

# Finance Python

May 31, 2021

```
[1]: import numpy as np
      from pandas_datareader import data as wb
      import matplotlib.pyplot as plt
```

```
/Users/alessandrosollazzo/opt/anaconda3/lib/python3.7/site-
packages/pandas_datareader/compat/__init__.py:7: FutureWarning:
pandas.util.testing is deprecated. Use the functions in the public API at
pandas.testing instead.
```

```
    from pandas.util.testing import assert_frame_equal
```

```
[2]: import panda as pd
```

```

↳
↳-----
↳
↳ModuleNotFoundError                                Traceback (most recent call↳
↳last)
↳
↳    <ipython-input-2-f542dbfa5144> in <module>
↳    ----> 1 import panda as pd
↳
↳    ~/opt/anaconda3/lib/python3.7/site-packages/panda/__init__.py in <module>
↳    ----> 1 from request import PandaRequest
↳          2 from models import Video, Cloud, Encoding, Profile, Notifications,↳
↳↳PandaDict
↳          3 from models import GroupRetriever, SingleRetriever
↳          4 from models import PandaError
↳          5 from upload_session import UploadSession
↳
↳ModuleNotFoundError: No module named 'request'
```

```
[3]: import pandas as pd
```

```
[1]: import pandas_datareader as web
```

```
/Users/alessandrosollazzo/opt/anaconda3/lib/python3.7/site-
packages/pandas_datareader/compat/__init__.py:7: FutureWarning:
pandas.util.testing is deprecated. Use the functions in the public API at
pandas.testing instead.
```

```
from pandas.util.testing import assert_frame_equal
```

```
[6]: PG=web.DataReader ('PG', data_source='yahoo', start='1995-1-1')
```

```
[7]: PG.head()
```

```
[7]:
```

	High	Low	Open	Close	Volume	Adj Close
Date						
1995-01-03	15.62500	15.43750	15.46875	15.59375	3318400.0	6.320252
1995-01-04	15.65625	15.31250	15.53125	15.46875	2218800.0	6.269589
1995-01-05	15.43750	15.21875	15.37500	15.25000	2319600.0	6.180927
1995-01-06	15.40625	15.15625	15.15625	15.28125	3438000.0	6.193593
1995-01-09	15.40625	15.18750	15.34375	15.21875	1795200.0	6.168259

```
[7]: PG.tail()
```

```
[7]:
```

	High	Low	Open	Close	Volume	\
Date						
2020-06-15	117.279999	113.760002	114.550003	116.690002	8786000.0	
2020-06-16	118.970001	116.930000	118.529999	118.129997	8165200.0	
2020-06-17	119.110001	117.440002	118.389999	117.930000	6320800.0	
2020-06-18	119.959999	117.370003	117.459999	119.279999	6274400.0	
2020-06-19	121.820000	118.830002	120.489998	118.919998	17506200.0	

```
Adj Close
```

Date	Adj Close
2020-06-15	116.690002
2020-06-16	118.129997
2020-06-17	117.930000
2020-06-18	119.279999
2020-06-19	118.919998

```
[11]: PG['simple_return']=(PG['Adj Close']/PG['Adj Close'].shift(1))-1
```

```
[13]: print PG['simple_return']
```

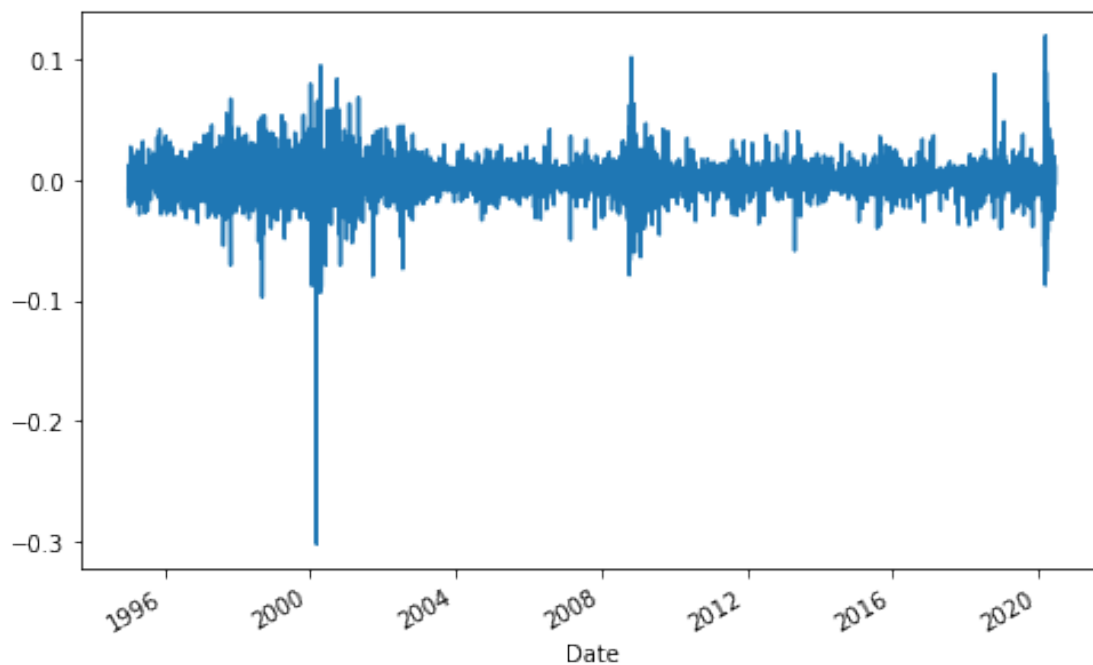
```
File "<ipython-input-13-488f3f1a7a51>", line 1
print PG['simple_return']
^
```

```
SyntaxError: Missing parentheses in call to 'print'. Did you mean
↳ print(PG['simple_return'])?
```

```
[14]: print(PG['simple_return'])
```

```
Date
1995-01-03      NaN
1995-01-04   -0.008016
1995-01-05   -0.014141
1995-01-06    0.002049
1995-01-09   -0.004090
...
2020-06-15    0.009254
2020-06-16    0.012340
2020-06-17   -0.001693
2020-06-18    0.011447
2020-06-19   -0.003018
Name: simple_return, Length: 6412, dtype: float64
```

```
[15]: PG['simple_return'].plot(figsize=(8, 5))
plt.show()
```



```
[16]: avg_returns_d=PG['simple_return'].mean()
avg_returns_d
```

```
[16]: 0.0005187078858490102
```

```
[17]: avg_returns_a=PG['simple_return'].mean()*250
      avg_returns_a
```

```
[17]: 0.12967697146225254
```

```
[18]: print str(round(avg_returns_a,5)*100)+'%
```

```
File "<ipython-input-18-90b7bbbf47cb>", line 1
print str(round(avg_returns_a,5)*100)+'%'
      ^
SyntaxError: invalid syntax
```

```
[20]: print str(round(avg_returns_a, 5)*100)+ '%'
```

```
File "<ipython-input-20-274dad27c35b>", line 1
print str(round(avg_returns_a, 5)*100)+ '%'
      ^
SyntaxError: invalid syntax
```

```
[21]: round ((avg_returns_a,2)*100)+ '%'
```

```

      ^
↳ -----

TypeError                                Traceback (most recent call↳
↳ last)

    <ipython-input-21-80bdac8ab425> in <module>
----> 1 round ((avg_returns_a,2)*100)+ '%'

TypeError: type tuple doesn't define __round__ method
```

```
[22]: round (avg_returns_a,2)+ '%'
```

```

      ^
↳ -----
```

```
TypeError                                Traceback (most recent call_
↳last)
```

```
<ipython-input-22-f2e2e74bb8fc> in <module>
----> 1 round (avg_returns_a,2)+ '%'
```

```
TypeError: unsupported operand type(s) for +: 'float' and 'str'
```

```
[24]: str(round (avg_returns_a,2)*100)+ '%'
```

```
[24]: '13.0%'
```

```
[29]: str(round (avg_returns_a,4)*100)+ '%'
```

```
[29]: '12.97%'
```

```
[30]: PG.head()
```

```
[30]:
```

	High	Low	Open	Close	Volume	Adj Close \
Date						
1995-01-03	15.62500	15.43750	15.46875	15.59375	3318400.0	8.313478
1995-01-04	15.65625	15.31250	15.53125	15.46875	2218800.0	8.246841
1995-01-05	15.43750	15.21875	15.37500	15.25000	2319600.0	8.130219
1995-01-06	15.40625	15.15625	15.15625	15.28125	3438000.0	8.146880
1995-01-09	15.40625	15.18750	15.34375	15.21875	1795200.0	8.113561

```
simple return  simple_return
```

Date	simple return	simple_return
1995-01-03	NaN	NaN
1995-01-04	-0.008016	-0.008016
1995-01-05	-0.014141	-0.014141
1995-01-06	0.002049	0.002049
1995-01-09	-0.004090	-0.004090

```
[31]: PG['log_return']=np.log(PG['Adj Close']/PG['Adj Close'].shift(1))
```

```
[32]: print PG['log_return']
```

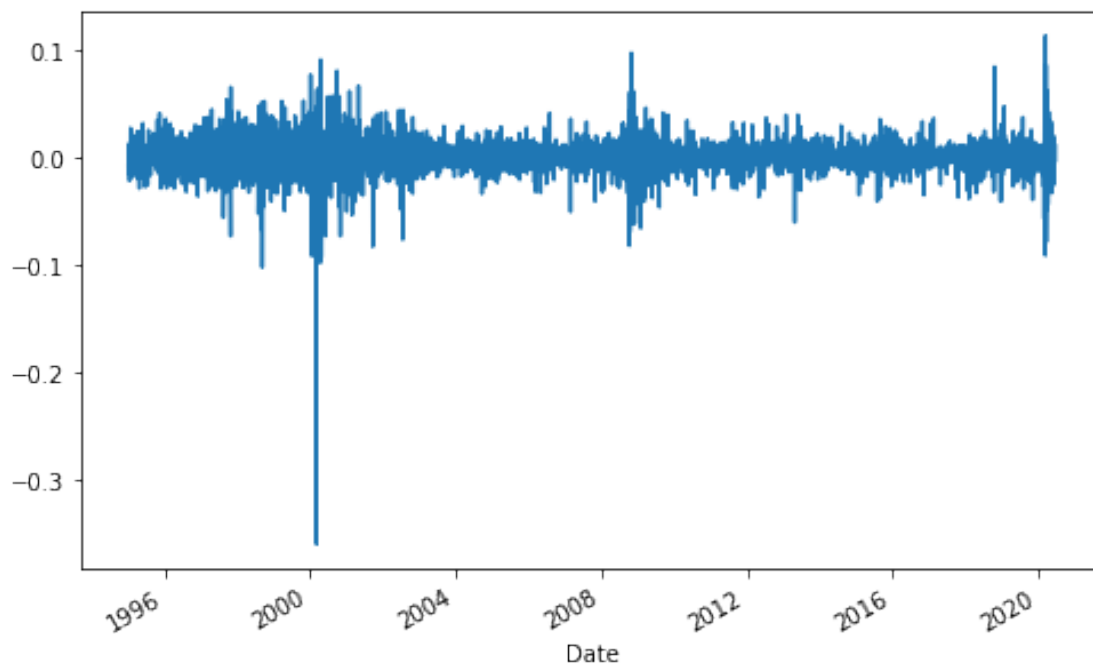
```
File "<ipython-input-32-21a66722a3eb>", line 1
print PG['log_return']
^
```

```
SyntaxError: Missing parentheses in call to 'print'. Did you mean_
↳print(PG['log_return'])?
```

```
[33]: print(PG['log_return'])
```

```
Date
1995-01-03      NaN
1995-01-04   -0.008048
1995-01-05   -0.014242
1995-01-06    0.002047
1995-01-09   -0.004098
...
2020-06-15    0.009212
2020-06-16    0.012265
2020-06-17   -0.001694
2020-06-18    0.011382
2020-06-19   -0.003023
Name: log_return, Length: 6412, dtype: float64
```

```
[36]: PG['log_return'].plot(figsize=(8, 5))
plt.show()
```



```
[37]: log_return_d=PG['log_return'].mean()
log_return_d
```

```
[37]: 0.00041500122804857776
```

```
[38]: log_return_a=PG['log_return'].mean()*250
log_return_a
```

```
[38]: 0.10375030701214444
```

```
[39]: str(round(log_return_a, 2)*100)+'%'
```

```
[39]: '10.0%'
```

```
[40]: PG.head()
```

```
[40]:
```

	High	Low	Open	Close	Volume	Adj Close \
Date						
1995-01-03	15.62500	15.43750	15.46875	15.59375	3318400.0	8.313478
1995-01-04	15.65625	15.31250	15.53125	15.46875	2218800.0	8.246841
1995-01-05	15.43750	15.21875	15.37500	15.25000	2319600.0	8.130219
1995-01-06	15.40625	15.15625	15.15625	15.28125	3438000.0	8.146880
1995-01-09	15.40625	15.18750	15.34375	15.21875	1795200.0	8.113561

	simple return	simple_return	log_return
Date			
1995-01-03	NaN	NaN	NaN
1995-01-04	-0.008016	-0.008016	-0.008048
1995-01-05	-0.014141	-0.014141	-0.014242
1995-01-06	0.002049	0.002049	0.002047
1995-01-09	-0.004090	-0.004090	-0.004098

```
[41]: tickers=['PG', 'MSFT', 'F', 'GE']
mydata = pd.DataFrame()
for t in tickers:
    mydata[t] = web.DataReader(t, data_source='yahoo', start='1995-1-1')['Adj_
↪Close']
```

```
[42]: mydata.info()
```

```
<class 'pandas.core.frame.DataFrame'>
DatetimeIndex: 6412 entries, 1995-01-03 to 2020-06-19
Data columns (total 4 columns):
#   Column  Non-Null Count  Dtype
---  -
0   PG      6412 non-null     float64
1   MSFT    6412 non-null     float64
2   F       6412 non-null     float64
3   GE      6412 non-null     float64
dtypes: float64(4)
memory usage: 250.5 KB
```

```
[43]: mydata.head()
```

```
[43]:
```

	PG	MSFT	F	GE
Date				
1995-01-03	8.313478	2.406834	3.262354	2.729230
1995-01-04	8.246841	2.424330	3.350130	2.729230
1995-01-05	8.130219	2.384342	3.320871	2.735919
1995-01-06	8.146880	2.424330	3.320871	2.722540
1995-01-09	8.113561	2.409335	3.379390	2.695783

```
[44]: mydata.tail()
```

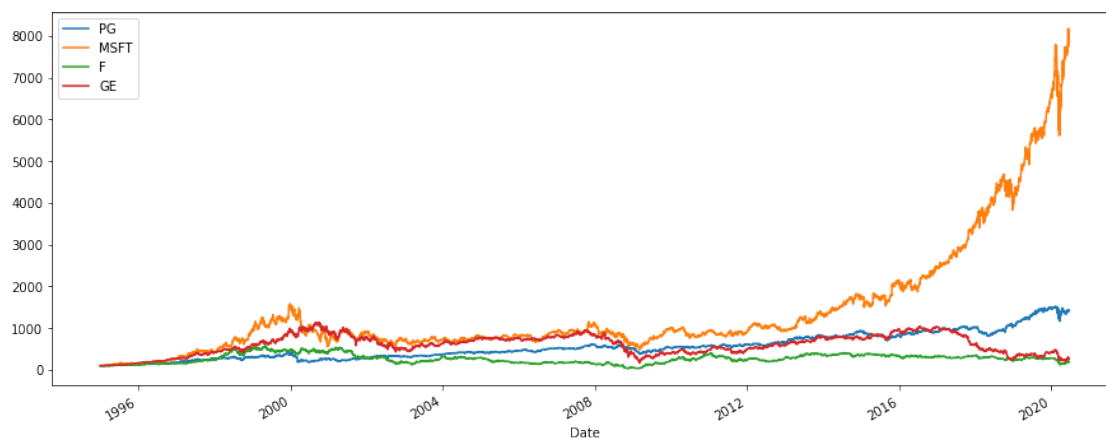
```
[44]:
```

	PG	MSFT	F	GE
Date				
2020-06-15	116.690002	188.940002	6.50	7.24
2020-06-16	118.129997	193.570007	6.55	7.47
2020-06-17	117.930000	194.240005	6.33	7.24
2020-06-18	119.279999	196.320007	6.33	7.28
2020-06-19	118.919998	195.149994	6.23	7.15

```
[45]: mydata.iloc[0]
```

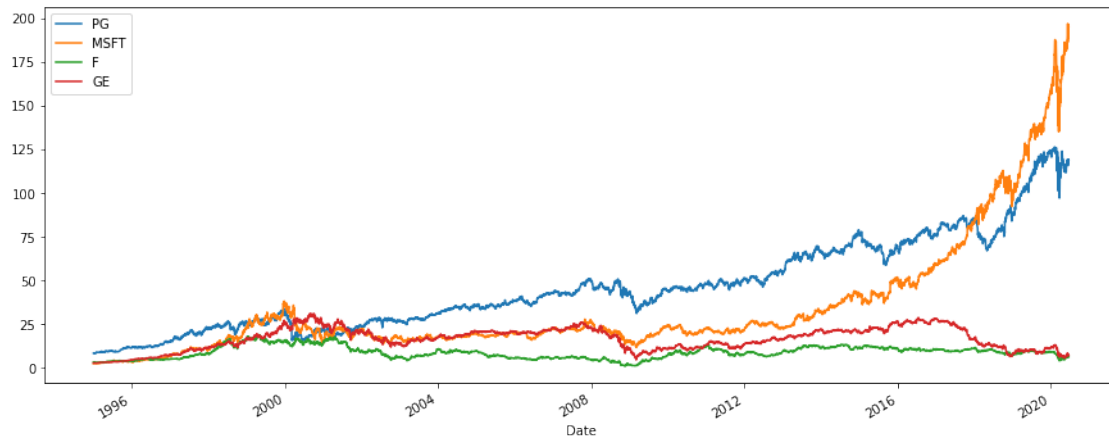
```
[45]: PG      8.313478
MSFT    2.406834
F        3.262354
GE       2.729230
Name: 1995-01-03 00:00:00, dtype: float64
```

```
[46]: (mydata/mydata.iloc[0]*100).plot(figsize=(15, 6));
plt.show()
```





```
[47]: mydata.plot(figsize=(15,6))  
plt.show()
```



```
[48]: mydata.loc[1995-01-03]
```

```
File "<ipython-input-48-a57165a38487>", line 1  
mydata.loc[1995-01-03]  
           ^
```

```
SyntaxError: invalid token
```

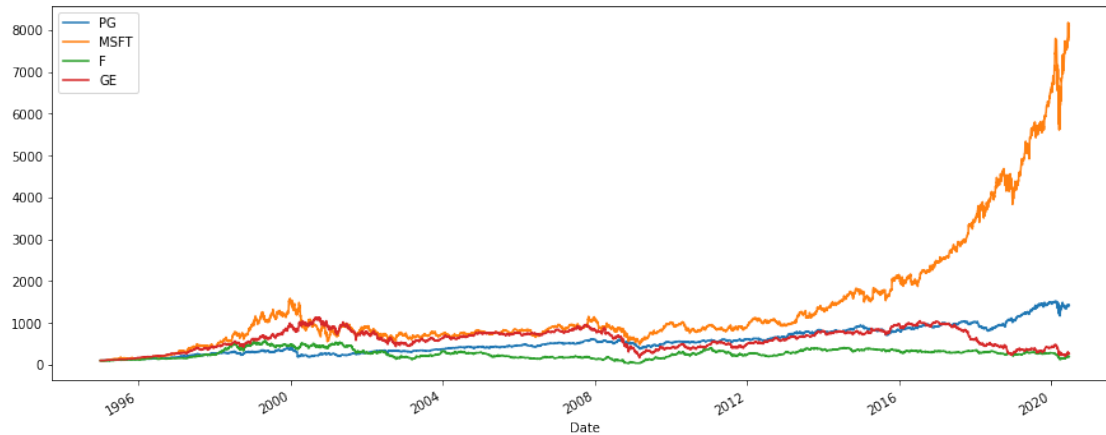
```
[49]: mydata.loc['1995-01-03']
```

```
[49]: PG      8.313478  
      MSFT    2.406834  
      F       3.262354  
      GE      2.729230  
      Name: 1995-01-03 00:00:00, dtype: float64
```

```
[50]: mydata.iloc[0]
```

```
[50]: PG      8.313478  
      MSFT    2.406834  
      F       3.262354  
      GE      2.729230  
      Name: 1995-01-03 00:00:00, dtype: float64
```

```
[51]: (mydata/mydata.iloc[0]*100).plot(figsize=(15, 6));  
plt.show()
```



```
[52]: returns=(mydata/mydata.shift(1))-1
```

```
[53]: returns.head
```

```
[53]: <bound method NDFrame.head of
```

	PG	MSFT	F	GE
Date				
1995-01-03	NaN	NaN	NaN	NaN
1995-01-04	-0.008016	0.007269	0.026906	0.000000
1995-01-05	-0.014141	-0.016495	-0.008734	0.002451
1995-01-06	0.002049	0.016771	0.000000	-0.004890
1995-01-09	-0.004090	-0.006185	0.017622	-0.009828
...	...	...	...	...
2020-06-15	0.009254	0.006392	0.006192	-0.001379
2020-06-16	0.012340	0.024505	0.007692	0.031768
2020-06-17	-0.001693	0.003461	-0.033588	-0.030790
2020-06-18	0.011447	0.010708	0.000000	0.005525
2020-06-19	-0.003018	-0.005960	-0.015798	-0.017857

```
[6412 rows x 4 columns]>
```

```
[54]: returns.head()
```

```
[54]:
```

	PG	MSFT	F	GE
Date				
1995-01-03	NaN	NaN	NaN	NaN
1995-01-04	-0.008016	0.007269	0.026906	0.000000
1995-01-05	-0.014141	-0.016495	-0.008734	0.002451
1995-01-06	0.002049	0.016771	0.000000	-0.004890
1995-01-09	-0.004090	-0.006185	0.017622	-0.009828

```
[57]: weights = np.array([0.25, 0.25, 0.25, 0.25])
```

```
[58]: np.dot(returns, weights)
```

```
[58]: array([          nan,  0.00653988, -0.00922976, ..., -0.01565235,  
          0.0069202 , -0.01065819])
```

```
[59]: annual_returns = returns.mean()*250
```

```
[60]: annual_returns
```

```
[60]: PG      0.129677  
      MSFT   0.221502  
      F      0.105312  
      GE     0.088126  
      dtype: float64
```

```
[61]: np.dot(returns, weights)
```

```
[61]: array([          nan,  0.00653988, -0.00922976, ..., -0.01565235,  
          0.0069202 , -0.01065819])
```

```
[63]: np.dot(annual_returns, weights)
```

```
[63]: 0.1361542098903807
```

```
[64]: pfolio_1=str(round(np.dot(annual_returns, weights),5)*100) + '%'
```

```
[66]: print (pfolio_1)
```

```
13.614999999999998%
```

```
[67]: weight_2=np.array([0.4,0.4,0.15,0.05])
```

```
[69]: pfolio_2= str(round(np.dot(annual_returns, weight_2), 5)*100)+'%'
```

```
[71]: print (pfolio_1)  
      print (pfolio_2)
```

```
13.614999999999998%  
16.067%
```

```
[74]: tickers = ['^GSPC','^IXIC','^GDAXI']  
      ind_data = pd.DataFrame()  
      for t in tickers:  
          ind_data[t] = web.DataReader(t, data_source = 'yahoo',  
          ↪start='1997-1-1')['Adj Close']
```

```
[75]: ind_data.head()
```

```
[75]:
```

	$\hat{GSPC}$	$\hat{IXIC}$	$\hat{GDAXI}$
Date			
1997-01-02	737.010010	1280.699951	2820.810059
1997-01-03	748.030029	1310.680054	2863.260010
1997-01-06	747.650024	1316.400024	2890.199951
1997-01-07	753.229980	1327.729980	2876.340088
1997-01-08	748.409973	1320.349976	2904.080078

```
[76]: ind_data.tail()
```

```
[76]:
```

	$\hat{GSPC}$	$\hat{IXIC}$	$\hat{GDAXI}$
Date			
2020-06-15	3066.590088	9726.019531	11911.349609
2020-06-16	3124.739990	9895.870117	12315.660156
2020-06-17	3113.489990	9910.530273	12382.139648
2020-06-18	3115.340088	9943.049805	12281.530273
2020-06-19	3097.739990	9946.120117	12330.759766

```
[77]: (ind_data/ind_data.iloc[0]*100).plot(figsize = (15, 6));
plt.show()
```



```
[78]: ind_returns=(ind_data/ind_data.shift(1))-1
ind_returns.tail()
```

```
[78]:
```

	$\hat{GSPC}$	$\hat{IXIC}$	$\hat{GDAXI}$
Date			
2020-06-15	0.008312	0.014309	-0.003174
2020-06-16	0.018962	0.017464	0.033943
2020-06-17	-0.003600	0.001481	0.005398
2020-06-18	0.000594	0.003281	-0.008125
2020-06-19	-0.005649	0.000309	0.004008

```
[79]: annual_ind_returns=ind_returns.mean()*250
```

```
[80]: annual_ind_returns
```

```
[80]: ^GSPC      0.080320  
      ^IXIC      0.118697  
      ^GDAXI      0.075806  
      dtype: float64
```

```
[81]: tickers=['PG','^GSPC','^DJI']  
      data_2=pd.DataFrame()  
      for t in tickers:  
          data_2[t]=web.DataReader(t,data_source='yahoo', start='2007-1-1') ['Adj_↵  
↵Close']
```

```
[82]: data_2.tail()
```

```
[82]:
```

	PG	^GSPC	^DJI
Date			
2020-06-15	116.690002	3066.590088	25763.160156
2020-06-16	118.129997	3124.739990	26289.980469
2020-06-17	117.930000	3113.489990	26119.609375
2020-06-18	119.279999	3115.340088	26080.099609
2020-06-19	118.919998	3097.739990	25871.460938

```
[83]: (data_2/data_2.iloc[0]*100).plot(figsize=(15,6));
```



```
[84]: data_2.tail()
```

```
[84]:
```

	PG	^GSPC	^DJI
Date			
2020-06-15	116.690002	3066.590088	25763.160156

2020-06-16	118.129997	3124.739990	26289.980469
2020-06-17	117.930000	3113.489990	26119.609375
2020-06-18	119.279999	3115.340088	26080.099609
2020-06-19	118.919998	3097.739990	25871.460938

```
[20]: tickers=['PG', 'BEI.DE']
      prova = pd.DataFrame()
      for t in tickers:
          prova[t] = web.DataReader(t, data_source='yahoo', start='2007-1-1')['Adj_
          ↪Close']
```

```
[21]: prova.tail()
```

```
[21]:
```

	PG	BEI.DE
Date		
2020-06-15	116.690002	96.379997
2020-06-16	118.129997	99.900002
2020-06-17	117.930000	100.250000
2020-06-18	119.279999	99.760002
2020-06-19	118.919998	101.250000

```
[22]: prova_returns=np.log(prova/prova.shift(1))
```

```
[23]: prova_returns
```

```
[23]:
```

	PG	BEI.DE
Date		
2007-01-03	NaN	NaN
2007-01-04	-0.007621	0.006544
2007-01-05	-0.008624	-0.020772
2007-01-08	0.002202	0.000202
2007-01-09	-0.002517	-0.022858
...	...	...
2020-06-15	0.009212	-0.019115
2020-06-16	0.012265	0.035871
2020-06-17	-0.001694	0.003497
2020-06-18	0.011382	-0.004900
2020-06-19	-0.003023	0.014825

[3390 rows x 2 columns]

```
[105]: prova_returns['PG'].mean()
```

```
[105]: 0.00029915224773705256
```

```
[106]: prova_returns['PG'].mean()*250
```

```
[106]: 0.07478806193426314
```

```
[107]: prova_returns['PG'].std()
```

```
[107]: 0.011965592497531945
```

```
[108]: prova_returns['PG'].std()*250**0.5
```

```
[108]: 0.18919262922811816
```

```
[113]: prova_returns['BEI.DE'].mean()
```

```
[113]: 0.000260295490819062
```

```
[111]: prova_returns['BEI.DE'].mean()*250
```

```
[111]: 0.0650738727047655
```

```
[112]: prova_returns['BEI.DE'].std()
```

```
[112]: 0.013801508704794914
```

```
[114]: prova_returns['BEI.DE'].std()*250**0.5
```

```
[114]: 0.2182210132689619
```

```
[25]: print (prova_returns['PG'].mean()*250)
      print (prova_returns['BEI.DE'].mean()*250)
```

```
0.07479513851801017
0.0650738727047655
```

```
[26]: prova_returns[['PG', 'BEI.DE']].mean()*250
```

```
[26]: PG          0.074795
      BEI.DE      0.065074
      dtype: float64
```

```
[120]: prova_returns[['PG', 'BEI.DE']].std()*250**0.5
```

```
[120]: PG          0.189193
      BEI.DE      0.218221
      dtype: float64
```

```
[123]: PG_Var = prova_returns ['PG'].var()
      PG_Var
```

```
[123]: 0.00014317540381699278
```

```
[124]: BEI_Var = prova_returns ['BEI.DE'].var()
```

```
[125]: BEI_Var
```

```
[125]: 0.00019048164252852978
```

```
[126]: PG_Var_a = prova_returns ['PG'].var()*250
```

```
[127]: PG_Var_a
```

```
[127]: 0.035793850954248196
```

```
[32]: BEI_Var_a = prova_returns ['BEI.DE'].var()*250
```

```
[33]: BEI_Var_a
```

```
[33]: 0.047620410632132446
```

```
[131]: cov_matrix=prova_returns.cov()  
cov_matrix
```

```
[131]:  
          PG    BEI.DE  
PG      0.000143  0.000045  
BEI.DE  0.000045  0.000190
```

```
[132]: cov_matrix_a=prova_returns.cov()*250
```

```
[133]: cov_matrix_a
```

```
[133]:  
          PG    BEI.DE  
PG      0.035794  0.011371  
BEI.DE  0.011371  0.047620
```

```
[134]: corr_matrix=prova_returns.corr()
```

```
[135]: corr_matrix
```

```
[135]:  
          PG    BEI.DE  
PG      1.000000  0.275006  
BEI.DE  0.275006  1.000000
```

```
[136]: corr_matrix_a=prova_returns.corr()*250  
corr_matrix_a
```



```
[136]:          PG      BEI.DE
PG      250.000000  68.751612
BEI.DE  68.751612  250.000000
```

```
[137]: #Calculating Portfolio Risk
```

```
[139]: #Equal Weighting scheme:
weights=np.array([0.5,0.5])
```

```
[40]: #Portfolio Variance
pfolio_var=np.dot(weights.T, np.dot(prova_returns.cov()*250,weights))
pfolio_var
```

```
[40]: 0.026539015888108423
```

```
[141]: #Portfolio Volatility
pfolio_vol=np.dot(weights.T, np.dot(prova_returns.cov()*250,weights))**0.5
pfolio_vol
```

```
[141]: 0.16290808293358686
```

```
[142]: print (str(round(pfolio_vol, 5)*100)+ '%')
```

```
16.291%
```

```
[1]: #Calculating Diversifiable and Non-Diversifiable Risk of a Portfolio
```

```
[27]: weights= np.array([0.5,0.5])
```

```
[3]: import pandas as pd
```

```
[8]: import pandas_datareader as web
```

```
[28]: weights= np.array([0.5,0.5])
```

```
[10]: import numpy as np
```

```
[11]: import matplotlib.pyplot as plt
```

```
[12]: weights= np.array([0.5,0.5])
```

```
[13]: weights[0]
```

```
[13]: 0.5
```

```
[14]: weights[1]
```

```
[14]: 0.5
```

```
[30]: PG_var_a=prova_returns[['PG']].var()*250
```

```
[31]: PG_var_a
```

```
[31]: PG      0.035794  
dtype: float64
```

```
[35]: BEI_var_a=prova_returns[['BEI.DE']].var()*250
```

```
[36]: BEI_var_a
```

```
[36]: BEI.DE    0.04762  
dtype: float64
```

```
[37]: float(PG_var_a)
```

```
[37]: 0.035793790587168625
```

```
[44]: PG_var_a=prova_returns['PG'].var()*250  
PG_var_a
```

```
[44]: 0.035793790587168625
```

```
[46]: BEI_var_a=prova_returns['BEI.DE'].var()*250  
BEI_var_a
```

```
[46]: 0.047620410632132446
```

```
[48]: dr=pfolio_var-(weights[0]**2*PG_var_a)-(weights[1]**2*BEI_var_a)
```

```
[49]: dr
```

```
[49]: 0.005685465583283155
```

```
[56]: print (str(round(dr*100,3)) + '%')
```

```
0.569%
```

```
[57]: n_dr_1=pfolio_var-dr
```

```
[58]: n_dr_1
```

```
[58]: 0.020853550304825268
```

```
[59]: import numpy as np
```

```
[60]: import pandas as pd
```

```
[61]: from scipy import stats
```

```
[62]: import statsmodels.api as sm
```

```
[63]: import matplotlib.pyplot as plt
```

```
[67]: data=pd.read_excel ("/Users/alessandrosollazzo/Desktop/original.xlsx")
```

```
[68]: data
```

```
[68]:
```

	House Price	House Size (sq.ft.)	State	Number of Rooms	\
0	1116000	1940	IN	8	
1	860000	1300	IN	5	
2	818400	1420	IN	6	
3	1000000	1680	IN	7	
4	640000	1270	IN	5	
5	1010000	1850	IN	7	
6	600000	1000	IN	4	
7	700000	1100	LA	4	
8	1100000	1600	LA	7	
9	570000	1000	NY	5	
10	860000	2150	NY	9	
11	1085000	1900	NY	9	
12	1250000	2200	NY	9	
13	850000	1100	TX	4	
14	640000	860	TX	4	
15	900000	1325	TX	6	
16	730000	1350	TX	6	
17	750000	1600	TX	6	
18	650000	950	TX	2	
19	680000	1250	TX	4	

	Year of Construction
0	2002
1	1992
2	1987
3	2000
4	1995
5	1998
6	2015
7	2014
8	2017
9	1997
10	1997
11	2000

12	2014
13	2017
14	1997
15	1997
16	2000
17	1992
18	1987
19	2000

```
[70]: data[['House Price','House Size (sq.ft.)']]
```

```
[70]:
```

	House Price	House Size (sq.ft.)
0	1116000	1940
1	860000	1300
2	818400	1420
3	1000000	1680
4	640000	1270
5	1010000	1850
6	600000	1000
7	700000	1100
8	1100000	1600
9	570000	1000
10	860000	2150
11	1085000	1900
12	1250000	2200
13	850000	1100
14	640000	860
15	900000	1325
16	730000	1350
17	750000	1600
18	650000	950
19	680000	1250

```
[71]: X=data ['House Size (sq.ft.)']
      Y=data['House Price']
```

```
[72]: X
```

```
[72]:
```

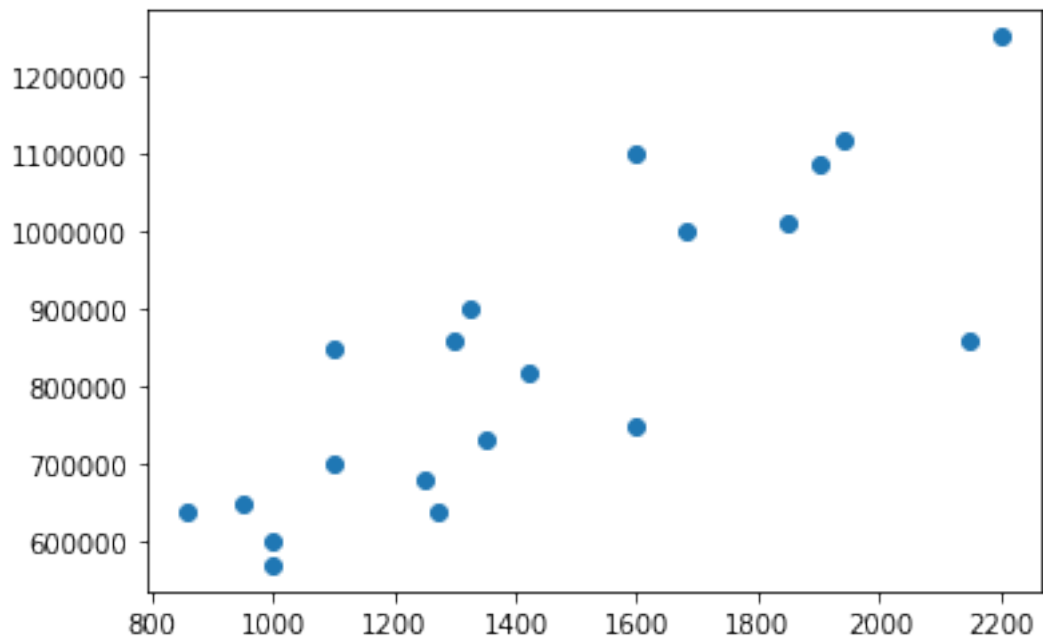
0	1940
1	1300
2	1420
3	1680
4	1270
5	1850
6	1000
7	1100
8	1600

```
9      1000
10     2150
11     1900
12     2200
13     1100
14      860
15     1325
16     1350
17     1600
18      950
19     1250
Name: House Size (sq.ft.), dtype: int64
```

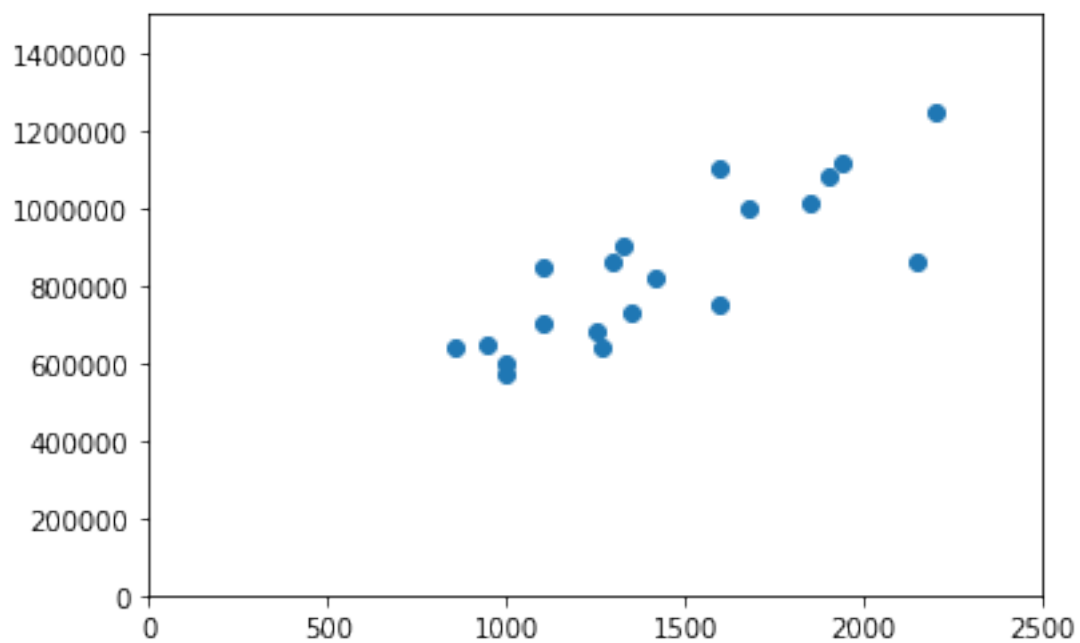
```
[73]: Y
```

```
[73]: 0      1116000
      1       860000
      2      818400
      3     1000000
      4      640000
      5     1010000
      6      600000
      7      700000
      8     1100000
      9      570000
     10      860000
     11     1085000
     12     1250000
     13      850000
     14      640000
     15      900000
     16      730000
     17      750000
     18      650000
     19      680000
Name: House Price, dtype: int64
```

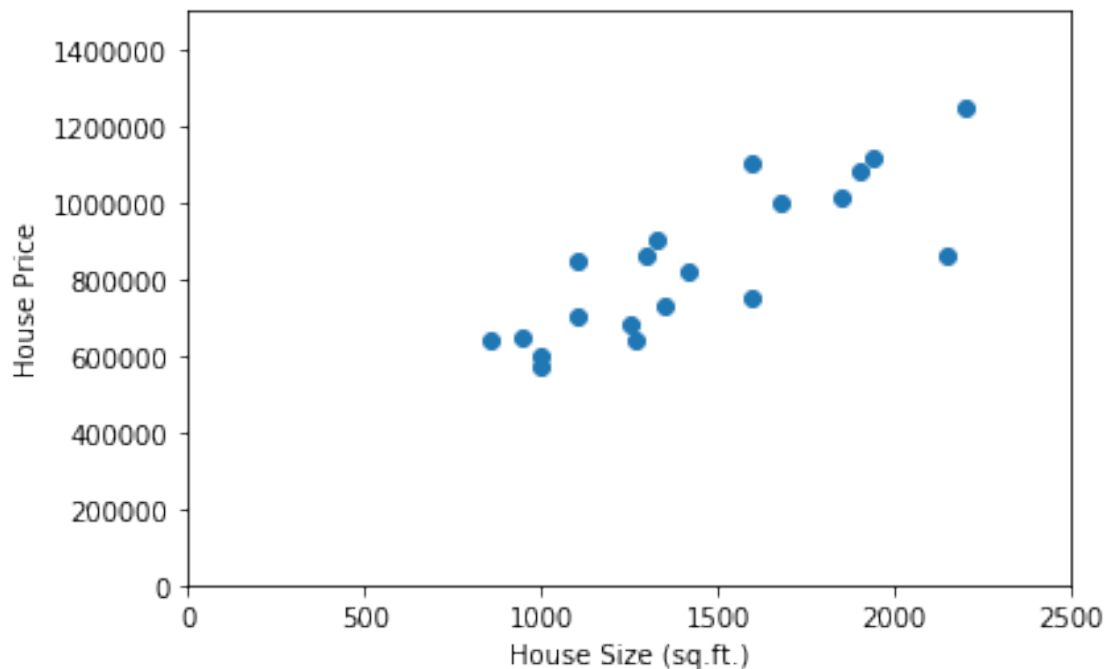
```
[74]: plt.scatter(X,Y)
      plt.show()
```



```
[75]: plt.scatter(X,Y)
plt.axis([0, 2500, 0, 1500000])
plt.show()
```



```
[76]: plt.scatter(X,Y)
plt.axis([0, 2500, 0, 1500000])
plt.ylabel('House Price')
plt.xlabel('House Size (sq.ft.)')
plt.show()
```



```
[77]: X1=sm.add_constant(X)
```

```
[78]: reg=sm.OLS(Y, X1).fit()
```

```
[79]: reg.summary()
```

```
[79]: <class 'statsmodels.iolib.summary.Summary'>
```

```
"""
```

#### OLS Regression Results

```
=====
Dep. Variable:          House Price    R-squared:                0.678
Model:                  OLS           Adj. R-squared:          0.660
Method:                 Least Squares  F-statistic:             37.95
Date:                  Mon, 22 Jun 2020  Prob (F-statistic):      8.13e-06
Time:                  18:08:31        Log-Likelihood:          -260.43
No. Observations:      20             AIC:                    524.9
Df Residuals:          18             BIC:                    526.8
Df Model:               1
Covariance Type:       nonrobust
```

```

=====
=====
              coef      std err          t      P>|t|      [0.025
0.975]
-----
-----
const              2.608e+05   9.76e+04     2.673     0.016   5.58e+04
4.66e+05
House Size (sq.ft.) 401.9163    65.243     6.160     0.000   264.846
538.987
=====
Omnibus:                1.238    Durbin-Watson:                1.810
Prob(Omnibus):          0.538    Jarque-Bera (JB):            0.715
Skew:                  -0.459    Prob(JB):                    0.699
Kurtosis:              2.884    Cond. No.                    5.66e+03
=====

```

Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The condition number is large, 5.66e+03. This might indicate that there are strong multicollinearity or other numerical problems.

"""

[80]: *#Expected Value of Y:*

260800 + 402\*1000

[81]: 260800 + 402\*1000

[81]: 662800

[83]: slope, intercept, r\_value, p\_value, std\_error = stats.linregress(X,Y)

[84]: slope

[84]: 401.91628631922595

[85]: intercept

[85]: 260806.2360560964

[86]: r\_value

[86]: 0.8235775534696924

[87]: r\_value\*\*2



```
[87]: 0.678279986579124
```

```
[88]: p_value
```

```
[88]: 8.129642377231308e-06
```

```
[89]: std_error
```

```
[89]: 65.24299510636492
```

```
[90]: #Efficient Frontier
```

```
[91]: import numpy as np
import pandas as pd
from pandas_datareader import data as web
import matplotlib.pyplot as plt
%matplotlib inline
```

```
[92]: assets=['PG', '^GSPC']
pf_data=pd.DataFrame()

for a in assets:
    pf_data[a]=web.DataReader(a,data_source='yahoo', start = '2010-1-1')['Adj_
↪Close']
```

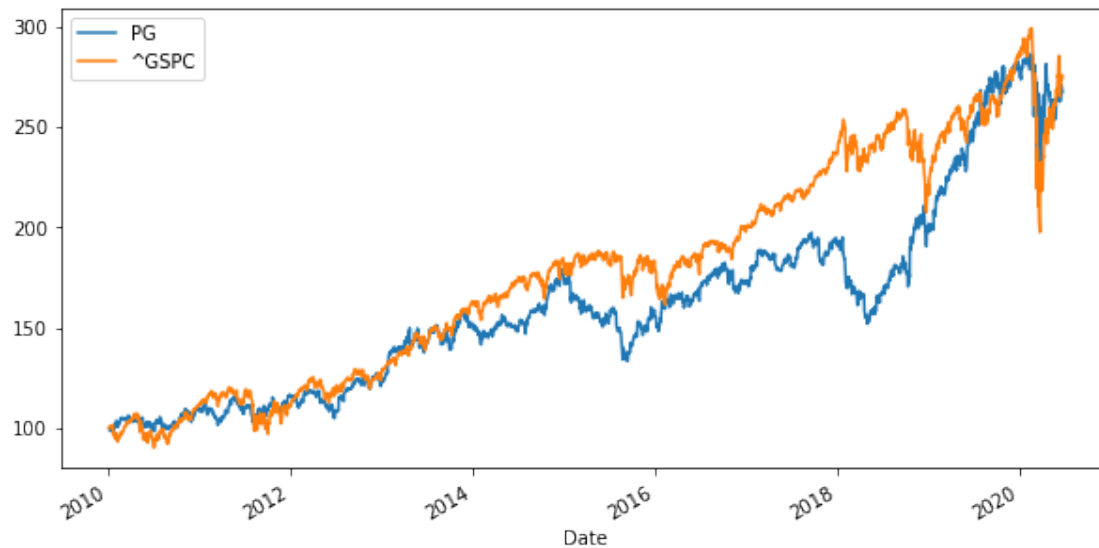
```
[94]: pf_data.tail()
```

```
[94]:
```

	PG	^GSPC
Date		
2020-06-16	118.129997	3124.739990
2020-06-17	117.930000	3113.489990
2020-06-18	119.279999	3115.340088
2020-06-19	118.919998	3097.739990
2020-06-22	117.750000	3117.860107

```
[96]: (pf_data/pf_data.iloc[0]*100).plot(figsize=(10, 5))
```

```
[96]: <matplotlib.axes._subplots.AxesSubplot at 0x7febc77e3210>
```



```
[ ]:
```

```
[97]: log_returns=np.log(pf_data/pf_data.shift(1))
```

```
[98]: log_returns.mean()*250
```

```
[98]: PG      0.093312
      ^GSPC   0.096079
      dtype: float64
```

```
[99]: log_returns.cov()*250
```

```
[99]:          PG      ^GSPC
PG      0.029550  0.017978
^GSPC   0.017978  0.030639
```

```
[101]: log_returns.corr()
```

```
[101]:          PG      ^GSPC
PG      1.000000  0.597484
^GSPC   0.597484  1.000000
```

```
[102]: num_assets=len(assets)
```

```
[103]: num_assets
```

```
[103]: 2
```

```

[104]: arr=np.random.random(2)
arr

[104]: array([0.92681938, 0.56287802])

[105]: weights=np.random.random(num_assets)

[106]: weights/=np.sum(weights)

[107]: weights

[107]: array([0.68965479, 0.31034521])

[108]: arr[0]+arr[1]

[108]: 1.489697399045426

[109]: weights[0]+weights[1]

[109]: 1.0

[110]: #Expected Portfolio Return

[112]: np.sum(weights*log_returns.mean())*250

[112]: 0.09417065781603497

[113]: np.dot(weights.T,np.dot(log_returns.cov()*250,weights))

[113]: 0.024701248606204557

[114]: np.sqrt(np.dot(weights.T,np.dot(log_returns.cov()*250,weights)))

[114]: 0.1571663087503316

[116]: pfolio_returns=[]
pfolio_volatilities=[]
for x in range (1000):
    weights=np.random.random(num_assets)
    weights/=np.sum(weights)
    pfolio_returns.append(np.sum(weights*log_returns.mean())*250)
    pfolio_volatilities.append(np.sqrt(np.dot(weights.T,np.dot(log_returns.
↪ cov()*250,weights))))

pfolio_returns=np.array(pfolio_returns)
pfolio_volatiltites=np.array(pfolio_volatilities)

```

```
pfolio_returns, pfolio_volatilities
```

```
[116]: (array([0.09560059, 0.09457073, 0.09527279, 0.09421889, 0.09346499,  
              0.09453736, 0.09482576, 0.09470435, 0.09379315, 0.09586867,  
              0.09366731, 0.09361216, 0.09427773, 0.0956426 , 0.09450247,  
              0.09509906, 0.09535168, 0.09509524, 0.09464568, 0.09426597,  
              0.09483238, 0.09458422, 0.09340929, 0.09470454, 0.09466953,  
              0.09603886, 0.09532372, 0.09352781, 0.095093 , 0.09542325,  
              0.09599244, 0.09538526, 0.09394972, 0.09516095, 0.09597559,  
              0.09361086, 0.09409965, 0.09588542, 0.09372767, 0.09395053,  
              0.09538051, 0.09450683, 0.09343882, 0.09562786, 0.09535451,  
              0.09594105, 0.09423206, 0.095123 , 0.0952852 , 0.09563569,  
              0.09583193, 0.09349934, 0.09410125, 0.09443747, 0.09479604,  
              0.0953479 , 0.09447449, 0.09542762, 0.09386007, 0.09490549,  
              0.09488146, 0.09555966, 0.09379803, 0.09392452, 0.09486269,  
              0.09497173, 0.09386709, 0.09335695, 0.09464032, 0.09371611,  
              0.09576502, 0.09333946, 0.09415869, 0.09560028, 0.09381995,  
              0.09569261, 0.09334741, 0.09419162, 0.09476275, 0.09449122,  
              0.09467136, 0.09542289, 0.09458963, 0.09511118, 0.09497702,  
              0.09367 , 0.09607735, 0.09455349, 0.09384961, 0.0941367 ,  
              0.09458645, 0.09480702, 0.09511152, 0.09381743, 0.09381165,  
              0.09406188, 0.09412077, 0.09516593, 0.09435748, 0.0943692 ,  
              0.09459019, 0.09405192, 0.09354372, 0.09471155, 0.09506413,  
              0.09468382, 0.09482836, 0.09470878, 0.09468423, 0.09334407,  
              0.09462403, 0.09429817, 0.094444843, 0.09441892, 0.094444931,  
              0.09373251, 0.09492435, 0.09382679, 0.09453852, 0.09347314,  
              0.09442133, 0.09514623, 0.09580872, 0.09537259, 0.09362725,  
              0.09332467, 0.09575327, 0.0939635 , 0.0933774 , 0.09487113,  
              0.09420953, 0.09473842, 0.09522466, 0.09360245, 0.09409441,  
              0.0945232 , 0.09426036, 0.09562779, 0.09484999, 0.0954985 ,  
              0.09600465, 0.09580403, 0.09432985, 0.09426775, 0.09578731,  
              0.09402261, 0.09354385, 0.09390005, 0.09416852, 0.09331696,  
              0.09494071, 0.09336879, 0.09434579, 0.09494284, 0.09454779,  
              0.09473965, 0.09528114, 0.09358425, 0.09545957, 0.09580045,  
              0.09370368, 0.0942451 , 0.09348772, 0.09571325, 0.09483538,  
              0.09465165, 0.09403779, 0.09371174, 0.09494763, 0.0946557 ,  
              0.09473866, 0.09454763, 0.09363299, 0.09440666, 0.09554985,  
              0.09372459, 0.09491579, 0.09479372, 0.09490603, 0.09448617,  
              0.09426497, 0.09506834, 0.09597165, 0.09466831, 0.0948345 ,  
              0.095598 , 0.09490794, 0.09467885, 0.09482733, 0.09430778,  
              0.09388452, 0.0941833 , 0.09480319, 0.09482704, 0.09483166,  
              0.09509057, 0.09364491, 0.09476545, 0.09548953, 0.09415353,  
              0.09550527, 0.09530421, 0.09458387, 0.09354876, 0.09451161,  
              0.09486543, 0.09453512, 0.09377775, 0.09437189, 0.09352002,  
              0.0959562 , 0.09438938, 0.09470043, 0.09518486, 0.09553177,  
              0.09359681, 0.09500778, 0.09506428, 0.09600106, 0.09566629,  
              0.09464552, 0.09485042, 0.09460678, 0.09438166, 0.09481292,
```

0.09423803, 0.09356546, 0.09437698, 0.09435717, 0.09437869,  
0.09481303, 0.09449171, 0.09430972, 0.09460732, 0.09553191,  
0.09546167, 0.09473337, 0.09432789, 0.09467944, 0.09462324,  
0.09526756, 0.09483637, 0.09558382, 0.09489111, 0.09398033,  
0.09578202, 0.0948634 , 0.09459256, 0.09445875, 0.09514919,  
0.09496428, 0.0958413 , 0.09439808, 0.09584979, 0.09486534,  
0.09469276, 0.09569968, 0.09444466, 0.09380509, 0.09475962,  
0.09506831, 0.09553872, 0.09564574, 0.0942259 , 0.09433833,  
0.09461849, 0.09467437, 0.09570759, 0.09411808, 0.09506523,  
0.09404612, 0.09471624, 0.09445415, 0.09471914, 0.09452893,  
0.0951889 , 0.09419634, 0.09539235, 0.09498545, 0.09389263,  
0.09482877, 0.09481596, 0.09498316, 0.09472813, 0.09435785,  
0.09453682, 0.09586364, 0.09423964, 0.09498459, 0.09474926,  
0.09453595, 0.09430449, 0.09413736, 0.09424614, 0.09366242,  
0.09467145, 0.09434941, 0.09515218, 0.09482906, 0.09459326,  
0.09449076, 0.09452294, 0.09476774, 0.0935524 , 0.0937 ,  
0.09473355, 0.09485895, 0.0950404 , 0.09507539, 0.09420116,  
0.09438484, 0.09419149, 0.09505918, 0.09495495, 0.09473871,  
0.09482658, 0.09430784, 0.09382664, 0.09482768, 0.09424511,  
0.09399444, 0.09544124, 0.09486196, 0.09522079, 0.09435505,  
0.09487176, 0.09462412, 0.09602129, 0.09386954, 0.09423516,  
0.09479473, 0.09515119, 0.09585169, 0.09431735, 0.09510718,  
0.09499307, 0.09333132, 0.09453344, 0.09479874, 0.0936654 ,  
0.09445205, 0.09470869, 0.0953035 , 0.09459969, 0.09565726,  
0.09373864, 0.09470249, 0.09466665, 0.09438727, 0.0960217 ,  
0.09331732, 0.09535988, 0.09435045, 0.09468354, 0.0954837 ,  
0.09407763, 0.09333941, 0.09546441, 0.09417555, 0.09569718,  
0.09559657, 0.09533504, 0.09481044, 0.09573911, 0.09474666,  
0.09363652, 0.0933125 , 0.09530799, 0.09516655, 0.09520845,  
0.09521502, 0.09528683, 0.09530614, 0.09482551, 0.09443718,  
0.09511975, 0.09340659, 0.09407747, 0.09520038, 0.09469809,  
0.09374609, 0.09494866, 0.09593475, 0.09377363, 0.09432521,  
0.0940583 , 0.0947183 , 0.09498863, 0.09465969, 0.09473607,  
0.09422115, 0.09478909, 0.09424622, 0.09423625, 0.09463608,  
0.09580799, 0.0947622 , 0.0942641 , 0.09556666, 0.09517263,  
0.09524058, 0.09463161, 0.09603559, 0.09528452, 0.09515566,  
0.09439066, 0.09360339, 0.09476386, 0.09500229, 0.09345873,  
0.09447614, 0.09529209, 0.09464621, 0.09381119, 0.09585759,  
0.09541107, 0.09432023, 0.09425936, 0.09531075, 0.09399517,  
0.09465208, 0.09434227, 0.09495394, 0.09364722, 0.09373139,  
0.09354985, 0.09448497, 0.09556413, 0.09410239, 0.09423214,  
0.0949336 , 0.09515413, 0.09463943, 0.09562285, 0.09516252,  
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0.15499638040092723,  
0.15920333207045528,  
0.15515364380985683,  
0.1550007464730745,  
0.15897082907968865,  
0.1576423585682393,  
0.1581786634555004,  
0.15657476859999567,  
0.16374049283841352,  
0.1682252465049379,  
0.15911970401921785,  
0.15501948093994208,  
0.1551049883121833,  
0.15557085239083931,  
0.15585899130026806,  
0.15517933062376663,  
0.15660672657988858,  
0.15894014724488228,  
0.15700072648311747,  
0.1691820015808929,  
0.15559736384916498,  
0.159409810448235,

```

0.16859278433499664,
0.15547280228732957,
0.1552789656794323,
0.15608991678702855,
0.1717898026013916,
0.15863220770681205,
0.16212531690264076,
0.15503003492037423,
0.16280507255907098,
0.1550524910888391,
0.16081913202097195,
0.1564957786695928,
0.1605216164117663,
0.16560722432848551,
0.1643186759419461,
0.15973757029303984,
0.1554185985673121,
0.16289134983974604,
0.15622733610588802,
0.15500082835972875,
0.1570078843359922,
0.1698877608609299,
0.15560845597223608,
0.16207829617653843,
0.1613910579999036,
0.15589571286744786,
0.15835261477080084])

```

```
[119]: portfolios=pd.DataFrame({'Return':pfolio_returns,'volatility':
↪pfolio_volatilities})
```

```
[120]: portfolios.head()
```

```
[120]:
```

	Return	volatility
0	0.095601	0.164274
1	0.094571	0.155036
2	0.095273	0.159118
3	0.094219	0.156740
4	0.093465	0.168357

```
[121]: portfolios.tail()
```

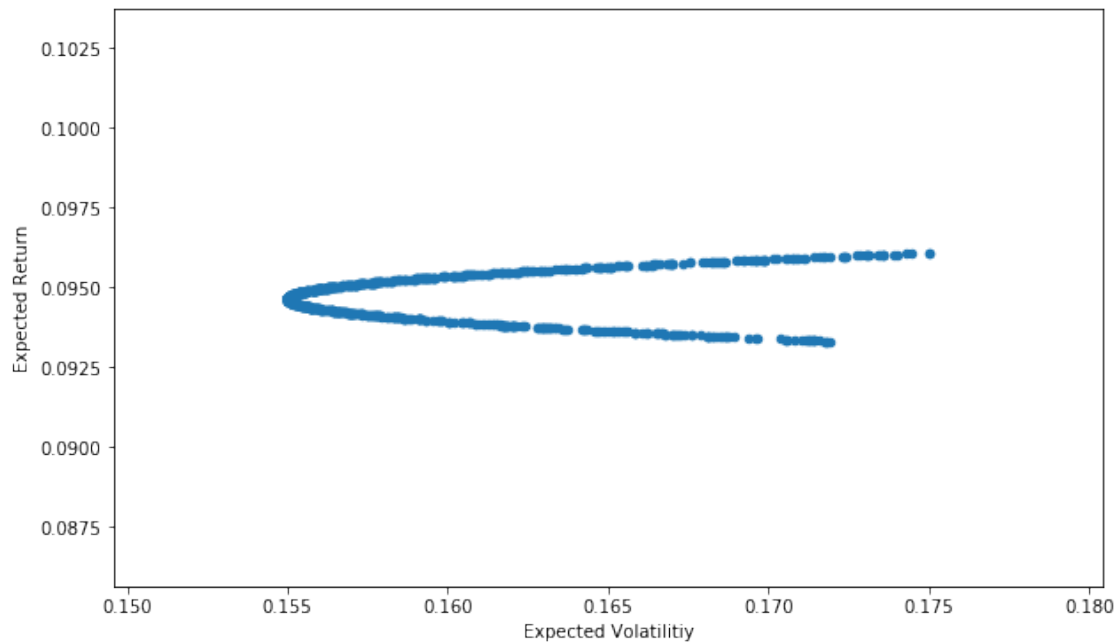
```
[121]:
```

	Return	volatility
995	0.094878	0.155608
996	0.095476	0.162078
997	0.095433	0.161391
998	0.094930	0.155896

999 0.094057 0.158353

```
[123]: portfolios.plot(x='volatility',y='Return',kind='scatter', figsize=(10, 6));  
plt.xlabel('Expected Volatilitiy')  
plt.ylabel('Expected Return')
```

```
[123]: Text(0, 0.5, 'Expected Return')
```



```
[ ]:
```

```
[125]: import numpy as np  
import pandas as pd  
from pandas_datareader import data as web  
  
tickers = ['PG', '^GSPC']  
data=pd.DataFrame()  
for t in tickers:  
    data[t]=web.DataReader(t,data_source='yahoo',  
        ↪start='2012-1-1',end='2016-12-31')['Adj Close']
```

```
[130]: sec_returns=np.log(data/data.shift(1))
```

```
[131]: cov=sec_returns.cov()*250  
cov
```

```
[131]:          PG      ^GSPC
PG      0.020409  0.010078
^GSPC   0.010078  0.016362
```

```
[132]: cov_with_market=cov.iloc[0,1]
```

```
[133]: cov_with_market
```

```
[133]: 0.010078060959412657
```

```
[134]: market_var=sec_returns['^GSPC'].var()*250
market_var
```

```
[134]: 0.016361631002308474
```

```
[135]: #BETA
```

```
[136]: PG_beta=cov_with_market/market_var
```

```
[137]: PG_beta
```

```
[137]: 0.6159569885172655
```

```
[138]: #CAPM
```

```
[139]: PG_er=0.025+PG_beta*0.05
PG_er
```

```
[139]: 0.05579784942586328
```

```
[140]: #SharpeRatio
```

```
[141]: Sharpe=(PG_er-0.025)/(sec_returns['PG'].std()*250**0.5)
Sharpe
```

```
[141]: 0.2155797835537668
```

```
[151]: import numpy as np
import pandas as pd
from scipy import stats
import statsmodels.api as sm
import matplotlib.pyplot as plt
data=pd.read_excel ("/Users/alessandrosollazzo/Desktop/original.xlsx")
data
```

```
[151]:      House Price  House Size (sq.ft.)  State  Number of Rooms  \
0          1116000             1940      IN              8
```

1	860000	1300	IN	5
2	818400	1420	IN	6
3	1000000	1680	IN	7
4	640000	1270	IN	5
5	1010000	1850	IN	7
6	600000	1000	IN	4
7	700000	1100	LA	4
8	1100000	1600	LA	7
9	570000	1000	NY	5
10	860000	2150	NY	9
11	1085000	1900	NY	9
12	1250000	2200	NY	9
13	850000	1100	TX	4
14	640000	860	TX	4
15	900000	1325	TX	6
16	730000	1350	TX	6
17	750000	1600	TX	6
18	650000	950	TX	2
19	680000	1250	TX	4

	Year of Construction
0	2002
1	1992
2	1987
3	2000
4	1995
5	1998
6	2015
7	2014
8	2017
9	1997
10	1997
11	2000
12	2014
13	2017
14	1997
15	1997
16	2000
17	1992
18	1987
19	2000

```
[158]: X=data[['House Size (sq.ft.)','Number of Rooms','Year of Construction']]
       Y=data['House Price']
```

```
[159]: X1=sm.add_constant(x)
```



```
[160]: from scipy import stats
import statsmodels.api as sm
```

```
[161]: X1=sm.add_constant(x)
```

```
[162]: data[['House Price','House Size (sq.ft.)','Year of Construction']]
```

```
[162]:
```

	House Price	House Size (sq.ft.)	Year of Construction
0	1116000	1940	2002
1	860000	1300	1992
2	818400	1420	1987
3	1000000	1680	2000
4	640000	1270	1995
5	1010000	1850	1998
6	600000	1000	2015
7	700000	1100	2014
8	1100000	1600	2017
9	570000	1000	1997
10	860000	2150	1997
11	1085000	1900	2000
12	1250000	2200	2014
13	850000	1100	2017
14	640000	860	1997
15	900000	1325	1997
16	730000	1350	2000
17	750000	1600	1992
18	650000	950	1987
19	680000	1250	2000

```
[163]: X=data[['House Size (sq.ft.)','Number of Rooms','Year of Construction']]
Y=data['House Price']
```

```
[164]: X1=sm.add_constant(x)
```

```
[165]: reg=sm.OLS(Y,X1).fit()
```

```
↳
↳-----
ValueError                                Traceback (most recent call↳
↳last)

<ipython-input-165-fb20264ce9ea> in <module>
----> 1 reg=sm.OLS(Y,X1).fit()
```

```

~/opt/anaconda3/lib/python3.7/site-packages/statsmodels/regression/
↳ linear_model.py in __init__(self, endog, exog, missing, hasconst, **kwargs)
    857         **kwargs):
    858         super(OLS, self).__init__(endog, exog, missing=missing,
--> 859                                 hasconst=hasconst, **kwargs)
    860         if "weights" in self._init_keys:
    861             self._init_keys.remove("weights")

```

```

~/opt/anaconda3/lib/python3.7/site-packages/statsmodels/regression/
↳ linear_model.py in __init__(self, endog, exog, weights, missing, hasconst,
↳ **kwargs)
    700         weights = weights.squeeze()
    701         super(WLS, self).__init__(endog, exog, missing=missing,
--> 702                                 weights=weights,
↳ hasconst=hasconst, **kwargs)
    703         nobs = self.exog.shape[0]
    704         weights = self.weights

```

```

~/opt/anaconda3/lib/python3.7/site-packages/statsmodels/regression/
↳ linear_model.py in __init__(self, endog, exog, **kwargs)
    188         """
    189         def __init__(self, endog, exog, **kwargs):
--> 190             super(RegressionModel, self).__init__(endog, exog, **kwargs)
    191             self._data_attr.extend(['pinv_wexog', 'wendog', 'wexog',
↳ 'weights'])
    192

```

```

~/opt/anaconda3/lib/python3.7/site-packages/statsmodels/base/model.py in
↳ __init__(self, endog, exog, **kwargs)
    234
    235     def __init__(self, endog, exog=None, **kwargs):
--> 236         super(LikelihoodModel, self).__init__(endog, exog, **kwargs)
    237         self.initialize()
    238

```

```

~/opt/anaconda3/lib/python3.7/site-packages/statsmodels/base/model.py in
↳ __init__(self, endog, exog, **kwargs)
    75         hasconst = kwargs.pop('hasconst', None)
    76         self.data = self._handle_data(endog, exog, missing, hasconst,
---> 77                                 **kwargs)
    78         self.k_constant = self.data.k_constant
    79         self.exog = self.data.exog

```

```

~/opt/anaconda3/lib/python3.7/site-packages/statsmodels/base/model.py in
↳ _handle_data(self, endog, exog, missing, hasconst, **kwargs)
    98
    99     def _handle_data(self, endog, exog, missing, hasconst, **kwargs):
--> 100         data = handle_data(endog, exog, missing, hasconst, **kwargs)
    101         # kwargs arrays could have changed, easier to just attach
↳ here
    102         for key in kwargs:

```

```

~/opt/anaconda3/lib/python3.7/site-packages/statsmodels/base/data.py in
↳ handle_data(endog, exog, missing, hasconst, **kwargs)
    670     klass = handle_data_class_factory(endog, exog)
    671     return klass(endog, exog=exog, missing=missing,
↳ hasconst=hasconst,
--> 672                     **kwargs)

```

```

~/opt/anaconda3/lib/python3.7/site-packages/statsmodels/base/data.py in
↳ __init__(self, endog, exog, missing, hasconst, **kwargs)
    81         self.orig_endog = endog
    82         self.orig_exog = exog
---> 83         self.endog, self.exog = self._convert_endog_exog(endog,
↳ exog)
    84
    85         self.const_idx = None

```

```

~/opt/anaconda3/lib/python3.7/site-packages/statsmodels/base/data.py in
↳ _convert_endog_exog(self, endog, exog)
    508         raise ValueError("Pandas data cast to numpy dtype of
↳ object. "
    509                             "Check input data with np.asarray(data).
↳ ")
--> 510         return super(PandasData, self)._convert_endog_exog(endog,
↳ exog)
    511
    512     @classmethod

```

```

~/opt/anaconda3/lib/python3.7/site-packages/statsmodels/base/data.py in
↳ _convert_endog_exog(self, endog, exog)
    324         xarr = xarr[:, None]
    325         if xarr.ndim != 2:
--> 326             raise ValueError("exog is not 1d or 2d")
    327

```

```
328         return yarr, xarr
```

```
ValueError: exog is not 1d or 2d
```

```
[166]: import numpy as np
```

```
[169]: import pandas as pd
```

```
[170]: from scipy import stats
```

```
[171]: import statsmodels.api as sm
```

```
[172]: import matplotlib.pyplot as plt
```

```
[173]: data=pd.read_excel ("/Users/alessandrosollazzo/Desktop/original.xlsx")
```

```
[174]: data
```

```
[174]:
```

	House Price	House Size (sq.ft.)	State	Number of Rooms	\
0	1116000	1940	IN	8	
1	860000	1300	IN	5	
2	818400	1420	IN	6	
3	1000000	1680	IN	7	
4	640000	1270	IN	5	
5	1010000	1850	IN	7	
6	600000	1000	IN	4	
7	700000	1100	LA	4	
8	1100000	1600	LA	7	
9	570000	1000	NY	5	
10	860000	2150	NY	9	
11	1085000	1900	NY	9	
12	1250000	2200	NY	9	
13	850000	1100	TX	4	
14	640000	860	TX	4	
15	900000	1325	TX	6	
16	730000	1350	TX	6	
17	750000	1600	TX	6	
18	650000	950	TX	2	
19	680000	1250	TX	4	

	Year of Construction
0	2002
1	1992
2	1987
3	2000

4	1995
5	1998
6	2015
7	2014
8	2017
9	1997
10	1997
11	2000
12	2014
13	2017
14	1997
15	1997
16	2000
17	1992
18	1987
19	2000

```
[175]: X=data[['House Size (sq.ft.)','Number of Rooms','Year of Construction']]
      Y=data['House Price']
```

```
[176]: X
```

```
[176]:
```

	House Size (sq.ft.)	Number of Rooms	Year of Construction
0	1940	8	2002
1	1300	5	1992
2	1420	6	1987
3	1680	7	2000
4	1270	5	1995
5	1850	7	1998
6	1000	4	2015
7	1100	4	2014
8	1600	7	2017
9	1000	5	1997
10	2150	9	1997
11	1900	9	2000
12	2200	9	2014
13	1100	4	2017
14	860	4	1997
15	1325	6	1997
16	1350	6	2000
17	1600	6	1992
18	950	2	1987
19	1250	4	2000

```
[178]: Y
```

```
[178]: 0    1116000
      1     860000
      2     818400
      3    1000000
      4     640000
      5    1010000
      6     600000
      7     700000
      8    1100000
      9     570000
     10     860000
     11    1085000
     12    1250000
     13     850000
     14     640000
     15     900000
     16     730000
     17     750000
     18     650000
     19     680000
      Name: House Price, dtype: int64
```

```
[179]: X1=sm.add_constant(x)
```

```
[180]: reg=sm.OLS(Y,X1).fit()
```

```

      □
↳ -----

ValueError                                Traceback (most recent call↳
↳ last)

<ipython-input-180-fb20264ce9ea> in <module>
----> 1 reg=sm.OLS(Y,X1).fit()

~/opt/anaconda3/lib/python3.7/site-packages/statsmodels/regression/
↳ linear_model.py in __init__(self, endog, exog, missing, hasconst, **kwargs)
      857         **kwargs):
      858         super(OLS, self).__init__(endog, exog, missing=missing,
--> 859                                hasconst=hasconst, **kwargs)
      860         if "weights" in self._init_keys:
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```

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    700         weights = weights.squeeze()
    701         super(WLS, self).__init__(endog, exog, missing=missing,
--> 702                                 weights=weights,
↳ hasconst=hasconst, **kwargs)
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```

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```

```

~/opt/anaconda3/lib/python3.7/site-packages/statsmodels/base/model.py in
↳ __init__(self, endog, exog, **kwargs)
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    235         def __init__(self, endog, exog=None, **kwargs):
--> 236             super(LikelihoodModel, self).__init__(endog, exog, **kwargs)
    237             self.initialize()
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```

~/opt/anaconda3/lib/python3.7/site-packages/statsmodels/base/model.py in
↳ __init__(self, endog, exog, **kwargs)
    75         hasconst = kwargs.pop('hasconst', None)
    76         self.data = self._handle_data(endog, exog, missing, hasconst,
---> 77                                     **kwargs)
    78         self.k_constant = self.data.k_constant
    79         self.exog = self.data.exog

```

```

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↳ _handle_data(self, endog, exog, missing, hasconst, **kwargs)
    98
    99         def _handle_data(self, endog, exog, missing, hasconst, **kwargs):
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    101             # kwargs arrays could have changed, easier to just attach
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    102             for key in kwargs:

```

```

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    671     return klass(endog, exog=exog, missing=missing,
↳ hasconst=hasconst,
--> 672                     **kwargs)

```

```

~/opt/anaconda3/lib/python3.7/site-packages/statsmodels/base/data.py in
↳ __init__(self, endog, exog, missing, hasconst, **kwargs)
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    82         self.orig_exog = exog
---> 83         self.endog, self.exog = self._convert_endog_exog(endog,
↳ exog)
    84
    85         self.const_idx = None

```

```

~/opt/anaconda3/lib/python3.7/site-packages/statsmodels/base/data.py in
↳ _convert_endog_exog(self, endog, exog)
    508         raise ValueError("Pandas data cast to numpy dtype of
↳ object. "
    509                             "Check input data with np.asarray(data).
↳ ")
--> 510         return super(PandasData, self)._convert_endog_exog(endog,
↳ exog)
    511
    512     @classmethod

```

```

~/opt/anaconda3/lib/python3.7/site-packages/statsmodels/base/data.py in
↳ _convert_endog_exog(self, endog, exog)
    324         xarr = xarr[:, None]
    325         if xarr.ndim != 2:
--> 326             raise ValueError("exog is not 1d or 2d")
    327
    328         return yarr, xarr

```

ValueError: exog is not 1d or 2d

[ ]: