AFL: Assignment 4

Due on November 15, 2018 at 11:55pm

Prof. Laura Pozzi

A. Romanelli

Problem 1

Prove that: Language $L1 = \{w|w \text{ has more 0s than 1s}\}$ is not regular.

Let's assume that language $L1$ is a regular language, then there must exist p such that all strings of length p can be pumped.

We'll choose a p such that the corresponding strings express the non regularity of language $L1$:

$$s = 1^p 0^{p+1}$$

1 1 1 . . . 1 1	0 0 0 . . . 0 0 0

We need to obey the three following conditions:

1. $xy^iz \in L1, \forall i \geq 0$,
2. $|y| > 0$,
3. $|xy| \leq p$

The third condition imposes that our y must consist of only 1s.
The second condition imposes that our y consists of at least one symbol. The first condition can now be no longer satisfied, as repeating the ones i times will cause the string to no longer belong to $L1$, since it will have more 1s than 0s.

Problem 2

Prove that: Language $L2 = \{w|w \text{ has even length and the first half of w has more 0s than the second half of w}\}$

We repeat the same process picking a string s which represents the non regularity of $L2$ and that can't possibly be pumped:

$$s = 1^p 0^p 1^{2p}$$

1 1 1 . . . 1 1 1	0 0 0 . . . 0 0 0	1 1 1 . . . 1 1 1	1 1 1 . . . 1 1 1

Because of the third condition, we know that y will be made up exclusively by 1s.
The second condition also reassures us that y must consist of at least a symbol.
To formally demonstrate how $xy^iz \notin L2$ we can imagine a very large i and $|y|$ as well.
Let $|y|$ be at its maximum, the closest to the condition $|xy| \leq p$ as possible, so that $y = 1^p$.
Now, pumping this string with $i \geq 1$ will result in the string of form:

$$s' = 1^i 0^p 1^{2p}$$

$$x = \epsilon, y = 1^i, z = 0^p 1^2 p$$

It's clearly visible that the first condition falls apart with $i \geq 2$, since it will result in an equal amount of zeros on both left and right side.
If instead of $y = 1^p$ we had picked $y = 1$ the result would be no different, i would simply need to be $i \geq 2p$ in order to achieve the same result. Hence we can definitely conclude that exists some $i \in \mathbb{N}|xy^iz \notin L2$, as we have already seen.