Mathematical Optimization

Federica Azzalini,
Alessandro Viol

September 29, 2022

Mancini, S., Gansterer, M., Hartl, R. F. (2021). The collaborative consistent vehicle routing problem with workload balance. European journal of operational research, 293(3), 955-965.

Introduction

- The logistics sector is one of the toughest markets to compete in
- To stand out companies need to improve usage of their resources and become more efficient

$$\downarrow$$

centrally organized multi-period collaborative vehicle routing

Mathematical Formulation

K set of carriers
I set of customers
A_k set of customers of carrier K
P set of periods
D set of depots (D_k represents depot associated to carrier K)
N set of nodes involved in the network $I \cup D$

Y_{ik} binary variable taking value 1 if customer i is assigned to carrier k and 0 otherwise

X_{ij}^{kp} binary variable taking value 1 if node j is visited immediately after node i by carrier k in period p and 0 otherwise

T_i^p non-negative variable representing visit time of node i on period p

L_i^p non-negative variable representing cumulative load at node i in period p

V_{kp}^{min} integer variable representing the minimum number of vehicles needed to fulfill the demand assigned to carrier k in period p.

Mathematical Formulation II

- Our goal $\rightarrow$ maximize the total profit: the sum of collected revenues reduced by total travel costs.

$$max \sum_{i \in I} \pi_i - \sum_{k \in K} \sum_{p \in P} \sum_{i \in N} \sum_{j \in N} c_{ij} X_{ij}^{kp}$$

- Model defined by less than 20 constraints

Constraints

- $\sum_{k \in K} Y_{ik} = 1 \quad \forall i \in I$ Each customer is assigned to one and only one carrier.
- $\sum_{i \in N} X_{ij}^{kp} \leq Y_{jk} \quad \forall j \in I, k \in K, p \in P$ Each customer can be visited by a carrier only if it has been assigned to it.
- $\sum_{j \in I} X_{jD_k}^{kp} \leq V_k \quad \forall k \in K, p \in P$ Each carrier can use up to V_k vehicles.

Constraints II

- $T_j^p \geq T_i^p + t_{ij} + s_i^p - T_{max}(1 - \sum_{k \in K} X_{ij}^{kp}) \quad \forall j \in I, i \in N, p \in P$ This constraints tracks arrival time at a customer.

- $T_j^p + t_{ij} \sum_{k \in K} X_{ji}^{kp} \leq T_{max} \quad \forall j \in I, i \in \cup_{k \in K} D_k, p \in P$ Route duration cannot exceed a maximum allowed value T_{max}.

Constraints III

- $L_j^p \geq L_i^p + q_j^p - Q_{max}\left(1 - \sum_{k \in K} X_{ij}^{kp}\right) \quad \forall j \in I, i \in N, p \in P$ This constraints tracks cumulative load at a customer.

- $L_j^p \leq Q_{max} \quad \forall j \in I, p \in P$ Each vehicle has maximum loading capacity Q_{max}.

Constraints IV

- $X_{ij}^{kp} = 0 \quad \forall j \in I, i \in D : i \neq D_k, p \in P, k \in K$ Only vehicles owned by a carrier can exit the depot of that carrier.
- $X_{ji}^{kp} = 0 \quad \forall j \in I, i \in D : i \neq D_k, p \in P, k \in K$
  Only vehicles owned by a carrier can enter the depot of that carrier.

Constraints V

- $\sum_{i \in N} \sum_{k \in K} X_{ij}^{kp} \geq q_j^p \frac{1}{Q_{max}} \quad \forall j \in I, p \in P$ If a customer requires some goods in a given period, they must be served in that period.

- $T_j^{p'} - T_j^{p''} \leq \delta \quad \forall j \in I, p', p'' \in P : q_j^{p'} > 0$ and $q_j^{p''} > 0$ Ensures arrival time consistency.

- $\sum_{j \in I} \pi_j Y_{jk} - \sum_{p \in P} \sum_{i \in N} \sum_{j \in I} c_{ij} X_{ij}^{kp} \geq R_k \quad \forall k \in K$ Each carrier's profit must be equal or higher than the profit obtainable without taking part in the coalition.

Constraints VI

- $\sum_{j \in I} Y_{jk} \geq |A_k| - \alpha_k \quad \forall k \in K$ Ensures workload balance, the number of customers assigned to a given carrier cannot be lower than a minimum value imposed by the carrier.
- $V_{kp}^{min} \geq \sum_{i \in I} \frac{q_i^p Y_{ik}}{Q_{max}^k} \; \forall k \in K, p \in P$ The minimum number of vehicles V_{kp}^{min} required to fulfill the demand of a carrier K in period P.

Constraints VII

- $\sum_{i \in N} X_{ij}^{kp} = \sum_{i \in N} X_{ji}^{kp} \quad \forall j \in I, k \in K, p \in P$ Flow balance constraints.

- $T_i^p = 0 \quad \forall i \in D, p \in P$ This constraint fixes the earliest starting time from the depot to 0.

- $L_i^p = 0 \quad \forall i \in D, p \in P$ This constraint fixes the cumulative load at the depot to 0.

Implementation

- To implement and solve the proposed model $\rightarrow$ Gurobi 9.5 with Accademic License for Python 3.9
- The problem instances have been provided by the authors

$$\downarrow$$

10 small-sized instances $\rightarrow$ 20 customers, 4 carriers and 4 periods

10 larger instances $\rightarrow$ 50 customers, 8 carriers and 5 periods

- Pandas library was used to import and manipulate each instance

Implementation II

Some data was missing $\rightarrow$ we deduced the missing values

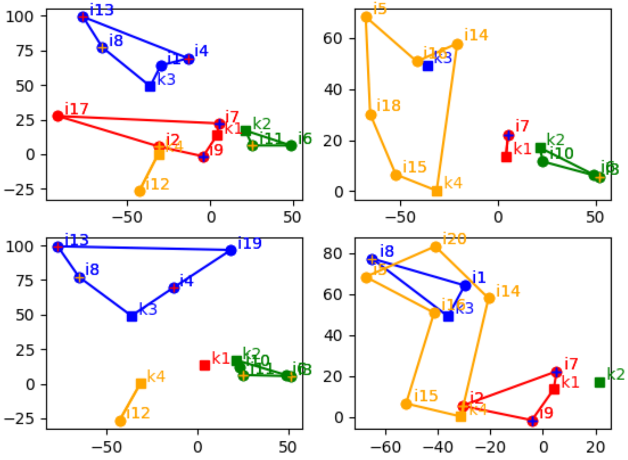
- $Q^k_{max} = Q_{max} \quad \forall k \in K$
- $c_{ij} = 0.1 \cdot d(i,j) \quad \forall i,j \in N$
- $t_{ij} = 0.7 \cdot d(i,j) \quad \forall i,j \in N$
- $s^p_i = 1 \quad \forall i \in D, p \in P$

We also added some constraints

- $X^{kp}_{ii} = 0 \quad \forall p \in P, k \in K, i \in N$ keeps a node i to be visited immediately after node i
- $T^p_i = 0 \quad \forall p \in P, i \in I : s^p_i = 0$ if a client doesn't need to be visited in a period p, then its visit time is set to 0
- $L^p_i = 0 \quad \forall p \in P, i \in I : q^p_i = 0$ if a client doesn't need to be visited in a period p, the its cumulative load is set to 0

The added constraints improve the performance.

Plot Example



Plot of the optimal solution for pr01_20

MH-pseudocode

Algorithm: MH

---

compute an initial solution running MIP with TL_{init} and keep the best solution obtained so far

for all for all pairs of carriers (k1, k2) belonging to the list of carriers combinations **do**

  fix the routing of all the other carriers

  run the model, with a time limit T_L only for k1 and k2 and customers currently assigned to them

  if the best solution obtained so far is better than the current solution **then**

    keep this solution as current best and restart exploring the list of carrier combinations

  end if

end for

MH*-pseudocode

Algorithm: MH*

compute an initial solution running the model with TL_{init} and keep the best solution obtained so far
set noimp $= 0$
while noimp $< N_{noimp}$ **do**
  randomly select two carriers k1 and k2
  fix the routing of all the other carriers
  run the MIP, with a time limit TL, only for k1 and k2 and cus- tomers currently assigned to them
  if the best solution obtained so far is better than the current solution **then**
    keep this solution as current best and set noimp $= 0$
  else
    set $noimp = noimp + 1$
  end if
end while

ILS-pseudocode

Algorithm: ILS

run MH
set $iter = 1$
while iter $< N_{iter}$ **do**
    apply a perturbation randomly selecting N_{pert} customers
    run the MIP with time limit TL_{pert} imposing that all the customers involved in the perturbation must be assigned to a different carrier than in the current best solution
    keep the best solution obtained so far as initial solution and run MH
    if the best solution obtained is better than the current best **then**
        keep it as current best
    end if
    set $iter = iter + 1$
end while

Scalability analysis

- Small instances case $\rightarrow$ 9504 variables and 7482 constraints (7522 in the case with valid inequalities).
- Large instances case $\rightarrow$ 135580 variables and 66516 constraints (66612 in the case with valid inequalities).

$$\downarrow$$

small increase in the dimensions of the instance causes a massive increase in complexity.

- Gurobi's MIP solver $\rightarrow$ no solution in reasonable time for the 50 clients instances.
- MH* and ILS $\rightarrow$ good feasible solutions in reasonable time for the 50 clients instances.

Performance and Results

We compare:

- MIP with valid inequalities vs MIP without valid inequalities in 10 minutes execution time

	MIP			MIP + Valid Inequalities		
Instance	Solution Value	Gap (%)	Time (s)	Solution Value	Gap (%)	Time (s)
Pr01_20	1731,14	2,80	**600,00**	1744,98	0,00	66,27
Pr02_20	1875,27	2,31	**600,00**	1882,18	0,00	93,16
Pr03_20	1732,56	1,01	**600,00**	1732,57	0,00	24,76
Pr04_20	1904,45	1,21	**600,00**	1910,04	0,00	46,96
Pr05_20	1884,14	1,90	**600,00**	1884,14	0,00	43,20
Pr06_20	1854,08	1,92	**600,00**	1857,38	0,00	56,75
Pr07_20	1667,93	2,18	**600,00**	1670,11	0,00	44,58
Pr08_20	1747,16	1,22	**600,00**	1747,16	0,00	25,15
Pr09_20	2018,92	1,62	**600,00**	2021,33	0,00	163,65
Pr10_20	1623,28	1,27	**600,00**	1623,38	0,00	24,99

MIP with the valid inequalities reaches optimal solution while MIP without the valid inequalities isn't able to.

Performance and Results II

- MH and ILS vs MIP with valid inequalities for small instances

	MIP			MH			ILS		
Instance	Solution Value	Gap (%)	Time (s)	Solution Value	Gap (%)	Time (s)	Solution Value	Gap (%)	Time (s)
Pr01_20	1744,98	0,00	66,27	1743,43	-0,09	24,68	1743,43	-0,09	70,12
Pr02_20	1882,18	0,00	93,16	1877,34	-0,26	17,91	1879,41	-0,15	78,06
Pr03_20	1732,57	0,00	24,76	1724,63	-0,46	15,90	1724,63	-0,46	68,89
Pr04_20	1910,04	0,00	46,96	1909,84	-0,01	12,33	1909,84	-0,01	60,38
Pr05_20	1884,14	0,00	43,20	1884,14	0,00	18,63	1883,65	-0,03	66,96
Pr06_20	1857,38	0,00	56,75	1857,38	0,00	19,56	1857,38	0,00	69,10
Pr07_20	1670,11	0,00	44,58	1665,33	-0,29	13,88	1667,86	-0,13	66,27
Pr08_20	1747,16	0,00	25,15	1747,16	0,00	14,07	1747,16	0,00	65,89
Pr09_20	2021,33	0,00	163,65	2021,33	0,00	27,25	2021,83	0,00	280,87
Pr10_20	1623,38	0,00	24,99	1623,38	0,00	11,71	1623,38	0,00	57,38

10 executions of ILS and average values considered.
MH doesn't always reaches an optimal solution but reaches good feasible solutions in shorter time than MIP.
ILS slightly improves MH's solutions but slower.

Performance and Results III

- MH* vs ILS for big instances

	MH*		ILS		
Instance	Solution Value	Time (s)	Solution Value	Time (s)	Gap (%)
Pr01_50	4706,00	128,00	4804,41	426,78	2,05
Pr02_50	4139,68	113,64	4120,00	495,57	-0,48
Pr03_50	4263,73	190,09	4265,58	487,45	0,04
Pr04_50	4567,45	128,15	4649,11	490,23	1,76
Pr05_50	4604,35	717,58	4618,99	507,16	0,32
Pr06_50	4157,73	184,42	4175,01	513,26	0,41
Pr07_50	4159,76	200,62	4160,42	430,44	0,02
Pr08_50	4605,52	165,54	4619,89	486,12	0,31
Pr09_50	4469,28	219,24	4537,82	453,39	1,51
Pr10_50	4149,23	242,02	4225,11	458,45	1,80

5 executions and average values considered
MH* obtains feasible solutions in reasonable times.
ILS improves by 0.77% the solution value in similar time.

Conclusion

- For small instances both MIP and MH reach good solutions in reasonable time
- For large instances both MH* and ILS are viable algorithms, MH* is faster but reaches worse solutions while ILS is slower but provides better solutions

We found some small mistakes in the paper:

- The variable T_i^p represents the arrival time at a node instead of a customer
- The constraint (9) is true for all $p \in P$ not only for all $j \in I$
- The constraint (19) is redundant as it is the same as constraint (18)
- Valid inequality (20) is actually a constraint

We thank you for your attention