Rocket Linear Dynamics

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1 Introduction

Tisserand plot can help understand the effects and potential gain of one (or multiple) flybys.

This analysis assumes that the secondary body (pedix "s") flies in a circular orbit about the primary body (pedix "p")

Also, we denotes distance from the primary body with upper-case R, wherease distances in the Sphere of Influence (SOI) of the secondary body with lower-case r.

We consider only the (R_P, R_P) Tisserand plot.

First, we show that we can cover the plot by iso-v_∞ contour lines.

In fact, each pair (R_P, R_A) corresponds to a unique pair (v_∞, α)

Let V_{sc} be the spacecraft velocity and V

$$V_{sc}^2 = V_s^2 + v_\infty^2 + 2V_s v_\infty \cos\alpha \tag{1}$$

with $V_s = \sqrt{\mu_p/r_s}$

$$a = (R_P + R_A)/2 \tag{2}$$

$$e = 1 - rp/a \tag{3}$$

$$\cos\nu = (p - r_s)/(er_s) \tag{4}$$

$$V_{sc,r} = \sqrt{\frac{\mu_p}{p}}e\sin\nu \tag{5}$$

$$V_{sc,t} = \sqrt{\frac{\mu_p}{p}}(1 + e\cos\nu) \tag{6}$$

$$V_{sc} = \sqrt{V_{sc,r}^2 + V_{sc,t}^2} \tag{7}$$

$$v_{\infty,r} = V_r \tag{8}$$

$$v_{\infty,t} = V_t - V_s \tag{9}$$

$$v_\infty = \sqrt{v_{\infty,r}^2 + v_{\infty,t}^2} \tag{10}$$

$$\alpha = \arccos\frac{V_{sc}^2 - V_s^2 - v_\infty^2}{2V_{sc}v_\infty} \tag{11}$$

The inverse transform follows the same reasoning

$$V_{sc}^2 = V_s^2 + v_\infty^2 + 2V_sv_\infty\cos\alpha \tag{12}$$

$$V_{sc,r} = v_\infty\sin\alpha \tag{13}$$

$$V_{sc,t} = V_s + v_\infty\cos\alpha \tag{14}$$

$$a = \frac{\mu_p/2}{\dfrac{V_{sc}^2}{2} - \dfrac{\mu_p}{R_s}} \tag{15}$$

$$h = R_sV_{sc,t} \tag{16}$$

$$e = \sqrt{1 - \frac{h^2}{\mu a}} \tag{17}$$

$$R_P = a(1 - e) \tag{18}$$

$$R_A = a(1 + e) \tag{19}$$

Using the inverse transform it is possible to draw lines (iso-contours) of constant v_∞, fixing $v_\infty = \bar{v}_\infty$ and varying $\alpha \in [0, \pi]$.

Iso-contours of *alpha* can also be plotted as dashed lines. These lines are obtained fixing the values of α (e.g., $\alpha = 0$, 15°, 30°, ..., 180°). and varying $v_\infty \in [0, v_{\infty_{\max}}]$.

FIGURA 1

As an alternative, one can draw lines of constant period (of the spacecraft orbit around the primary body), for a given $n:m$ resonance. These iso-contours appears a straight lines in the (R_A, R_P). In fact, $R_P + R_A = 2a$, wuth a constant if the period is constat, being $a =^3\sqrt{\left(\frac{T}{2\pi}\right)^2\mu_p}$.

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For a resontant orbit, one hase $T_{sc} : T_s = n : m$, hence

$$a_{sc} = a_s \left(\frac{n}{m}\right)^{2/3} \tag{20}$$

Thus, the iso-resonant lines are found for a specific choice of $n : m$ calculating the pairs (R_A, R_P) varying the eccentricity from 0 to 1 (or $e_{\max}$.

$$R_P = a_s \left(\frac{n}{m}\right)^{2/3} (1 - e) \tag{21}$$

$$R_A = a_s \left(\frac{n}{m}\right)^{2/3} (1 + e) \tag{22}$$

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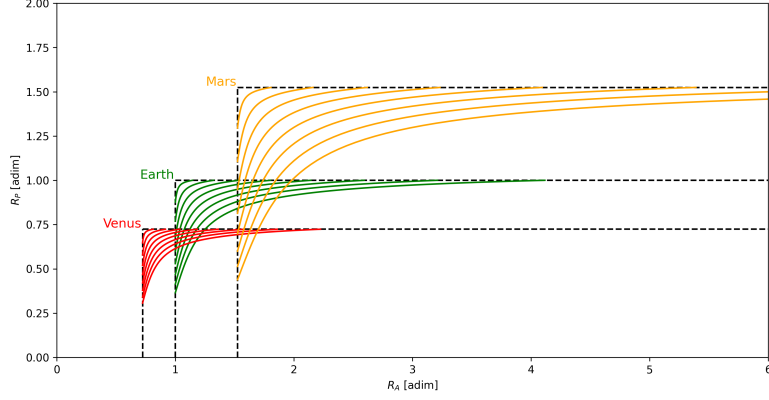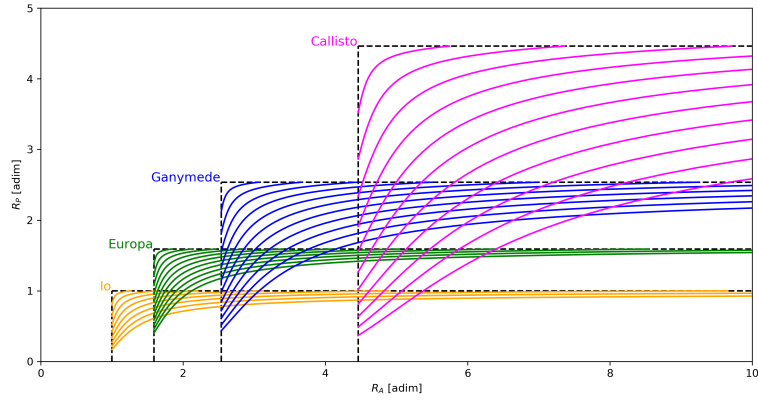Figure 1: Tisserand plot for the Solar system.

Figure 2: Tisserand plot for the Jovian system.

The effect of a flyby corrsponds to an instantaneous change of velocity. This means a jump on the Tisserand plot. Note that the v_∞ is not altered by a flyby, thus the points before and after the flyby must be located on the same iso-v_∞ contour line.

The amount of displacement along the v_∞ contour line depends on the change of the pump angle between the conditions after (α_+) and before (α_-) the flyby. Their difference is equal to the deflection angle δ

$$\boldsymbol{v}_{\infty-}^T \cdot bmv_{\infty+} = v_\infty^2 \cos \delta \tag{23}$$
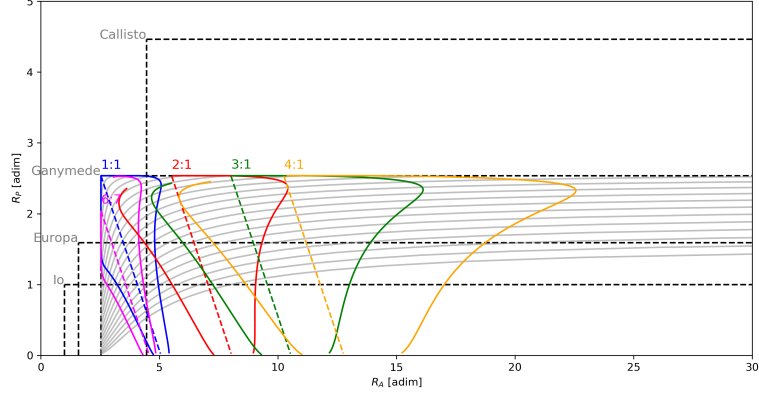
figura del triangolo di velocita'

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Figure 3: Tisserand plot for the Solar system.

The deflection angle δ, can be found as

$$\sin \frac{\delta}{2} = \frac{\mu_s / r_{flyby}}{v_\infty^2 + \mu_s / r_{flyby}} \tag{24}$$

where r_{flyby} is the pericenter radius of the planetocentric hyperbola, that is, the flyby radius.

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