

$$\alpha \in R \implies \alpha_{\perp 0} = \alpha(\alpha_{\perp 0}) = (\alpha \alpha_0)_{\perp 0} \in S$$