Electronics and Communications in Japan, Part 3, Vol. 84, No. 12, 2001
Translated from Denshi Joho Tsushin Gakkai Ronbunshi, Vol. J83-A, No. 6, June 2000, pp. 677–685

Blind Deconvolution of Blurred Images by Use of ICA

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SUMMARY

Independent Component Analysis is a new data analysis method, and its computation algorithms and applications have been widely studied recently. Most applications, however, are in the field of one-dimensional data analysis, such as sound data analysis, and few applications for two-dimensional data (e.g., image data) have been studied. In this paper, a new application of ICA for two-dimensional data analysis is given, namely, image restoration of blurred images. The proposed method can restore the original image without knowing the blurring process. © 2001 Scripta Technica, Electron Comm Jpn Pt 3, 84(12): 1–9, 2001

Key words: Independent component analysis (ICA); image restoration; blurred image.

1. Introduction

Independent Component Analysis (ICA) is a new data analysis method [1], and its computation algorithms [2, 7–9, 13] and applications have been widely studied recently. Its application fields include blind source separation which realizes a kind of *cocktail party effect*, a self-organization model of early vision [3], and an analysis of brain activity by applying ICA to electroencephalograms [11].

Most applications, however, are in the field of one-dimensional data analysis, such as sound data analysis, and few applications for two-dimensional data (e.g., image data) have been studied.

In this paper, a new application of ICA for two-dimensional data analysis is given, namely, image restoration of blurred images. The proposed method can restore the original image without knowing the blurring process.

This shows that some tasks which do not seem to be in the framework of ICA can be solved by ICA when transforming them properly, and also shows the applicability of ICA to a real-world problem.

2. Independent Component Analysis

2.1. Framework of ICA

When several mixtures, $x_1(t), x_2(t), \ldots, x_m(t)$, of statistically independent information sources, $s_1(t), s_2(t), \ldots, s_l(t)$, of zero mean are observed, ICA estimates the original information sources $s_i(t)$. If the observation and the original information sources are represented as vectors $\boldsymbol{x}(t) = (x_1(t), x_2(t), \ldots, x_m(t))^T$ and $\boldsymbol{s}(t) = (s_1(t), s_2(t), \ldots, s_l(t))^T$, the observation process is given as follows:

$$\boldsymbol{x}(t) = A\boldsymbol{s}(t) \tag{1}$$

We call $m \times l$ matrix $A(m \geq l)$ a mixing matrix, and assume that it is of full column rank.

If mixing matrix A is known, the least-squares estimation of the original information sources $\boldsymbol{s}(t)$ can be obtained by using the pseudo inverse A^+ of A. If mixing matrix A is unknown, however, the estimation of the original signal seems to be impossible. ICA can realize this estimation by using the statistical independence of the original information sources even when both $\boldsymbol{s}(t)$ and A are unknown.

2.2. ICA algorithm

Suppose we have n observations $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n$, and let X be an $m \times n$ data matrix which has $\boldsymbol{x}_i$ as a column vector. Let S be an $l \times n$ information source matrix, and A be a mixing matrix. The row vectors of X are called data series, and the row vectors of S are independent components. ICA assumes that the mean of information source must be zero, thus the mean value of each data series of X also should be zero. The mean values of the data series are set to 0 by extracting the mean values from them.

The goal of ICA is to decompose data matrix X into a product of two matrices, A and S:

$$X = AS \tag{2}$$

S and A are full rank, thus this decomposition is a *maximum rank decomposition* of X [10]. When a singular value decomposition of X is given as $X = UDV^T$, the maximum rank decomposition of X can be represented as

$$X = \hat{U}\hat{D}W^{-1}W\hat{V}^T \tag{3}$$
$$= \left(\frac{1}{\sqrt{n}}\hat{U}\hat{D}W^{-1}\right)\cdot(W\cdot\sqrt{n}\hat{V}^T) \tag{4}$$

where $\hat{D} = \mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_l)$ is a diagonal matrix of only positive singular values, and $\hat{U}$ and $\hat{V}$ are $m \times l$ and $l \times n$ matrices which are composed by selecting eigenvectors of U and V corresponding to $\hat{D}$. W is an arbitrary $l \times l$ nonsingular matrix. This representation gives candidate mixing matrices and information source matrices as follows:

$$A = \frac{1}{\sqrt{n}}\hat{U}\hat{D}W^{-1} \tag{5}$$
$$S = W\cdot\sqrt{n}\hat{V}^T \tag{6}$$

The derivation of Eq. (4) by SVD is called *whitening* or *sphering* in ICA, and is a principal component analysis in the data analysis field.

The maximum rank decomposition has an ambiguity originating from matrix W, thus this matrix W is selected so that the row vectors of S (independent components) are as statistically independent of each other as possible. This is the essential part of the ICA algorithm. The selection of W is done as follows.

First, W should be an orthogonal matrix, since the covariance matrix of row vectors of S is an identity matrix I:

$$\frac{1}{n}SS^T = W\hat{V}^T\hat{V}W^T = WW^T = I \tag{7}$$

Next, the matrix W which maximizes the statistical independence of the row vectors of S is sought. Several criteria have been proposed to achieve this goal, for example, the minimum mutual information method [13] and the maximum entropy method [2]. Oja and his group showed that ICA can be achieved by maximizing/minimizing the fourth cumulant κ_4 of the row vectors of S [9]. In this paper this criterion is used. Oja's algorithm is a kind of stochastic gradient algorithm and the updating rule of W is as follows:

$$W_{k+1} = W_k + \mathrm{diag}(\mu_i)(\tanh \boldsymbol{y}_k)\boldsymbol{v}_k^T \tag{8}$$

where $\boldsymbol{v}_k$ is a column vector $\sqrt{n}\hat{V}^T$ and $\boldsymbol{y}_k = W\boldsymbol{v}_k$. μ_i is a learning parameter. This iteration makes W nonorthogonal; thus it is transformed into an orthogonal matrix by the Gram–Schmidt orthogonalization method at every updating step.

3. ICA and Differentiation

In this section the relation between ICA and differentiation is considered.

When a function $f(x)$ is given, the Taylor expansion of its shift $f(x + d)$ is

$$f(x+d) = f(x) + df'(x) + \ldots \tag{9}$$

This equation shows that $f(x + d)$ is approximately equal to a linear sum of $f(x)$ and $f'(x)$. Thus, if ICA is applied to sampled data from $f(x)$ and $f(x + d)$, $f'(x)$ may be obtained under the assumption that the sampled data are statistically independent even if the shift distance is not known.

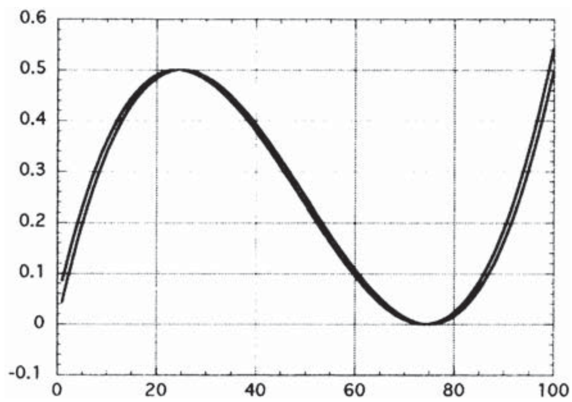

Fig. 1. Third-order polynomial and its shift.

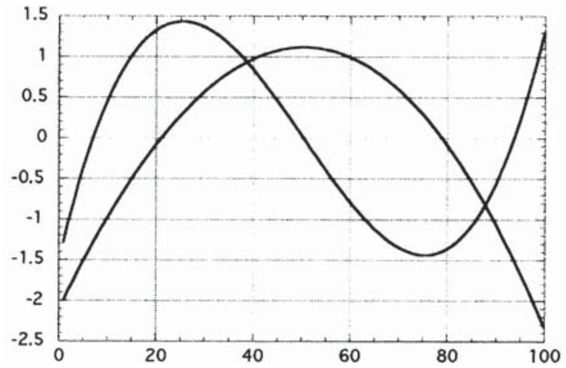

Fig. 2. ICA application result of polynomials in Fig. 1.

Here a simple example is given. Figure 1 shows a third-order polynomial and its small shift. When ICA is applied to sampled data of these functions, two independent components are obtained as shown in Fig. 2. One component of Fig. 2 is the original third-order polynomial and the other component is a second-order polynomial, which is considered to be a first derivative of the original function.

This example shows that differentiation of a function may be realized when applying ICA to the original function and its shift, if the sampled data can be supposed to be statistically independent. This process is nothing but an approximation of the differentiation process by a difference process.

4. Image Restoration

Image restoration is a method of recovering an original image from the blurred and noisy image [12]. Many restoration algorithms have been proposed, such as the Wiener filter, generalized inverse filter, constrained least-squares filter, and projection filter.

The blurring process is modeled as a convolution of an original image with a blurring function (a point spread function) and additive noise. Thus, the image restoration algorithm usually needs to know some information about the blurring function in advance. The above-mentioned methods all require this kind of information. Recently some although to recover an image without knowing the blurring function have been described [4]. However, they need to suppose some a priori knowledge, for example, symmetric shape of the blurring function.

The estimation of a blurring function is not easy except, for example, when the blurring is caused by a uniform camera movement or a long-exposure film influenced by atmospheric scattering. In these cases the physical models of the blurring process are known and it may be easy to estimate the blurring function analytically. However, in general, the blurring process cannot be known.

In this paper it is shown that deblurring can be realized without knowing the blurring process when ICA is used.

5. Image Restoration by ICA (1)

Shift-invariant blurring of an image is modeled as a convolution of original image $f(x, y)$ with a blurring function $h(x, y)$. The discrete convolution of $f(x, y)$ and $h(x, y)$ is represented as follows. This equation shows that discrete convolution is a kind of weighted sum of shifted images:

$$g(x,y) = \sum_{s=-M}^{M} \sum_{t=-M}^{M} h(s,t) f(x-s, y-t) \tag{10}$$

According to the discussion in Section 3, a shifted image is considered to be a weighted sum of the original image and its derivatives if the size M of the blurring function is sufficiently small compared to the size of the image. Thus, a blurred image which is a weighted sum of shifted images is also a weighted sum of the original image and its derivatives. Actually, $g(x, y)$ can be decomposed as follows:

$$\begin{aligned} g(x,y) &= \sum_{s=-M}^{M} \sum_{t=-M}^{M} h(s,t)[f - sf_x - tf_y + \frac{1}{2}s^2 f_{xx} + \ldots] \\ &= [\sum_{s=-M}^{M} \sum_{t=-M}^{M} h(s,t)]f - [\sum_{s=-M}^{M} \sum_{t=-M}^{M} sh(s,t)]f_x \\ &\quad -[\sum_{s=-M}^{M} \sum_{t=-M}^{M} th(s,t)]f_y + \ldots \\ &= a_1 f + a_2 f_x + a_3 f_y + \ldots \end{aligned} \tag{11}$$

When the blurring process changes, coefficients $a_1, a_2, \ldots$ change.

This discussion leads to the possibility that an original image (and its derivatives) can be restored when ICA is applied to several blurred images which undergo different blurring processes (Fig. 3).

Note that images are transformed into vectors by scanning them from left to right and from top to bottom when applying ICA to images.

5.1. Evaluation of independence between an original image and its derivatives

The proposed image restoration is possible only when an original image and its derivatives are independent,

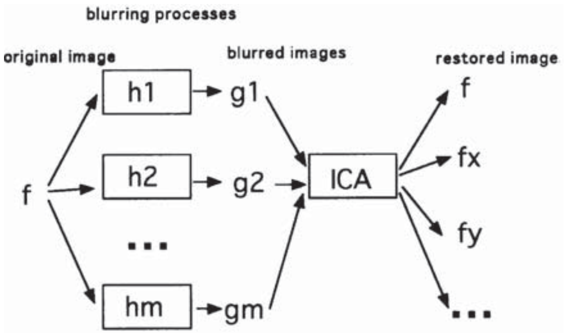

Fig. 3. ICA deblurring process for multiple blurred images.

at least approximately. In this section this point is considered experimentally using several natural images.

The following measure ρ is introduced in order to evaluate the probabilistic independence between two data, X_1 and X_2 [5]:

$$\rho(X_1, X_2) = \frac{I(X_1; X_2)}{H(X_1)} \tag{12}$$

where $I(X_1; X_2)$ is the mutual information between X_1 and X_2, and $H(X_1)$ is an entropy of X_1. The measure ρ is an asymmetric measure for X_1 and X_2 since $\rho(X_1, X_2) \neq \rho(X_2, X_1)$. The range of this measure is $0 \leq \rho \leq 1$. When X_1 and X_2 are independent, $\rho = 0$, and when they are dependent, $\rho = 1$.

To evaluate the independence between an image and its derivatives, first-order derivative images (f_xAf_y) and second-order derivative images (f_{xx}Af_{xy}Af_{yy}) of an image f are computed, and the independence measure ρ for paired images from these six images are computed. Since ρ is an asymmetric measure, both $\rho(X_1, X_2)$ and $\rho(X_2, X_1)$ are computed and their average is used as a measure. This computation is done for each image from five real images in Fig. 4. The mean and standard deviation of all calculated measure values are 0.064 and 0.025, respectively.

The independence measure is also calculated for paired images from five images in Fig. 4. The mean and standard deviation of all calculated measure values are 0.049 and 0.011, respectively, in this case.

This experiment shows that measure ρ of the independence between an image and its derivative image or between two different derivative images is rather small, and is only slightly greater than the measure of independence between the original images. Thus, the probabilistic independence between an image and its derivative image is considered to be high when comparing it with the independence value between the real images captured independently.

5.2. Image restoration by multiple blurred images

The image restoration experiment based on the proposed method is given in this section.

Figure 5(a) shows the original image, and Figs. 5(b) and 5(c) are the x- and y-derivative image, respectively. The size of the images is 180×180. The independence measures between an original image and its derivative image and between two different derivative images are calculated as in the previous section. The average and standard deviation of these values are 0.072 and 0.016, respectively. Ten different blurring processes (10 random 5×5 masks) are prepared, and are applied to the original images. Figure 5(d) is one of these 10 blurred images. The other blurred images are similar, and thus are not shown here.

ICA is applied to 10 blurred images. The obtained result is shown in Fig. 6 as 10 independent components. When applying ICA, the dimension of the whitening output is set $L = 10$ and a learning parameter $|\mu_i| = 0.0003$, and 5×10^5 learning steps are used. The learning parameter setting is a very important factor for fast and stable convergence in nonlinear optimization methods. In this experiment the parameter was set from the standpoint of stable conver-

Fig. 4. Images for independence evaluation.

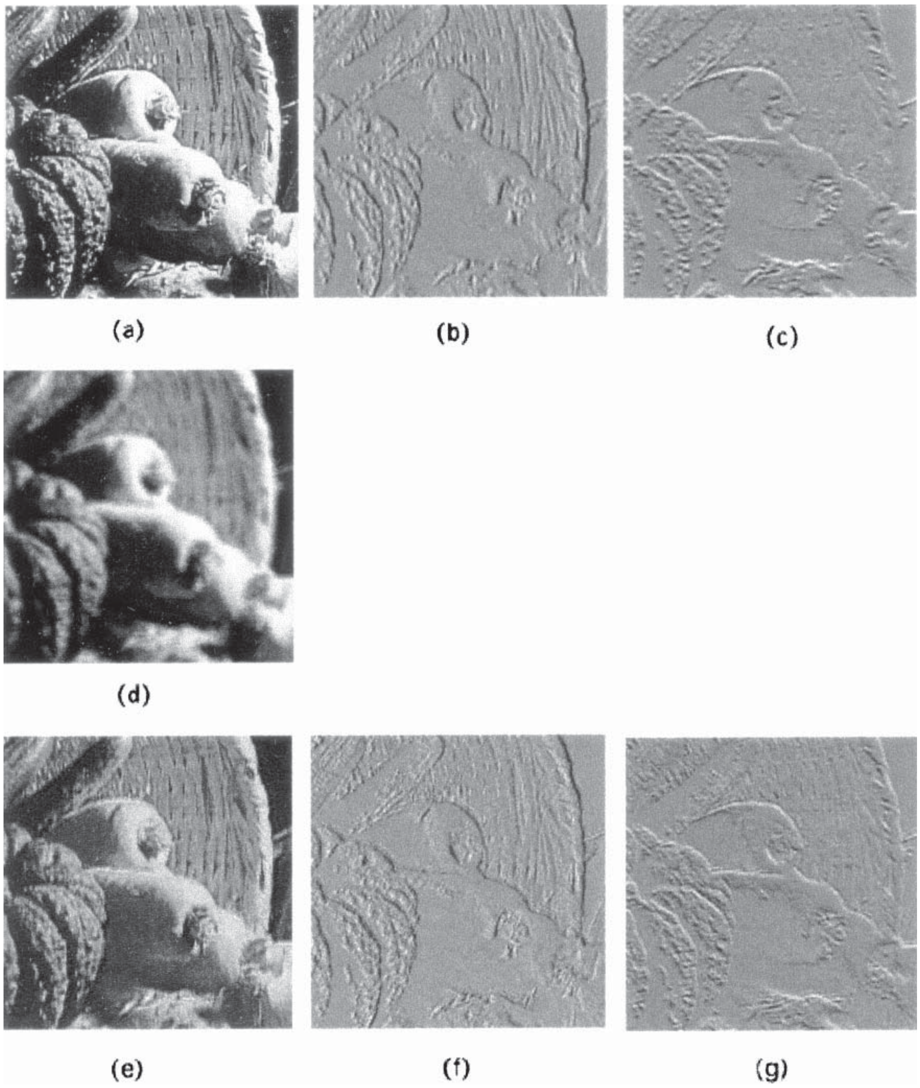

Fig. 5. Deblurring experiment of multiple blurred images. (a) Original image; (b) x-derivative image of the original image; (c) y-derivative image of the original image; (d) blurred image; (e) restored image; (f) restored image of x-derivative image; (g) restored image of y-derivative image.

gence rather than fast convergence, since the purpose of this experiment is to show the empirical possibility of the proposed method. For this purpose several preliminary experiments were performed to investigate the convergence behavior of the 4th cumulant of the independent components, and the learning parameter was set to make the convergence stable and smooth.

The sign of the learning parameter μ_i must be set positive when minimizing the 4th cumulant of the independent component, or set negative when maximizing it. Since the 4th cumulant of an original image is usually negative and that of a derivative image is usually positive (this is true of the images used in the experiment), the learning parameter for the first independent component (the restored image) was set positive and those of the remaining components negative.

The obtained independent components are shown in panels (e), (f), and (g) of Fig. 5. [Here (f) and (g) are "negated" images of the components obtained by reversing them.] They are components which have high correlation with the original image, x-derivative image, and y-derivative image, respectively. The correlation coefficient between the original image (a) and the restored image (e) is 0.949, that between (b) and (f) is 0.745, and that between

(c) and (g) is 0.790. On the other hand, the correlation coefficient between the blurred image (d) and the original image is 0.8979. This result shows that ICA can give a good restoration without knowing the blurring process, and in addition can give images similar to derivative images as additional independent components.

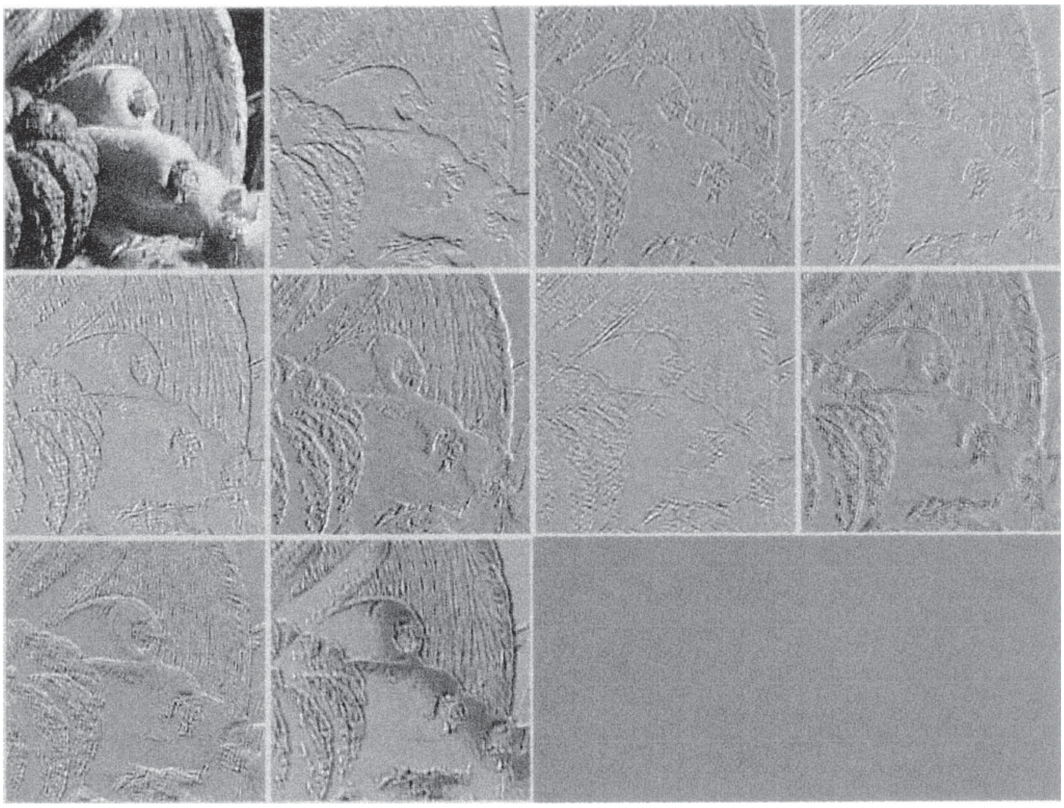

Fig. 6. Independent components given by experiment 1.

6. Image Restoration by ICA (2)

In ordinary image restoration tasks, it is very rare that several blurred images which are observed through different blurring processes are given. Thus, a restoration algorithm applicable to a single blurred image is needed. This can become possible if shifted blurred images are used.

As shown in the previous section, a blurred image can be represented as a weighted sum of an original image and its derivative images:

$$g(x, y) = a_1 f + a_2 f_x + a_3 f_y + \dots \tag{13}$$

A shifted blurred image is an image which is obtained by shifting a blurred original image in the x- or y-direction. The Taylor expansion of a shifted blurred image indicates, as in the previous section, that this image is also represented by a weighted sum of an original image and its derivative images, since

$$\begin{aligned} g(x+d, y) &= a_1 f(x+d, y) + a_2 f_x(x+d, y) \\ &\quad + a_3 f_y(x+d, y) + \dots \\ &= a_1[f + df_x + \frac{1}{2}d^2 f_{xx} + \dots] \\ &\quad + a_2[f_x + df_{xx} + \dots] \\ &\quad + a_3[f_y + df_{yx} + \dots] + \dots \\ &= a_1 f + [a_1 d + a_2] f_x + a_3 f_y \\ &\quad + [\frac{1}{2}a_1 d^2 + a_2 d + \dots] f_{xx} + \dots \\ &= b_1 f + b_2 f_x + b_3 f_y + b_4 f_{xx} + \dots \end{aligned} \tag{14}$$

When the shift direction and distance change, coefficients $b_1, b_2, \dots$ of the equation above change.

This discussion leads to the possibility that image restoration can be achieved by applying ICA to several different shifted blurred images (Fig. 7). When the shift

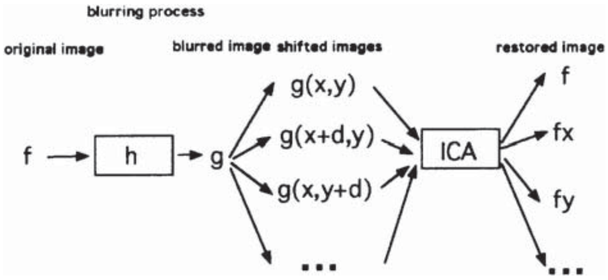

Fig. 7. ICA deblurring process for shifted blurred images.

directions are constrained in the $m \times m$ local area, this restoration is a kind of $m \times m$ local regression.

6.1. Image restoration by a single blurred image

Here an image restoration experiment using a shifted blurred image is given.

Figure 8(a) is an original (unblurred) 180 × 180 image, and Fig. 8(b) is a blurred image given by applying a Gaussian convolution mask (σ= 1) to the original image. The independence measures ρ are computed for the original image and its derivatives. The mean value and standard deviation are 0.086 and 0.021, respectively.

The set of shifted blurred images was generated by shifting the blurred image in 49 directions (7 × 7 local area). The dimension of whitening output was set as $L = 40$ and the learning parameter was $|\mu_i| = 0.0003$; 5×10^5 learning steps were used. The sign of the learning parameter was set positive when obtaining the first independent component (unblurred image), and set negative otherwise.

The obtained restored image is shown in Fig. 8(c). Though the result is not so good as in the previous experiment (image restoration by multiple blurred images), it shows that the proposed method can achieve a good restoration. The correlation between the original image Fig. 8(a) and the restored image Fig. 8(c) is 0.885, and the correlation between Fig. 8(a) and the blurred image Fig. 8(b) is 0.813.

The image restoration by shifted blurred images is considered to be $m \times m$ local regression. In order to know the regression coefficients, the pseudoinverse $A^+ = \sqrt{n}\,W\hat{D}^{-1}\hat{U}^T$ of mixing matrix A in Eq. (5) was computed. Since we have

$$S = A^+ X \tag{15}$$

each row vector of A^+ corresponds to the regression coefficients, that is, the image restoration mask.

The 7 × 7 restoration mask of the experiment above is shown in Fig. 9(a). The same mask was drawn by Mathematica using the ContourPlot routine [Fig. 9(b)] in order to enhance the characteristics of the mask. It is well known that the Mexican-hat restoration mask is effective for Gaussian blurring processes. The figure shows that the obtained mask is similar to the Mexican-hat restoration mask.

6.2. Evaluation of restoration accuracy

Here we evaluate the relation between the size of blurring and the restoration accuracy.

The original image (180 × 180) shown in Fig. 5(a) was prepared, and several Gaussian masks with various standard deviations $0.9 \le \sigma \le 1.6$ were applied to it. The

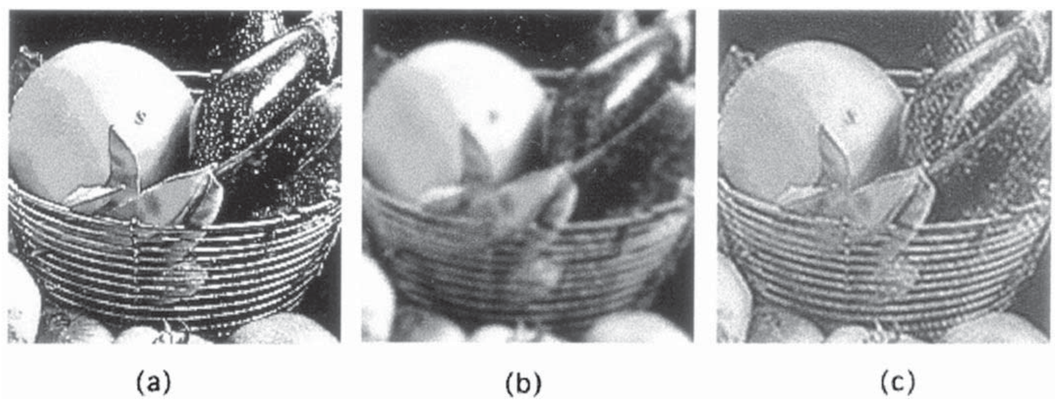

Fig. 8. Deblurring experiment of shifted blurred images. (a) Original image; (b) blurred image; (c) restored image.

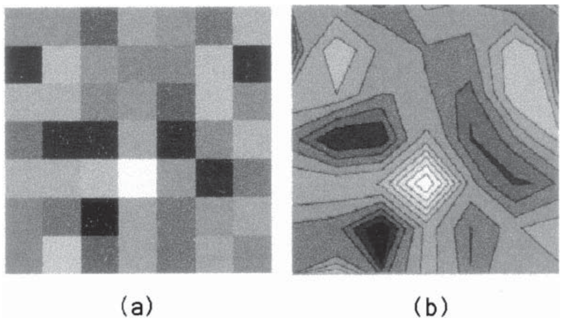

Fig. 9. Restoration mask. (a) Image representation; (b) contour plot.

proposed restoration method was applied to the obtained blurred images. This experiment was performed for three shifting patterns: (a) 9 shift directions (3×3 local regression), (b) 25 shift directions (5×5 local regression), and (c) 49 shift directions (7×7 local regression). The other learning parameters were the same as in the previous experiments.

The result is shown in Fig. 10. The horizontal axis of Fig. 10 corresponds to the blurring size (σ), and the vertical axis shows the correlation between the restored image and the original image; "3," "5," and "7" on the graph correspond to the three shift patterns explained above. The correlation between the original image and the blurred image is also plotted. The legend in this case is "blurred image."

This result shows that:

1. Restoration accuracy degrades in proportion to the size of blurring.
2. When the number of shift directions is small (Case "3"), restoration quality is good in the case of weak blurring. However, when the blurring becomes stronger, the restoration quality degrades, and meaningful restoration is impossible when $\sigma > 1.4$.
3. When the number of shift directions is large (Cases "5" and "7"), the restoration quality is almost the same.

If we consider 2σ as the blurring size for the Gaussian blurring mask, it is 5×5 when $\sigma = 1.0$ and 7×7 when $\sigma = 1.5$. In Fig. 10, shift patterns (b) and (c) achieve comparable restoration quality, although the local regression area of (b) is smaller than the estimated size of blurring. This indicates that the restoration quality is not degraded very much if the local regression area is smaller than the blurring size. In other words, the restoration quality does not improve if a larger local regression area is used. This limits the applicability of the proposed method.

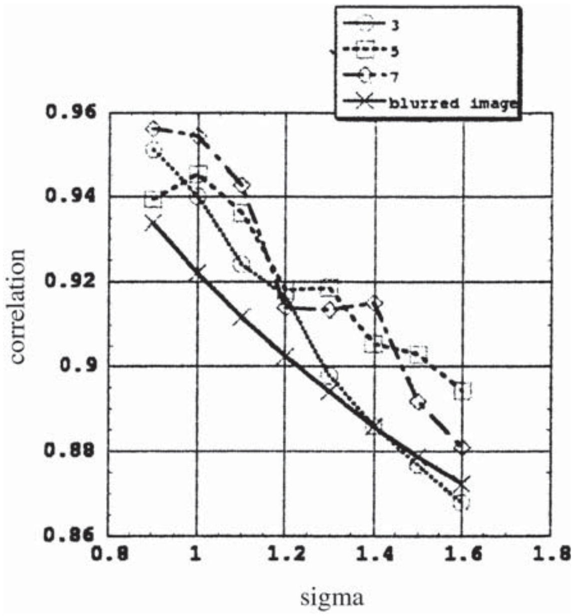

Fig. 10. Evaluation of the restoration performance.

7. Conclusions

A new image restoration method using ICA has been proposed. This method is based on the conjecture that in some conditions, the derivatives of a function can be estimated by applying ICA to its several shifted functions.

In the history of pattern recognition, it is well known that robust pattern recognition can be realized if derivative images are used with the original image. For example, this is the case in the character recognition field [6]. In this case, derivative images are used as a kind of "features" of the original image, and it is unnecessary to know the blurring process itself in advance. On the other hand, image restoration needs to know both the derivative images and their content in a blurred image. Thus, if the blurring process is not known, this kind of technique is not applicable. The proposed method overcomes this difficulty by using ICA.

In this paper an ICA algorithm proposed by Oja has been used. The learning equation of this algorithm is equivalent to that of Sejnowski's maximum entropy algorithm. This means that the restored image is obtained from the nonlinear transformation (e.g., by a sigmoidal function) having maximum entropy. In the restoration process, the constraint that the restored image should be a weighted sum of principal components of shifted images is used.

Many methods have been proposed for image restoration, and one such method is the maximum entropy method (MEM) [12]. MEM estimates the original image by maximizing the entropy of the restored image. In the restoration process of the algorithm, the distance between the observed image and the restored image blurred by a known blurring process is used as a constraint. MEM and the proposed method are very similar, since the estimation in both methods is done by maximizing some entropy. The biggest difference between them is that MEM is applicable only when the blurring process is known and, on the other hand, the proposed method is available even when it is unknown.

MEM has some relation to MAP estimation [14]; thus, research in this direction is also interesting for the proposed method.

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