Strong:

$$\begin{cases} \frac{dc}{dt} - \nabla\cdot\left(D\nabla c\right) - \alpha c(1-c) = 0 \\ D\nabla c\cdot \bar{n} = 0 \\ c(t=0) = c_0 \end{cases}$$

Weak:

Given $V = H^1(\Omega)$ (since $\Gamma_D = \emptyset$)

Find $c \in V$, s.t $\forall v \in V$

$$\begin{cases} \int_\Omega \frac{\partial c}{\partial t} v + a(c,v) - \overset{r_L}{\alpha c v} + \overset{r_N}{\alpha c^2 v}\, d\bar{x} = \cancel{F(v)}^{\,0 \text{ for } F = \phi_N = 0} \qquad \forall v \in V \\ c(t=0) = c_0 \end{cases}$$

with: $a(c,v) = D\cdot\nabla c\cdot\nabla v$

Def: $\sqrt{D}$ s.t. $\sqrt{D}\sqrt{D} = D$???

$\underbrace{\text{Why use}}_{\text{papers}}$ $a(c,v) = \sqrt{D}\cdot\nabla c\sqrt{D}\nabla v$???

Semi-Discrete:

$$R(c_h, v_h) = \int_\Omega \frac{\partial c_h}{\partial t} v_h\, d\bar{x} + \int_\Omega a(c_h, v_h)\, d\bar{x} - \int_\Omega \overset{r_L}{\alpha c_h v_h}\, d\bar{x} + \int_\Omega \overset{r_N}{\alpha c_h^2 v_h}\, d\bar{x} - \overset{F(v)}{0}$$

$$= \int_\Omega \frac{\partial c_h}{\partial t} v_h\, d\bar{x} + \int_\Omega b(c_h, v_h)\, d\bar{x} = 0$$

Full-Discr.:

- Partition $(0,T]$ into intervals (t_n, t_{n+1}) for $n = 0,1,\dots,N$

  $t_0 = 0$, $t_{N+1} = T$, $t_{n+1} - t_n = \Delta t\ \forall n$

- Denote $c_h^n \sim c_h(t_n)$

$$R^{n+1}(c_h^{n+1}, v_h) = \int_\Omega \frac{c_h^{n+1} - c_h^n}{\Delta t} v_h\, dx + \int_\Omega \underbrace{b(c_h^{n+1}, v_h)}_{\leftarrow\ \text{NON-LINEAR}}\, dx = 0$$

- Compute derivative

$$d(c_h, d_h, v_h) = R(c+d, v) - R(c, v) =$$

$$= \int_\Omega \frac{d_h}{\Delta t} v\, d\bar{x} + \int_\Omega a(c_h + d, v_h)\, d\bar{x} - \int_\Omega \overset{r_L}{\alpha(c_h + d_h) v_h}\, d\bar{x} + \int_\Omega \overset{r_N}{\alpha(c_h + d_h)^2 v_h}\, dx$$

$$- \int_\Omega a(c_h, v_h)\, d\bar{x} + \int_\Omega \alpha c_h v_h\, d\bar{x} - \int_\Omega \alpha c_h^2 v_h\, d\bar{x} =$$

$$= \int_\Omega \frac{d_h}{\Delta t} v_h + \int_\Omega a(d_h, v_h)\, d\bar{x} - \int_\Omega \alpha d_h v_h + \int_\Omega \alpha\left(\cancel{c_h^2} + 2 d_h c_h + d_h^2\right) v_h\, d\bar{x} =$$

$$= \int_\Omega \frac{d_h}{\Delta t} v_h + \int_\Omega a(d_h, v_h)\, d\bar{x} - \int_\Omega \alpha d_h v_h + \int_\Omega 2\alpha d_h c_h v_h\, d\bar{x} + \cancel{o(d_h^2)}$$

IGNORE

- Steps:

1) Compute $\delta^{(k)}$ by solving $\underbrace{d(c_h^{(k)}, \delta_h^{(k)}, v_h) = -R(c_h^{(k)}, v_h)}_{\text{LINEAR}}$
2) Update $c^{(k+1)} = c^{(k)} + \delta^{(k)}$

LHS: $\int_\Omega \frac{d_h}{\Delta t} v_h + \int_\Omega a(d_h, v_h)\, d\bar{x} - \int_\Omega \alpha d_h v_h + \int_\Omega 2\alpha d_h c_h^{n+1(k)} v_h\, d\bar{x}$ ← use solution_loc

RHS: $\int_\Omega \frac{c_h^{n+1(k)} - c_h^{n(k)}}{\Delta t} v_h\, dx + \int_\Omega b(c_h^{n+1(k)}, v_h)\, dx = 0$ ← NON-LINEAR, use solution_loc

Fem-Elements:

$\delta_h = \varphi_j$; $v_h = \varphi_i$; $c_h = c_h^{(k)}$ (solution_loc)

Time-Discretiz. ⇒ θ-METHOD