Automata and Formal Languages

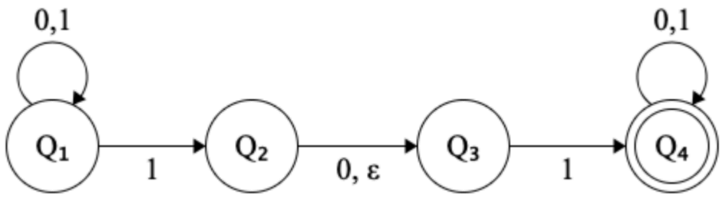
Formalising a NFA

Different only in the transition function

For a DFA: $Q \times \Sigma \mapsto Q$

$Q \times \Sigma_\epsilon \mapsto P(Q), \qquad \Sigma_\epsilon := \Sigma \cup \{\epsilon\}$

An NFA is a 5-tuple as a DFA, the only definition changing is the transition function.



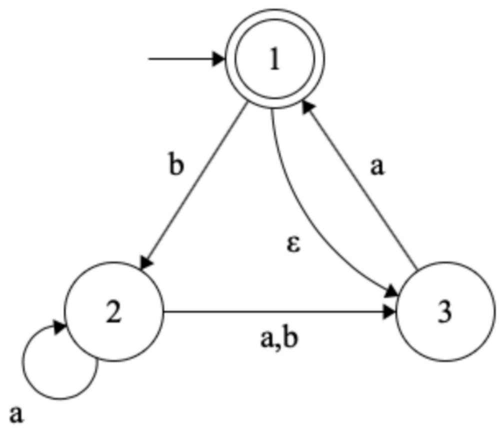
$\delta := Q \times \Sigma_\epsilon = P(Q)$

δ	0	1	ϵ
Q_1	$\{Q_1\}$	$\{Q_1, Q_2\}$	$\varnothing$
Q_2	$\{Q_3\}$	$\varnothing$	$\{Q_1\}$
Q_3	$\varnothing$	$\{Q_4\}$	$\varnothing$
Q_4	$\{Q_4\}$	$\{Q_4\}$	$\varnothing$

Equivalence of DFAs and NFAs

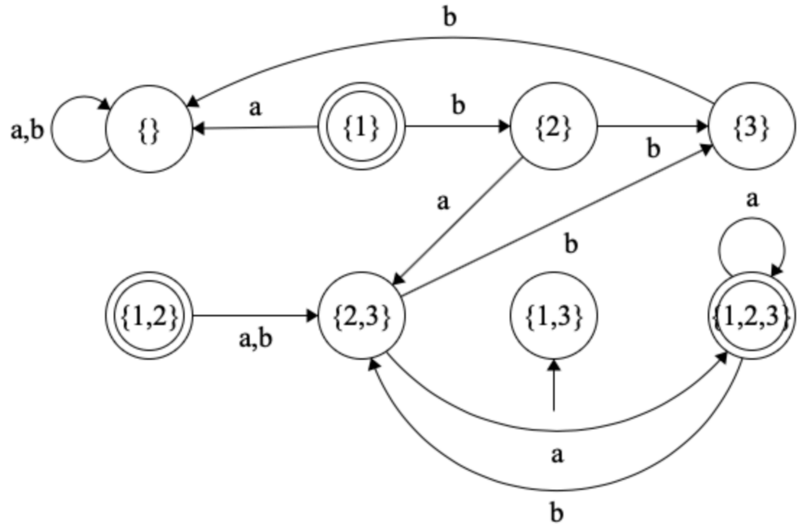
They recognise the same class of languages: every NFA has an equivalent DFA.

Given the following NFA:



How to construct a DFA from the previous NFA?

Set of states: how many? 2^m, $m = |Q|$



Given $N = (Q, \Sigma, \delta, q_0, F)$

Construct $M = (Q', \Sigma, \delta', q'_0, F')$

Such that $L(M) = L(N)$ at first, let's not consider ϵ arrows

$$Q' = P(Q)$$

$$\delta'(R, a) = \bigcup_{r \in R} \delta(r, a) \qquad \text{for } R \in Q' \wedge a \in \Sigma$$

$$\delta'(R, a) = \{q \in Q \,|\, q \in \delta(r, a) \text{ for some } r \in R\}$$

$$q'_0 = \{q_0\}$$

$$F' = \{R \in Q' \,|\, R \text{ contains an accepting state of } N\}$$

Considering ϵ arrows:

$E(R) = \{q \,|\, q$ can be reached from R by travelling along 0 or more epsilon arrows$\}$

The same construction as before but:

$$\delta'(R, a) = \{q \in Q \,|\, q \in E(\delta(r, a)) \text{ for some } r \in R\}$$

$$q'_0 = E(q_0)$$

$M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$

$M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$

$N = (Q, \Sigma, \delta, q_0, F)$

Union: $L(N) = L(M_1) \cup L(M_2)$

$Q = Q_1 \cup Q_2 \cup \{q_0\}$

q_0 is the initial state of N

$F = F_1 \cup F_2$

$$\delta(q, \sigma) = \begin{cases} \delta_1(q, \sigma) & \text{if } q \in Q_1 \\ \delta_2(q, \sigma) & \text{if } q \in Q_2 \\ \{q_1, q_2\} & \text{if } q = q_0 \wedge \sigma = \epsilon \\ \varnothing & \text{if } q = q_0 \wedge \sigma \neq \epsilon \end{cases}$$

Concatenation: $L(N) = L(M_1) \cdot L(M_2)$

$Q = Q_1 \cup Q_2$

$q_0 = q_1$

$F = F_2$

$$\delta(q, \sigma) = \begin{cases} \delta_1(q, \sigma) & \text{if } q \in Q_1 \wedge q \notin F_1 \\ \{q_2\} \cup \delta_1(q, \epsilon) & \text{if } q \in F_1 \wedge \sigma = \epsilon \\ \delta_1(q, \sigma) & \text{if } q \in F_1 \wedge \sigma \neq \epsilon \\ \delta_2(q, \sigma) & \text{if } q \in Q_2 \end{cases}$$

$L = \{w \mid w \text{ ends in } 0001 \cup w \text{ has odd length}\}$