation effectively acts as a CNOT_n gate

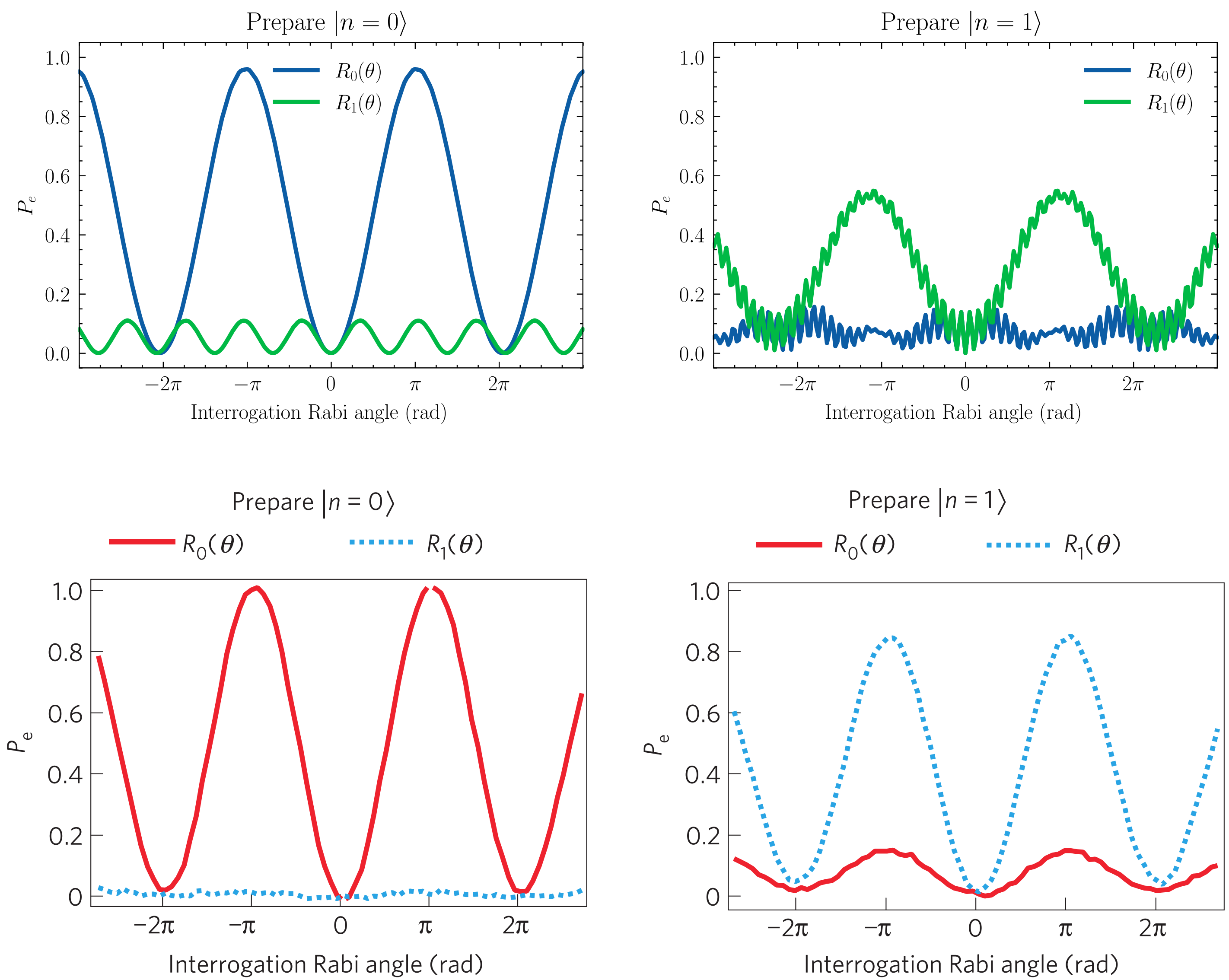
$$H_{\mathrm{drive}}(t) = \hbar\Omega_{\mathrm{R}} \cos(\omega_d t + \varphi)(b + b^\dagger)$$

$$H_{\mathrm{drive}}^{\mathrm{RWA}}(t) = \frac{\hbar\Omega_{\mathrm{R}}}{2}(e^{i\varphi}b + e^{-i\varphi}b^\dagger)$$



Probability $R_n(\theta)$ of measuring the qubit in the excited state varying the '*interrogation pulse*' duration $\theta = \Omega_R t$.
A π-pulse results in the qubit flip only if there are exactly n photons in the storage cavity

CNOT$_n$ operation: dispersive case



Numerical simulation using a detuning of $\Delta_s/g_s = 5$ and RWA. The dressed frequencies are $\omega_{ge}^0 = 5.43$ GHz and $\omega_{ge}^1 = 5.38$ GHz, with separation of 53 MHz (experimetally $\sim$ 65 MHz)

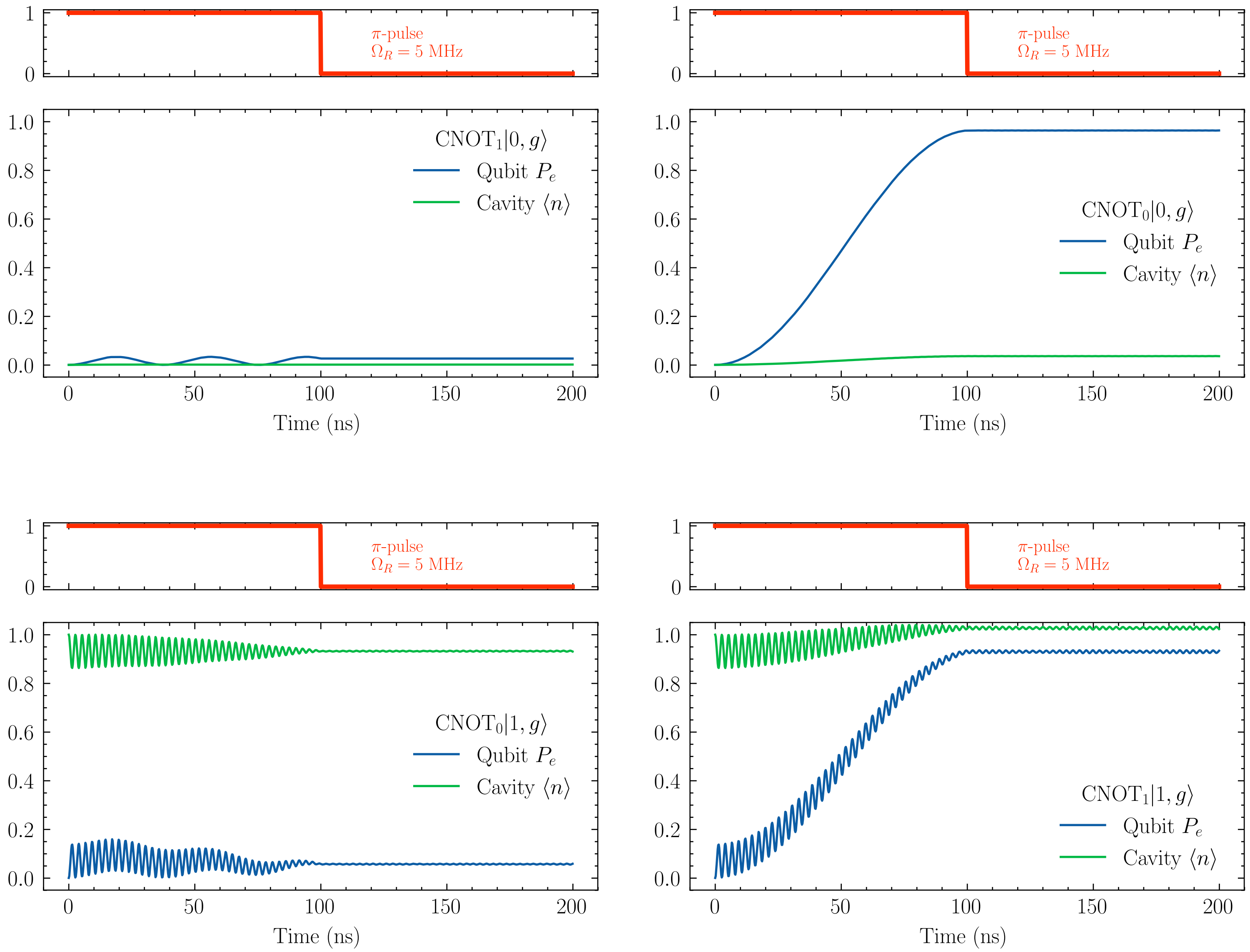
In the rotating frame, the hamiltonian become

$$H_0^{\mathrm{RF}} = \delta_s a^\dagger a + \delta_q b^\dagger b + \frac{\alpha}{2} b^\dagger b^\dagger b b + g(a^\dagger b + a b^\dagger)$$

$$H_{\mathrm{drive}}^{\mathrm{RF}}(t) \approx \frac{\Omega_{\mathrm{R}}(t)}{2}\,(b^\dagger + b)$$

with $\delta_{s/q} = \omega_{s/q} - \omega_d$ and Rabi frequency Ω_{R}. The pulse duration is $t_{\mathrm{pulse}} = \theta/\Omega_{\mathrm{R}}$

$\mathrm{CNOT_n}$ operation: dispersive case

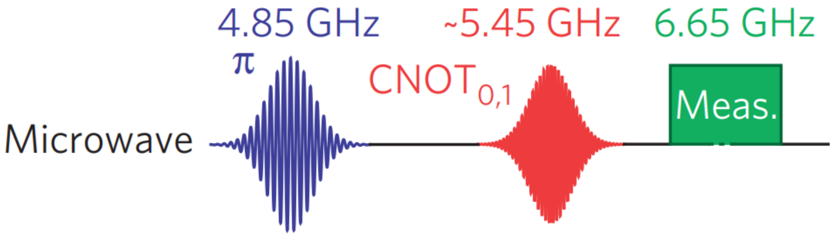


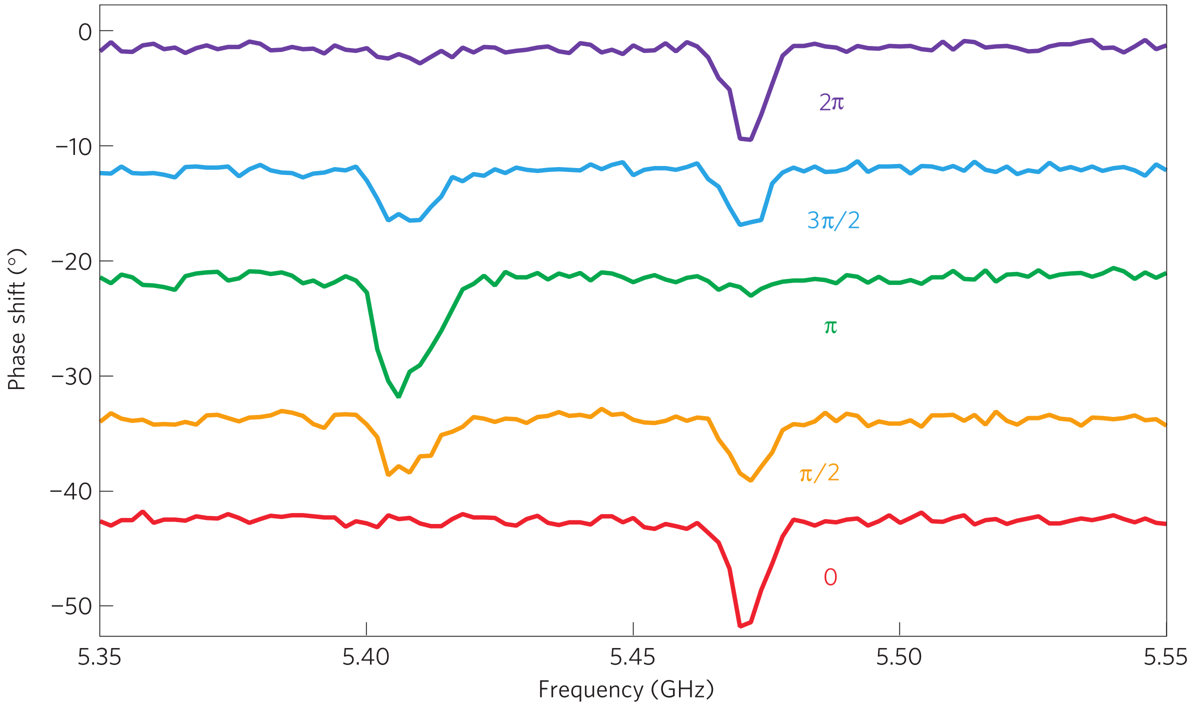
The π-pulse is used to interrogate the system, asking the question: *«Are there exactly n photons in the cavity?»*

$$\mathrm{CNOT}_n|m,g\rangle = \begin{cases} |m,g\rangle \text{ if } n \neq m \\ |m,e\rangle \text{ if } n = m \end{cases}$$

If the answer is affirmative, the qubit is flipped to its excited state. Because the cavity remains in its initial state, further interrogations can be made

Measurament



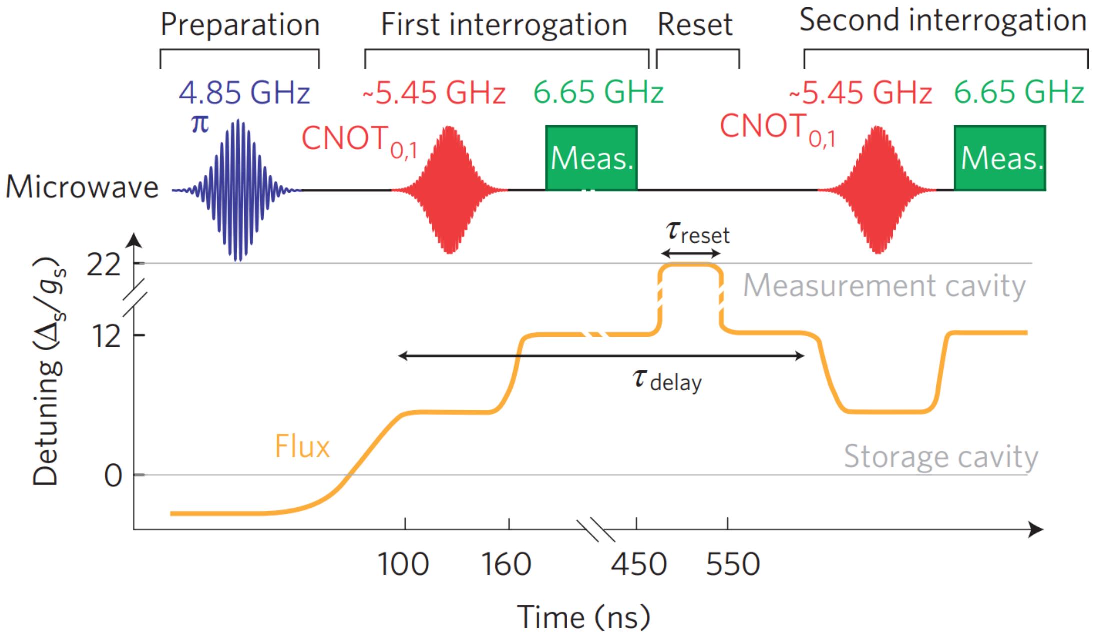


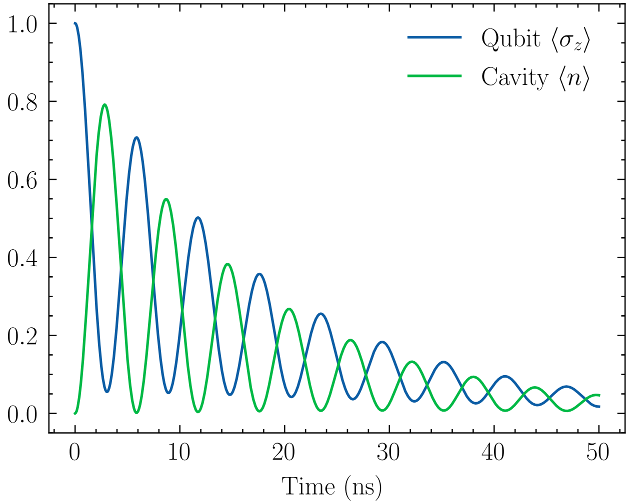
During the preparation phase it is possible to prepare the cavity in any arbitrary superposition of $|0, g\rangle$ and $|1, g\rangle$ by tuning the angle of the initial pulse

A pulsed signal to the measurement cavity allows to determine the number of photon in the cavity. The reflected signal's phase shows distinct dips at these specific frequencies:

$$\omega_q^{n=0} \approx 5.47\ \mathrm{GHz} \qquad \omega_q^{n=1} \approx 5.41\ \mathrm{GHz}$$

Reset of the qubit





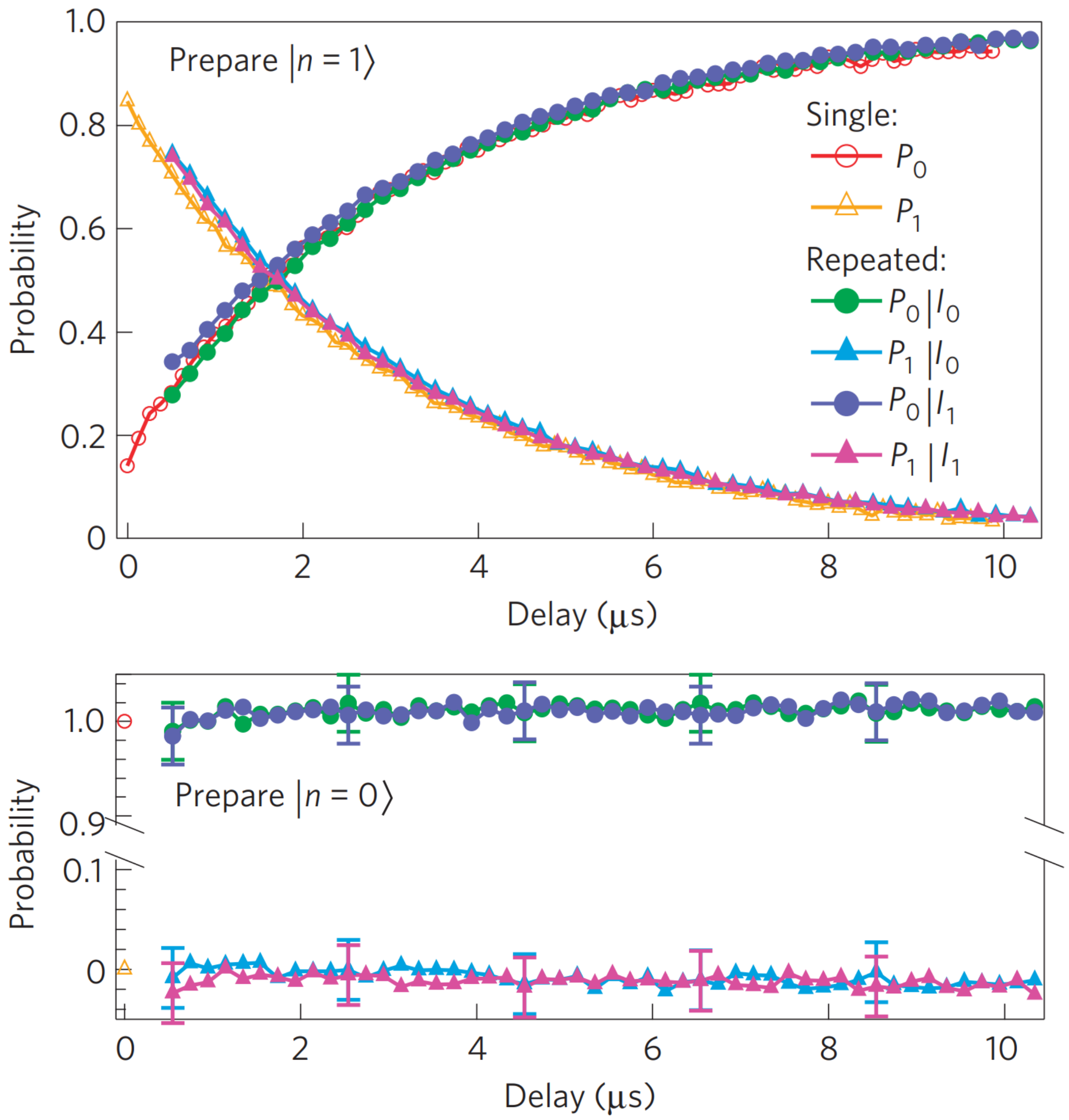
Evolution of the excited qubit state $|0, e\rangle$ coupled with the measurement cavity. For the *QuTip* simulation, a Lindblad operators $L = \sqrt{\kappa_m} a$ is used

After the first interrogation the qubit may remain in the excited state for some time. To repeat the protocol, a delay of at least $\tau_{\text{reset}} = 50\ \text{ns}$ is needed to reset the qubit with probability $\sim 98\%$

The qubit decay fast through the measurement cavity, which have a decay rate of $\kappa_m/2\pi = 20\ \text{MHz}$

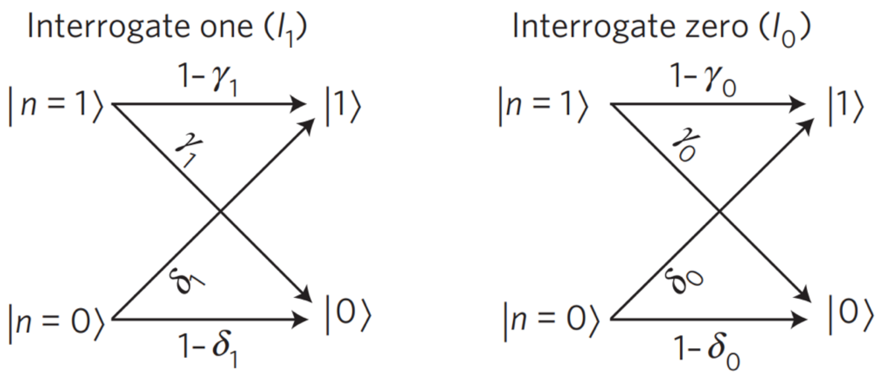
In total, a single interrogation process takes $\sim 550\ \text{ns}$

Results



→ **Single and repeated measurements are nearly overlapped: the protocol is highly QND**

→ **Not significant deviation from $P_0 + P_1 = 1$ suggests negligible population for $n \geq 2$ cavity states**



$\gamma_1 = 10\%$	false positive	$\gamma_0 = 1\%$
$\delta_1 = 3\%$	false negative	$\delta_0 = 7\%$