Automata and Formal Languages

Regular Languages

- A language is called **regular language** if some finite automaton recognises it
- $L(M) = \{w \mid w$ is the empty string or ends in $0\}$

Formal Definition of Computation

- Let M be a finite automaton and
- Let $w = w_1 w_2 \ldots w_n$ be a string of length n
- **M accepts string w if**:

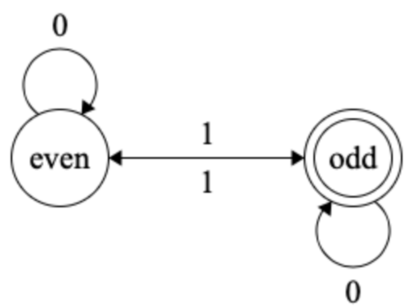
  a sequence of states $r_o, r_1, \ldots, r_n$ exists in Q with the following conditions:

  1. $r_0 = q_0$
  2. $\delta(r_1, w_{i+1}) = r_{i+1}$ for $i = 0, \ldots, n-1$
  3. $r_n \in F$

  then the ***string is accepted***.
- M recognises language A if $A = \{w \mid M$ accepts $w\}$

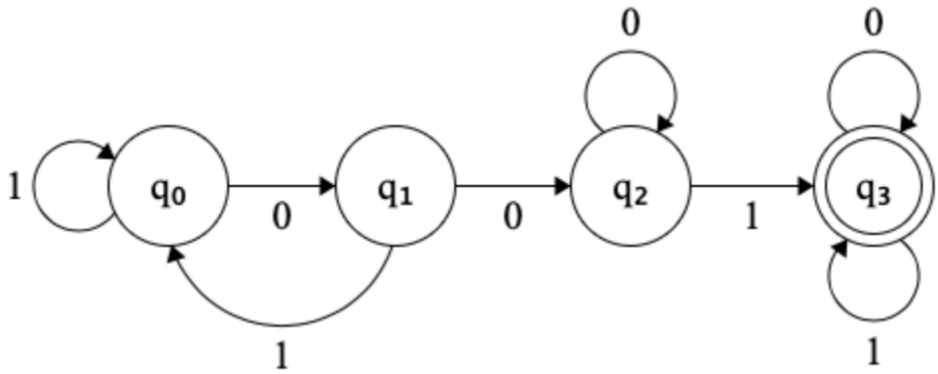
Designing Finite Automata

- A FA which recognises a language $A = \{w \mid w$ has an odd number of 1s$\}$

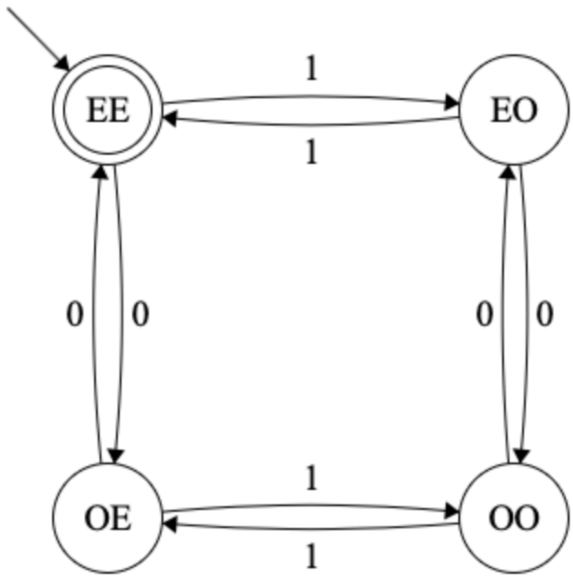


Design an FA that recognises the regular language of all strings that contain string 001 as a substring:

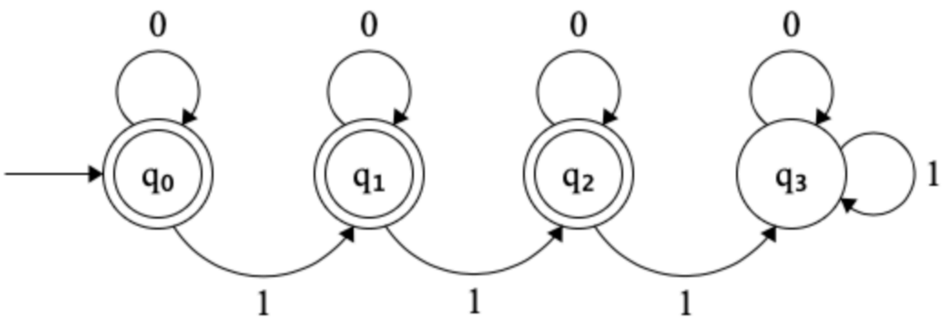
$L(M) = \{w \,|\, w \text{ is } x001y \text{ for some } x \text{ and } y\}$



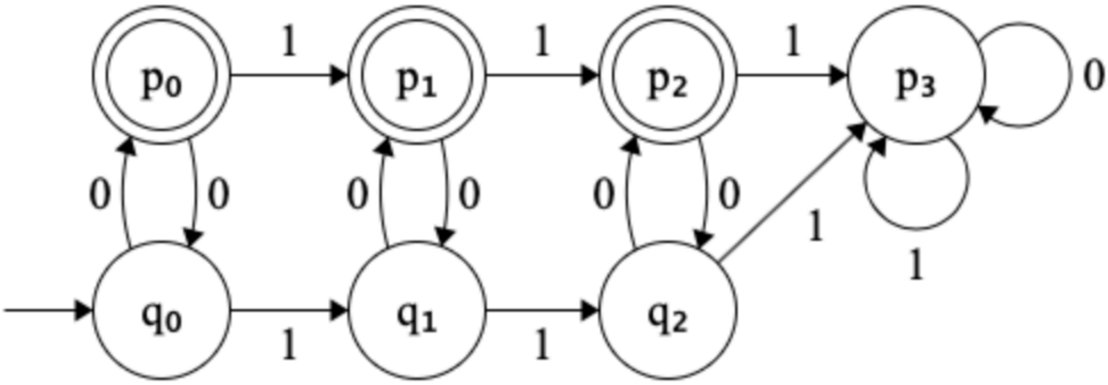
$L(M) = \{w \,|\, w \text{ has an even number of 0s and an even number of 1s}\}$



$L(M) = \{w \,|\, w \text{ contains at most two 1s}\}$

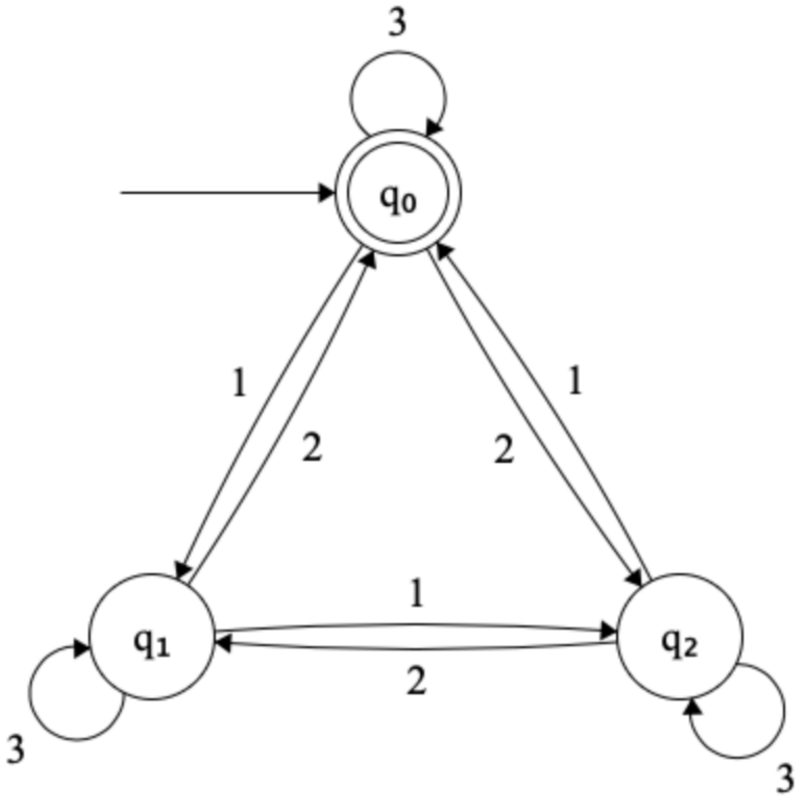


$L(M) = \{w \mid w$ contains at most two 1s, and has an odd number of 0s$\}$

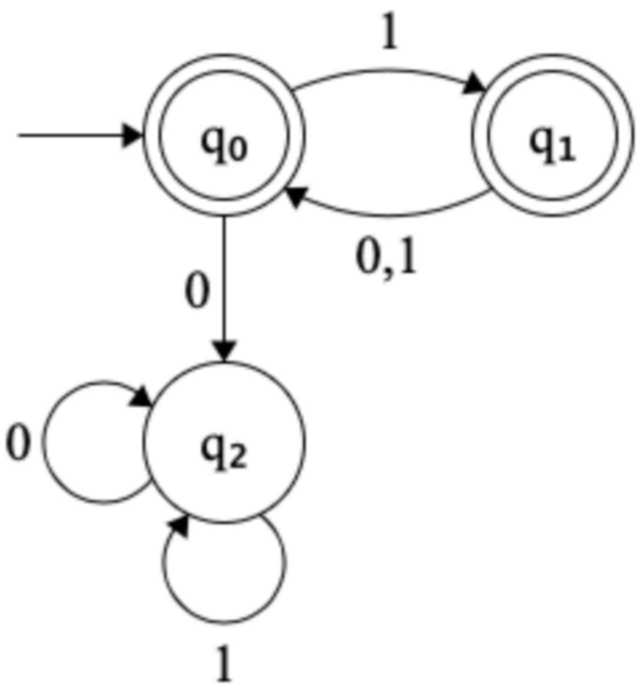


$L(M) = \{w \mid$ the sum of the numerical values of the symbols of w is a multiple of $3\}$

$\Sigma = \{0,1,2\}$



$L(M) = \{w \mid$ every odd position of w is a 1$\}$



$L(M) = \{w \mid w$ contains an even number of occurrences of substring 00$\}]$