$$\alpha \in \mathbb{R} \implies \alpha \underline{v}_0 = \alpha(\alpha_0 \underline{u}) = (\alpha \alpha_0) \underline{u} \in S$$