

Tisserand Plot

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1 Introduction

Tisserand plot can help understand the effects and benefits of (one or multiple) flybys.

This analysis assumes that the secondary body (pedix “s”) flies in a circular orbit of radius a_s about the primary body (pedix “p”).

Also, we denote distance from the primary body with upper-case R , whereas distances in the Sphere of Influence (SOI) of the secondary body with lower-case r .

In this document we will only consider the (R_P, R_P) Tisserand plot, which is the simplest one to deal with elliptic orbits about the primary body. Note that hyperbolic orbits are not allowed in this plot, since the apocenter radius R_A does not exist. In that case, other kinds of Tisserand plots could be used such as the $R_P - \mathcal{E}$ (that is pericenter radius vs period) plot.

First, we show that we can cover the plot by iso- v_∞ contour lines. Indeed, each pair (R_P, R_A) corresponds to a unique pair (v_∞, α) .

Let V_{sc} be the spacecraft velocity and V

$$V_{sc}^2 = V_s^2 + v_\infty^2 + 2V_s v_\infty \cos \alpha \quad (1)$$

with $V_s = \sqrt{\mu_p/r_s}$

$$a = (R_P + R_A)/2 \quad (2)$$

$$e = 1 - rp/a \quad (3)$$

$$\cos \nu = (p - r_s)/(er_s) \quad (4)$$

$$V_{sc,r} = \sqrt{\frac{\mu_p}{p}} e \sin \nu \quad (5)$$

$$V_{sc,t} = \sqrt{\frac{\mu_p}{p}} (1 + e \cos \nu) \quad (6)$$

$$V_{sc} = \sqrt{V_{sc,r}^2 + V_{sc,t}^2} \quad (7)$$

$$v_{\infty,r} = V_r \quad (8)$$

$$v_{\infty,t} = V_t - V_s \quad (9)$$

$$v_{\infty} = \sqrt{v_{\infty,r}^2 + v_{\infty,t}^2} \quad (10)$$

$$\alpha = \arccos \frac{V_{sc}^2 - V_s^2 - v_{\infty}^2}{2V_{sc}v_{\infty}} \quad (11)$$

The inverse transform follows the same reasoning

$$V_{sc}^2 = V_s^2 + v_{\infty}^2 + 2V_s v_{\infty} \cos \alpha \quad (12)$$

$$V_{sc,r} = v_{\infty} \sin \alpha \quad (13)$$

$$V_{sc,t} = V_s + v_{\infty} \cos \alpha \quad (14)$$

$$a = \frac{\mu_p/2}{\frac{V_{sc}^2}{2} - \frac{\mu_p}{R_s}} \quad (15)$$

$$h = R_s V_{sc,t} \quad (16)$$

$$e = \sqrt{1 - \frac{h^2}{\mu a}} \quad (17)$$

$$R_P = a(1 - e) \quad (18)$$

$$R_A = a(1 + e) \quad (19)$$

Using the inverse transform it is possible to draw lines (iso-contours) of constant v_{∞} , fixing $v_{\infty} = \bar{v}_{\infty}$ and varying $\alpha \in [0, \pi]$.

Iso-contours of *alpha* can also be plotted as dashed lines. These lines are obtained fixing the values of α (e.g., $\alpha = 0, 15^\circ, 30^\circ, \dots, 180^\circ$). and varying $v_{\infty} \in [0, v_{\infty \max}]$.

FIGURA 1

As an alternative, one can draw lines of constant period (of the spacecraft orbit around the primary body), for a given $n : m$ resonance. These iso-contours appears a straight lines in the (R_A, R_P) . In fact, $R_P + R_A = 2a$, wuth a constant if the period is constatat, being $a = \sqrt[3]{\left(\frac{T}{2\pi}\right)^2 \mu_p}$.

For a resonant orbit, one has $T_{sc} : T_s = n : m$, hence

$$a_{sc} = a_s \left(\frac{n}{m} \right)^{2/3} \quad (20)$$

Thus, the iso-resonant lines are found for a specific choice of $n : m$ calculating the pairs (R_A, R_P) varying the eccentricity from 0 to 1 (or e_{\max}).

$$R_P = a_s \left(\frac{n}{m} \right)^{2/3} (1 - e) \quad (21)$$

$$R_A = a_s \left(\frac{n}{m} \right)^{2/3} (1 + e) \quad (22)$$

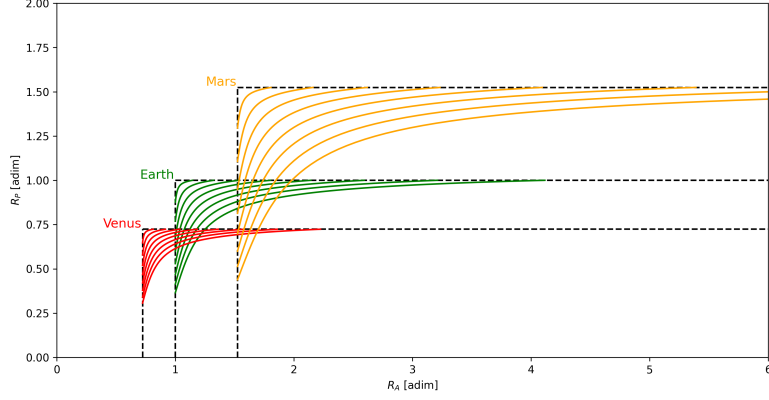


Figure 1: Tisserand plot for the Solar system.

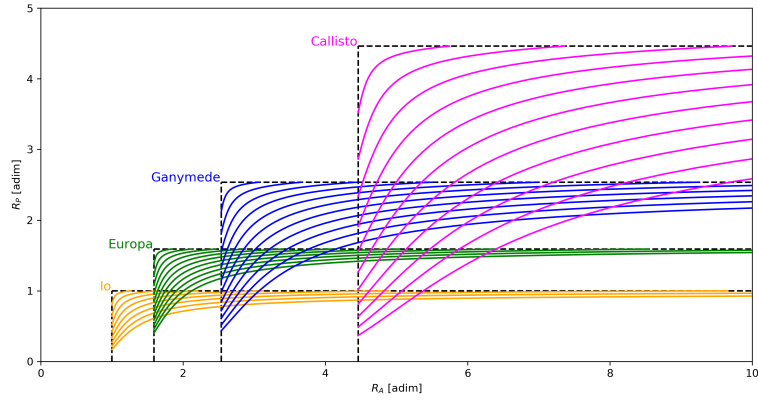


Figure 2: Tisserand plot for the Jovian system.

The effect of a flyby corresponds to an instantaneous change of velocity. This means a jump on the Tisserand plot. Note that the v_∞ is not altered by a flyby, thus the points before and after the flyby must be located on the same iso- v_∞ contour line.

The amount of displacement along the v_∞ contour line depends on the change of the pump angle between the conditions after (α_+) and before (α_-) the flyby. Their difference is equal to the deflection angle δ

$$\mathbf{v}_{\infty-}^T \cdot \mathbf{v}_{\infty+} = v_\infty^2 \cos \delta \quad (23)$$

figura del triangolo di velocita'

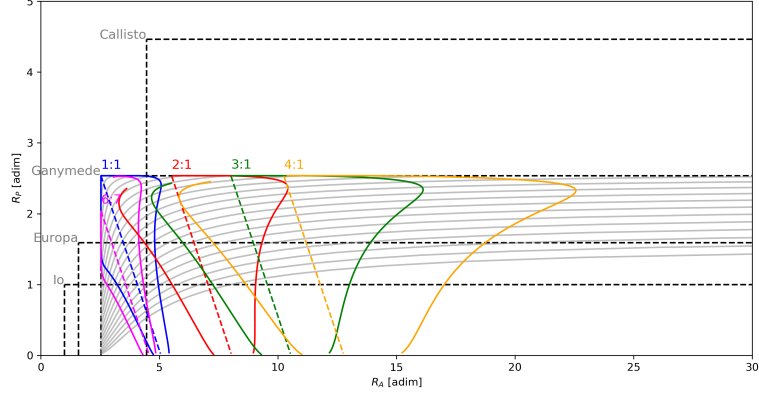


Figure 3: Tisserand plot for the Solar system.

The deflection angle δ , can be found as

$$\sin \frac{\delta}{2} = \frac{\mu_s / r_{flyby}}{v_\infty^2 + \mu_s / r_{flyby}} \quad (24)$$

where r_{flyby} is the pericenter radius of the planetocentric hyperbola, that is, the flyby radius.