Academic Year 2023-2024

Politecnico di Milano

GROUP N.14

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Contents

1 Original text of the project

FINAL PROJECT - SII

Consider a simplified insurance company whose assets and liabilities sides are characterized as follows:

ASSETS

- there is a single fund made of equity (80%) and property (20%), $F_t = EQ_t + PR_t$
- at the beginning (t=0) the value of the fund is equal to the invested premium $F_0 = C_0 = 100{,}000$
- equity features
  - listed in the regulated markets in the EEA
  - no dividend yield
  - to be simulated with a Risk Neutral GBM (sigma=20%) and a time varying instantaneous rate r
- property features
  - listed in the regulated markets in the EEA
  - no dividend yield
  - to be simulated with a Risk Neutral GBM (sigma=10%) and a time varying instantaneous rate r

LIABILITIES

- contract terms
  - whole Life policy
  - benefits
    - in case of lapse, the beneficiary gets the value of the fund at the time of lapse, with 20 euros of penalties applied
    - in case of death, the beneficiary gets the maximum between the invested premium and the value of the fund
  - others
    - Regular Deduction, RD of 2.20%
    - Commissions to the distribution channels, COMM (or trailing) of 1.40%
- model points
  - just 1 model point
  - male with insured aged x=60 at the beginning of the contract
- operating assumptions
  - mortality: rates derived from the life table SI2022 (https://demo.istat.it/index_e.php)
  - lapse: flat annual rates l_t=15%
  - expenses: constant unitary (i.e. per policy) cost of 50 euros per year, that grows following the inflation pattern
- economic assumption
  - risk free: rate r derived from the yield curve (EIOPA IT without VA 31.03.24)
  - inflation: flat annual rate of 2%

Other specifications:

- time horizon for the projection: 50 years.
  In case of outstanding portfolio in T=50, let all the people leave the contract with a massive surrender
- the interest rates dynamic is deterministic, while the equity and property ones are stochastic.

QUESTIONS

1. code a Matlab/Python script to compute the Basic Solvency Capital Requirement via Standard Formula and provide comments on the results obtained. The risks to be considered are:
   - Market Interest
   - Market equity
   - Market property
   - Life mortality
   - Life lapse
   - Life cat
   - Expense
2. Split the BEL value into its main PV components: premiums (=0), death benefits, lapse benefits, expenses, and commissions
3. Replicate the same calculations in an Excel spread sheet using a deterministic projection.
   - Do the results differ from 1? If so, what is the reason behind?
   - For the base case only
     i. calculate the Macaulay duration of the liabilities;
     ii. calculate the sources of profit for the insurance company, deriving its PVFP
     iii. check the magnitude of leakage by verifying the equation MVA = BEL + PVFP (i.e. MVA=BEL+PVFP+LEAK)
     iv. sense check the PVFP using a proxy calculation, based on the annual profit and the duration of the contract
4. Open questions:
   - what happens to the asset and liabilities when the risk-free rate increases/decreases with a parallel shift of, say, 100bps? Describe the effects for all the BEL components;
   - what happens to the liabilities if the insured age increases? What if there were two model points, one male and one female?

2 Summary tables

Results	MVA	BEL	BoF	d_BoF	dur_L
BASE	100000,00	94848,29	5151,71	0	5,58
IR_UP	100000,00	94798,94	5177,74	0	5,58
IR_DW	100000,00	95163,33	4886,32	265,39	5,60
Mkt_Equity	64600,00	61379,54	3220,46	1931,26	5,58
Mkt_Prop	95000,00	90091,77	4908,23	243,48	5,58
Mortality	100000,00	94699,63	5300,37	0	5,54
CAT	100000,00	94862,06	5137,94	13,77	5,57
Lapse_up	100000,00	96149,41	3850,59	1301,12	4,00
Lapse_dw	100000,00	92169,55	7830,45	0	8,96
Lapse_mass	100000,00	96569,08	3430,92	1720,79	3,38
Expenses	100000,00	94880,95	5119,05	32,66	5,58
BCR	3185,34				

Table 1 : Basic Solvency Capital Requirement via Matlab

BEL components	Death	Lapse	Expense	Commissions
BASE	5466,93	81259,25	300,36	7848,39
IR_UP	5463,52	81212,32	279,34	7843,76
IR_DW	5750,88	81246,55	318,65	7847,25
Equity_shock	3529,70	52461,44	300,36	5067,51
Property_shock	5189,68	77148,56	300,36	7451,43
Mortality shock	6160,68	80515,43	297,82	7789,08
CAT shock	5579,89	81090,16	299,98	7834,16
Lapse up	2840,07	87478,65	209,17	5619,11
Lapse down	14573,81	64523,05	504,71	12543,18
Lapse mass	2894,33	88740,34	182,18	4813,81
Expenses	5464,37	81230,57	350,14	7845,61

Table 2 : BEL value splitted into its main PV components via Matlab

Results	MVA	BEL	BoF	d_BoF	dur_L
BASE	100000,00	94827,92	5172,08		5,575502
IR_UP	100000,00	94807,99	5192,01	0	5,574867
IR_DW	100000,00	94862,32	5137,677	34,40284	5,595824
Mkt_Equity	64600,00	61359,44	3240,564	1931,515	5,575951
Mkt_Prop	95000,00	90100,73	4899,267	272,8129	5,575545
Mortality	100000,00	94734,68	5265,317	0	5,532037
CAT	100000,00	94812,45	5187,554	0	5,569339
Lapse_up	100000,00	96145,43	3854,568	1317,512	4,000222
Lapse_dw	100000,00	92142,57	7857,431	0	8,948763
Lapse_mass	100000,00	96596,88	3403,124	1768,956	3,377965
Expenses	100000,00	94877,64	5122,358	49,72167	5,576545
BCR	3135,535124				

Table 3 : Basic Solvency Capital Requirement via Excel

BEL components	Death	Lapse	Expense	Commissions
BASE	5533,648	81154,63	300,1072	7839,531265
IR_UP	5533,648	81155,69	279,1241	7839,531265
IR_DW	5823,128	80905,31	318,3768	7815,512123
Equity_shock	3574,737	52420,25	300,1072	5064,337197
Property_shock	5256,966	77096,11	300,1072	7447,554701
Mortality shock	6237,34	80418,67	297,5378	7781,133606
CAT shock	5650,813	81032,1	299,7284	7829,80803
Lapse up	2881,535	87437,57	209,0683	5617,262996
Lapse down	14689,77	64423,04	503,8738	12525,88554
Lapse mass	2929,578	88676,08	182,0516	4809,163611
Expenses	5533,648	81154,63	349,8289	7839,531265

Table 4 : BEL value splitted into its main PV components via Excel

3 Formulas adopted during the computations

The final aim of our project is to compute the Basic Solvency Capital Requirement via Standard Formula. As a first step, we took the risk free rate spot curve from EIOPA IT considering the "without VA" term structure of 31-03-2024, taking into account a time horizon of 50 years. We then derived the spot and forward discount factor and the forward rate at 1 year using the known expression for the continuous formulation of rates.
For the stochastic simulation of the fund in the Matlab, we constructed a function capable of simulating a number of paths for each of the two components, since the two respective volatilities were different, adding only in the end since we could assume them to be independent. The formula we used for the Geometric Brownian Motion is the following one:

$$F(t) = \frac{F(0)}{(1-RD)^t} * \exp\left(\left(r - \frac{\sigma^2}{2}\right) * t + \sigma W(t)\right)$$

The time step we took was of 1 year as it was fitting perfectly with the requests, while for the number of simulation we opted for N=1e6 after considering different order of magnitude that were leading to insignificant changes for a bigger N, while the changes in the result were too big for a smaller N. At every time step we multiplied by a factor (1-RD), where RD stands for "Regular Deduction" , to be able to take into account the withholding happening every year.

The next important step was to import the probability of death within 1 year which we took from ISTAT, taking the "Probability of death (per thousand) qx" from the "Life tables of the resident population Italy - Males 2023" for the ages between 60 and 110 years. We then divided every value for 1000 to obtain the so called "qx". We then built what we called the "contract prob", a vector with the probabilities to be still in the in the contract at year t defined as:

$$\prod_{i=0}^{t-1} (1 - qx(i))^i * (1 - LT(i))^i$$

where LT stands for the annual lapse rate for the contract. We then computed each one of the four components of the Best Estimate Liabilities (death, lapse, expenses and commissions). For the "Death Liabilities", for every time step, we took:

$$\max(100'000, F(t) * qx(t) * (1 - LT(t))) * contractprob(t) * DFspot(t)$$

where the first term is the actualized money to be given at time t, assuming the man survived until time t-1, died during year t and didn't lapse during that same year.

For the "Lapse Liabilities" for every time step we did:

$$(F(t) - 20) * (1 - qx(t)) * L(t) * contractprob(t) * DFspot(t)$$

where the first term is the actualized money to be given at time t minus the commissions for the lapse, assuming the man survived until time t-1, didn't die during year t but lapsed during that same year. For the " Liabilities" we simply did the growth of 50 euros in time due to inflation actualizing every value and multiplying for the contract probability.
Last, we computed the "Commission Liabilities" where we did:

$$\frac{F(t) * COMM}{1-\mathrm{RD}}$$

where COMM=0.014 and RD=0.02
We then did the sum of all the components for each liability to find our value of the different liabilities and finally summing them to obtain our total liabilities.

Now, given the value of the BEL, we were able to compute the Basic Own Funds ("BOF") and its difference from the base case in every scenario as:

$$\textit{BOF=Asset-Liabilities}$$
$$\textit{DeltaBOF} = \textit{max(0 , BOF base case - BOF stressed scenario)}$$

where for Asset we mean F(0)=100'000 euros.

Another passage required was the computation of the Macaulay Duration of our liabilities. The formula we adopted is simply a weighted average of our total liabilities for each year, with weights equal to the time t.

The final step was the computation of BSCR. The latter is calculated as:

$$(1) \qquad BSCR = \sqrt{SCR_{Life} * CorrelationMatrix * SCR_{Market}}$$

where the SCRs we take into account are just the ones regarding Market risk and Life risk. These two are multiplied for the matrix here disposed:

$$(\mathbf{a}) \qquad \begin{bmatrix} 1 & 0.25 \\ 0.25 & 1 \end{bmatrix}$$

To perform this computation we calculated the SCR in the two different cases as it follows:

- SCR Market comes from the following formula

$$(2) \qquad SCR_{\text{Market}} = \sqrt{\sum_{i,j} SCR_i \times \text{Correlation Matrix}(i,j) \times SCR_j}$$

  where the correlation matrix is:

$$(\mathbf{b}) \qquad \begin{bmatrix} 1 & A & A \\ A & 1 & 0.75 \\ A & 0.75 & 1 \end{bmatrix}$$

  with A=0 in case the DeltaBOF is bigger in the scenario called "Interest Up" than "Interest Down", A=0.5 otherwise.

- SCR Life comes from the following formula:

$$(3) \qquad SCR_{\text{Life}} = \sqrt{\sum_{i,j} SCR_i \times \text{Correlation Matrix}(i,j) \times SCR_j}$$

  The correlation matrix is:

$$(\mathbf{c}) \qquad \begin{bmatrix} 1 & 0 & 0.25 & 0.5 \\ 0 & 1 & 0.25 & 0.25 \\ 0.25 & 0.25 & 1 & 0.25 \\ 0.5 & 0.25 & 0.25 & 1 \end{bmatrix}$$

Further explications about the BSCR formulas can be found in chapter 5 where we've included the different risk scenario discussed in chapter 4.

4 Discussion of the different risk scenarios

In this section we will provide some comments about the results that we've obtained when considering different kind of risk exposure for our company.

4.1 Market interest

In order to deal with market interest risk, we've considered shifted EIOPA interest rates, both in the up and down directions.
This has an influence both on the value of the asset, since the GBM's forward rate parameter is changing, and on the value of the liabilities, since every component, a part from the expenses, has some dependence on the asset.
In particular, what we've obtained is that in the case of increased risk-free rate there are less liabilities, so the BOF is bigger than the base case and DeltaBOF is null, while In the second case we have opposite results, with a positive DeltaBOF.
This means that the company has a market interest risk exposure only in the down case

4.2 Market equity

The Market Equity risk has been addressed by considering a shock on the stock's value over the entire period. The magnitude of this shock depends on a classification of the fund's equity provided by Solvency II regulations: in this case, we had a Type 1 equity, so we've applied a shock of 39%. Moreover, we've also included the symmetry adjustment, taken from the EIOPA tables of March 2024, which was 5.25%:

$$E(0)_{shocked} = E(0) * (1–0.39–0.0525)$$

With such a value, obviously both asset and liabilities decrease, but with a stronger effect on the asset value: this lead to a lower BOF with respect to the base scenario, and therefore to a positive DeltaBOF.

4.3 Market property

Similarly, we've considered a shock of 25% in the market price of property.
The signs of the results are analogous to the previous case, but with quantities that are closer to the base scenario since the value of property represents only the 20% of the asset value and because the shock is lower.

4.4 Life mortality

Life mortality risk refers to an increase of the annual death probabilities.
In particular, we've shocked up by 15% the mortality rates (making sure they stayed below 1). The impact of this modification on the liabilities is really weak since there's obviously an increase in the death component, but also a decrease in the other components and these two effects almost compensate.
The observed BOF is very similar to the base case's one, just slightly bigger, and the DeltaBOF is null.

4.5 Life lapse

For what concerns the lapse risk, we've evaluated three possible cases and considered the one with maximum DeltaBOF when determining the SCR.

The first one is the lapse up scenario, in which the annual lapse rate has been incremented by 50%, becoming 22.5% (also in this case had to ensure that it didn't exceed 100%).
As we could expect, the lapseBEL value is higher since, on average, the payments to the policyholders have

been anticipated and they are discounted via higher factors.
The other BEL component decrase, but not enough to compensate the lapseBEL, so overall we have a bigger value of BEL which implies a BOF lower than the base case and a positive DeltaBOF.

The second scenario is the lapse down. In this case, we've decreased the annual lapse rate by 50%, making sure that the variation remained below the 20%.
The results are symmetric with respect to the lapse up scenario: lapseBEL is lower but the other components are remarkably higher leading to a null DeltaBOF.

The third and last case is the lapse mass scenario, in which only the lapse rate referring to the first year has been increased by 0.4, leaving the others unchanged.
The results that we've obtained go in the same direction of the lapse up ones, but the effect is amplified since the first component of the lapseBEL is the one associated to a bigger discount factor, so this massive initial increase of the lapse rate results more impactful than the constant up shock of 50%.
In view of these considerations, we can say that the lapse mass scenario is the one to which we have greater exposure, and therefore is the one that we've considered in the computation of the capital requirement.

4.6 Life cat

The risk of catastrophy has been assessed by considering an up shift of 0.15% only on the death probability of the first year.
Similarly to what we've concluded about the life mortality scenario, we can say that the rise of the deathBEL is compensated by the decrease of the other BEL components. The DeltaBOF is null also in this case, so we don't have a risk exposure to catastrophic events happening in the first period.

4.7 Life Expense

The last kind of risk to be considered was the expense one, which refers to the fact that the future expenses of the company could be more consistent due to the rise of inflation or to specific operational costs.
To assess this point, we've considered a rise of 10% in the constant annual expenses, while 1% has been added to the inflation rate.
As we could easily predict, there's an increase in the expensesBEL while all the other components remain unchanged: this implies that the DeltaBOF is positive, but it has a low value since expenses represent by far the less impactful term of BEL. So there's actually a risk exposure, but not so relevant.

5 BSCR computation

After investigating all the risk scenarios, we finally computed the Basic Solvency Capital Requirement of our company by using the formulas described in the first section.
In particular, looking at formula (2) the components of SCR_{market} are SCR_{interest}, SCR_{equity} and SCR_{property}.

As we've anticipated, SCR_{interest} is the maximum between $DeltaBOF_{\text{IRup}}$ and $DeltaBOF_{\text{IRdown}}$: in our case it's the second one and the proper correlation matrix is (b) with $A = 0.5$. SCR_{equity} and SCR_{property} are simply $DeltaBOF_{\text{equity}}$ and $DeltaBOF_{\text{property}}$.

It's important to notice that the equity component is way bigger than the other two, so it gives the main contribution to SCR_{market}.

On the other hand, formula (3) includes $SCR_{\text{Mortality}}$, SCR_{Lapse}, SCR_{Expenses}, and SCR_{Cat}.

All these components are given by to the corresponding $DeltaBOF$'s, a part from the SCR_{lapse}, which is the maximum between $DeltaBOF_{\text{lapseUP}}$, $DeltaBOF_{\text{lapseDOWN}}$ and $DeltaBOF_{\text{lapseMASS}}$.

Knowing that the lapse mass scenario is the one towards which our company has the biggest exposure, this last value is the one that appears in our formula. Furthermore,SCR_{Lapse} is the predominant one in determining SCR_{Life}.
Once we've obtained the market and life components of SCR, we were able to actually compute the value of BSCR by applying formula (1) with the correlation matrix (a).

6 Deterministic computations and comments

6.1 Differences in Matlab and Excel results

In our analysis, Matlab and Excel evaluations slightly differ from one another.
This is due to the fact that the deterministic system's behavior is fully determined by the initial conditions and the set of equations governing its dynamics (explained before), while our stochastic model accounts for variability of the solutions.

On the other hand we discovered that the outcomes yielded by Excel and Matlab exhibit remarkable congruences, a phenomenon we attribute to the mitigating influence of the mean across all simulations of Brownian Motion on the effects of stochastic component. Consequently, employing identical deterministic formulae and a consistent varying instantaneous rate 'r' for both cases seemingly attenuates disparities in outcomes, thus leveling discrepancies.

6.2 Base Case calculations

Regarding the base case only, the analysis reveals the following:

- Maculay Duration of the liabilities:

$$DU_{liab} = \frac{\sum_{t=0}^{50} t * Liab(t)}{Liab_{tot}} = 5.5755 \text{ years}$$

- Present Value of Future Profits
  Future Profits at each time step:

$$FP(t) = (F(t) * RD - (Expenses(t) + Commisions(t))) * ContractProb$$

  Total value of Future Profits:

$$PVFP = \sum_{t=0}^{50} FP(t) * DFspot(t) = 3908.6012$$

- Check for the Leakage magnitude:

$$Leakage = 1263.4785$$

  with amount of leakage calculated as the difference between MVA and the sum of Liabilities and PVFP

- Proxy Calculation of PVFP:
  Total Future Profits

$$FP = \sum_{t=0}^{50} FP(t) = 4518.3803$$

$$PVFPproxy = FP * DF_{spot}^{equivalent} = 3930.9909$$

  where $DF_{\text{spot}}^{\text{equivalent}}$ is the discount factor corresponding to the Macaulay Duration of the liabilities. There are two sources of approximation in this formula: the main one is that we're using the Duration of the liabilities which is smaller than the one of the Future Profits; the second one is that an interpolation has been made when considering $DF_{\text{spot}}^{\text{equivalent}}$, since the Duration is between two different time steps.

7 Open question

7.1 What happens to the asset and liabilities when the risk-free rate increases/decreases with a parallel shift of, say, 100bps? Describe the effects for all the BEL components.

As we've already underlined in the discussion of interest risk, a change in the risk free rate influences not only the evolution of the value of the asset through the GBM, but also every component of the liabilities a part from the expenses, the only one that is independent of the value of F(t).

In particular, given that in the expression of the payment for the death case there is a minimum level fixed at 100'000 euros, this will have an impact only in the 100bps decrease scenario.

Indeed, in the Up case there would be a compensation between the evolution of the assets accordingly to the risk free rate and the discount factor.

This would lead in the Down case to a more likely increase in the death BEL and a small decrease as a consequence in the lapse BEL, while keeping mostly unchanged the death, commission and lapse BEL in the Up case.

Lastly, the expenses BEL will increase in the down case and will decrease in the up case due to a different discount factor computation, not affected by the evolution of the fund.

7.2 What happens to the liabilities if the insured age increases? What if there were two model points, one male and one female?

If the insured age increases, there's obviously an increment of the annual death probabilities qx. The most trivial effect is that the death component of the liabilities becomes bigger, but also the other three components are affected: the lapse BEL will likely decrease due to a higher mortality recorded every year, while the expenses and commission BEL will be smaller because of a reduction of the probability to still be in the contract every year.

On the other hand, if we consider two model points, there will be a decrease in the annual death probabilities thanks to the fact that female's mortality rates are lower. This will imply an opposite effect on the liabilities with respect to the previous case: the death BEL will likely decrease while the other components, as a consequence, will increase slightly.

8 Matlab Code

```
%%%%%%%%%%
% GROUP 14
%%%%%%%%%%

% Facconi Michele
% Spelta Anna
% Pigato Alessandro

% AY 2023-2024

%%%%%%%%%%%%%%
%%%%%%%%%%%%%%

clear all
clc
close all

% Numerical data
F0=100000;
E0=0.8*F0;
P0=0.2*F0;
sigma_eq=0.2;
sigma_pr=0.1;
rho=0;
Cov_function=[sigma_eq^2 2*rho*sigma_eq*sigma_pr; 2*rho*sigma_eq*sigma_pr
    sigma_pr^2];
T = 50;
RD=0.022;
COMM=0.014;
expenses_base= 50;
inflation_rate=0.02;
symm_adj = 0.0525;

% Risk free rates (Base, Up, Down)
RFR_spot_table=xlsread('EIOPA_RFR_20240331_Term_Structures.xlsx',3);
RFR_spot_vector=RFR_spot_table(8:58,18);

RFR_spot_table_UP=xlsread('EIOPA_RFR_20240331_Term_Structures.xlsx',5);
RFR_spot_vector_UP=RFR_spot_table_UP(8:58,18);

RFR_spot_table_DOWN=xlsread('EIOPA_RFR_20240331_Term_Structures.xlsx',6);
RFR_spot_vector_DOWN=RFR_spot_table_DOWN(8:58,18);

% Life Table
Life_table_read=xlsread('Life tables of the resident population_Males_2022.
    xlsx');
Life_table=Life_table_read(61:end, 4);
```

```
Total_Life_table=Life_table_read(3:end, 4);

% Computation of discount factor spot and forward
% Base
for t=1:T+1
    DF_spot(t)=exp(-RFR_spot_vector(t).*t);
end

for t=1:T
     DF_forward(t)=DF_spot(t+1)/DF_spot(t);
end

for t=2:T
      RFR_fwd_vector(t)=-log(DF_forward(t-1));
end
RFR_fwd_vector(1)=RFR_spot_vector(1);

% Up
for t=1:T+1
    DF_spot_UP(t)=exp(-RFR_spot_vector_UP(t).*t);
end

for t=1:T
     DF_forward_UP(t)=DF_spot_UP(t+1)/DF_spot_UP(t);
end
for t=2:T
      RFR_fwd_vector_UP(t)=-log(DF_forward_UP(t-1));
end
RFR_fwd_vector_UP(1)=RFR_spot_vector_UP(1);

% Down
for t=1:T+1
    DF_spot_DOWN(t)=exp(-RFR_spot_vector_DOWN(t).*t);
end

for t=1:T
     DF_forward_DOWN(t)=DF_spot_DOWN(t+1)/DF_spot_DOWN(t);
end

for t=2:T
      RFR_fwd_vector_DOWN(t)=-log(DF_forward_DOWN(t-1));
end
RFR_fwd_vector_DOWN(1)=RFR_spot_vector_DOWN(1);

% vector of annual lapse rates
LT=ones(1,T).*0.15;

% vector of death probabilities
qx=Life_table(1:T)'/1000;
```

```
% Evolution of the asset F(t)
delta_t=1;
Nx=1e6;
E=GBM(E0, RFR_fwd_vector, sigma_eq, delta_t, Nx, RD, T);
P=GBM(P0, RFR_fwd_vector, sigma_pr, delta_t, Nx, RD, T);
F=E+P;

% Computation of Best Estimate Liabilities (BEL) in different risk
% scenarios
%% BASE

[liabilities_interest, M_duration, death_BEL, lapse_BEL, commission_BEL,
    expenses_BEL]=Liabilities_Computation(T, RFR_fwd_vector,E0, P0, delta_t,
    Nx, RD, COMM, expenses_base, inflation_rate, qx, LT, DF_spot);

BOF=F0-liabilities_interest;

%% INTEREST UP
[liabilities_interest_up, M_duration_UP, death_BEL_UP, lapse_BEL_UP,
    commission_BEL_UP, expenses_BEL_UP]=Liabilities_Computation(T,
    RFR_fwd_vector_UP, E0, P0, delta_t, Nx, RD, COMM, expenses_base,
    inflation_rate, qx, LT, DF_spot_UP);

BOF_Interest_UP=F0-liabilities_interest_up;

Delta_BOF_Interest_UP=max(0, BOF-BOF_Interest_UP);

%% INTEREST DOWN
[liabilities_interest_down, M_duration_DOWN, death_BEL_DOWN, lapse_BEL_DOWN,
    commission_BEL_DOWN, expenses_BEL_DOWN]=Liabilities_Computation(T,
    RFR_fwd_vector_DOWN, E0, P0, delta_t, Nx, RD, COMM, expenses_base,
    inflation_rate, qx, LT, DF_spot_DOWN);

BOF_Interest_DOWN=F0-liabilities_interest_down;

Delta_BOF_Interest_DOWN=max(0, BOF-BOF_Interest_DOWN);


%% MARKET EQUITY
E0_equity=(1-0.39-symm_adj)*E0;

[liabilities_equity, M_duration_equity, death_BEL_equity, lapse_BEL_equity,
    commission_BEL_equity, expenses_BEL_equity]=Liabilities_Computation(T,
    RFR_fwd_vector, E0_equity, P0, delta_t, Nx, RD, COMM, expenses_base,
    inflation_rate, qx, LT, DF_spot);

BOF_equity=(E0_equity+P0)-liabilities_equity;

```

```
Delta_BOF_equity=max(0, BOF-BOF_equity);

%% MARKET PROPERTY
P0_property=(1-0.25)*P0;

[liabilities_property, M_duration_property, death_BEL_property,
    lapse_BEL_property, commission_BEL_property, expenses_BEL_property]=
    Liabilities_Computation(T, RFR_fwd_vector, E0, P0_property, delta_t, Nx,
    RD, COMM, expenses_base, inflation_rate, qx, LT,DF_spot);

BOF_property=(E0+P0_property)-liabilities_property;

Delta_BOF_property=max(0, BOF-BOF_property);


%% LIFE MORTALITY
qx_mortality=qx.*1.15;

[liabilities_mortality, M_duration_mortality, death_BEL_mortality,
    lapse_BEL_mortality, commission_BEL_mortality, expenses_BEL_mortality]=
    Liabilities_Computation(T, RFR_fwd_vector, E0, P0, delta_t, Nx, RD, COMM,
     expenses_base, inflation_rate, qx_mortality,LT, DF_spot);

BOF_mortality=F0-liabilities_mortality;

Delta_BOF_mortality=max(0, BOF-BOF_mortality);

%% LIFE LAPSE UP
LT_lUP=min(1.5*LT,1);

[liabilities_lapse_UP, M_duration_lapse_UP, death_BEL_lapse_UP,
    lapse_BEL_lapse_UP, commission_BEL_lapse_UP, expenses_BEL_lapse_UP]=
    Liabilities_Computation(T, RFR_fwd_vector, E0, P0, delta_t, Nx, RD, COMM,
     expenses_base, inflation_rate, qx, LT_lUP, DF_spot);

BOF_lapse_UP=F0-liabilities_lapse_UP;

Delta_BOF_lapse_UP=max(0, BOF-BOF_lapse_UP);

%% LIFE LAPSE DOWN
LT_lDOWN=max(0.5*LT,LT-0.2);

[liabilities_lapse_DOWN, M_duration_lapse_DOWN, death_BEL_lapse_DOWN,
    lapse_BEL_lapse_DOWN, commission_BEL_lapse_DOWN, expenses_BEL_lapse_DOWN
    ]=Liabilities_Computation(T, RFR_fwd_vector, E0, P0, delta_t, Nx, RD,
    COMM, expenses_base, inflation_rate, qx, LT_lDOWN, DF_spot);

BOF_lapse_DOWN=F0-liabilities_lapse_DOWN;

```

```
Delta_BOF_lapse_DOWN=max(0, BOF-BOF_lapse_DOWN);

%% LIFE LAPSE MASS
LT_lMASS=[];
LT_lMASS=LT;
LT_lMASS(1)=LT_lMASS(1)+0.4;

[liabilities_lapse_MASS, M_duration_lapse_MASS, death_BEL_lapse_MASS,
    lapse_BEL_lapse_MASS, commission_BEL_lapse_MASS, expenses_BEL_lapse_MASS
    ]=Liabilities_Computation(T, RFR_fwd_vector, E0, P0, delta_t, Nx, RD,
    COMM, expenses_base, inflation_rate, qx, LT_lMASS, DF_spot);

BOF_lapse_MASS=F0-liabilities_lapse_MASS;

Delta_BOF_lapse_MASS=max(0, BOF-BOF_lapse_MASS);

%% LIFE CAT
qx_cat=qx;
qx_cat(1)=qx(1)+0.0015;

[liabilities_CAT, M_duration_CAT, death_BEL_CAT, lapse_BEL_CAT,
    commission_BEL_CAT, expenses_BEL_CAT]=Liabilities_Computation(T,
    RFR_fwd_vector, E0, P0, delta_t, Nx, RD, COMM, expenses_base,
    inflation_rate, qx_cat, LT, DF_spot);

BOF_CAT=F0-liabilities_CAT;

Delta_BOF_CAT=max(0, BOF-BOF_CAT);

%% EXPENSE
new_inflation=inflation_rate+0.01;
new_expenses=expenses_base*1.1;

[liabilities_EXP, M_duration_EXP, death_BEL_EXP, lapse_BEL_EXP,
    commission_BEL_EXP, expenses_BEL_EXP]=Liabilities_Computation(T,
    RFR_fwd_vector, E0, P0, delta_t, Nx, RD, COMM, new_expenses,
    new_inflation, qx, LT, DF_spot);

BOF_EXP=F0-liabilities_EXP;

Delta_BOF_EXP=max(0, BOF-BOF_EXP);

%% BSCR

% SCR Market
SCR_Interest=max(Delta_BOF_Interest_UP, Delta_BOF_Interest_DOWN);
SCR_Equity=Delta_BOF_equity;
SCR_Property=Delta_BOF_property;
if Delta_BOF_Interest_UP>Delta_BOF_Interest_DOWN
```

```
  Corr_mat_Market=[1 0 0; 0 1 0.75; 0 0.75 1];
else
    Corr_mat_Market=[1 0.5 0.5; 0.5 1 0.75; 0.5 0.75 1];
end
Market_vector=[SCR_Interest SCR_Equity SCR_Property];
SCR_Market=sqrt(Market_vector*Corr_mat_Market*Market_vector');

% SCR Life
SCR_Mortality=Delta_BOF_mortality;
SCR_Lapse=max([Delta_BOF_lapse_UP Delta_BOF_lapse_DOWN Delta_BOF_lapse_MASS
    ]);
SCR_Cat=Delta_BOF_CAT;
SCR_Expenses=Delta_BOF_EXP;
Corr_mat_Life=[1 0 0.25 0.25; 0 1 0.5 0.25; 0.25 0.5 1 0.25; 0.25 0.25 0.25
    1];
Life_vector=[ SCR_Mortality SCR_Lapse SCR_Expenses SCR_Cat];
SCR_Life=sqrt(Life_vector*Corr_mat_Life*Life_vector');

% BSCR
Corr_mat=[1 0.25; 0.25 1];
BSCR_vector=[SCR_Market SCR_Life];
BSCR=sqrt(BSCR_vector*Corr_mat*BSCR_vector')
```

```
function X=GBM(F0, RFR_fwd, Sigma, delta_t, Nx, RD, T)

% Function simulating a Geometric Brownian Motion
% F0        --> initial condition
% RFR_fwd   --> Risk free Forward Ratet
% Sigma     --> volatility
% delta_t   --> time step
% Nx        --> number of simulations
% RD        --> regular deduction parameter
% T         --> time horizon

% Initializing
x=zeros(Nx, (T*delta_t)+1);
x(:,1)=F0;

% Computing the stochastic part
dJ=randn(Nx, T*delta_t);

% Simulation
for i=1:T*delta_t
    x(:, i+1)= (1-RD).*x(:, i).*exp((RFR_fwd(i)*delta_t-Sigma^2/2*delta_t)+
        Sigma*sqrt(delta_t)*dJ(:,i));
end

X=mean(x,1);
end
```

```
function [liabilities_total, M_duration, death_BEL, lapse_BEL, commission_BEL, expenses_BEL]=Liabilities_Computation(T, frw_rates, E0, P0, delta_t, Nx, RD, COMM, expenses_base, inflation_rate, qx, LT, DF_spot)

% Function that calculate the Best Estimate liabilities
% It also restitutes the Macaulay Duration of the liabilities and each BEL
% component

% T              ---> time horizon
% frw_rates      ---> forward rates
% E0             ---> initial condition (equity)
% P0             ---> initial condition (property)
% delta_t        ---> time step
% Nx             ---> number of simulations
% RD             ---> regular deduction
% COMM           ---> commissions
% expenses_base  ---> constant value of annual expenses
% inflation_rate ---> inflation rate
% qx             ---> vector of death probabilities
% LT             ---> vector of annual lapse rates
% DF_spot        ---> discount factor spot

sigma_eq=0.2;
sigma_pr=0.1;
F0=E0+P0;
lapse_commission=20;

% GBM simulations for equity and property
E = GBM(E0, frw_rates, sigma_eq, delta_t, Nx, RD, T);

P = GBM(P0, frw_rates, sigma_pr, delta_t, Nx, RD, T);

F = E + P;

% Liabilities computations
contract_prob=cumprod([1; (1-qx(1:end-1))'.*(1-LT(1:end-1))']);
for i=1:T
     commission(i)=(F(i+1)/(1-RD)*COMM);
     expenses(i)=expenses_base*((1+inflation_rate)^i);
     death(i)=(max(F0, F(i+1)))*qx(i)*(1-LT(i));
     if i==T
          lapse(i)=(F(i+1)-lapse_commission)*(1-qx(i));
     else
      lapse(i)=(F(i+1)-lapse_commission)*(1-qx(i))*LT(i);
     end

end

```

```
for i=1:T
  liabilities(i)=contract_prob(i)*(death(i)+lapse(i)+expenses(i)+commission(i))*DF_spot(i);
end

liabilities_total=sum(liabilities);

% Macauley Duration
M_duration=(liabilities*[1:T]')/liabilities_total;

% BEL components
death_BEL=sum(contract_prob(1:T)'.*death.*DF_spot(1:50))
lapse_BEL=sum(contract_prob(1:T)'.*lapse.*DF_spot(1:50))
commission_BEL=sum(contract_prob(1:T)'.*commission.*DF_spot(1:50))
expenses_BEL=sum(contract_prob(1:T)'.*expenses.*DF_spot(1:50))


end
```